

1. [Preface](#)
2. Electric Charge and Electric Field
 1. [Introduction to Electric Charge and Electric Field](#)
 2. [Static Electricity and Charge: Conservation of Charge](#)
 3. [Conductors and Insulators](#)
 4. [Coulomb's Law](#)
 5. [Electric Field: Concept of a Field Revisited](#)
 6. [Electric Field Lines: Multiple Charges](#)
 7. [Conductors and Electric Fields in Static Equilibrium](#)
 8. [Electric Forces in Biology](#)
 9. [Applications of Electrostatics](#)
3. Electric Potential and Electric Field
 1. [Introduction to Electric Potential and Electric Energy](#)
 2. [Electric Potential Energy: Potential Difference](#)
 3. [Electric Potential in a Uniform Electric Field](#)
 4. [Electrical Potential Due to a Point Charge](#)
 5. [Equipotential Lines](#)
 6. [Capacitors and Dielectrics](#)
 7. [Capacitors in Series and Parallel](#)
 8. [Energy Stored in Capacitors](#)
4. Electric Current, Resistance, and Ohm's Law
 1. [Introduction to Electric Current, Resistance, and Ohm's Law](#)
 2. [Current](#)
 3. [Ohm's Law: Resistance and Simple Circuits](#)
 4. [Resistance and Resistivity](#)
 5. [Electric Power and Energy](#)
 6. [Alternating Current versus Direct Current](#)
 7. [Electric Hazards and the Human Body](#)
 8. [Nerve Conduction–Electrocardiograms](#)
5. Circuits and DC Instruments

1. [Introduction to Circuits and DC Instruments](#)
 2. [Resistors in Series and Parallel](#)
 3. [Electromotive Force: Terminal Voltage](#)
 4. [Kirchhoff's Rules](#)
 5. [DC Voltmeters and Ammeters](#)
 6. [Null Measurements](#)
 7. [DC Circuits Containing Resistors and Capacitors](#)
6. Magnetism
1. [Introduction to Magnetism](#)
 2. [Magnets](#)
 3. [Ferromagnets and Electromagnets](#)
 4. [Magnetic Fields and Magnetic Field Lines](#)
 5. [Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field](#)
 6. [Force on a Moving Charge in a Magnetic Field: Examples and Applications](#)
 7. [Magnetic Fields Produced by Currents: Ampere's Law](#)
 8. [The Hall Effect](#)
 9. [Magnetic Force on a Current-Carrying Conductor](#)
 10. [Magnetic Force between Two Parallel Conductors](#)
 11. [Torque on a Current Loop: Motors and Meters](#)
 12. [More Applications of Magnetism](#)
7. Electromagnetic Induction, AC Circuits, and Electrical Technologies
1. [Introduction to Electromagnetic Induction, AC Circuits and Electrical Technologies](#)
 2. [Induced Emf and Magnetic Flux](#)
 3. [Faraday's Law of Induction: Lenz's Law](#)
 4. [Motional Emf](#)
 5. [Eddy Currents and Magnetic Damping](#)
 6. [Electric Generators](#)
 7. [Back Emf](#)

8. [Transformers](#)
9. [Electrical Safety: Systems and Devices](#)
10. [Inductance](#)
11. [RL Circuits](#)
12. [Reactance, Inductive and Capacitive](#)
13. [RLC Series AC Circuits](#)
8. Electromagnetic Waves
 1. [Introduction to Electromagnetic Waves](#)
 2. [Maxwell's Equations: Electromagnetic Waves Predicted and Observed](#)
 3. [Production of Electromagnetic Waves](#)
 4. [Energy in Electromagnetic Waves](#)
9. Fluid Statics
 1. [Introduction to Fluid Statics](#)
 2. [What Is a Fluid?](#)
 3. [Density](#)
 4. [Pressure](#)
 5. [Variation of Pressure with Depth in a Fluid](#)
 6. [Pascal's Principle](#)
 7. [Gauge Pressure, Absolute Pressure, and Pressure Measurement](#)
 8. [Archimedes' Principle](#)
 9. [Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action](#)
 10. [Pressures in the Body](#)
10. Fluid Dynamics and Its Biological and Medical Applications
 1. [Introduction to Fluid Dynamics and Its Biological and Medical Applications](#)
 2. [Flow Rate and Its Relation to Velocity](#)
 3. [Bernoulli's Equation](#)
 4. [The Most General Applications of Bernoulli's Equation](#)
 5. [Viscosity and Laminar Flow; Poiseuille's Law](#)

6. [Motion of an Object in a Viscous Fluid](#)
7. [Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes](#)
11. Temperature, Kinetic Theory, and the Gas Laws
 1. [Introduction to Temperature, Kinetic Theory, and the Gas Laws](#)
 2. [Temperature](#)
 3. [Thermal Expansion of Solids and Liquids](#)
 4. [The Ideal Gas Law](#)
 5. [Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature](#)
 6. [Phase Changes](#)
 7. [Humidity, Evaporation, and Boiling](#)
12. Atomic Physics
 1. [Introduction to Atomic Physics](#)
 2. [Discovery of the Atom](#)
 3. [Discovery of the Parts of the Atom: Electrons and Nuclei](#)
 4. [Bohr's Theory of the Hydrogen Atom](#)
 5. [X Rays: Atomic Origins and Applications](#)
 6. [Applications of Atomic Excitations and De-Excitations](#)
 7. [The Wave Nature of Matter Causes Quantization](#)
 8. [Patterns in Spectra Reveal More Quantization](#)
 9. [Quantum Numbers and Rules](#)
 10. [The Pauli Exclusion Principle](#)
13. Introduction to Quantum Physics
 1. [Introduction to Quantum Physics](#)
 2. [Quantization of Energy](#)
 3. [The Photoelectric Effect](#)
 4. [Photon Energies and the Electromagnetic Spectrum](#)
 5. [Photon Momentum](#)
 6. [The Particle-Wave Duality](#)
 7. [The Wave Nature of Matter](#)

8. [Probability: The Heisenberg Uncertainty Principle](#)
9. [The Particle-Wave Duality Reviewed](#)
14. Radioactivity and Nuclear Physics
 1. [Introduction to Radioactivity and Nuclear Physics](#)
 2. [Nuclear Radioactivity](#)
 3. [Radiation Detection and Detectors](#)
 4. [Substructure of the Nucleus](#)
 5. [Nuclear Decay and Conservation Laws](#)
 6. [Half-Life and Activity](#)
 7. [Binding Energy](#)
 8. [Tunneling](#)
15. Medical Applications of Nuclear Physics
 1. [Introduction to Applications of Nuclear Physics](#)
 2. [Medical Imaging and Diagnostics](#)
 3. [Biological Effects of Ionizing Radiation](#)
 4. [Therapeutic Uses of Ionizing Radiation](#)
 5. [Food Irradiation](#)
 6. [Fusion](#)
 7. [Fission](#)
 8. [Nuclear Weapons](#)
16. Frontiers of Physics
 1. [Introduction to Frontiers of Physics](#)
 2. [Cosmology and Particle Physics](#)
 3. [General Relativity and Quantum Gravity](#)
 4. [Complexity and Chaos](#)
 5. [High-temperature Superconductors](#)
 6. [Some Questions We Know to Ask](#)
17. [Atomic Masses](#)
18. [Selected Radioactive Isotopes](#)
19. [Useful Information](#)
20. [Glossary of Key Symbols and Notation](#)

Preface

Welcome to *College Physics*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

About OpenStax

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012, and our library has since scaled to over 20 books for college and AP courses used by hundreds of thousands of students. Our adaptive learning technology, designed to improve learning outcomes through personalized educational paths, is being piloted in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

About OpenStax Resources

Customization

College Physics is licensed under a Creative Commons Attribution 4.0 International (CC BY) license, which means that you can distribute, remix, and build upon the content, as long as you provide attribution to OpenStax and its content contributors.

Because our books are openly licensed, you are free to use the entire book or pick and choose the sections that are most relevant to the needs of your course. Feel free to remix the content by assigning your students certain chapters and sections in your syllabus, in the order that you prefer. You can even provide a direct link in your syllabus to the sections in the web view of your book.

Instructors also have the option of creating a customized version of their OpenStax book. The custom version can be made available to students in low-cost print or digital form through their campus bookstore. Visit your book page on openstax.org for more information.

Errata

All OpenStax textbooks undergo a rigorous review process. However, like any professional-grade textbook, errors sometimes occur. Since our books are web based, we can make updates periodically when deemed pedagogically necessary. If you have a correction to suggest, submit it through the link on your book page on openstax.org. Subject matter experts review all errata suggestions. OpenStax is committed to remaining transparent about all updates, so you will also find a list of past errata changes on your book page on openstax.org.

Format

You can access this textbook for free in web view or PDF through openstax.org, and in low-cost print and iBooks editions.

About *College Physics*

College Physics meets standard scope and sequence requirements for a two-semester introductory algebra-based physics course. The text is grounded in real-world examples to help students grasp fundamental physics concepts. It requires knowledge of algebra and some trigonometry, but not calculus. *College Physics* includes learning objectives, concept questions, links to labs and simulations, and ample practice opportunities for traditional physics application problems.

Coverage and Scope

College Physics is organized such that topics are introduced conceptually with a steady progression to precise definitions and analytical applications. The analytical aspect (problem solving) is tied back to the conceptual before moving on to another topic. Each introductory chapter, for example, opens with an engaging photograph relevant to the subject of the chapter and interesting applications that are easy for most students to visualize.

Chapter 1: Introduction: The Nature of Science and Physics

Chapter 2: Kinematics

Chapter 3: Two-Dimensional Kinematics

Chapter 4: Dynamics: Force and Newton's Laws of Motion

Chapter 5: Further Applications of Newton's Laws: Friction, Drag, and Elasticity

Chapter 6: Uniform Circular Motion and Gravitation

Chapter 7: Work, Energy, and Energy Resources

Chapter 8: Linear Momentum and Collisions

Chapter 9: Statics and Torque

Chapter 10: Rotational Motion and Angular Momentum

Chapter 11: Fluid Statics

Chapter 12: Fluid Dynamics and Its Biological and Medical Applications

Chapter 13: Temperature, Kinetic Theory, and the Gas Laws

Chapter 14: Heat and Heat Transfer Methods

Chapter 15: Thermodynamics

Chapter 16: Oscillatory Motion and Waves

Chapter 17: Physics of Hearing

Chapter 18: Electric Charge and Electric Field

Chapter 19: Electric Potential and Electric Field

Chapter 20: Electric Current, Resistance, and Ohm's Law

Chapter 21: Circuits and DC Instruments

Chapter 22: Magnetism

Chapter 23: Electromagnetic Induction, AC Circuits, and Electrical Technologies

Chapter 24: Electromagnetic Waves

Chapter 25: Geometric Optics

Chapter 26: Vision and Optical Instruments

Chapter 27: Wave Optics

Chapter 28: Special Relativity

Chapter 29: Introduction to Quantum Physics
Chapter 30: Atomic Physics
Chapter 31: Radioactivity and Nuclear Physics
Chapter 32: Medical Applications of Nuclear Physics
Chapter 33: Particle Physics
Chapter 34: Frontiers of Physics
Appendix A: Atomic Masses
Appendix B: Selected Radioactive Isotopes
Appendix C: Useful Information
Appendix D: Glossary of Key Symbols and Notation

Concepts and Calculations

The ability to calculate does not guarantee conceptual understanding. In order to unify conceptual, analytical, and calculation skills within the learning process, we have integrated Strategies and Discussions throughout the text.

Modern Perspective

The chapters on modern physics are more complete than many other texts on the market, with an entire chapter devoted to medical applications of nuclear physics and another to particle physics. The final chapter of the text, “Frontiers of Physics,” is devoted to the most exciting endeavors in physics. It ends with a module titled “Some Questions We Know to Ask.”

Key Features

Modularity

This textbook is organized as a collection of modules that can be rearranged and modified to suit the needs of a particular professor or class. That being said, modules often contain references to content in other modules, as most topics in physics cannot be discussed in isolation.

Learning Objectives

Every module begins with a set of learning objectives. These objectives are designed to guide the instructor in deciding what content to include or assign, and to guide the student with respect to what he or she can expect to learn. After completing the module and end-of-module exercises, students should be able to demonstrate mastery of the learning objectives.

Call-Outs

Key definitions, concepts, and equations are called out with a special design treatment. Call-outs are designed to catch readers' attention, to make it clear that a specific term, concept, or equation is particularly important, and to provide easy reference for a student reviewing content.

Key Terms

Key terms are in bold and are followed by a definition in context. Definitions of key terms are also listed in the Glossary, which appears at the end of the module.

Worked Examples

Worked examples have four distinct parts to promote both analytical and conceptual skills. Worked examples are introduced in words, always using some application that should be of interest. This is followed by a Strategy section that emphasizes the concepts involved and how solving the problem

relates to those concepts. This is followed by the mathematical Solution and Discussion.

Many worked examples contain multiple-part problems to help the students learn how to approach normal situations, in which problems tend to have multiple parts. Finally, worked examples employ the techniques of the problem-solving strategies so that students can see how those strategies succeed in practice as well as in theory.

Problem-Solving Strategies

Problem-solving strategies are first presented in a special section and subsequently appear at crucial points in the text where students can benefit most from them. Problem-solving strategies have a logical structure that is reinforced in the worked examples and supported in certain places by line drawings that illustrate various steps.

Misconception Alerts

Students come to physics with preconceptions from everyday experiences and from previous courses. Some of these preconceptions are misconceptions, and many are very common among students and the general public. Some are inadvertently picked up through misunderstandings of lectures and texts. The Misconception Alerts feature is designed to point these out and correct them explicitly.

Take-Home Investigations

Take Home Investigations provide the opportunity for students to apply or explore what they have learned with a hands-on activity.

Things Great and Small

In these special topic essays, macroscopic phenomena (such as air pressure) are explained with submicroscopic phenomena (such as atoms bouncing off walls). These essays support the modern perspective by describing aspects of modern physics before they are formally treated in later chapters. Connections are also made between apparently disparate phenomena.

Simulations

Where applicable, students are directed to the interactive PHeT physics simulations developed by the University of Colorado. There they can further explore the physics concepts they have learned about in the module.

Summary

Module summaries are thorough and functional and present all important definitions and equations. Students are able to find the definitions of all terms and symbols as well as their physical relationships. The structure of the summary makes plain the fundamental principles of the module or collection and serves as a useful study guide.

Glossary

At the end of every module or chapter is a Glossary containing definitions of all of the key terms in the module or chapter.

End-of-Module Problems

At the end of every chapter is a set of Conceptual Questions and/or skills-based Problems & Exercises. Conceptual Questions challenge students' ability to explain what they have learned conceptually, independent of the mathematical details. Problems & Exercises challenge students to apply both concepts and skills to solve mathematical physics problems. Online,

every other problem includes an answer that students can reveal immediately by clicking on a “Show Solution” button.

In addition to traditional skills-based problems, there are three special types of end-of-module problems: Integrated Concept Problems, Unreasonable Results Problems, and Construct Your Own Problems. All of these problems are indicated with a subtitle preceding the problem.

Integrated Concept Problems

In Integrated Concept Problems, students are asked to apply what they have learned about two or more concepts to arrive at a solution to a problem. These problems require a higher level of thinking because, before solving a problem, students have to recognize the combination of strategies required to solve it.

Unreasonable Results

In Unreasonable Results Problems, students are challenged to not only apply concepts and skills to solve a problem, but also to analyze the answer with respect to how likely or realistic it really is. These problems contain a premise that produces an unreasonable answer and are designed to further emphasize that properly applied physics must describe nature accurately and is not simply the process of solving equations.

Construct Your Own Problem

These problems require students to construct the details of a problem, justify their starting assumptions, show specific steps in the problem’s solution, and finally discuss the meaning of the result. These types of problems relate well to both conceptual and analytical aspects of physics, emphasizing that physics must describe nature. Often they involve an integration of topics from more than one chapter. Unlike other problems, solutions are not provided since there is no single correct answer.

Instructors should feel free to direct students regarding the level and scope of their considerations. Whether the problem is solved and described correctly will depend on initial assumptions.

Additional Resources

Student and Instructor Resources

We've compiled additional resources for both students and instructors, including Getting Started Guides, an instructor solution manual, and PowerPoint slides. Instructor resources require a verified instructor account, which can be requested on your openstax.org log-in. Take advantage of these resources to supplement your OpenStax book.

Partner Resources

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on openstax.org.

About the Authors

Senior Contributing Authors

Paul Peter Urone, Professor Emeritus at California State University, Sacramento

Roger Hinrichs, State University of New York, College at Oswego

Contributing Authors

Kim Dirks, University of Auckland
Manjula Sharma, University of Sydney

Reviewers

Matthew Adams, Crafton Hills College, San Bernardino Community
College District
Erik Christensen, South Florida Community College
Douglas Ingram, Texas Christian University
Eric Kincanon, Gonzaga University
Lee H. LaRue, Paris Junior College
Chuck Pearson, Virginia Intermont College
Marc Sher, College of William and Mary
Ulrich Zurcher, Cleveland State University

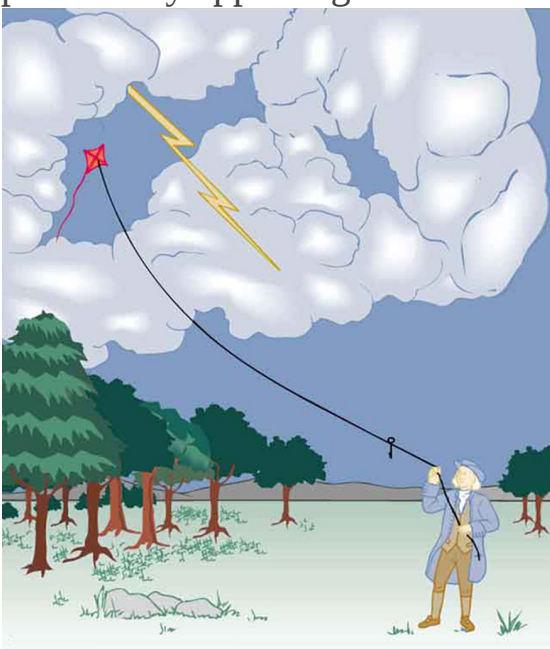
Introduction to Electric Charge and Electric Field

class="introduction"

Static electricity from this plastic slide causes the child's hair to stand on end. The sliding motion stripped electrons away from the child's body, leaving an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma/Wikimedia Commons)



The image of American politician and scientist Benjamin Franklin (1706–1790) flying a kite in a thunderstorm is familiar to every schoolchild. (See [\[link\]](#).) In this experiment, Franklin demonstrated a connection between lightning and **static electricity**. Sparks were drawn from a key hung on a kite string during an electrical storm. These sparks were like those produced by static electricity, such as the spark that jumps from your finger to a metal doorknob after you walk across a wool carpet. What Franklin demonstrated in his dangerous experiment was a connection between phenomena on two different scales: one the grand power of an electrical storm, the other an effect of more human proportions. Connections like this one reveal the underlying unity of the laws of nature, an aspect we humans find particularly appealing.



When Benjamin Franklin demonstrated that lightning was related to static electricity, he made a connection that is now part of the evidence that all directly experienced forces except the gravitational force are manifestations of the electromagnetic force.

Much has been written about Franklin. His experiments were only part of the life of a man who was a scientist, inventor, revolutionary, statesman, and writer. Franklin's experiments were not performed in isolation, nor were they the only ones to reveal connections.

For example, the Italian scientist Luigi Galvani (1737–1798) performed a series of experiments in which static electricity was used to stimulate contractions of leg muscles of dead frogs, an effect already known in humans subjected to static discharges. But Galvani also found that if he joined two metal wires (say copper and zinc) end to end and touched the other ends to muscles, he produced the same effect in frogs as static discharge. Alessandro Volta (1745–1827), partly inspired by Galvani's work, experimented with various combinations of metals and developed the battery.

During the same era, other scientists made progress in discovering fundamental connections. The periodic table was developed as the systematic properties of the elements were discovered. This influenced the development and refinement of the concept of atoms as the basis of matter. Such submicroscopic descriptions of matter also help explain a great deal more.

Atomic and molecular interactions, such as the forces of friction, cohesion, and adhesion, are now known to be manifestations of the **electromagnetic force**. Static electricity is just one aspect of the electromagnetic force, which also includes moving electricity and magnetism.

All the macroscopic forces that we experience directly, such as the sensations of touch and the tension in a rope, are due to the electromagnetic force, one of the four fundamental forces in nature. The gravitational force, another fundamental force, is actually sensed through the electromagnetic interaction of molecules, such as between those in our feet and those on the top of a bathroom scale. (The other two fundamental forces, the strong nuclear force and the weak nuclear force, cannot be sensed on the human scale.)

This chapter begins the study of electromagnetic phenomena at a fundamental level. The next several chapters will cover static electricity, moving electricity, and magnetism—collectively known as electromagnetism. In this chapter, we begin with the study of electric phenomena due to charges that are at least temporarily stationary, called electrostatics, or static electricity.

Glossary

static electricity

a buildup of electric charge on the surface of an object

electromagnetic force

one of the four fundamental forces of nature; the electromagnetic force consists of static electricity, moving electricity and magnetism

Static Electricity and Charge: Conservation of Charge

- Define electric charge, and describe how the two types of charge interact.
- Describe three common situations that generate static electricity.
- State the law of conservation of charge.



Borneo amber was mined in Sabah, Malaysia, from shale-sandstone-mudstone veins.

When a piece of amber is rubbed with a piece of silk, the amber gains more electrons, giving it a net negative charge.

At the same time, the silk, having lost electrons, becomes positively charged. (credit: Sebakoamber, Wikimedia Commons)

What makes plastic wrap cling? Static electricity. Not only are applications of static electricity common these days, its existence has been known since ancient times. The first record of its effects dates to ancient Greeks who noted more than 500 years B.C. that polishing amber temporarily enabled it

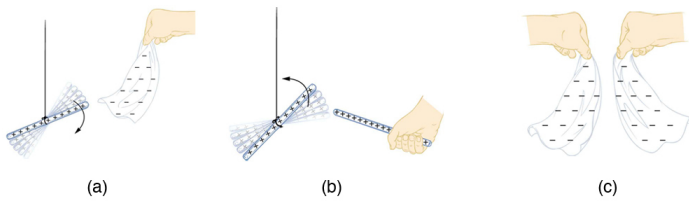
to attract bits of straw (see [\[link\]](#)). The very word *electric* derives from the Greek word for amber (*electron*).

Many of the characteristics of static electricity can be explored by rubbing things together. Rubbing creates the spark you get from walking across a wool carpet, for example. Static cling generated in a clothes dryer and the attraction of straw to recently polished amber also result from rubbing. Similarly, lightning results from air movements under certain weather conditions. You can also rub a balloon on your hair, and the static electricity created can then make the balloon cling to a wall. We also have to be cautious of static electricity, especially in dry climates. When we pump gasoline, we are warned to discharge ourselves (after sliding across the seat) on a metal surface before grabbing the gas nozzle. Attendants in hospital operating rooms must wear booties with aluminum foil on the bottoms to avoid creating sparks which may ignite the oxygen being used.

Some of the most basic characteristics of static electricity include:

- The effects of static electricity are explained by a physical quantity not previously introduced, called electric charge.
- There are only two types of charge, one called positive and the other called negative.
- Like charges repel, whereas unlike charges attract.
- The force between charges decreases with distance.

How do we know there are two types of **electric charge**? When various materials are rubbed together in controlled ways, certain combinations of materials always produce one type of charge on one material and the opposite type on the other. By convention, we call one type of charge “positive”, and the other type “negative.” For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Since the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, since each rod has positive charge on it. Similarly, two silk cloths so rubbed will repel, since both cloths have negative charge. [\[link\]](#) shows how these simple materials can be used to explore the nature of the force between charges.



A glass rod becomes positively charged when rubbed with silk, while the silk becomes negatively charged.

(a) The glass rod is attracted to the silk because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

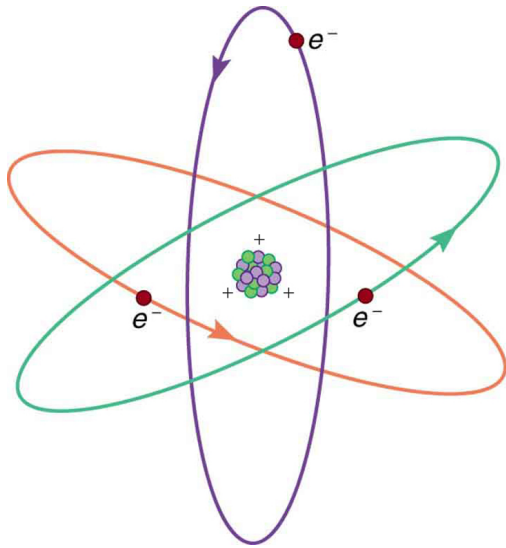
More sophisticated questions arise. Where do these charges come from? Can you create or destroy charge? Is there a smallest unit of charge? Exactly how does the force depend on the amount of charge and the distance between charges? Such questions obviously occurred to Benjamin Franklin and other early researchers, and they interest us even today.

Charge Carried by Electrons and Protons

Franklin wrote in his letters and books that he could see the effects of electric charge but did not understand what caused the phenomenon. Today we have the advantage of knowing that normal matter is made of atoms, and that atoms contain positive and negative charges, usually in equal amounts.

[\[link\]](#) shows a simple model of an atom with negative **electrons** orbiting its positive nucleus. The nucleus is positive due to the presence of positively charged **protons**. Nearly all charge in nature is due to electrons and protons, which are two of the three building blocks of most matter. (The third is the neutron, which is neutral, carrying no charge.) Other charge-carrying particles are observed in cosmic rays and nuclear decay, and are created in

particle accelerators. All but the electron and proton survive only a short time and are quite rare by comparison.



This simplified (and not to scale) view of an atom is called the planetary model of the atom.

Negative electrons orbit a much heavier positive nucleus, as the planets orbit the much heavier sun. There the similarity ends, because forces in the atom are electromagnetic, whereas those in the planetary system are gravitational.

Normal macroscopic amounts of matter contain immense numbers of atoms and molecules and, hence, even greater numbers of individual

negative and positive charges.

The charges of electrons and protons are identical in magnitude but opposite in sign. Furthermore, all charged objects in nature are integral multiples of this basic quantity of charge, meaning that all charges are made of combinations of a basic unit of charge. Usually, charges are formed by combinations of electrons and protons. The magnitude of this basic charge is

Equation:

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

The symbol q is commonly used for charge and the subscript e indicates the charge of a single electron (or proton).

The SI unit of charge is the coulomb (C). The number of protons needed to make a charge of 1.00 C is

Equation:

$$1.00 \text{ C} \times \frac{1 \text{ proton}}{1.60 \times 10^{-19} \text{ C}} = 6.25 \times 10^{18} \text{ protons}.$$

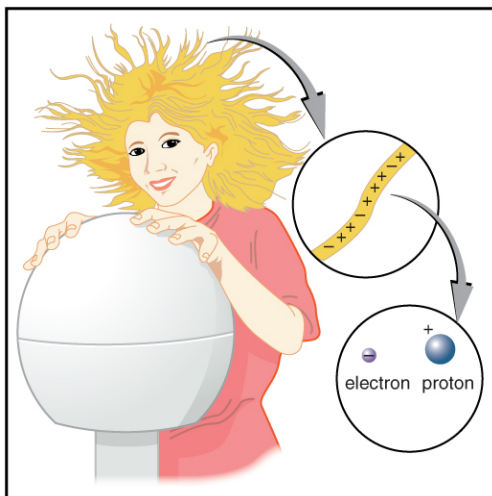
Similarly, 6.25×10^{18} electrons have a combined charge of -1.00 coulomb. Just as there is a smallest bit of an element (an atom), there is a smallest bit of charge. There is no directly observed charge smaller than $|q_e|$ (see [Things Great and Small: The Submicroscopic Origin of Charge](#)), and all observed charges are integral multiples of $|q_e|$.

Note:

Things Great and Small: The Submicroscopic Origin of Charge

With the exception of exotic, short-lived particles, all charge in nature is carried by electrons and protons. Electrons carry the charge we have named negative. Protons carry an equal-magnitude charge that we call positive. (See [\[link\]](#).) Electron and proton charges are considered fundamental building blocks, since all other charges are integral multiples of those carried by electrons and protons. Electrons and protons are also two of the three fundamental building blocks of ordinary matter. The neutron is the third and has zero total charge.

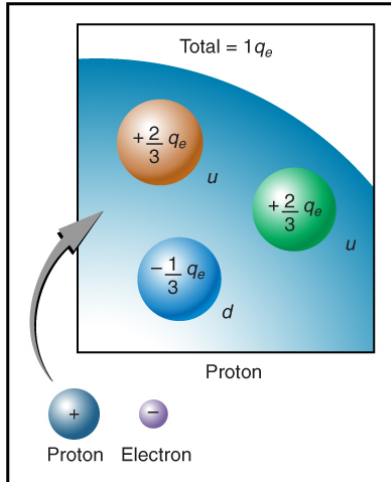
[\[link\]](#) shows a person touching a Van de Graaff generator and receiving excess positive charge. The expanded view of a hair shows the existence of both types of charges but an excess of positive. The repulsion of these positive like charges causes the strands of hair to repel other strands of hair and to stand up. The further blowup shows an artist's conception of an electron and a proton perhaps found in an atom in a strand of hair.



When this person touches a Van de Graaff generator, she receives an excess of positive charge, causing her hair to stand on end. The charges in

one hair are shown. An artist's conception of an electron and a proton illustrate the particles carrying the negative and positive charges. We cannot really see these particles with visible light because they are so small (the electron seems to be an infinitesimal point), but we know a great deal about their measurable properties, such as the charges they carry.

The electron seems to have no substructure; in contrast, when the substructure of protons is explored by scattering extremely energetic electrons from them, it appears that there are point-like particles inside the proton. These sub-particles, named quarks, have never been directly observed, but they are believed to carry fractional charges as seen in [\[link\]](#). Charges on electrons and protons and all other directly observable particles are unitary, but these quark substructures carry charges of either $-\frac{1}{3}$ or $+\frac{2}{3}$. There are continuing attempts to observe fractional charge directly and to learn of the properties of quarks, which are perhaps the ultimate substructure of matter.



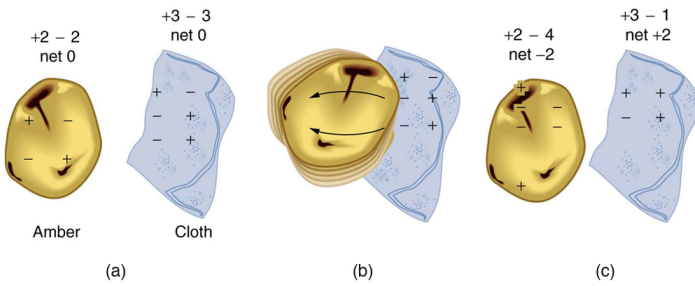
Artist's conception of fractional quark charges inside a proton. A group of three quark charges add up to the single positive charge on the proton:

$$-\frac{1}{3}q_e + \frac{2}{3}q_e + \frac{2}{3}q_e = +1q_e$$

.

Separation of Charge in Atoms

Charges in atoms and molecules can be separated—for example, by rubbing materials together. Some atoms and molecules have a greater affinity for electrons than others and will become negatively charged by close contact in rubbing, leaving the other material positively charged. (See [link](#).) Positive charge can similarly be induced by rubbing. Methods other than rubbing can also separate charges. Batteries, for example, use combinations of substances that interact in such a way as to separate charges. Chemical interactions may transfer negative charge from one substance to the other, making one battery terminal negative and leaving the first one positive.



When materials are rubbed together, charges can be separated, particularly if one material has a greater affinity for electrons than another. (a) Both the amber and cloth are originally neutral, with equal positive and negative charges. Only a tiny fraction of the charges are involved, and only a few of them are shown here. (b) When rubbed together, some negative charge is transferred to the amber, leaving the cloth with a net positive charge. (c) When separated, the amber and cloth now have net charges, but the absolute value of the net positive and negative charges will be equal.

No charge is actually created or destroyed when charges are separated as we have been discussing. Rather, existing charges are moved about. In fact, in all situations the total amount of charge is always constant. This universally obeyed law of nature is called the **law of conservation of charge**.

Note:

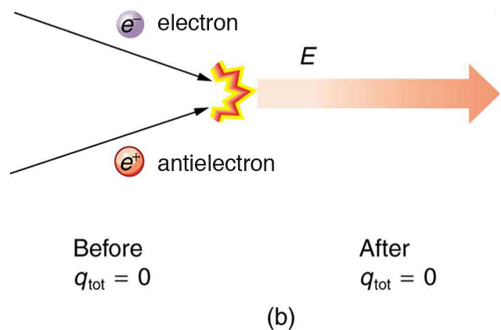
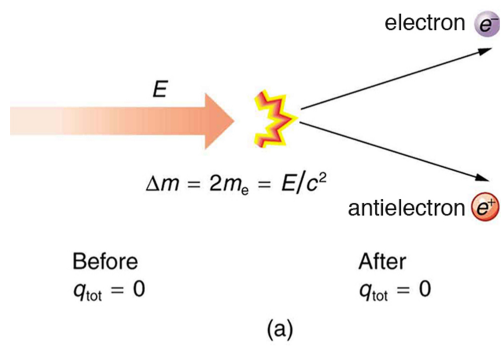
Law of Conservation of Charge

Total charge is constant in any process.

In more exotic situations, such as in particle accelerators, mass, Δm , can be created from energy in the amount $\Delta m = \frac{E}{c^2}$. Sometimes, the created mass is charged, such as when an electron is created. Whenever a charged particle is created, another having an opposite charge is always created along with it, so that the total charge created is zero. Usually, the two particles are “matter-antimatter” counterparts. For example, an antielectron would usually be created at the same time as an electron. The antielectron has a positive charge (it is called a positron), and so the total charge created is zero. (See [\[link\]](#).) All particles have antimatter counterparts with opposite signs. When matter and antimatter counterparts are brought together, they completely annihilate one another. By annihilate, we mean that the mass of the two particles is converted to energy E , again obeying the relationship $\Delta m = \frac{E}{c^2}$. Since the two particles have equal and opposite charge, the total charge is zero before and after the annihilation; thus, total charge is conserved.

Note:**Making Connections: Conservation Laws**

Only a limited number of physical quantities are universally conserved. Charge is one—energy, momentum, and angular momentum are others. Because they are conserved, these physical quantities are used to explain more phenomena and form more connections than other, less basic quantities. We find that conserved quantities give us great insight into the rules followed by nature and hints to the organization of nature. Discoveries of conservation laws have led to further discoveries, such as the weak nuclear force and the quark substructure of protons and other particles.



(a) When enough energy is present, it can be converted into matter. Here the matter created is an electron–antielectron pair. (m_e is the electron’s mass.) The total charge before and after this event is zero. (b) When matter and antimatter collide, they annihilate each other; the total charge is conserved at zero before and after the annihilation.

The law of conservation of charge is absolute—it has never been observed to be violated. Charge, then, is a special physical quantity, joining a very

short list of other quantities in nature that are always conserved. Other conserved quantities include energy, momentum, and angular momentum.

Note:

PhET Explorations: Balloons and Static Electricity

Why does a balloon stick to your sweater? Rub a balloon on a sweater, then let go of the balloon and it flies over and sticks to the sweater. View the charges in the sweater, balloons, and the wall.

https://phet.colorado.edu/sims/html/balloons-and-static-electricity/latest/balloons-and-static-electricity_en.html

Section Summary

- There are only two types of charge, which we call positive and negative.
- Like charges repel, unlike charges attract, and the force between charges decreases with the square of the distance.
- The vast majority of positive charge in nature is carried by protons, while the vast majority of negative charge is carried by electrons.
- The electric charge of one electron is equal in magnitude and opposite in sign to the charge of one proton.
- An ion is an atom or molecule that has nonzero total charge due to having unequal numbers of electrons and protons.
- The SI unit for charge is the coulomb (C), with protons and electrons having charges of opposite sign but equal magnitude; the magnitude of this basic charge $|q_e|$ is

Equation:

$$|q_e| = 1.60 \times 10^{-19} \text{ C}.$$

- Whenever charge is created or destroyed, equal amounts of positive and negative are involved.
- Most often, existing charges are separated from neutral objects to obtain some net charge.

- Both positive and negative charges exist in neutral objects and can be separated by rubbing one object with another. For macroscopic objects, negatively charged means an excess of electrons and positively charged means a depletion of electrons.
- The law of conservation of charge ensures that whenever a charge is created, an equal charge of the opposite sign is created at the same time.

Conceptual Questions

Exercise:

Problem:

There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electricity?

Exercise:

Problem:

Why do most objects tend to contain nearly equal numbers of positive and negative charges?

Problems & Exercises

Exercise:

Problem:

Common static electricity involves charges ranging from nanocoulombs to microcoulombs. (a) How many electrons are needed to form a charge of -2.00 nC (b) How many electrons must be removed from a neutral object to leave a net charge of $0.500 \mu\text{C}$?

Solution:

(a) 1.25×10^{10}

(b) 3.13×10^{12}

Exercise:

Problem:

If 1.80×10^{20} electrons move through a pocket calculator during a full day's operation, how many coulombs of charge moved through it?

Exercise:

Problem:

To start a car engine, the car battery moves 3.75×10^{21} electrons through the starter motor. How many coulombs of charge were moved?

Solution:

-600 C

Exercise:

Problem:

A certain lightning bolt moves 40.0 C of charge. How many fundamental units of charge $|q_e|$ is this?

Glossary

electric charge

a physical property of an object that causes it to be attracted toward or repelled from another charged object; each charged object generates and is influenced by a force called an electromagnetic force

law of conservation of charge

states that whenever a charge is created, an equal amount of charge with the opposite sign is created simultaneously

electron

a particle orbiting the nucleus of an atom and carrying the smallest unit of negative charge

proton

a particle in the nucleus of an atom and carrying a positive charge equal in magnitude and opposite in sign to the amount of negative charge carried by an electron

Conductors and Insulators

- Define conductor and insulator, explain the difference, and give examples of each.
- Describe three methods for charging an object.
- Explain what happens to an electric force as you move farther from the source.
- Define polarization.

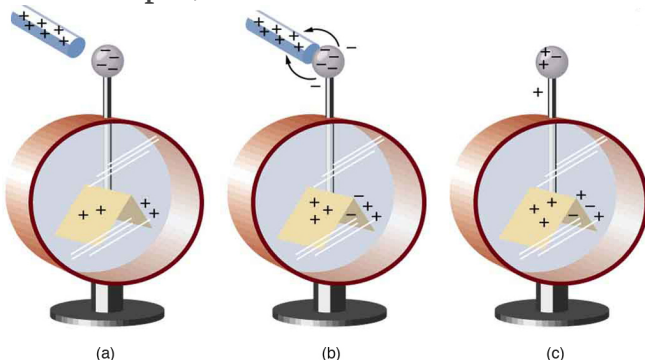


This power adapter uses metal wires and connectors to conduct electricity from the wall socket to a laptop computer. The conducting wires allow electrons to move freely through the cables, which are shielded by rubber and plastic. These materials act as insulators that don't allow electric charge to escape outward. (credit: Evan-Amos, Wikimedia Commons)

Some substances, such as metals and salty water, allow charges to move through them with relative ease. Some of the electrons in metals and similar conductors are not bound to individual atoms or sites in the material. These **free electrons** can move through the material much as air moves through loose sand. Any substance that has free electrons and allows charge to move

relatively freely through it is called a **conductor**. The moving electrons may collide with fixed atoms and molecules, losing some energy, but they can move in a conductor. Superconductors allow the movement of charge without any loss of energy. Salty water and other similar conducting materials contain free ions that can move through them. An ion is an atom or molecule having a positive or negative (nonzero) total charge. In other words, the total number of electrons is not equal to the total number of protons.

Other substances, such as glass, do not allow charges to move through them. These are called **insulators**. Electrons and ions in insulators are bound in the structure and cannot move easily—as much as 10^{23} times more slowly than in conductors. Pure water and dry table salt are insulators, for example, whereas molten salt and salty water are conductors.



An electroscope is a favorite instrument in physics demonstrations and student laboratories. It is typically made with gold foil leaves hung from a (conducting) metal stem and is insulated from the room air in a glass-walled container. (a) A positively charged glass rod is brought near the tip of the electroscope, attracting electrons to the top and leaving a net positive charge on the leaves. Like charges in the light flexible gold leaves

repel, separating them. (b) When the rod is touched against the ball, electrons are attracted and transferred, reducing the net charge on the glass rod but leaving the electroscope positively charged. (c) The excess charges are evenly distributed in the stem and leaves of the electroscope once the glass rod is removed.

Charging by Contact

[\[link\]](#) shows an electroscope being charged by touching it with a positively charged glass rod. Because the glass rod is an insulator, it must actually touch the electroscope to transfer charge to or from it. (Note that the extra positive charges reside on the surface of the glass rod as a result of rubbing it with silk before starting the experiment.) Since only electrons move in metals, we see that they are attracted to the top of the electroscope. There, some are transferred to the positive rod by touch, leaving the electroscope with a net positive charge.

Electrostatic repulsion in the leaves of the charged electroscope separates them. The electrostatic force has a horizontal component that results in the leaves moving apart as well as a vertical component that is balanced by the gravitational force. Similarly, the electroscope can be negatively charged by contact with a negatively charged object.

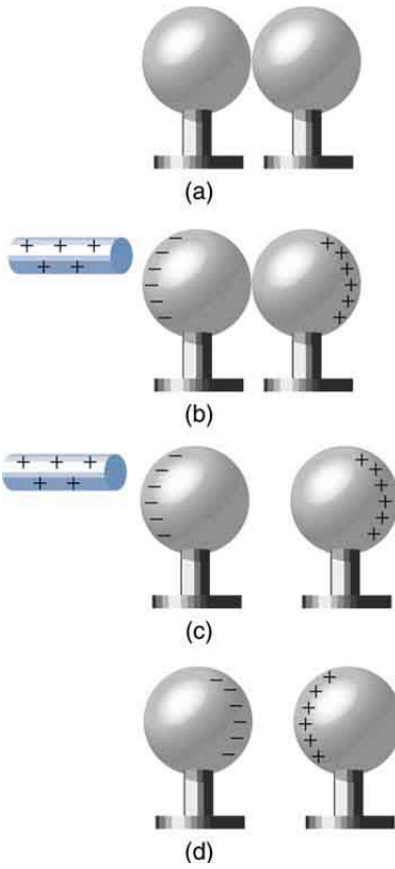
Charging by Induction

It is not necessary to transfer excess charge directly to an object in order to charge it. [\[link\]](#) shows a method of **induction** wherein a charge is created in a nearby object, without direct contact. Here we see two neutral metal spheres in contact with one another but insulated from the rest of the world.

A positively charged rod is brought near one of them, attracting negative charge to that side, leaving the other sphere positively charged.

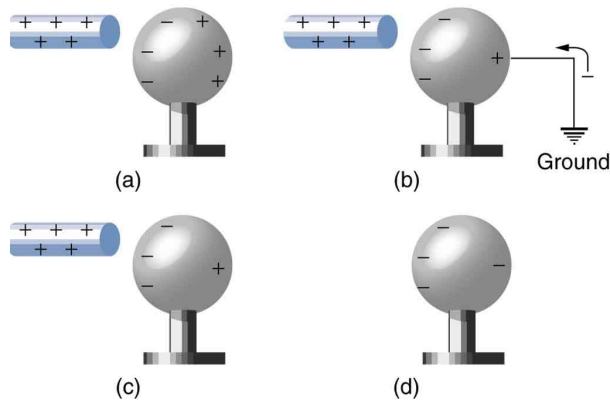
This is an example of induced **polarization** of neutral objects. Polarization is the separation of charges in an object that remains neutral. If the spheres are now separated (before the rod is pulled away), each sphere will have a net charge. Note that the object closest to the charged rod receives an opposite charge when charged by induction. Note also that no charge is removed from the charged rod, so that this process can be repeated without depleting the supply of excess charge.

Another method of charging by induction is shown in [\[link\]](#). The neutral metal sphere is polarized when a charged rod is brought near it. The sphere is then grounded, meaning that a conducting wire is run from the sphere to the ground. Since the earth is large and most ground is a good conductor, it can supply or accept excess charge easily. In this case, electrons are attracted to the sphere through a wire called the ground wire, because it supplies a conducting path to the ground. The ground connection is broken before the charged rod is removed, leaving the sphere with an excess charge opposite to that of the rod. Again, an opposite charge is achieved when charging by induction and the charged rod loses none of its excess charge.



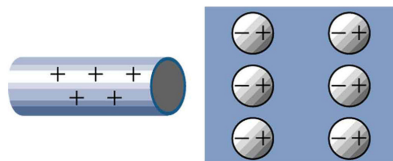
Charging by induction. (a) Two uncharged or neutral metal spheres are in contact with each other but insulated from the rest of the world. (b) A positively charged glass rod is brought near the sphere on the left, attracting negative charge and leaving the other sphere positively charged. (c) The

spheres are separated before the rod is removed, thus separating negative and positive charge. (d) The spheres retain net charges after the inducing rod is removed—without ever having been touched by a charged object.

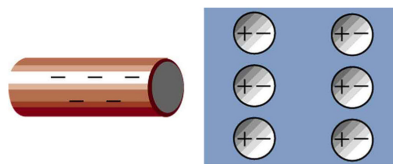


Charging by induction, using a ground connection. (a) A positively charged rod is brought near a neutral metal sphere, polarizing it. (b) The sphere is grounded, allowing electrons to be attracted from the earth's ample supply. (c) The ground connection is broken. (d) The positive rod is

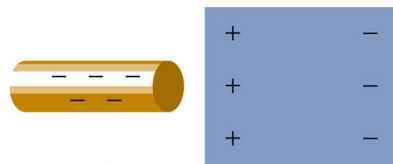
removed, leaving the sphere
with an induced negative
charge.



(a)



(b)



(c)

Both positive and
negative objects
attract a neutral
object by polarizing
its molecules. (a) A
positive object
brought near a
neutral insulator
polarizes its
molecules. There is
a slight shift in the
distribution of the
electrons orbiting
the molecule, with

unlike charges being brought nearer and like charges moved away. Since the electrostatic force decreases with distance, there is a net attraction. (b) A negative object produces the opposite polarization, but again attracts the neutral object. (c) The same effect occurs for a conductor; since the unlike charges are closer, there is a net attraction.

Neutral objects can be attracted to any charged object. The pieces of straw attracted to polished amber are neutral, for example. If you run a plastic comb through your hair, the charged comb can pick up neutral pieces of paper. [\[link\]](#) shows how the polarization of atoms and molecules in neutral objects results in their attraction to a charged object.

When a charged rod is brought near a neutral substance, an insulator in this case, the distribution of charge in atoms and molecules is shifted slightly. Opposite charge is attracted nearer the external charged rod, while like charge is repelled. Since the electrostatic force decreases with distance, the repulsion of like charges is weaker than the attraction of unlike charges, and so there is a net attraction. Thus a positively charged glass rod attracts neutral pieces of paper, as will a negatively charged rubber rod. Some

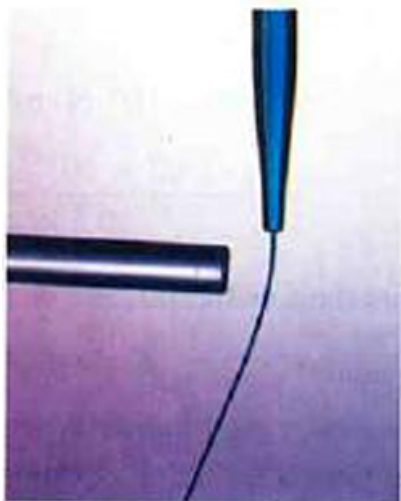
molecules, like water, are polar molecules. Polar molecules have a natural or inherent separation of charge, although they are neutral overall. Polar molecules are particularly affected by other charged objects and show greater polarization effects than molecules with naturally uniform charge distributions.

Exercise:

Check Your Understanding

Problem:

Can you explain the attraction of water to the charged rod in the figure below?



Solution:

Answer

Water molecules are polarized, giving them slightly positive and slightly negative sides. This makes water even more susceptible to a charged rod's attraction. As the water flows downward, due to the force of gravity, the charged conductor exerts a net attraction to the opposite charges in the stream of water, pulling it closer.

Note:

PhET Explorations: John Travoltage

Make sparks fly with John Travoltage. Wiggle Johnnie's foot and he picks up charges from the carpet. Bring his hand close to the door knob and get rid of the excess charge.

https://phet.colorado.edu/sims/html/john-travoltage/latest/john-travoltage_en.html

Section Summary

- Polarization is the separation of positive and negative charges in a neutral object.
- A conductor is a substance that allows charge to flow freely through its atomic structure.
- An insulator holds charge within its atomic structure.
- Objects with like charges repel each other, while those with unlike charges attract each other.
- A conducting object is said to be grounded if it is connected to the Earth through a conductor. Grounding allows transfer of charge to and from the earth's large reservoir.
- Objects can be charged by contact with another charged object and obtain the same sign charge.
- If an object is temporarily grounded, it can be charged by induction, and obtains the opposite sign charge.
- Polarized objects have their positive and negative charges concentrated in different areas, giving them a non-symmetrical charge.
- Polar molecules have an inherent separation of charge.

Conceptual Questions

Exercise:

Problem:

An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while attempting to place a large negative charge on himself, his clothes fly off. Explain.

Exercise:**Problem:**

If you have charged an electroscope by contact with a positively charged object, describe how you could use it to determine the charge of other objects. Specifically, what would the leaves of the electroscope do if other charged objects were brought near its knob?

Exercise:**Problem:**

When a glass rod is rubbed with silk, it becomes positive and the silk becomes negative—yet both attract dust. Does the dust have a third type of charge that is attracted to both positive and negative? Explain.

Exercise:**Problem:**

Why does a car always attract dust right after it is polished? (Note that car wax and car tires are insulators.)

Exercise:**Problem:**

Describe how a positively charged object can be used to give another object a negative charge. What is the name of this process?

Exercise:**Problem:**

What is grounding? What effect does it have on a charged conductor? On a charged insulator?

Problems & Exercises**Exercise:**

Problem:

Suppose a speck of dust in an electrostatic precipitator has 1.0000×10^{12} protons in it and has a net charge of -5.00 nC (a very large charge for a small speck). How many electrons does it have?

Solution:

$$1.03 \times 10^{12}$$

Exercise:**Problem:**

An amoeba has 1.00×10^{16} protons and a net charge of 0.300 pC. (a) How many fewer electrons are there than protons? (b) If you paired them up, what fraction of the protons would have no electrons?

Exercise:**Problem:**

A 50.0 g ball of copper has a net charge of $2.00 \mu\text{C}$. What fraction of the copper's electrons has been removed? (Each copper atom has 29 protons, and copper has an atomic mass of 63.5.)

Solution:

$$9.09 \times 10^{-13}$$

Exercise:**Problem:**

What net charge would you place on a 100 g piece of sulfur if you put an extra electron on 1 in 10^{12} of its atoms? (Sulfur has an atomic mass of 32.1.)

Exercise:

Problem:

How many coulombs of positive charge are there in 4.00 kg of plutonium, given its atomic mass is 244 and that each plutonium atom has 94 protons?

Solution:

$$1.48 \times 10^8 \text{ C}$$

Glossary

free electron

an electron that is free to move away from its atomic orbit

conductor

a material that allows electrons to move separately from their atomic orbits

insulator

a material that holds electrons securely within their atomic orbits

grounded

when a conductor is connected to the Earth, allowing charge to freely flow to and from Earth's unlimited reservoir

induction

the process by which an electrically charged object brought near a neutral object creates a charge in that object

polarization

slight shifting of positive and negative charges to opposite sides of an atom or molecule

electrostatic repulsion

the phenomenon of two objects with like charges repelling each other

Coulomb's Law

- State Coulomb's law in terms of how the electrostatic force changes with the distance between two objects.
- Calculate the electrostatic force between two charged point forces, such as electrons or protons.
- Compare the electrostatic force to the gravitational attraction for a proton and an electron; for a human and the Earth.



This NASA image of Arp 87 shows the result of a strong gravitational attraction between two galaxies. In contrast, at the subatomic level, the electrostatic attraction between two objects, such as an electron and a proton, is far greater than their mutual attraction due to gravity. (credit: NASA/HST)

Through the work of scientists in the late 18th century, the main features of the **electrostatic force**—the existence of two types of charge, the observation that like charges repel, unlike charges attract, and the decrease of force with distance—were eventually refined, and expressed as a mathematical formula. The mathematical formula for the electrostatic force is called **Coulomb's law** after the French physicist Charles Coulomb (1736–1806), who performed experiments and first proposed a formula to calculate it.

Note:

Coulomb's Law

Equation:

$$F = k \frac{|q_1 q_2|}{r^2}.$$

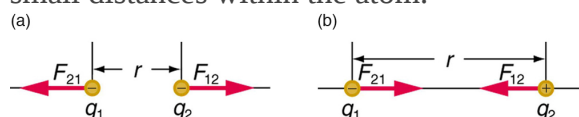
Coulomb's law calculates the magnitude of the force F between two point charges, q_1 and q_2 , separated by a distance r . In SI units, the constant k is equal to

Equation:

$$k = 8.988 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2} \approx 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}.$$

The electrostatic force is a vector quantity and is expressed in units of newtons. The force is understood to be along the line joining the two charges. (See [\[link\]](#).)

Although the formula for Coulomb's law is simple, it was no mean task to prove it. The experiments Coulomb did, with the primitive equipment then available, were difficult. Modern experiments have verified Coulomb's law to great precision. For example, it has been shown that the force is inversely proportional to distance between two objects squared ($F \propto 1/r^2$) to an accuracy of 1 part in 10^{16} . No exceptions have ever been found, even at the small distances within the atom.



The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given

by Coulomb's law. Note that

Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force on q_1 is equal in magnitude and opposite in direction to the force it exerts on q_2 .

(a) Like charges. (b) Unlike charges.

Example:

How Strong is the Coulomb Force Relative to the Gravitational Force?

Compare the electrostatic force between an electron and proton separated by 0.530×10^{-10} m with the gravitational force between them. This distance is their average separation in a hydrogen atom.

Strategy

To compare the two forces, we first compute the electrostatic force using Coulomb's law, $F = k \frac{|q_1 q_2|}{r^2}$. We then calculate the gravitational force using Newton's universal law of

gravitation. Finally, we take a ratio to see how the forces compare in magnitude.

Solution

Entering the given and known information about the charges and separation of the electron and proton into the expression of Coulomb's law yields

Equation:

$$F = k \frac{|q_1 q_2|}{r^2}$$

Equation:

$$= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(1.60 \times 10^{-19} \text{ C})(1.60 \times 10^{-19} \text{ C})}{(0.530 \times 10^{-10} \text{ m})^2}$$

Thus the Coulomb force is

Equation:

$$F = 8.19 \times 10^{-8} \text{ N}.$$

The charges are opposite in sign, so this is an attractive force. This is a very large force for an electron—it would cause an acceleration of $8.99 \times 10^{22} \text{ m/s}^2$ (verification is left as an end-of-section problem). The gravitational force is given by Newton's law of gravitation as:

Equation:

$$F_G = G \frac{mM}{r^2},$$

where $G = 6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$. Here m and M represent the electron and proton masses, which can be found in the appendices. Entering values for the knowns yields

Equation:

$$F_G = (6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2) \times \frac{(9.11 \times 10^{-31} \text{ kg})(1.67 \times 10^{-27} \text{ kg})}{(0.530 \times 10^{-10} \text{ m})^2} = 3.61 \times 10^{-47} \text{ N}$$

This is also an attractive force, although it is traditionally shown as positive since gravitational force is always attractive. The ratio of the magnitude of the electrostatic force to gravitational force in this case is, thus,

Equation:

$$\frac{F}{F_G} = 2.27 \times 10^{39}.$$

Discussion

This is a remarkably large ratio! Note that this will be the ratio of electrostatic force to gravitational force for an electron and a proton at any distance (taking the ratio before entering numerical values shows that the distance cancels). This ratio gives some indication

of just how much larger the Coulomb force is than the gravitational force between two of the most common particles in nature.

As the example implies, gravitational force is completely negligible on a small scale, where the interactions of individual charged particles are important. On a large scale, such as between the Earth and a person, the reverse is true. Most objects are nearly electrically neutral, and so attractive and repulsive **Coulomb forces** nearly cancel. Gravitational force on a large scale dominates interactions between large objects because it is always attractive, while Coulomb forces tend to cancel.

Section Summary

- Frenchman Charles Coulomb was the first to publish the mathematical equation that describes the electrostatic force between two objects.
- Coulomb's law gives the magnitude of the force between point charges. It is **Equation:**

$$F = k \frac{|q_1 q_2|}{r^2},$$

where q_1 and q_2 are two point charges separated by a distance r , and $k \approx 8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$

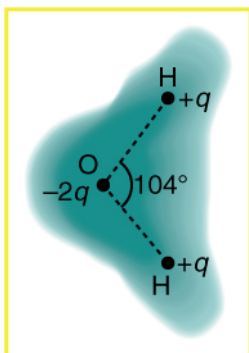
- This Coulomb force is extremely basic, since most charges are due to point-like particles. It is responsible for all electrostatic effects and underlies most macroscopic forces.
- The Coulomb force is extraordinarily strong compared with the gravitational force, another basic force—but unlike gravitational force it can cancel, since it can be either attractive or repulsive.
- The electrostatic force between two subatomic particles is far greater than the gravitational force between the same two particles.

Conceptual Questions

Exercise:

Problem:

[\[link\]](#) shows the charge distribution in a water molecule, which is called a polar molecule because it has an inherent separation of charge. Given water's polar character, explain what effect humidity has on removing excess charge from objects.



Schematic representation of the outer electron cloud of a neutral water molecule. The electrons spend more time near the oxygen than the hydrogens, giving a permanent charge separation as shown. Water is thus a *polar molecule*. It is more easily affected by electrostatic forces than molecules with uniform charge distributions.

Exercise:**Problem:**

Using [\[link\]](#), explain, in terms of Coulomb's law, why a polar molecule (such as in [\[link\]](#)) is attracted by both positive and negative charges.

Exercise:

Problem:

Given the polar character of water molecules, explain how ions in the air form nucleation centers for rain droplets.

Problems & Exercises

Exercise:

Problem:

What is the repulsive force between two pith balls that are 8.00 cm apart and have equal charges of -30.0 nC ?

Exercise:

Problem:

(a) How strong is the attractive force between a glass rod with a $0.700\text{ }\mu\text{C}$ charge and a silk cloth with a $-0.600\text{ }\mu\text{C}$ charge, which are 12.0 cm apart, using the approximation that they act like point charges? (b) Discuss how the answer to this problem might be affected if the charges are distributed over some area and do not act like point charges.

Solution:

(a) 0.263 N

(b) If the charges are distributed over some area, there will be a concentration of charge along the side closest to the oppositely charged object. This effect will increase the net force.

Exercise:

Problem:

Two point charges exert a 5.00 N force on each other. What will the force become if the distance between them is increased by a factor of three?

Exercise:

Problem:

Two point charges are brought closer together, increasing the force between them by a factor of 25. By what factor was their separation decreased?

Solution:

The separation decreased by a factor of 5.

Exercise:**Problem:**

How far apart must two point charges of 75.0 nC (typical of static electricity) be to have a force of 1.00 N between them?

Exercise:**Problem:**

If two equal charges each of 1 C each are separated in air by a distance of 1 km, what is the magnitude of the force acting between them? You will see that even at a distance as large as 1 km, the repulsive force is substantial because 1 C is a very significant amount of charge.

Exercise:**Problem:**

A test charge of $+2\ \mu\text{C}$ is placed halfway between a charge of $+6\ \mu\text{C}$ and another of $+4\ \mu\text{C}$ separated by 10 cm. (a) What is the magnitude of the force on the test charge? (b) What is the direction of this force (away from or toward the $+6\ \mu\text{C}$ charge)?

Exercise:**Problem:**

Bare free charges do not remain stationary when close together. To illustrate this, calculate the acceleration of two isolated protons separated by 2.00 nm (a typical distance between gas atoms). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

Solution:

$$\begin{aligned} F &= k \frac{|q_1 q_2|}{r^2} = ma \Rightarrow a = \frac{kq^2}{mr^2} \\ &= \frac{(9.00 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1.60 \times 10^{-19} \text{ m})^2}{(1.67 \times 10^{-27} \text{ kg})(2.00 \times 10^{-9} \text{ m})^2} \\ &= 3.45 \times 10^{16} \text{ m/s}^2 \end{aligned}$$

Exercise:**Problem:**

(a) By what factor must you change the distance between two point charges to change the force between them by a factor of 10? (b) Explain how the distance can either increase or decrease by this factor and still cause a factor of 10 change in the force.

Solution:

(a) 3.2

(b) If the distance increases by 3.2, then the force will decrease by a factor of 10 ; if the distance decreases by 3.2, then the force will increase by a factor of 10. Either way, the force changes by a factor of 10.

Exercise:

Problem:

Suppose you have a total charge q_{tot} that you can split in any manner. Once split, the separation distance is fixed. How do you split the charge to achieve the greatest force?

Exercise:

Problem:

(a) Common transparent tape becomes charged when pulled from a dispenser. If one piece is placed above another, the repulsive force can be great enough to support the top piece's weight. Assuming equal point charges (only an approximation), calculate the magnitude of the charge if electrostatic force is great enough to support the weight of a 10.0 mg piece of tape held 1.00 cm above another. (b) Discuss whether the magnitude of this charge is consistent with what is typical of static electricity.

Solution:

(a) $1.04 \times 10^{-9} \text{ C}$

(b) This charge is approximately 1 nC, which is consistent with the magnitude of charge typical for static electricity

Exercise:

Problem:

(a) Find the ratio of the electrostatic to gravitational force between two electrons. (b) What is this ratio for two protons? (c) Why is the ratio different for electrons and protons?

Exercise:

Problem:

At what distance is the electrostatic force between two protons equal to the weight of one proton?

Exercise:

Problem:

A certain five cent coin contains 5.00 g of nickel. What fraction of the nickel atoms' electrons, removed and placed 1.00 m above it, would support the weight of this coin? The atomic mass of nickel is 58.7, and each nickel atom contains 28 electrons and 28 protons.

Solution:

$$1.02 \times 10^{-11}$$

Exercise:**Problem:**

(a) Two point charges totaling $8.00 \mu\text{C}$ exert a repulsive force of 0.150 N on one another when separated by 0.500 m. What is the charge on each? (b) What is the charge on each if the force is attractive?

Exercise:**Problem:**

Point charges of $5.00 \mu\text{C}$ and $-3.00 \mu\text{C}$ are placed 0.250 m apart. (a) Where can a third charge be placed so that the net force on it is zero? (b) What if both charges are positive?

Solution:

- a. 0.859 m beyond negative charge on line connecting two charges
- b. 0.109 m from lesser charge on line connecting two charges

Exercise:**Problem:**

Two point charges q_1 and q_2 are 3.00 m apart, and their total charge is $20 \mu\text{C}$. (a) If the force of repulsion between them is 0.075N, what are magnitudes of the two charges? (b) If one charge attracts the other with a force of 0.525N, what are the magnitudes of the two charges? Note that you may need to solve a quadratic equation to reach your answer.

Glossary**Coulomb's law**

the mathematical equation calculating the electrostatic force vector between two charged particles

Coulomb force

another term for the electrostatic force

electrostatic force

the amount and direction of attraction or repulsion between two charged bodies

Electric Field: Concept of a Field Revisited

- Describe a force field and calculate the strength of an electric field due to a point charge.
- Calculate the force exerted on a test charge by an electric field.
- Explain the relationship between electrical force (F) on a test charge and electrical field strength (E).

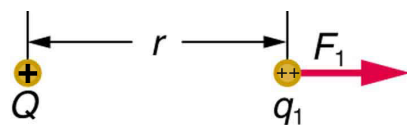
Contact forces, such as between a baseball and a bat, are explained on the small scale by the interaction of the charges in atoms and molecules in close proximity. They interact through forces that include the **Coulomb force**. Action at a distance is a force between objects that are not close enough for their atoms to “touch.” That is, they are separated by more than a few atomic diameters.

For example, a charged rubber comb attracts neutral bits of paper from a distance via the Coulomb force. It is very useful to think of an object being surrounded in space by a **force field**. The force field carries the force to another object (called a test object) some distance away.

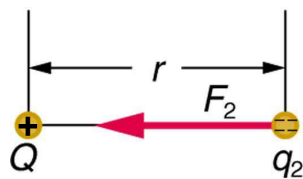
Concept of a Field

A field is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding the earth (and all other masses) represents the gravitational force that would be experienced if another mass were placed at a given point within the field.

In the same way, the Coulomb force field surrounding any charge extends throughout space. Using Coulomb’s law, $F = k|q_1q_2|/r^2$, its magnitude is given by the equation $F = k|qQ|/r^2$, for a **point charge** (a particle having a charge Q) acting on a **test charge** q at a distance r (see [\[link\]](#)). Both the magnitude and direction of the Coulomb force field depend on Q and the test charge q .



(a)



(b)

The Coulomb force field due to a positive charge Q is shown acting on two different charges. Both charges are the same distance from Q . (a) Since q_1 is positive, the force F_1 acting on it is repulsive. (b) The charge q_2 is negative and greater in magnitude than q_1 , and so the force F_2 acting on it is attractive and stronger than F_1 . The Coulomb force field is thus not unique at any point in space, because it depends on the test charges q_1 and q_2

as well as the
charge Q .

To simplify things, we would prefer to have a field that depends only on Q and not on the test charge q . The electric field is defined in such a manner that it represents only the charge creating it and is unique at every point in space. Specifically, the electric field E is defined to be the ratio of the Coulomb force to the test charge:

Equation:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the electrostatic force (or Coulomb force) exerted on a positive test charge q . It is understood that $\mathbf{E}$ is in the same direction as $\mathbf{F}$. It is also assumed that q is so small that it does not alter the charge distribution creating the electric field. The units of electric field are newtons per coulomb (N/C). If the electric field is known, then the electrostatic force on any charge q is simply obtained by multiplying charge times electric field, or $\mathbf{F} = q\mathbf{E}$. Consider the electric field due to a point charge Q . According to Coulomb's law, the force it exerts on a test charge q is $F = k|qQ|/r^2$. Thus the magnitude of the electric field, E , for a point charge is

Equation:

$$E = \left| \frac{F}{q} \right| = k \left| \frac{qQ}{qr^2} \right| = k \frac{|Q|}{r^2}.$$

Since the test charge cancels, we see that

Equation:

$$E = k \frac{|Q|}{r^2}.$$

The electric field is thus seen to depend only on the charge Q and the distance r ; it is completely independent of the test charge q .

Example:

Calculating the Electric Field of a Point Charge

Calculate the strength and direction of the electric field E due to a point charge of 2.00 nC (nano-Coulombs) at a distance of 5.00 mm from the charge.

Strategy

We can find the electric field created by a point charge by using the equation $E = kQ/r^2$.

Solution

Here $Q = 2.00 \times 10^{-9}$ C and $r = 5.00 \times 10^{-3}$ m. Entering those values into the above equation gives

Equation:

$$\begin{aligned} E &= k \frac{Q}{r^2} \\ &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \times \frac{(2.00 \times 10^{-9} \text{ C})}{(5.00 \times 10^{-3} \text{ m})^2} \\ &= 7.19 \times 10^5 \text{ N/C}. \end{aligned}$$

Discussion

This **electric field strength** is the same at any point 5.00 mm away from the charge Q that creates the field. It is positive, meaning that it has a direction pointing away from the charge Q .

Example:

Calculating the Force Exerted on a Point Charge by an Electric Field

What force does the electric field found in the previous example exert on a point charge of $-0.250 \mu\text{C}$?

Strategy

Since we know the electric field strength and the charge in the field, the force on that charge can be calculated using the definition of electric field

$\mathbf{E} = \mathbf{F}/q$ rearranged to $\mathbf{F} = q\mathbf{E}$.

Solution

The magnitude of the force on a charge $q = -0.250 \mu\text{C}$ exerted by a field of strength $E = 7.20 \times 10^5 \text{ N/C}$ is thus,

Equation:

$$\begin{aligned} F &= -qE \\ &= (0.250 \times 10^{-6} \text{ C})(7.20 \times 10^5 \text{ N/C}) \\ &= 0.180 \text{ N.} \end{aligned}$$

Because q is negative, the force is directed opposite to the direction of the field.

Discussion

The force is attractive, as expected for unlike charges. (The field was created by a positive charge and here acts on a negative charge.) The charges in this example are typical of common static electricity, and the modest attractive force obtained is similar to forces experienced in static cling and similar situations.

Note:

PhET Explorations: Electric Field of Dreams

Play ball! Add charges to the Field of Dreams and see how they react to the electric field. Turn on a background electric field and adjust the direction and magnitude.

<https://archive.cnx.org/specials/ca9a78b4-06a7-11e6-b638-3bb71d1f0b42/electric-field-of-dreams/#sim-electric-field-of-dreams>

Section Summary

- The electrostatic force field surrounding a charged object extends out into space in all directions.
- The electrostatic force exerted by a point charge on a test charge at a distance r depends on the charge of both charges, as well as the

distance between the two.

- The electric field $\mathbf{E}$ is defined to be
Equation:

$$\mathbf{E} = \frac{\mathbf{F}}{q},$$

where $\mathbf{F}$ is the Coulomb or electrostatic force exerted on a small positive test charge q . $\mathbf{E}$ has units of N/C.

- The magnitude of the electric field $\mathbf{E}$ created by a point charge Q is
Equation:

$$\mathbf{E} = k \frac{|Q|}{r^2}.$$

where r is the distance from Q . The electric field $\mathbf{E}$ is a vector and fields due to multiple charges add like vectors.

Conceptual Questions

Exercise:

Problem:

Why must the test charge q in the definition of the electric field be vanishingly small?

Exercise:

Problem:

Are the direction and magnitude of the Coulomb force unique at a given point in space? What about the electric field?

Problem Exercises

Exercise:

Problem:

What is the magnitude and direction of an electric field that exerts a 2.00×10^{-5} N upward force on a $-1.75 \mu\text{C}$ charge?

Exercise:**Problem:**

What is the magnitude and direction of the force exerted on a $3.50 \mu\text{C}$ charge by a 250 N/C electric field that points due east?

Solution:

$$8.75 \times 10^{-4} \text{ N}$$

Exercise:**Problem:**

Calculate the magnitude of the electric field 2.00 m from a point charge of 5.00 mC (such as found on the terminal of a Van de Graaff).

Exercise:**Problem:**

(a) What magnitude point charge creates a 10,000 N/C electric field at a distance of 0.250 m? (b) How large is the field at 10.0 m?

Solution:

(a) $6.94 \times 10^{-8} \text{ C}$

(b) 6.25 N/C

Exercise:

Problem:

Calculate the initial (from rest) acceleration of a proton in a $5.00 \times 10^6 \text{ N/C}$ electric field (such as created by a research Van de Graaff). Explicitly show how you follow the steps in the Problem-Solving Strategy for electrostatics.

Exercise:**Problem:**

(a) Find the magnitude and direction of an electric field that exerts a $4.80 \times 10^{-17} \text{ N}$ westward force on an electron. (b) What magnitude and direction force does this field exert on a proton?

Solution:

(a) 300 N/C (east)

(b) $4.80 \times 10^{-17} \text{ N}$ (east)

Glossary**field**

a map of the amount and direction of a force acting on other objects, extending out into space

point charge

A charged particle, designated Q , generating an electric field

test charge

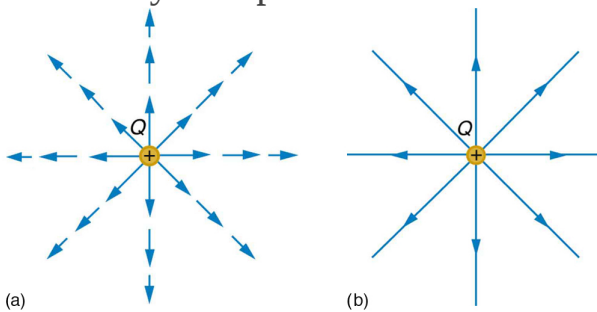
A particle (designated q) with either a positive or negative charge set down within an electric field generated by a point charge

Electric Field Lines: Multiple Charges

- Calculate the total force (magnitude and direction) exerted on a test charge from more than one charge
- Describe an electric field diagram of a positive point charge; of a negative point charge with twice the magnitude of positive charge
- Draw the electric field lines between two points of the same charge; between two points of opposite charge.

Drawings using lines to represent **electric fields** around charged objects are very useful in visualizing field strength and direction. Since the electric field has both magnitude and direction, it is a vector. Like all **vectors**, the electric field can be represented by an arrow that has length proportional to its magnitude and that points in the correct direction. (We have used arrows extensively to represent force vectors, for example.)

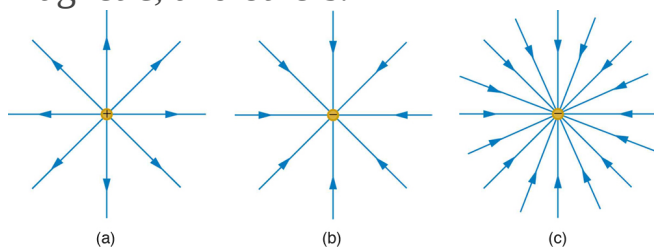
[\[link\]](#) shows two pictorial representations of the same electric field created by a positive point charge Q . [\[link\]](#) (b) shows the standard representation using continuous lines. [\[link\]](#) (a) shows numerous individual arrows with each arrow representing the force on a test charge q . Field lines are essentially a map of infinitesimal force vectors.



Two equivalent representations of the electric field due to a positive charge Q . (a) Arrows representing the electric field's magnitude and direction. (b) In the standard representation, the arrows are replaced by continuous field lines having the same direction at any point

as the electric field. The closeness of the lines is directly related to the strength of the electric field. A test charge placed anywhere will feel a force in the direction of the field line; this force will have a strength proportional to the density of the lines (being greater near the charge, for example).

Note that the electric field is defined for a positive test charge q , so that the field lines point away from a positive charge and toward a negative charge. (See [\[link\]](#).) The electric field strength is exactly proportional to the number of field lines per unit area, since the magnitude of the electric field for a point charge is $E = k|Q|/r^2$ and area is proportional to r^2 . This pictorial representation, in which field lines represent the direction and their closeness (that is, their areal density or the number of lines crossing a unit area) represents strength, is used for all fields: electrostatic, gravitational, magnetic, and others.



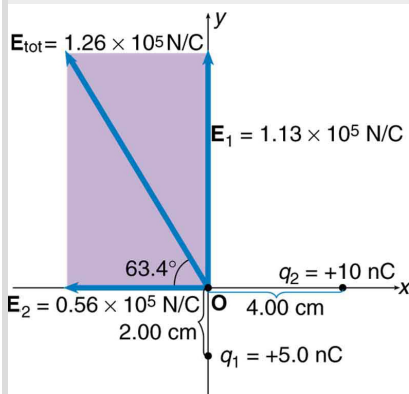
The electric field surrounding three different point charges. (a) A positive charge. (b) A negative charge of equal magnitude. (c) A larger negative charge.

In many situations, there are multiple charges. The total electric field created by multiple charges is the vector sum of the individual fields created by each charge. The following example shows how to add electric field vectors.

Example:

Adding Electric Fields

Find the magnitude and direction of the total electric field due to the two point charges, q_1 and q_2 , at the origin of the coordinate system as shown in [\[link\]](#).



The electric fields $\mathbf{E}_1$ and $\mathbf{E}_2$ at the origin O add to $\mathbf{E}_{\text{tot}}$.

Strategy

Since the electric field is a vector (having magnitude and direction), we add electric fields with the same vector techniques used for other types of vectors. We first must find the electric field due to each charge at the point of interest, which is the origin of the coordinate system (O) in this instance. We pretend that there is a positive test charge, q , at point O , which allows us to determine the direction of the fields $\mathbf{E}_1$ and $\mathbf{E}_2$. Once those fields are found, the total field can be determined using **vector addition**.

Solution

The electric field strength at the origin due to q_1 is labeled E_1 and is calculated:

Equation:

$$E_1 = k \frac{q_1}{r_1^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \frac{(5.00 \times 10^{-9} \text{ C})}{(2.00 \times 10^{-2} \text{ m})^2}$$
$$E_1 = 1.124 \times 10^5 \text{ N/C}.$$

Similarly, E_2 is

Equation:

$$E_2 = k \frac{q_2}{r_2^2} = \left(8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2 \right) \frac{(10.0 \times 10^{-9} \text{ C})}{(4.00 \times 10^{-2} \text{ m})^2}$$
$$E_2 = 0.5619 \times 10^5 \text{ N/C}.$$

Four digits have been retained in this solution to illustrate that E_1 is exactly twice the magnitude of E_2 . Now arrows are drawn to represent the magnitudes and directions of $\mathbf{E}_1$ and $\mathbf{E}_2$. (See [\[link\]](#).) The direction of the electric field is that of the force on a positive charge so both arrows point directly away from the positive charges that create them. The arrow for $\mathbf{E}_1$ is exactly twice the length of that for $\mathbf{E}_2$. The arrows form a right triangle in this case and can be added using the Pythagorean theorem. The magnitude of the total field E_{tot} is

Equation:

$$E_{\text{tot}} = (E_1^2 + E_2^2)^{1/2}$$
$$= \{(1.124 \times 10^5 \text{ N/C})^2 + (0.5619 \times 10^5 \text{ N/C})^2\}^{1/2}$$
$$= 1.26 \times 10^5 \text{ N/C}.$$

The direction is

Equation:

$$\begin{aligned}
 \theta &= \tan^{-1} \left(\frac{E_1}{E_2} \right) \\
 &= \tan^{-1} \left(\frac{1.124 \times 10^5 \text{ N/C}}{0.5619 \times 10^5 \text{ N/C}} \right) \\
 &= 63.4^\circ,
 \end{aligned}$$

or 63.4° above the x -axis.

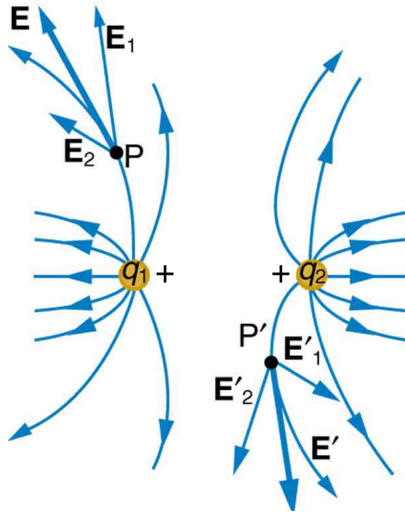
Discussion

In cases where the electric field vectors to be added are not perpendicular, vector components or graphical techniques can be used. The total electric field found in this example is the total electric field at only one point in space. To find the total electric field due to these two charges over an entire region, the same technique must be repeated for each point in the region. This impossibly lengthy task (there are an infinite number of points in space) can be avoided by calculating the total field at representative points and using some of the unifying features noted next.

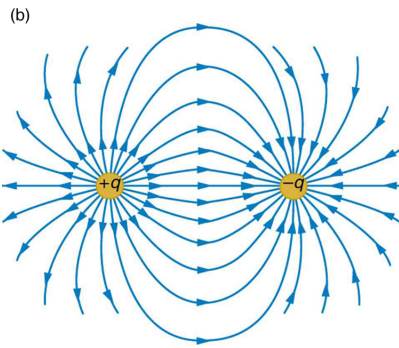
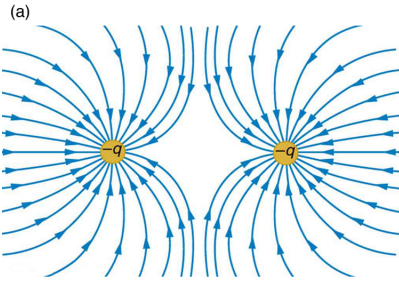
[\[link\]](#) shows how the electric field from two point charges can be drawn by finding the total field at representative points and drawing electric field lines consistent with those points. While the electric fields from multiple charges are more complex than those of single charges, some simple features are easily noticed.

For example, the field is weaker between like charges, as shown by the lines being farther apart in that region. (This is because the fields from each charge exert opposing forces on any charge placed between them.) (See [\[link\]](#) and [\[link\]](#)(a).) Furthermore, at a great distance from two like charges, the field becomes identical to the field from a single, larger charge.

[\[link\]](#)(b) shows the electric field of two unlike charges. The field is stronger between the charges. In that region, the fields from each charge are in the same direction, and so their strengths add. The field of two unlike charges is weak at large distances, because the fields of the individual charges are in opposite directions and so their strengths subtract. At very large distances, the field of two unlike charges looks like that of a smaller single charge.



Two positive point charges q_1 and q_2 produce the resultant electric field shown. The field is calculated at representative points and then smooth field lines drawn following the rules outlined in the text.



(a) Two negative charges produce the fields shown. It is very similar to the field produced by two positive charges, except that the directions are reversed. The field is clearly weaker between the charges. The individual forces on a test charge in that region are in opposite directions.

(b) Two opposite charges produce the field shown, which is stronger in the region between the charges.

We use electric field lines to visualize and analyze electric fields (the lines are a pictorial tool, not a physical entity in themselves). The properties of electric field lines for any charge distribution can be summarized as follows:

1. Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
2. The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
3. The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
4. The direction of the electric field is tangent to the field line at any point in space.
5. Field lines can never cross.

The last property means that the field is unique at any point. The field line represents the direction of the field; so if they crossed, the field would have two directions at that location (an impossibility if the field is unique).

Note:

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

[Click here
for the
simulation](#)

.

Section Summary

- Drawings of electric field lines are useful visual tools. The properties of electric field lines for any charge distribution are that:
- Field lines must begin on positive charges and terminate on negative charges, or at infinity in the hypothetical case of isolated charges.
- The number of field lines leaving a positive charge or entering a negative charge is proportional to the magnitude of the charge.
- The strength of the field is proportional to the closeness of the field lines—more precisely, it is proportional to the number of lines per unit area perpendicular to the lines.
- The direction of the electric field is tangent to the field line at any point in space.
- Field lines can never cross.

Conceptual Questions

Exercise:

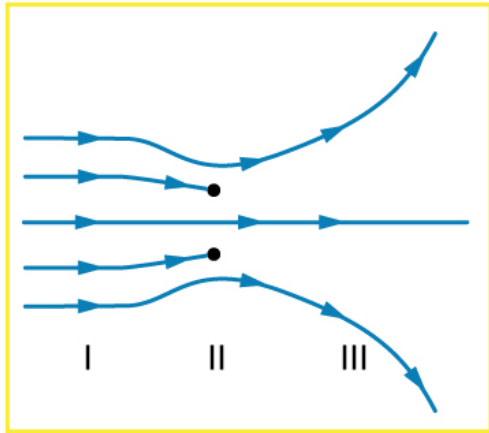
Problem:

Compare and contrast the Coulomb force field and the electric field. To do this, make a list of five properties for the Coulomb force field analogous to the five properties listed for electric field lines. Compare each item in your list of Coulomb force field properties with those of the electric field—are they the same or different? (For example, electric field lines cannot cross. Is the same true for Coulomb field lines?)

Exercise:

Problem:

[\[link\]](#) shows an electric field extending over three regions, labeled I, II, and III. Answer the following questions. (a) Are there any isolated charges? If so, in what region and what are their signs? (b) Where is the field strongest? (c) Where is it weakest? (d) Where is the field the most uniform?



Problem Exercises

Exercise:

Problem:

(a) Sketch the electric field lines near a point charge $+q$. (b) Do the same for a point charge $-3.00q$.

Exercise:

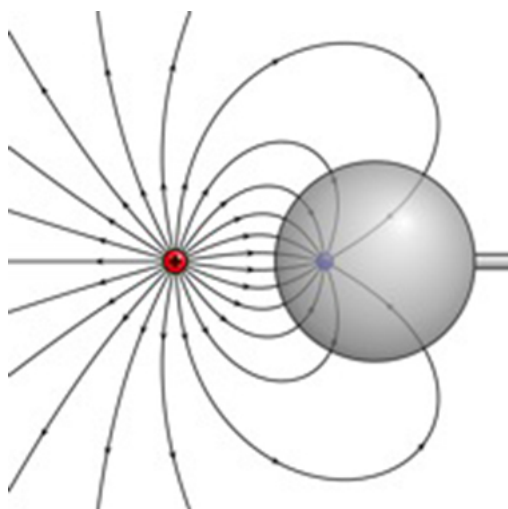
Problem:

Sketch the electric field lines a long distance from the charge distributions shown in [\[link\]](#) (a) and (b)

Exercise:

Problem:

[\[link\]](#) shows the electric field lines near two charges q_1 and q_2 . What is the ratio of their magnitudes? (b) Sketch the electric field lines a long distance from the charges shown in the figure.



The electric field near two charges.

Exercise:

Problem:

Sketch the electric field lines in the vicinity of two opposite charges, where the negative charge is three times greater in magnitude than the positive. (See [\[link\]](#) for a similar situation).

Glossary

electric field

a three-dimensional map of the electric force extended out into space from a point charge

electric field lines

a series of lines drawn from a point charge representing the magnitude and direction of force exerted by that charge

vector

a quantity with both magnitude and direction

vector addition

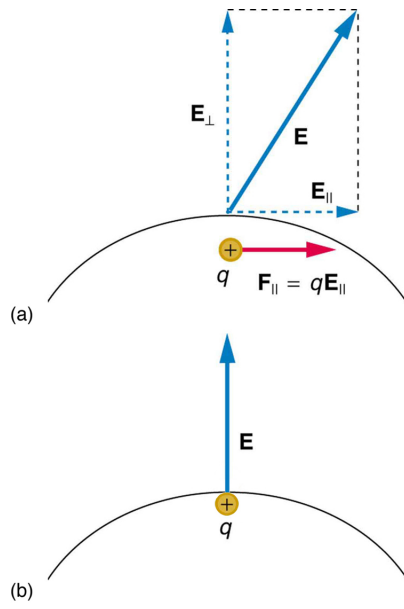
mathematical combination of two or more vectors, including their magnitudes, directions, and positions

Conductors and Electric Fields in Static Equilibrium

- List the three properties of a conductor in electrostatic equilibrium.
- Explain the effect of an electric field on free charges in a conductor.
- Explain why no electric field may exist inside a conductor.
- Describe the electric field surrounding Earth.
- Explain what happens to an electric field applied to an irregular conductor.
- Describe how a lightning rod works.
- Explain how a metal car may protect passengers inside from the dangerous electric fields caused by a downed line touching the car.

Conductors contain **free charges** that move easily. When excess charge is placed on a conductor or the conductor is put into a static electric field, charges in the conductor quickly respond to reach a steady state called **electrostatic equilibrium**.

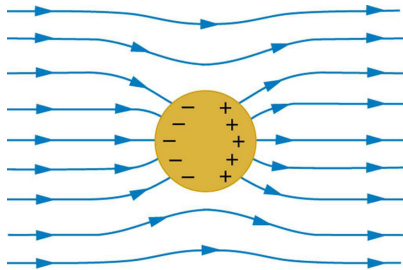
[\[link\]](#) shows the effect of an electric field on free charges in a conductor. The free charges move until the field is perpendicular to the conductor's surface. There can be no component of the field parallel to the surface in electrostatic equilibrium, since, if there were, it would produce further movement of charge. A positive free charge is shown, but free charges can be either positive or negative and are, in fact, negative in metals. The motion of a positive charge is equivalent to the motion of a negative charge in the opposite direction.



When an electric field $\mathbf{E}$ is applied to a conductor, free charges inside the conductor move until the field is perpendicular to the surface. (a) The electric field is a vector quantity, with both parallel and perpendicular components. The parallel component ($\mathbf{E}_\parallel$) exerts a force ($\mathbf{F}_\parallel$) on the free charge q , which moves the charge until $\mathbf{F}_\parallel = 0$. (b) The resulting field is perpendicular to the surface. The free charge has

been brought to the conductor's surface, leaving electrostatic forces in equilibrium.

A conductor placed in an **electric field** will be **polarized**. [\[link\]](#) shows the result of placing a neutral conductor in an originally uniform electric field. The field becomes stronger near the conductor but entirely disappears inside it.



This illustration shows a spherical conductor in static equilibrium with an originally uniform electric field. Free charges move within the conductor, polarizing it, until the electric field lines are perpendicular to the surface. The field lines end on excess negative charge on one section of the surface and begin

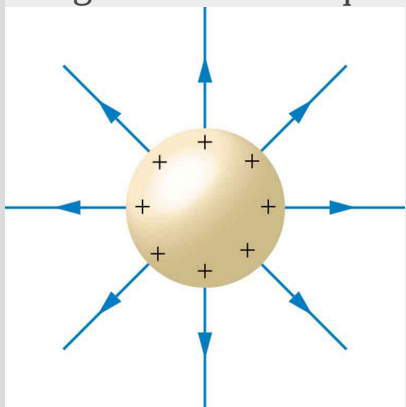
again on excess positive charge on the opposite side.

No electric field exists inside the conductor, since free charges in the conductor would continue moving in response to any field until it was neutralized.

Note:

Misconception Alert: Electric Field inside a Conductor

Excess charges placed on a spherical conductor repel and move until they are evenly distributed, as shown in [\[link\]](#). Excess charge is forced to the surface until the field inside the conductor is zero. Outside the conductor, the field is exactly the same as if the conductor were replaced by a point charge at its center equal to the excess charge.



The mutual repulsion of excess positive charges on

a spherical conductor distributes them uniformly on its surface. The resulting electric field is perpendicular to the surface and zero inside. Outside the conductor, the field is identical to that of a point charge at the center equal to the excess charge.

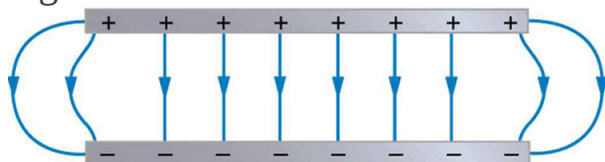
Note:

Properties of a Conductor in Electrostatic Equilibrium

1. The electric field is zero inside a conductor.
2. Just outside a conductor, the electric field lines are perpendicular to its surface, ending or beginning on charges on the surface.
3. Any excess charge resides entirely on the surface or surfaces of a conductor.

The properties of a conductor are consistent with the situations already discussed and can be used to analyze any conductor in electrostatic equilibrium. This can lead to some interesting new insights, such as described below.

How can a very uniform electric field be created? Consider a system of two metal plates with opposite charges on them, as shown in [\[link\]](#). The properties of conductors in electrostatic equilibrium indicate that the electric field between the plates will be uniform in strength and direction. Except near the edges, the excess charges distribute themselves uniformly, producing field lines that are uniformly spaced (hence uniform in strength) and perpendicular to the surfaces (hence uniform in direction, since the plates are flat). The edge effects are less important when the plates are close together.



Two metal plates with equal,
but opposite, excess charges.

The field between them is
uniform in strength and
direction except near the edges.

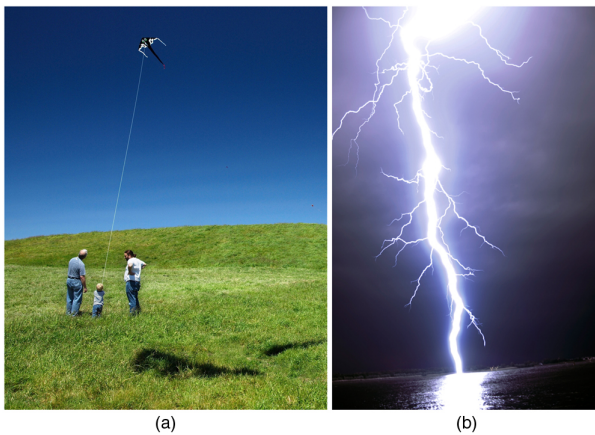
One use of such a field is to
produce uniform acceleration of
charges between the plates,
such as in the electron gun of a
TV tube.

Earth's Electric Field

A near uniform electric field of approximately 150 N/C , directed downward, surrounds Earth, with the magnitude increasing slightly as we get closer to the surface. What causes the electric field? At around 100 km above the surface of Earth we have a layer of charged particles, called the **ionosphere**. The ionosphere is responsible for a range of phenomena including the electric field surrounding Earth. In fair weather the ionosphere is positive and the Earth largely negative, maintaining the electric field ([\[link\]](#)(a)).

In storm conditions clouds form and localized electric fields can be larger and reversed in direction ([link](#)(b)). The exact charge distributions depend on the local conditions, and variations of [link](#)(b) are possible.

If the electric field is sufficiently large, the insulating properties of the surrounding material break down and it becomes conducting. For air this occurs at around 3×10^6 N/C. Air ionizes ions and electrons recombine, and we get discharge in the form of lightning sparks and corona discharge.



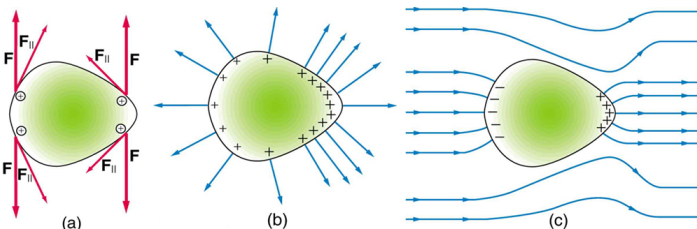
Earth's electric field. (a) Fair weather field. Earth and the ionosphere (a layer of charged particles) are both conductors. They produce a uniform electric field of about 150 N/C. (credit: D. H. Parks) (b) Storm fields. In the presence of storm clouds, the local electric fields can be larger. At very high fields, the insulating properties of the air break down and lightning can occur. (credit: Jan-Joost Verhoef)

Electric Fields on Uneven Surfaces

So far we have considered excess charges on a smooth, symmetrical conductor surface. What happens if a conductor has sharp corners or is pointed? Excess charges on a nonuniform conductor become concentrated at the sharpest points. Additionally, excess charge may move on or off the conductor at the sharpest points.

To see how and why this happens, consider the charged conductor in [\[link\]](#). The electrostatic repulsion of like charges is most effective in moving them apart on the flattest surface, and so they become least concentrated there. This is because the forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surfaces are different. The component parallel to the surface is greatest on the flattest surface and, hence, more effective in moving the charge.

The same effect is produced on a conductor by an externally applied electric field, as seen in [\[link\]](#) (c). Since the field lines must be perpendicular to the surface, more of them are concentrated on the most curved parts.



Excess charge on a nonuniform conductor becomes most concentrated at the location of greatest curvature.

- (a) The forces between identical pairs of charges at either end of the conductor are identical, but the components of the forces parallel to the surface are different. It is $\mathbf{F}_{\parallel}$ that moves the charges apart once they

have reached the surface. (b) $\mathbf{F}_{\parallel}$ is smallest at the more pointed end, the charges are left closer together, producing the electric field shown. (c)

An uncharged conductor in an originally uniform electric field is polarized, with the most concentrated charge at its most pointed end.

Applications of Conductors

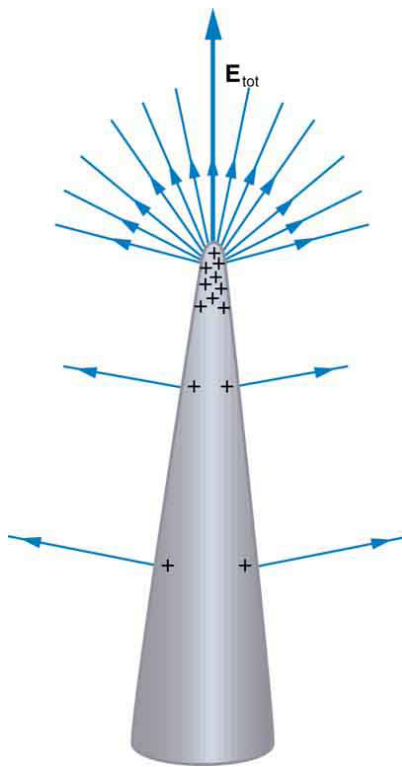
On a very sharply curved surface, such as shown in [\[link\]](#), the charges are so concentrated at the point that the resulting electric field can be great enough to remove them from the surface. This can be useful.

Lightning rods work best when they are most pointed. The large charges created in storm clouds induce an opposite charge on a building that can result in a lightning bolt hitting the building. The induced charge is bled away continually by a lightning rod, preventing the more dramatic lightning strike.

Of course, we sometimes wish to prevent the transfer of charge rather than to facilitate it. In that case, the conductor should be very smooth and have as large a radius of curvature as possible. (See [\[link\]](#).) Smooth surfaces are used on high-voltage transmission lines, for example, to avoid leakage of charge into the air.

Another device that makes use of some of these principles is a **Faraday cage**. This is a metal shield that encloses a volume. All electrical charges will reside on the outside surface of this shield, and there will be no electrical field inside. A Faraday cage is used to prohibit stray electrical fields in the environment from interfering with sensitive measurements, such as the electrical signals inside a nerve cell.

During electrical storms if you are driving a car, it is best to stay inside the car as its metal body acts as a Faraday cage with zero electrical field inside. If in the vicinity of a lightning strike, its effect is felt on the outside of the car and the inside is unaffected, provided you remain totally inside. This is also true if an active (“hot”) electrical wire was broken (in a storm or an accident) and fell on your car.



A very pointed conductor has a large charge concentration at the point. The electric field is very strong at the point and can exert a force large enough to transfer charge on or off the conductor.

Lightning rods are used to prevent the buildup of large excess charges on structures and, thus, are pointed.



(a) A lightning rod is pointed to facilitate the transfer of charge. (credit: Romaine, Wikimedia Commons) (b) This Van de Graaff generator has a smooth surface with a large radius of curvature to prevent the transfer of charge and allow a large voltage to be generated. The mutual repulsion of like charges is evident in the person's hair while touching the metal sphere. (credit: Jon 'ShakataGaNai' Davis/Wikimedia Commons).

Section Summary

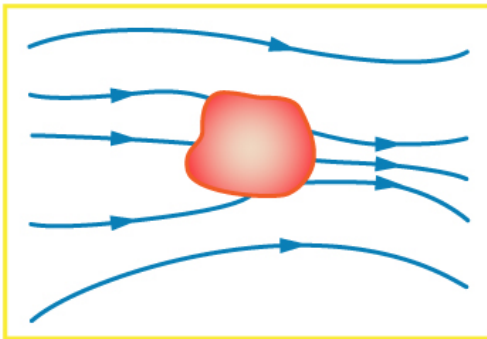
- A conductor allows free charges to move about within it.
- The electrical forces around a conductor will cause free charges to move around inside the conductor until static equilibrium is reached.
- Any excess charge will collect along the surface of a conductor.
- Conductors with sharp corners or points will collect more charge at those points.
- A lightning rod is a conductor with sharply pointed ends that collect excess charge on the building caused by an electrical storm and allow it to dissipate back into the air.
- Electrical storms result when the electrical field of Earth's surface in certain locations becomes more strongly charged, due to changes in the insulating effect of the air.
- A Faraday cage acts like a shield around an object, preventing electric charge from penetrating inside.

Conceptual Questions

Exercise:

Problem:

Is the object in [\[link\]](#) a conductor or an insulator? Justify your answer.



Exercise:

Problem:

If the electric field lines in the figure above were perpendicular to the object, would it necessarily be a conductor? Explain.

Exercise:

Problem:

The discussion of the electric field between two parallel conducting plates, in this module states that edge effects are less important if the plates are close together. What does close mean? That is, is the actual plate separation crucial, or is the ratio of plate separation to plate area crucial?

Exercise:**Problem:**

Would the self-created electric field at the end of a pointed conductor, such as a lightning rod, remove positive or negative charge from the conductor? Would the same sign charge be removed from a neutral pointed conductor by the application of a similar externally created electric field? (The answers to both questions have implications for charge transfer utilizing points.)

Exercise:**Problem:**

Why is a golfer with a metal club over her shoulder vulnerable to lightning in an open fairway? Would she be any safer under a tree?

Exercise:**Problem:**

Can the belt of a Van de Graaff accelerator be a conductor? Explain.

Exercise:**Problem:**

Are you relatively safe from lightning inside an automobile? Give two reasons.

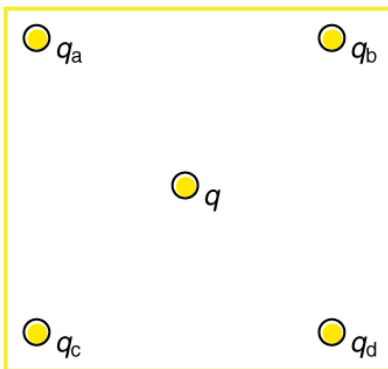
Exercise:

Problem:

Discuss pros and cons of a lightning rod being grounded versus simply being attached to a building.

Exercise:**Problem:**

Using the symmetry of the arrangement, show that the net Coulomb force on the charge q at the center of the square below ([link](#)) is zero if the charges on the four corners are exactly equal.



Four point charges q_a , q_b , q_c , and q_d lie on the corners of a square and q is located at its center.

Exercise:

Problem:

(a) Using the symmetry of the arrangement, show that the electric field at the center of the square in [\[link\]](#) is zero if the charges on the four corners are exactly equal. (b) Show that this is also true for any combination of charges in which $q_a = q_d$ and $q_b = q_c$

Exercise:**Problem:**

(a) What is the direction of the total Coulomb force on q in [\[link\]](#) if q is negative, $q_a = q_c$ and both are negative, and $q_b = q_c$ and both are positive? (b) What is the direction of the electric field at the center of the square in this situation?

Exercise:**Problem:**

Considering [\[link\]](#), suppose that $q_a = q_d$ and $q_b = q_c$. First show that q is in static equilibrium. (You may neglect the gravitational force.) Then discuss whether the equilibrium is stable or unstable, noting that this may depend on the signs of the charges and the direction of displacement of q from the center of the square.

Exercise:**Problem:**

If $q_a = 0$ in [\[link\]](#), under what conditions will there be no net Coulomb force on q ?

Exercise:**Problem:**

In regions of low humidity, one develops a special “grip” when opening car doors, or touching metal door knobs. This involves placing as much of the hand on the device as possible, not just the ends of one’s fingers. Discuss the induced charge and explain why this is done.

Exercise:**Problem:**

Tollbooth stations on roadways and bridges usually have a piece of wire stuck in the pavement before them that will touch a car as it approaches. Why is this done?

Exercise:**Problem:**

Suppose a woman carries an excess charge. To maintain her charged status can she be standing on ground wearing just any pair of shoes? How would you discharge her? What are the consequences if she simply walks away?

Problems & Exercises**Exercise:****Problem:**

Sketch the electric field lines in the vicinity of the conductor in [\[link\]](#) given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?

**Exercise:****Problem:**

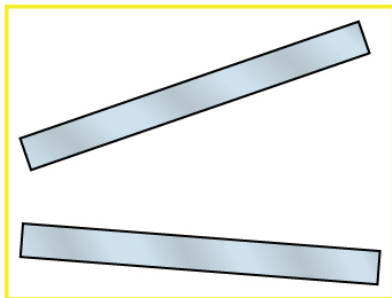
Sketch the electric field lines in the vicinity of the conductor in [\[link\]](#) given the field was originally uniform and parallel to the object's long axis. Is the resulting field small near the long side of the object?



Exercise:

Problem:

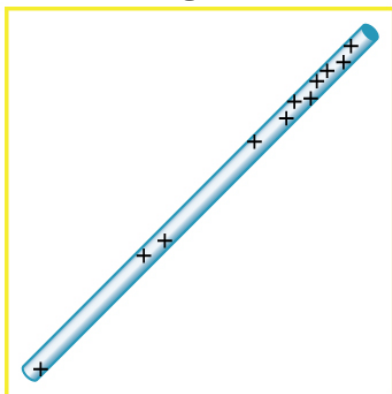
Sketch the electric field between the two conducting plates shown in [\[link\]](#), given the top plate is positive and an equal amount of negative charge is on the bottom plate. Be certain to indicate the distribution of charge on the plates.



Exercise:

Problem:

Sketch the electric field lines in the vicinity of the charged insulator in [\[link\]](#) noting its nonuniform charge distribution.



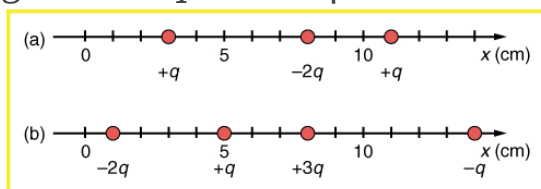
A charged
insulating rod such
as might be used in

a classroom
demonstration.

Exercise:

Problem:

What is the force on the charge located at $x = 8.00$ cm in [\[link\]](#)(a) given that $q = 1.00 \mu\text{C}$?



(a) Point charges located at 3.00, 8.00, and 11.0 cm along the x -axis. (b) Point charges located at 1.00, 5.00, 8.00, and 14.0 cm along the x -axis.

Exercise:

Problem:

(a) Find the total electric field at $x = 1.00$ cm in [\[link\]](#)(b) given that $q = 5.00 \text{ nC}$. (b) Find the total electric field at $x = 11.00$ cm in [\[link\]](#)(b). (c) If the charges are allowed to move and eventually be brought to rest by friction, what will the final charge configuration be? (That is, will there be a single charge, double charge, etc., and what will its value(s) be?)

Solution:

(a) $E_{x=1.00 \text{ cm}} = -\infty$

(b) $2.12 \times 10^5 \text{ N/C}$

(c) one charge of $+q$

Exercise:

Problem:

(a) Find the electric field at $x = 5.00 \text{ cm}$ in [\[link\]](#)(a), given that $q = 1.00 \mu\text{C}$. (b) At what position between 3.00 and 8.00 cm is the total electric field the same as that for $-2q$ alone? (c) Can the electric field be zero anywhere between 0.00 and 8.00 cm ? (d) At very large positive or negative values of x , the electric field approaches zero in both (a) and (b). In which does it most rapidly approach zero and why? (e) At what position to the right of 11.0 cm is the total electric field zero, other than at infinity? (Hint: A graphing calculator can yield considerable insight in this problem.)

Exercise:

Problem:

(a) Find the total Coulomb force on a charge of 2.00 nC located at $x = 4.00 \text{ cm}$ in [\[link\]](#) (b), given that $q = 1.00 \mu\text{C}$. (b) Find the x -position at which the electric field is zero in [\[link\]](#) (b).

Solution:

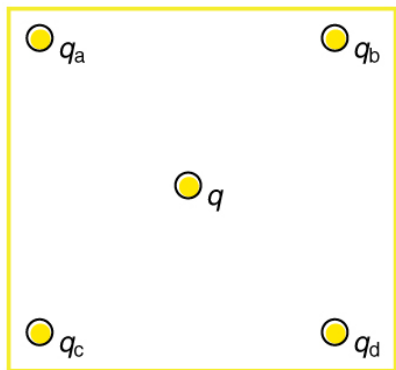
(a) 0.252 N to the left

(b) $x = 6.07 \text{ cm}$

Exercise:

Problem:

Using the symmetry of the arrangement, determine the direction of the force on q in the figure below, given that $q_a = q_b = +7.50 \mu\text{C}$ and $q_c = q_d = -7.50 \mu\text{C}$. (b) Calculate the magnitude of the force on the charge q , given that the square is 10.0 cm on a side and $q = 2.00 \mu\text{C}$.



Exercise:

Problem:

(a) Using the symmetry of the arrangement, determine the direction of the electric field at the center of the square in [\[link\]](#), given that $q_a = q_b = -1.00 \mu\text{C}$ and $q_c = q_d = +1.00 \mu\text{C}$. (b) Calculate the magnitude of the electric field at the location of q , given that the square is 5.00 cm on a side.

Solution:

(a) The electric field at the center of the square will be straight up, since q_a and q_b are positive and q_c and q_d are negative and all have the same magnitude.

(b) $2.04 \times 10^7 \text{ N/C}$ (upward)

Exercise:

Problem:

Find the electric field at the location of q_a in [\[link\]](#) given that $q_b = q_c = q_d = +2.00 \text{ nC}$, $q = -1.00 \text{ nC}$, and the square is 20.0 cm on a side.

Exercise:

Problem:

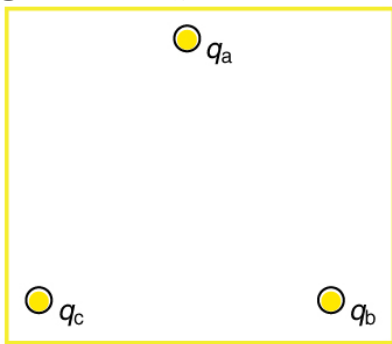
Find the total Coulomb force on the charge q in [\[link\]](#), given that $q = 1.00 \mu\text{C}$, $q_a = 2.00 \mu\text{C}$, $q_b = -3.00 \mu\text{C}$, $q_c = -4.00 \mu\text{C}$, and $q_d = +1.00 \mu\text{C}$. The square is 50.0 cm on a side.

Solution:

0.102 N, in the $-y$ direction

Exercise:**Problem:**

(a) Find the electric field at the location of q_a in [\[link\]](#), given that $q_b = +10.00 \mu\text{C}$ and $q_c = -5.00 \mu\text{C}$. (b) What is the force on q_a , given that $q_a = +1.50 \text{ nC}$?



Point charges
located at the
corners of an
equilateral triangle
25.0 cm on a side.

Exercise:

Problem:

(a) Find the electric field at the center of the triangular configuration of charges in [\[link\]](#), given that $q_a = +2.50 \text{ nC}$, $q_b = -8.00 \text{ nC}$, and $q_c = +1.50 \text{ nC}$. (b) Is there any combination of charges, other than $q_a = q_b = q_c$, that will produce a zero strength electric field at the center of the triangular configuration?

Solution:

(a) $\vec{E} = 4.36 \times 10^3 \text{ N/C}$, 35.0° , below the horizontal.

(b) No

Glossary**conductor**

an object with properties that allow charges to move about freely within it

free charge

an electrical charge (either positive or negative) which can move about separately from its base molecule

electrostatic equilibrium

an electrostatically balanced state in which all free electrical charges have stopped moving about

polarized

a state in which the positive and negative charges within an object have collected in separate locations

ionosphere

a layer of charged particles located around 100 km above the surface of Earth, which is responsible for a range of phenomena including the electric field surrounding Earth

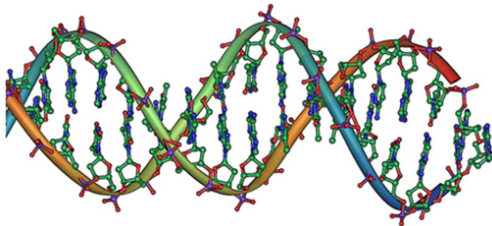
Faraday cage

a metal shield which prevents electric charge from penetrating its surface

Electric Forces in Biology

- Describe how a water molecule is polar.
- Explain electrostatic screening by a water molecule within a living cell.

Classical electrostatics has an important role to play in modern molecular biology. Large molecules such as proteins, nucleic acids, and so on—so important to life—are usually electrically charged. DNA itself is highly charged; it is the electrostatic force that not only holds the molecule together but gives the molecule structure and strength. [\[link\]](#) is a schematic of the DNA double helix.



DNA is a highly charged molecule. The DNA double helix shows the two coiled strands each containing a row of nitrogenous bases, which “code” the genetic information needed by a living organism. The strands are connected by bonds between pairs of bases. While pairing combinations between certain bases are fixed (C-G and A-T), the sequence of nucleotides in the strand varies. (credit: Jerome Walker)

The four nucleotide bases are given the symbols A (adenine), C (cytosine), G (guanine), and T (thymine). The order of the four bases varies in each strand, but the pairing between bases is always the same. C and G are always paired and A and T are always paired, which helps to preserve the order of bases in cell division (mitosis) so as to pass on the correct genetic information. Since the Coulomb force drops with distance ($F \propto 1/r^2$), the distances between the base pairs must be small enough that the electrostatic force is sufficient to hold them together.

DNA is a highly charged molecule, with about $2q_e$ (fundamental charge) per 0.3×10^{-9} m. The distance separating the two strands that make up the DNA structure is about 1 nm, while the distance separating the individual atoms within each base is about 0.3 nm.

One might wonder why electrostatic forces do not play a larger role in biology than they do if we have so many charged molecules. The reason is that the electrostatic force is “diluted” due to **screening** between molecules. This is due to the presence of other charges in the cell.

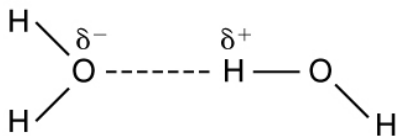
Polarity of Water Molecules

The best example of this charge screening is the water molecule, represented as H_2O . Water is a strongly **polar molecule**. Its 10 electrons (8 from the oxygen atom and 2 from the two hydrogen atoms) tend to remain closer to the oxygen nucleus than the hydrogen nuclei. This creates two centers of equal and opposite charges—what is called a **dipole**, as illustrated in [\[link\]](#). The magnitude of the dipole is called the dipole moment.

These two centers of charge will terminate some of the electric field lines coming from a free charge, as on a DNA molecule. This results in a reduction in the strength of the **Coulomb interaction**. One might say that screening makes the Coulomb force a short range force rather than long range.

Other ions of importance in biology that can reduce or screen Coulomb interactions are Na^+ , and K^+ , and Cl^- . These ions are located both inside and outside of living cells. The movement of these ions through cell membranes is crucial to the motion of nerve impulses through nerve axons.

Recent studies of electrostatics in biology seem to show that electric fields in cells can be extended over larger distances, in spite of screening, by “microtubules” within the cell. These microtubules are hollow tubes composed of proteins that guide the movement of chromosomes when cells divide, the motion of other organisms within the cell, and provide mechanisms for motion of some cells (as motors).



This schematic shows water (H_2O) as a polar molecule. Unequal sharing of electrons between the oxygen (O) and hydrogen (H) atoms leads to a net separation of positive and negative charge—forming a dipole.

The symbols δ^- and δ^+ indicate that the oxygen side of the H_2O molecule tends to be more negative, while the hydrogen ends tend

to be more positive.

This leads to an attraction of opposite charges between molecules.

Section Summary

- Many molecules in living organisms, such as DNA, carry a charge.
- An uneven distribution of the positive and negative charges within a polar molecule produces a dipole.
- The effect of a Coulomb field generated by a charged object may be reduced or blocked by other nearby charged objects.
- Biological systems contain water, and because water molecules are polar, they have a strong effect on other molecules in living systems.

Conceptual Question

Exercise:

Problem:

A cell membrane is a thin layer enveloping a cell. The thickness of the membrane is much less than the size of the cell. In a static situation the membrane has a charge distribution of $-2.5 \times 10^{-6} \text{C/m}^2$ on its inner surface and $+2.5 \times 10^{-6} \text{C/m}^2$ on its outer surface. Draw a diagram of the cell and the surrounding cell membrane. Include on this diagram the charge distribution and the corresponding electric field. Is there any electric field inside the cell? Is there any electric field outside the cell?

Glossary

dipole

a molecule's lack of symmetrical charge distribution, causing one side to be more positive and another to be more negative

polar molecule

a molecule with an asymmetrical distribution of positive and negative charge

screening

the dilution or blocking of an electrostatic force on a charged object by the presence of other charges nearby

Coulomb interaction

the interaction between two charged particles generated by the Coulomb forces they exert on one another

Applications of Electrostatics

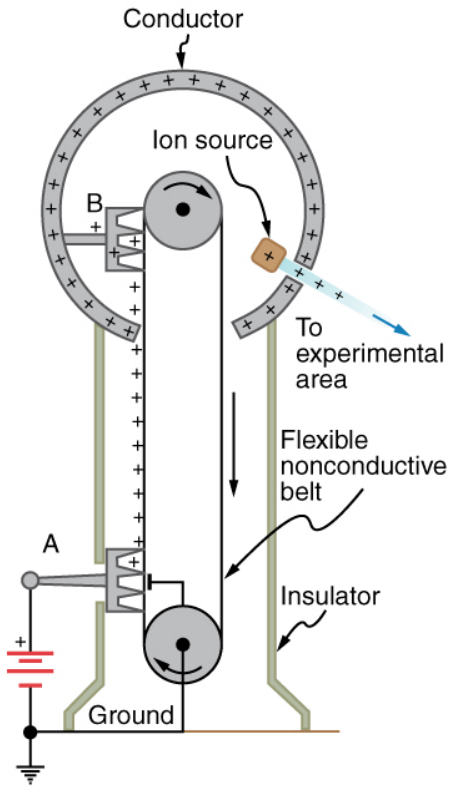
- Name several real-world applications of the study of electrostatics.

The study of **electrostatics** has proven useful in many areas. This module covers just a few of the many applications of electrostatics.

The Van de Graaff Generator

Van de Graaff generators (or Van de Graaffs) are not only spectacular devices used to demonstrate high voltage due to static electricity—they are also used for serious research. The first was built by Robert Van de Graaff in 1931 (based on original suggestions by Lord Kelvin) for use in nuclear physics research. [\[link\]](#) shows a schematic of a large research version. Van de Graaffs utilize both smooth and pointed surfaces, and conductors and insulators to generate large static charges and, hence, large voltages.

A very large excess charge can be deposited on the sphere, because it moves quickly to the outer surface. Practical limits arise because the large electric fields polarize and eventually ionize surrounding materials, creating free charges that neutralize excess charge or allow it to escape. Nevertheless, voltages of 15 million volts are well within practical limits.



Schematic of Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not

remain inside the conducting sphere but moves to its outside surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities.

Note:

Take-Home Experiment: Electrostatics and Humidity

Rub a comb through your hair and use it to lift pieces of paper. It may help to tear the pieces of paper rather than cut them neatly. Repeat the exercise in your bathroom after you have had a long shower and the air in the bathroom is moist. Is it easier to get electrostatic effects in dry or moist air? Why would torn paper be more attractive to the comb than cut paper? Explain your observations.

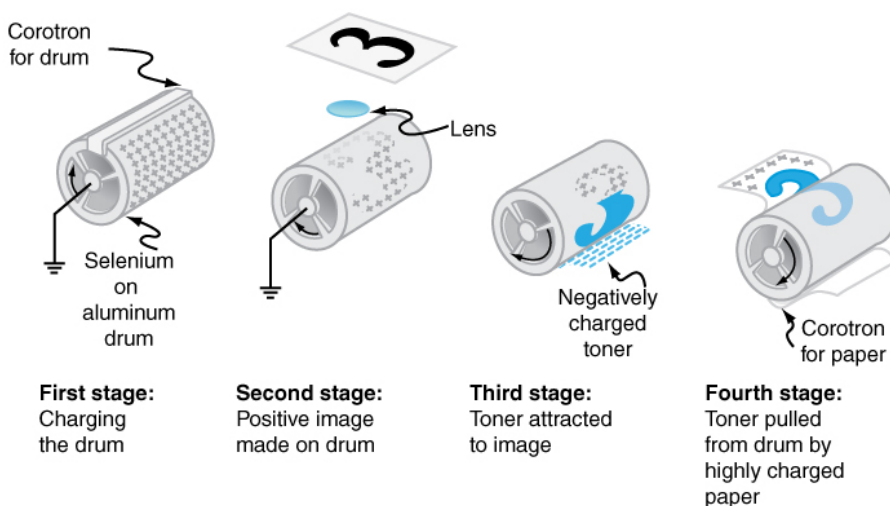
Xerography

Most copy machines use an electrostatic process called **xerography**—a word coined from the Greek words *xeros* for dry and *graphos* for writing. The heart of the process is shown in simplified form in [\[link\]](#).

A selenium-coated aluminum drum is sprayed with positive charge from points on a device called a corotron. Selenium is a substance with an interesting property—it is a **photoconductor**. That is, selenium is an insulator when in the dark and a conductor when exposed to light.

In the first stage of the xerography process, the conducting aluminum drum is **grounded** so that a negative charge is induced under the thin layer of uniformly positively charged selenium. In the second stage, the surface of the drum is exposed to the image of whatever is to be copied. Where the image is light, the selenium becomes conducting, and the positive charge is neutralized. In dark areas, the positive charge remains, and so the image has been transferred to the drum.

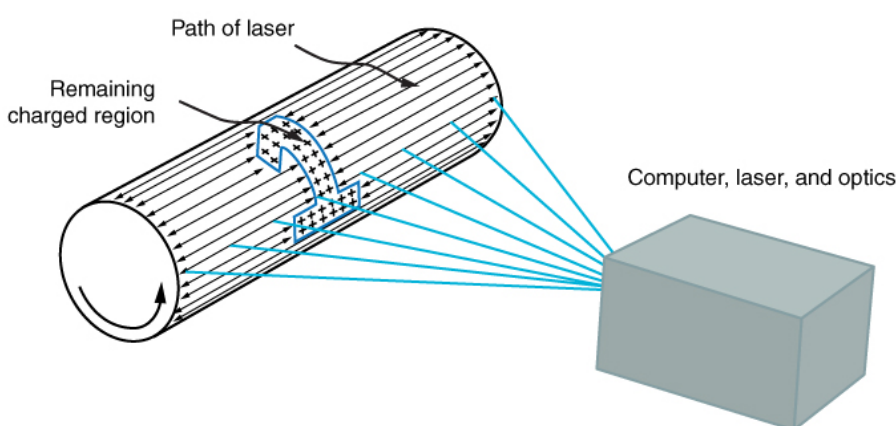
The third stage takes a dry black powder, called toner, and sprays it with a negative charge so that it will be attracted to the positive regions of the drum. Next, a blank piece of paper is given a greater positive charge than on the drum so that it will pull the toner from the drum. Finally, the paper and electrostatically held toner are passed through heated pressure rollers, which melt and permanently adhere the toner within the fibers of the paper.



Xerography is a dry copying process based on electrostatics. The major steps in the process are the charging of the photoconducting drum, transfer of an image creating a positive charge duplicate, attraction of toner to the charged parts of the drum, and transfer of toner to the paper. Not shown are heat treatment of the paper and cleansing of the drum for the next copy.

Laser Printers

Laser printers use the xerographic process to make high-quality images on paper, employing a laser to produce an image on the photoconducting drum as shown in [\[link\]](#). In its most common application, the laser printer receives output from a computer, and it can achieve high-quality output because of the precision with which laser light can be controlled. Many laser printers do significant information processing, such as making sophisticated letters or fonts, and may contain a computer more powerful than the one giving them the raw data to be printed.

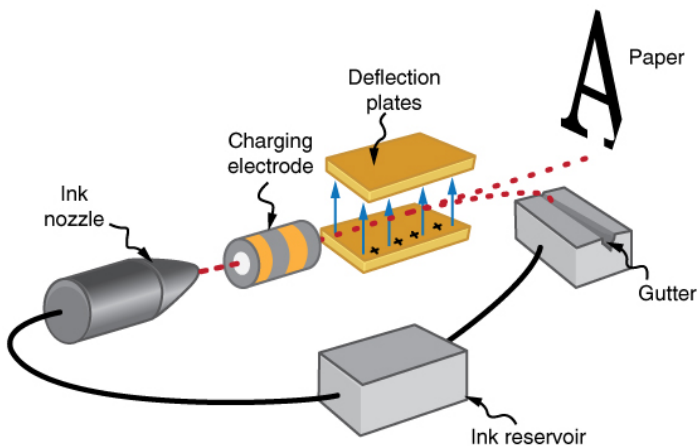


In a laser printer, a laser beam is scanned across a photoconducting drum, leaving a positive charge image. The other steps for charging the drum and transferring the image to paper are the same as in xerography. Laser light can be very precisely controlled, enabling laser printers to produce high-quality images.

Ink Jet Printers and Electrostatic Painting

The **ink jet printer**, commonly used to print computer-generated text and graphics, also employs electrostatics. A nozzle makes a fine spray of tiny ink droplets, which are then given an electrostatic charge. (See [\[link\]](#).)

Once charged, the droplets can be directed, using pairs of charged plates, with great precision to form letters and images on paper. Ink jet printers can produce color images by using a black jet and three other jets with primary colors, usually cyan, magenta, and yellow, much as a color television produces color. (This is more difficult with xerography, requiring multiple drums and toners.)



The nozzle of an ink-jet printer produces small ink droplets, which are sprayed with electrostatic charge. Various computer-driven devices are then used to direct the droplets to the correct positions on a page.

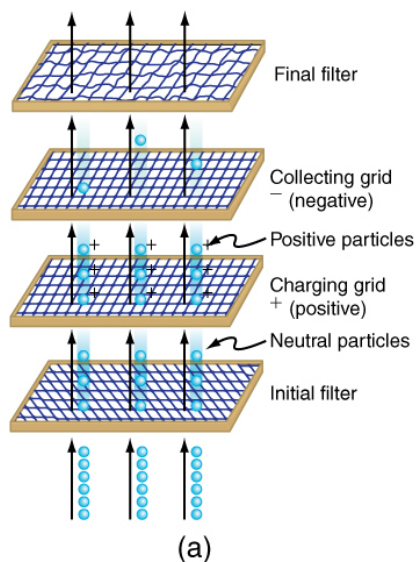
Electrostatic painting employs electrostatic charge to spray paint onto odd-shaped surfaces. Mutual repulsion of like charges causes the paint to fly away from its source. Surface tension forms drops, which are then attracted by unlike charges to the surface to be painted. Electrostatic painting can reach those hard-to-get at places, applying an even coat in a controlled

manner. If the object is a conductor, the electric field is perpendicular to the surface, tending to bring the drops in perpendicularly. Corners and points on conductors will receive extra paint. Felt can similarly be applied.

Smoke Precipitators and Electrostatic Air Cleaning

Another important application of electrostatics is found in air cleaners, both large and small. The electrostatic part of the process places excess (usually positive) charge on smoke, dust, pollen, and other particles in the air and then passes the air through an oppositely charged grid that attracts and retains the charged particles. (See [\[link\]](#).)

Large **electrostatic precipitators** are used industrially to remove over 99% of the particles from stack gas emissions associated with the burning of coal and oil. Home precipitators, often in conjunction with the home heating and air conditioning system, are very effective in removing polluting particles, irritants, and allergens.



(a) Schematic of an electrostatic precipitator. Air is passed through grids of opposite charge. The first grid charges airborne particles, while the second attracts and collects them. (b) The dramatic effect of

electrostatic precipitators is seen by the absence of smoke from this power plant. (credit: Cmdalgleish, Wikimedia Commons)

Note:

Problem-Solving Strategies for Electrostatics

1. Examine the situation to determine if static electricity is involved. This may concern separated stationary charges, the forces among them, and the electric fields they create.
2. Identify the system of interest. This includes noting the number, locations, and types of charges involved.
3. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful. Determine whether the Coulomb force is to be considered directly—if so, it may be useful to draw a free-body diagram, using electric field lines.
4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is important to distinguish the Coulomb force F from the electric field E , for example.
5. Solve the appropriate equation for the quantity to be determined (the unknown) or draw the field lines as requested.
6. Examine the answer to see if it is reasonable: Does it make sense? Are units correct and the numbers involved reasonable?

Integrated Concepts

The Integrated Concepts exercises for this module involve concepts such as electric charges, electric fields, and several other topics. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. The electric field exerts force on charges, for example, and hence the relevance of [Dynamics: Force and Newton's Laws of Motion](#). The following topics are involved in some or all of the problems labeled “Integrated Concepts”:

- [Kinematics](#)
- [Two-Dimensional Kinematics](#)
- [Dynamics: Force and Newton's Laws of Motion](#)
- [Uniform Circular Motion and Gravitation](#)
- [Statics and Torque](#)
- [Fluid Statics](#)

The following worked example illustrates how this strategy is applied to an Integrated Concept problem:

Example:

Acceleration of a Charged Drop of Gasoline

If steps are not taken to ground a gasoline pump, static electricity can be placed on gasoline when filling your car's tank. Suppose a tiny drop of gasoline has a mass of 4.00×10^{-15} kg and is given a positive charge of 3.20×10^{-19} C. (a) Find the weight of the drop. (b) Calculate the electric force on the drop if there is an upward electric field of strength 3.00×10^5 N/C due to other static electricity in the vicinity. (c) Calculate the drop's acceleration.

Strategy

To solve an integrated concept problem, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for weight. This is a topic of dynamics and is defined in [Dynamics: Force and Newton's Laws of Motion](#). Part (b) deals with electric force on a charge, a topic of [Electric Charge and Electric Field](#). Part (c) asks for acceleration, knowing forces and mass. These are part of Newton's laws, also found in [Dynamics: Force and Newton's Laws of Motion](#).

The following solutions to each part of the example illustrate how the specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

Weight is mass times the acceleration due to gravity, as first expressed in

Equation:

$$w = mg.$$

Entering the given mass and the average acceleration due to gravity yields

Equation:

$$w = (4.00 \times 10^{-15} \text{ kg})(9.80 \text{ m/s}^2) = 3.92 \times 10^{-14} \text{ N}.$$

Discussion for (a)

This is a small weight, consistent with the small mass of the drop.

Solution for (b)

The force an electric field exerts on a charge is given by rearranging the following equation:

Equation:

$$F = qE.$$

Here we are given the charge ($3.20 \times 10^{-19} \text{ C}$ is twice the fundamental unit of charge) and the electric field strength, and so the electric force is found to be

Equation:

$$F = (3.20 \times 10^{-19} \text{ C})(3.00 \times 10^5 \text{ N/C}) = 9.60 \times 10^{-14} \text{ N}.$$

Discussion for (b)

While this is a small force, it is greater than the weight of the drop.

Solution for (c)

The acceleration can be found using Newton's second law, provided we can identify all of the external forces acting on the drop. We assume only the drop's weight and the electric force are significant. Since the drop has a positive charge and the electric field is given to be upward, the electric force is upward. We thus have a one-dimensional (vertical direction) problem, and we can state Newton's second law as

Equation:

$$a = \frac{F_{\text{net}}}{m}.$$

where $F_{\text{net}} = F - w$. Entering this and the known values into the expression for Newton's second law yields

Equation:

$$\begin{aligned} a &= \frac{F-w}{m} \\ &= \frac{9.60 \times 10^{-14} \text{ N} - 3.92 \times 10^{-14} \text{ N}}{4.00 \times 10^{-15} \text{ kg}} \\ &= 14.2 \text{ m/s}^2. \end{aligned}$$

Discussion for (c)

This is an upward acceleration great enough to carry the drop to places where you might not wish to have gasoline.

This worked example illustrates how to apply problem-solving strategies to situations that include topics in different chapters. The first step is to identify the physical principles involved in the problem. The second step is to solve for the unknown using familiar problem-solving strategies. These are found throughout the text, and many worked examples show how to use them for single topics. In this integrated concepts example, you can see how to apply them across several topics. You will find these techniques useful in applications of physics outside a physics course, such as in your profession, in other science disciplines, and in everyday life. The following problems will build your skills in the broad application of physical principles.

Note:

Unreasonable Results

The Unreasonable Results exercises for this module have results that are unreasonable because some premise is unreasonable or because certain of the premises are inconsistent with one another. Physical principles applied correctly then produce unreasonable results. The purpose of these problems is to give practice in assessing whether nature is being accurately described, and if it is not to trace the source of difficulty.

Note:**Problem-Solving Strategy**

To determine if an answer is reasonable, and to determine the cause if it is not, do the following.

1. Solve the problem using strategies as outlined above. Use the format followed in the worked examples in the text to solve the problem as usual.
2. Check to see if the answer is reasonable. Is it too large or too small, or does it have the wrong sign, improper units, and so on?
3. If the answer is unreasonable, look for what specifically could cause the identified difficulty. Usually, the manner in which the answer is unreasonable is an indication of the difficulty. For example, an extremely large Coulomb force could be due to the assumption of an excessively large separated charge.

Section Summary

- Electrostatics is the study of electric fields in static equilibrium.
- In addition to research using equipment such as a Van de Graaff generator, many practical applications of electrostatics exist, including photocopiers, laser printers, ink-jet printers and electrostatic air filters.

Problems & Exercises**Exercise:**

Problem:

(a) What is the electric field 5.00 m from the center of the terminal of a Van de Graaff with a 3.00 mC charge, noting that the field is equivalent to that of a point charge at the center of the terminal? (b) At this distance, what force does the field exert on a $2.00\ \mu\text{C}$ charge on the Van de Graaff's belt?

Exercise:**Problem:**

(a) What is the direction and magnitude of an electric field that supports the weight of a free electron near the surface of Earth? (b) Discuss what the small value for this field implies regarding the relative strength of the gravitational and electrostatic forces.

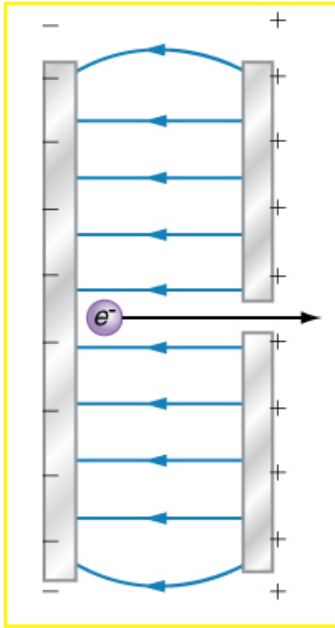
Solution:

(a) $5.58 \times 10^{-11}\ \text{N/C}$

(b) the coulomb force is extraordinarily stronger than gravity

Exercise:**Problem:**

A simple and common technique for accelerating electrons is shown in [\[link\]](#), where there is a uniform electric field between two plates. Electrons are released, usually from a hot filament, near the negative plate, and there is a small hole in the positive plate that allows the electrons to continue moving. (a) Calculate the acceleration of the electron if the field strength is $2.50 \times 10^4\ \text{N/C}$. (b) Explain why the electron will not be pulled back to the positive plate once it moves through the hole.



Parallel
conducting
plates with
opposite charges
on them create a
relatively
uniform electric
field used to
accelerate
electrons to the
right. Those that
go through the
hole can be used
to make a TV or
computer screen
glow or to
produce X-rays.

Exercise:

Problem:

Earth has a net charge that produces an electric field of approximately 150 N/C downward at its surface. (a) What is the magnitude and sign of the excess charge, noting the electric field of a conducting sphere is equivalent to a point charge at its center? (b) What acceleration will the field produce on a free electron near Earth's surface? (c) What mass object with a single extra electron will have its weight supported by this field?

Solution:

(a) $-6.76 \times 10^5 \text{ C}$

(b) $2.63 \times 10^{13} \text{ m/s}^2$ (upward)

(c) $2.45 \times 10^{-18} \text{ kg}$

Exercise:**Problem:**

Point charges of $25.0 \mu\text{C}$ and $45.0 \mu\text{C}$ are placed 0.500 m apart. (a) At what point along the line between them is the electric field zero? (b) What is the electric field halfway between them?

Exercise:**Problem:**

What can you say about two charges q_1 and q_2 , if the electric field one-fourth of the way from q_1 to q_2 is zero?

Solution:

The charge q_2 is 9 times greater than q_1 .

Exercise:**Problem: Integrated Concepts**

Calculate the angular velocity ω of an electron orbiting a proton in the hydrogen atom, given the radius of the orbit is 0.530×10^{-10} m. You may assume that the proton is stationary and the centripetal force is supplied by Coulomb attraction.

Exercise:

Problem: Integrated Concepts

An electron has an initial velocity of 5.00×10^6 m/s in a uniform 2.00×10^5 N/C strength electric field. The field accelerates the electron in the direction opposite to its initial velocity. (a) What is the direction of the electric field? (b) How far does the electron travel before coming to rest? (c) How long does it take the electron to come to rest? (d) What is the electron's velocity when it returns to its starting point?

Exercise:

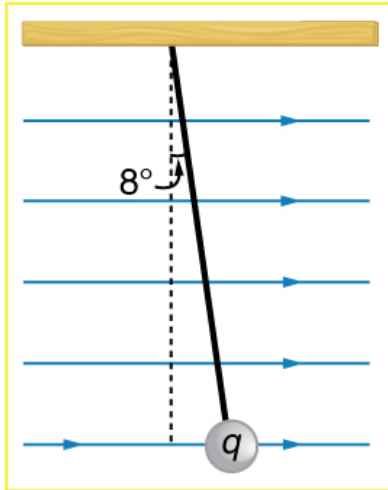
Problem: Integrated Concepts

The practical limit to an electric field in air is about 3.00×10^6 N/C. Above this strength, sparking takes place because air begins to ionize and charges flow, reducing the field. (a) Calculate the distance a free proton must travel in this field to reach 3.00% of the speed of light, starting from rest. (b) Is this practical in air, or must it occur in a vacuum?

Exercise:

Problem: Integrated Concepts

A 5.00 g charged insulating ball hangs on a 30.0 cm long string in a uniform horizontal electric field as shown in [\[link\]](#). Given the charge on the ball is $1.00 \mu\text{C}$, find the strength of the field.

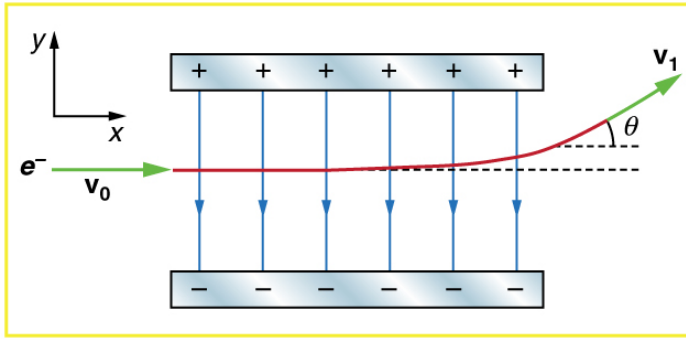


A horizontal electric field causes the charged ball to hang at an angle of 8.00° .

Exercise:

Problem: Integrated Concepts

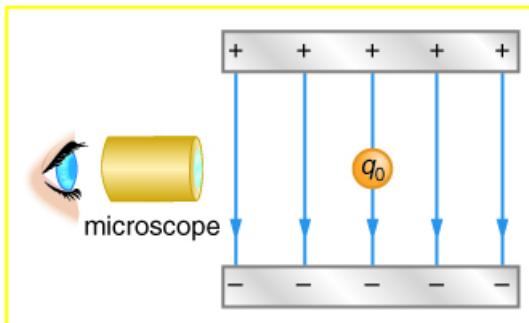
[\[link\]](#) shows an electron passing between two charged metal plates that create an 100 N/C vertical electric field perpendicular to the electron's original horizontal velocity. (These can be used to change the electron's direction, such as in an oscilloscope.) The initial speed of the electron is $3.00 \times 10^6 \text{ m/s}$, and the horizontal distance it travels in the uniform field is 4.00 cm . (a) What is its vertical deflection? (b) What is the vertical component of its final velocity? (c) At what angle does it exit? Neglect any edge effects.



Exercise:

Problem: Integrated Concepts

The classic Millikan oil drop experiment was the first to obtain an accurate measurement of the charge on an electron. In it, oil drops were suspended against the gravitational force by a vertical electric field. (See [\[link\]](#).) Given the oil drop to be $1.00 \mu\text{m}$ in radius and have a density of 920 kg/m^3 : (a) Find the weight of the drop. (b) If the drop has a single excess electron, find the electric field strength needed to balance its weight.



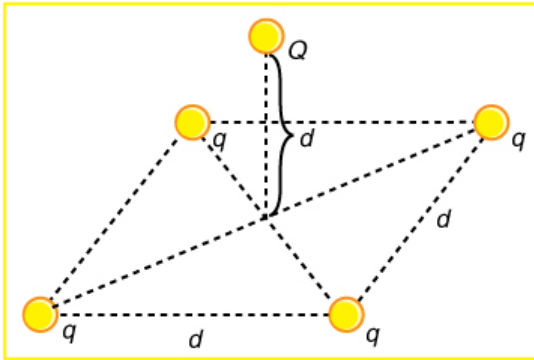
In the Millikan oil drop experiment, small drops can be suspended in an electric field by the force exerted on a single excess electron. Classically, this experiment was used to determine the electron charge q_e by

measuring the electric field
and mass of the drop.

Exercise:

Problem: Integrated Concepts

(a) In [\[link\]](#), four equal charges q lie on the corners of a square. A fifth charge Q is on a mass m directly above the center of the square, at a height equal to the length d of one side of the square. Determine the magnitude of q in terms of Q , m , and d , if the Coulomb force is to equal the weight of m . (b) Is this equilibrium stable or unstable? Discuss.



Four equal charges on the
corners of a horizontal
square support the weight of
a fifth charge located
directly above the center of
the square.

Exercise:

Problem: Unreasonable Results

(a) Calculate the electric field strength near a 10.0 cm diameter
conducting sphere that has 1.00 C of excess charge on it. (b) What is

unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Unreasonable Results

(a) Two 0.500 g raindrops in a thunderhead are 1.00 cm apart when they each acquire 1.00 mC charges. Find their acceleration. (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

Exercise:

Problem: Unreasonable Results

A wrecking yard inventor wants to pick up cars by charging a 0.400 m diameter ball and inducing an equal and opposite charge on the car. If a car has a 1000 kg mass and the ball is to be able to lift it from a distance of 1.00 m: (a) What minimum charge must be used? (b) What is the electric field near the surface of the ball? (c) Why are these results unreasonable? (d) Which premise or assumption is responsible?

Exercise:

Problem: Construct Your Own Problem

Consider two insulating balls with evenly distributed equal and opposite charges on their surfaces, held with a certain distance between the centers of the balls. Construct a problem in which you calculate the electric field (magnitude and direction) due to the balls at various points along a line running through the centers of the balls and extending to infinity on either side. Choose interesting points and comment on the meaning of the field at those points. For example, at what points might the field be just that due to one ball and where does the field become negligibly small? Among the things to be considered are the magnitudes of the charges and the distance between the centers of the balls. Your instructor may wish for you to consider the electric

field off axis or for a more complex array of charges, such as those in a water molecule.

Exercise:

Problem: Construct Your Own Problem

Consider identical spherical conducting space ships in deep space where gravitational fields from other bodies are negligible compared to the gravitational attraction between the ships. Construct a problem in which you place identical excess charges on the space ships to exactly counter their gravitational attraction. Calculate the amount of excess charge needed. Examine whether that charge depends on the distance between the centers of the ships, the masses of the ships, or any other factors. Discuss whether this would be an easy, difficult, or even impossible thing to do in practice.

Glossary

Van de Graaff generator

a machine that produces a large amount of excess charge, used for experiments with high voltage

electrostatics

the study of electric forces that are static or slow-moving

photoconductor

a substance that is an insulator until it is exposed to light, when it becomes a conductor

xerography

a dry copying process based on electrostatics

grounded

connected to the ground with a conductor, so that charge flows freely to and from the Earth to the grounded object

laser printer

uses a laser to create a photoconductive image on a drum, which attracts dry ink particles that are then rolled onto a sheet of paper to print a high-quality copy of the image

ink-jet printer

small ink droplets sprayed with an electric charge are controlled by electrostatic plates to create images on paper

electrostatic precipitators

filters that apply charges to particles in the air, then attract those charges to a filter, removing them from the airstream

Introduction to Electric Potential and Electric Energy

class="introduction"

Automated
external
defibrillato
r unit
(AED)
(credit:
U.S.
Defense
Department
photo/Tech.
Sgt.
Suzanne
M. Day)



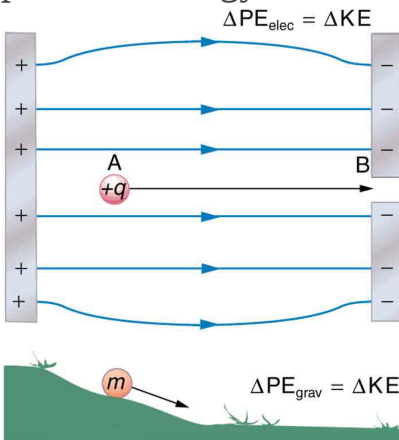
In [Electric Charge and Electric Field](#), we just scratched the surface (or at least rubbed it) of electrical phenomena. Two of the most familiar aspects of

electricity are its energy and *voltage*. We know, for example, that great amounts of electrical energy can be stored in batteries, are transmitted cross-country through power lines, and may jump from clouds to explode the sap of trees. In a similar manner, at molecular levels, *ions* cross cell membranes and transfer information. We also know about voltages associated with electricity. Batteries are typically a few volts, the outlets in your home produce 120 volts, and power lines can be as high as hundreds of thousands of volts. But energy and voltage are not the same thing. A motorcycle battery, for example, is small and would not be very successful in replacing the much larger car battery, yet each has the same voltage. In this chapter, we shall examine the relationship between voltage and electrical energy and begin to explore some of the many applications of electricity.

Electric Potential Energy: Potential Difference

- Define electric potential and electric potential energy.
- Describe the relationship between potential difference and electrical potential energy.
- Explain electron volt and its usage in submicroscopic process.
- Determine electric potential energy given potential difference and amount of charge.

When a free positive charge q is accelerated by an electric field, such as shown in [\[link\]](#), it is given kinetic energy. The process is analogous to an object being accelerated by a gravitational field. It is as if the charge is going down an electrical hill where its electric potential energy is converted to kinetic energy. Let us explore the work done on a charge q by the electric field in this process, so that we may develop a definition of electric potential energy.



A charge accelerated by an electric field is analogous to a mass going down a hill. In both cases potential energy is converted to another form. Work is done by a force, but since this force

is conservative, we
can write
 $W = -\Delta PE.$

The electrostatic or Coulomb force is conservative, which means that the work done on q is independent of the path taken. This is exactly analogous to the gravitational force in the absence of dissipative forces such as friction. When a force is conservative, it is possible to define a potential energy associated with the force, and it is usually easier to deal with the potential energy (because it depends only on position) than to calculate the work directly.

We use the letters PE to denote electric potential energy, which has units of joules (J). The change in potential energy, ΔPE , is crucial, since the work done by a conservative force is the negative of the change in potential energy; that is, $W = -\Delta PE$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE . There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Note:

Potential Energy

$W = -\Delta PE$. For example, work W done to accelerate a positive charge from rest is positive and results from a loss in PE, or a negative ΔPE . There must be a minus sign in front of ΔPE to make W positive. PE can be found at any point by taking one point as a reference and calculating the work needed to move a charge to the other point.

Gravitational potential energy and electric potential energy are quite analogous. Potential energy accounts for work done by a conservative force and gives added insight regarding energy and energy transformation

without the necessity of dealing with the force directly. It is much more common, for example, to use the concept of voltage (related to electric potential energy) than to deal with the Coulomb force directly.

Calculating the work directly is generally difficult, since $W = Fd \cos \theta$ and the direction and magnitude of F can be complex for multiple charges, for odd-shaped objects, and along arbitrary paths. But we do know that, since $F = qE$, the work, and hence ΔPE , is proportional to the test charge q . To have a physical quantity that is independent of test charge, we define **electric potential** V (or simply potential, since electric is understood) to be the potential energy per unit charge:

Equation:

$$V = \frac{PE}{q}.$$

Note:

Electric Potential

This is the electric potential energy per unit charge.

Equation:

$$V = \frac{PE}{q}$$

Since PE is proportional to q , the dependence on q cancels. Thus V does not depend on q . The change in potential energy ΔPE is crucial, and so we are concerned with the difference in potential or potential difference ΔV between two points, where

Equation:

$$\Delta V = V_B - V_A = \frac{\Delta PE}{q}.$$

The **potential difference** between points A and B, $V_B - V_A$, is thus defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

Equation:

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

Note:

Potential Difference

The potential difference between points A and B, $V_B - V_A$, is defined to be the change in potential energy of a charge q moved from A to B, divided by the charge. Units of potential difference are joules per coulomb, given the name volt (V) after Alessandro Volta.

Equation:

$$1 \text{ V} = 1 \frac{\text{J}}{\text{C}}$$

The familiar term **voltage** is the common name for potential difference. Keep in mind that whenever a voltage is quoted, it is understood to be the potential difference between two points. For example, every battery has two terminals, and its voltage is the potential difference between them. More fundamentally, the point you choose to be zero volts is arbitrary. This is analogous to the fact that gravitational potential energy has an arbitrary zero, such as sea level or perhaps a lecture hall floor.

In summary, the relationship between potential difference (or voltage) and electrical potential energy is given by

Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

Note:**Potential Difference and Electrical Potential Energy**

The relationship between potential difference (or voltage) and electrical potential energy is given by

Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

The second equation is equivalent to the first.

Voltage is not the same as energy. Voltage is the energy per unit charge. Thus a motorcycle battery and a car battery can both have the same voltage (more precisely, the same potential difference between battery terminals), yet one stores much more energy than the other since $\Delta \text{PE} = q\Delta V$. The car battery can move more charge than the motorcycle battery, although both are 12 V batteries.

Example:**Calculating Energy**

Suppose you have a 12.0 V motorcycle battery that can move 5000 C of charge, and a 12.0 V car battery that can move 60,000 C of charge. How much energy does each deliver? (Assume that the numerical value of each charge is accurate to three significant figures.)

Strategy

To say we have a 12.0 V battery means that its terminals have a 12.0 V potential difference. When such a battery moves charge, it puts the charge

through a potential difference of 12.0 V, and the charge is given a change in potential energy equal to $\Delta PE = q\Delta V$.

So to find the energy output, we multiply the charge moved by the potential difference.

Solution

For the motorcycle battery, $q = 5000 \text{ C}$ and $\Delta V = 12.0 \text{ V}$. The total energy delivered by the motorcycle battery is

Equation:

$$\begin{aligned}\Delta PE_{\text{cycle}} &= (5000 \text{ C})(12.0 \text{ V}) \\ &= (5000 \text{ C})(12.0 \text{ J/C}) \\ &= 6.00 \times 10^4 \text{ J.}\end{aligned}$$

Similarly, for the car battery, $q = 60,000 \text{ C}$ and

Equation:

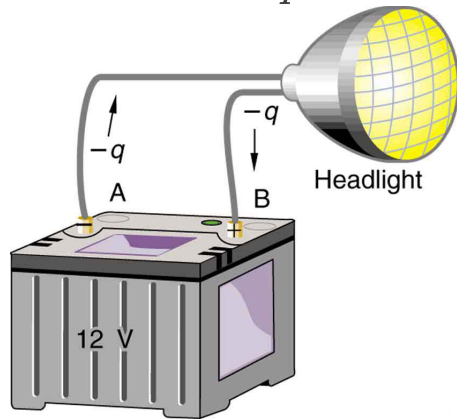
$$\begin{aligned}\Delta PE_{\text{car}} &= (60,000 \text{ C})(12.0 \text{ V}) \\ &= 7.20 \times 10^5 \text{ J.}\end{aligned}$$

Discussion

While voltage and energy are related, they are not the same thing. The voltages of the batteries are identical, but the energy supplied by each is quite different. Note also that as a battery is discharged, some of its energy is used internally and its terminal voltage drops, such as when headlights dim because of a low car battery. The energy supplied by the battery is still calculated as in this example, but not all of the energy is available for external use.

Note that the energies calculated in the previous example are absolute values. The change in potential energy for the battery is negative, since it loses energy. These batteries, like many electrical systems, actually move negative charge—electrons in particular. The batteries repel electrons from their negative terminals (A) through whatever circuitry is involved and attract them to their positive terminals (B) as shown in [\[link\]](#). The change in potential is $\Delta V = V_B - V_A = +12 \text{ V}$ and the charge q is negative, so that

$\Delta PE = q\Delta V$ is negative, meaning the potential energy of the battery has decreased when q has moved from A to B.



A battery moves negative charge from its negative terminal through a headlight to its positive terminal.

Appropriate combinations of chemicals in the battery separate charges so that the negative terminal has an excess of negative charge, which is repelled by it and attracted to the excess positive charge on the other terminal. In terms of potential, the positive terminal is at a higher voltage than the negative. Inside the battery, both positive and negative charges move.

Example:**How Many Electrons Move through a Headlight Each Second?**

When a 12.0 V car battery runs a single 30.0 W headlight, how many electrons pass through it each second?

Strategy

To find the number of electrons, we must first find the charge that moved in 1.00 s. The charge moved is related to voltage and energy through the equation $\Delta PE = q\Delta V$. A 30.0 W lamp uses 30.0 joules per second. Since the battery loses energy, we have $\Delta PE = -30.0$ J and, since the electrons are going from the negative terminal to the positive, we see that $\Delta V = +12.0$ V.

Solution

To find the charge q moved, we solve the equation $\Delta PE = q\Delta V$:

Equation:

$$q = \frac{\Delta PE}{\Delta V}.$$

Entering the values for ΔPE and ΔV , we get

Equation:

$$q = \frac{-30.0 \text{ J}}{+12.0 \text{ V}} = \frac{-30.0 \text{ J}}{+12.0 \text{ J/C}} = -2.50 \text{ C}.$$

The number of electrons n_e is the total charge divided by the charge per electron. That is,

Equation:

$$n_e = \frac{-2.50 \text{ C}}{-1.60 \times 10^{-19} \text{ C/e}^-} = 1.56 \times 10^{19} \text{ electrons}.$$

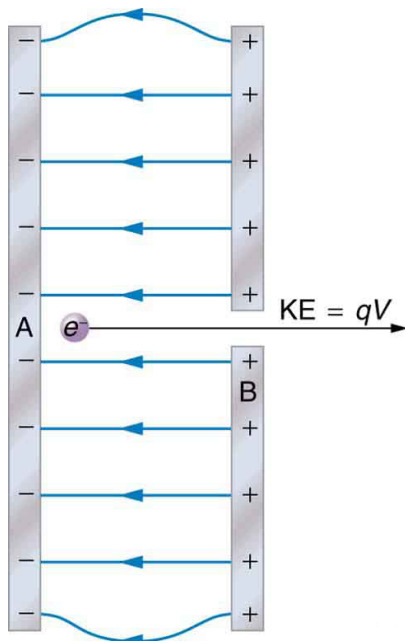
Discussion

This is a very large number. It is no wonder that we do not ordinarily observe individual electrons with so many being present in ordinary

systems. In fact, electricity had been in use for many decades before it was determined that the moving charges in many circumstances were negative. Positive charge moving in the opposite direction of negative charge often produces identical effects; this makes it difficult to determine which is moving or whether both are moving.

The Electron Volt

The energy per electron is very small in macroscopic situations like that in the previous example—a tiny fraction of a joule. But on a submicroscopic scale, such energy per particle (electron, proton, or ion) can be of great importance. For example, even a tiny fraction of a joule can be great enough for these particles to destroy organic molecules and harm living tissue. The particle may do its damage by direct collision, or it may create harmful x rays, which can also inflict damage. It is useful to have an energy unit related to submicroscopic effects. [\[link\]](#) shows a situation related to the definition of such an energy unit. An electron is accelerated between two charged metal plates as it might be in an old-model television tube or oscilloscope. The electron is given kinetic energy that is later converted to another form—light in the television tube, for example. (Note that downhill for the electron is uphill for a positive charge.) Since energy is related to voltage by $\Delta PE = q\Delta V$, we can think of the joule as a coulomb-volt.



A typical electron gun accelerates electrons using a potential difference between two metal plates. The energy of the electron in electron volts is numerically the same as the voltage between the plates.

For example, a 5000 V potential difference produces 5000 eV electrons.

On the submicroscopic scale, it is more convenient to define an energy unit called the **electron volt** (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form, **Equation:**

$$\begin{aligned}
 1 \text{ eV} &= (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) \\
 &= 1.60 \times 10^{-19} \text{ J}.
 \end{aligned}$$

Note:

Electron Volt

On the submicroscopic scale, it is more convenient to define an energy unit called the electron volt (eV), which is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

Equation:

$$\begin{aligned}
 1 \text{ eV} &= (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) \\
 &= 1.60 \times 10^{-19} \text{ J}.
 \end{aligned}$$

An electron accelerated through a potential difference of 1 V is given an energy of 1 eV. It follows that an electron accelerated through 50 V is given 50 eV. A potential difference of 100,000 V (100 kV) will give an electron an energy of 100,000 eV (100 keV), and so on. Similarly, an ion with a double positive charge accelerated through 100 V will be given 200 eV of energy. These simple relationships between accelerating voltage and particle charges make the electron volt a simple and convenient energy unit in such circumstances.

Note:

Connections: Energy Units

The electron volt (eV) is the most common energy unit for submicroscopic processes. This will be particularly noticeable in the chapters on modern physics. Energy is so important to so many subjects that there is a tendency to define a special energy unit for each major topic. There are, for example,

calories for food energy, kilowatt-hours for electrical energy, and therms for natural gas energy.

The electron volt is commonly employed in submicroscopic processes—chemical valence energies and molecular and nuclear binding energies are among the quantities often expressed in electron volts. For example, about 5 eV of energy is required to break up certain organic molecules. If a proton is accelerated from rest through a potential difference of 30 kV, it is given an energy of 30 keV (30,000 eV) and it can break up as many as 6000 of these molecules ($30,000 \text{ eV} \div 5 \text{ eV per molecule} = 6000 \text{ molecules}$). Nuclear decay energies are on the order of 1 MeV (1,000,000 eV) per event and can, thus, produce significant biological damage.

Conservation of Energy

The total energy of a system is conserved if there is no net addition (or subtraction) of work or heat transfer. For conservative forces, such as the electrostatic force, conservation of energy states that mechanical energy is a constant.

Mechanical energy is the sum of the kinetic energy and potential energy of a system; that is, $KE + PE = \text{constant}$. A loss of PE of a charged particle becomes an increase in its KE. Here PE is the electric potential energy. Conservation of energy is stated in equation form as

Equation:

$$KE + PE = \text{constant}$$

or

Equation:

$$KE_i + PE_i = KE_f + PE_f,$$

where i and f stand for initial and final conditions. As we have found many times before, considering energy can give us insights and facilitate problem solving.

Example:

Electrical Potential Energy Converted to Kinetic Energy

Calculate the final speed of a free electron accelerated from rest through a potential difference of 100 V. (Assume that this numerical value is accurate to three significant figures.)

Strategy

We have a system with only conservative forces. Assuming the electron is accelerated in a vacuum, and neglecting the gravitational force (we will check on this assumption later), all of the electrical potential energy is converted into kinetic energy. We can identify the initial and final forms of energy to be $KE_i = 0$, $KE_f = \frac{1}{2}mv^2$, $PE_i = qV$, and $PE_f = 0$.

Solution

Conservation of energy states that

Equation:

$$KE_i + PE_i = KE_f + PE_f.$$

Entering the forms identified above, we obtain

Equation:

$$qV = \frac{mv^2}{2}.$$

We solve this for v :

Equation:

$$v = \sqrt{\frac{2qV}{m}}.$$

Entering values for q , V , and m gives

Equation:

$$\begin{aligned}
 v &= \sqrt{\frac{2(-1.60 \times 10^{-19} \text{ C})(-100 \text{ J/C})}{9.11 \times 10^{-31} \text{ kg}}} \\
 &= 5.93 \times 10^6 \text{ m/s.}
 \end{aligned}$$

Discussion

Note that both the charge and the initial voltage are negative, as in [\[link\]](#). From the discussions in [Electric Charge and Electric Field](#), we know that electrostatic forces on small particles are generally very large compared with the gravitational force. The large final speed confirms that the gravitational force is indeed negligible here. The large speed also indicates how easy it is to accelerate electrons with small voltages because of their very small mass. Voltages much higher than the 100 V in this problem are typically used in electron guns. Those higher voltages produce electron speeds so great that relativistic effects must be taken into account. That is why a low voltage is considered (accurately) in this example.

Section Summary

- Electric potential is potential energy per unit charge.
- The potential difference between points A and B, $V_B - V_A$, defined to be the change in potential energy of a charge q moved from A to B, is equal to the change in potential energy divided by the charge. Potential difference is commonly called voltage, represented by the symbol ΔV .

Equation:

$$\Delta V = \frac{\Delta \text{PE}}{q} \text{ and } \Delta \text{PE} = q\Delta V.$$

- An electron volt is the energy given to a fundamental charge accelerated through a potential difference of 1 V. In equation form,

Equation:

$$\begin{aligned}
 1 \text{ eV} &= (1.60 \times 10^{-19} \text{ C})(1 \text{ V}) = (1.60 \times 10^{-19} \text{ C})(1 \text{ J/C}) \\
 &= 1.60 \times 10^{-19} \text{ J.}
 \end{aligned}$$

- Mechanical energy is the sum of the kinetic energy and potential energy of a system, that is, $KE + PE$. This sum is a constant.

Conceptual Questions

Exercise:

Problem:

Voltage is the common word for potential difference. Which term is more descriptive, voltage or potential difference?

Exercise:

Problem:

If the voltage between two points is zero, can a test charge be moved between them with zero net work being done? Can this necessarily be done without exerting a force? Explain.

Exercise:

Problem:

What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential energy?

Exercise:

Problem: Voltages are always measured between two points. Why?

Exercise:

Problem:

How are units of volts and electron volts related? How do they differ?

Problems & Exercises

Exercise:

Problem:

Find the ratio of speeds of an electron and a negative hydrogen ion (one having an extra electron) accelerated through the same voltage, assuming non-relativistic final speeds. Take the mass of the hydrogen ion to be 1.67×10^{-27} kg.

Solution:

42.8

Exercise:**Problem:**

An evacuated tube uses an accelerating voltage of 40 kV to accelerate electrons to hit a copper plate and produce x rays. Non-relativistically, what would be the maximum speed of these electrons?

Exercise:**Problem:**

A bare helium nucleus has two positive charges and a mass of 6.64×10^{-27} kg. (a) Calculate its kinetic energy in joules at 2.00% of the speed of light. (b) What is this in electron volts? (c) What voltage would be needed to obtain this energy?

Exercise:**Problem: Integrated Concepts**

Singly charged gas ions are accelerated from rest through a voltage of 13.0 V. At what temperature will the average kinetic energy of gas molecules be the same as that given these ions?

Solution:

1.00×10^5 K

Exercise:**Problem: Integrated Concepts**

The temperature near the center of the Sun is thought to be 15 million degrees Celsius (1.5×10^7 °C). Through what voltage must a singly charged ion be accelerated to have the same energy as the average kinetic energy of ions at this temperature?

Exercise:**Problem: Integrated Concepts**

(a) What is the average power output of a heart defibrillator that dissipates 400 J of energy in 10.0 ms? (b) Considering the high-power output, why doesn't the defibrillator produce serious burns?

Solution:

(a) 4×10^4 W

(b) A defibrillator does not cause serious burns because the skin conducts electricity well at high voltages, like those used in defibrillators. The gel used aids in the transfer of energy to the body, and the skin doesn't absorb the energy, but rather lets it pass through to the heart.

Exercise:**Problem: Integrated Concepts**

A lightning bolt strikes a tree, moving 20.0 C of charge through a potential difference of 1.00×10^2 MV. (a) What energy was dissipated? (b) What mass of water could be raised from 15°C to the boiling point and then boiled by this energy? (c) Discuss the damage that could be caused to the tree by the expansion of the boiling steam.

Exercise:

Problem: Integrated Concepts

A 12.0 V battery-operated bottle warmer heats 50.0 g of glass, 2.50×10^2 g of baby formula, and 2.00×10^2 g of aluminum from 20.0°C to 90.0°C. (a) How much charge is moved by the battery? (b) How many electrons per second flow if it takes 5.00 min to warm the formula? (Hint: Assume that the specific heat of baby formula is about the same as the specific heat of water.)

Solution:

(a) 7.40×10^3 C

(b) 1.54×10^{20} electrons per second

Exercise:**Problem: Integrated Concepts**

A battery-operated car utilizes a 12.0 V system. Find the charge the batteries must be able to move in order to accelerate the 750 kg car from rest to 25.0 m/s, make it climb a 2.00×10^2 m high hill, and then cause it to travel at a constant 25.0 m/s by exerting a 5.00×10^2 N force for an hour.

Solution:

3.89×10^6 C

Exercise:**Problem: Integrated Concepts**

Fusion probability is greatly enhanced when appropriate nuclei are brought close together, but mutual Coulomb repulsion must be overcome. This can be done using the kinetic energy of high-temperature gas ions or by accelerating the nuclei toward one another.

(a) Calculate the potential energy of two singly charged nuclei separated by 1.00×10^{-12} m by finding the voltage of one at that distance and multiplying by the charge of the other. (b) At what temperature will atoms of a gas have an average kinetic energy equal to this needed electrical potential energy?

Exercise:

Problem: Unreasonable Results

(a) Find the voltage near a 10.0 cm diameter metal sphere that has 8.00 C of excess positive charge on it. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Solution:

(a) 1.44×10^{12} V

(b) This voltage is very high. A 10.0 cm diameter sphere could never maintain this voltage; it would discharge.

(c) An 8.00 C charge is more charge than can reasonably be accumulated on a sphere of that size.

Exercise:

Problem: Construct Your Own Problem

Consider a battery used to supply energy to a cellular phone. Construct a problem in which you determine the energy that must be supplied by the battery, and then calculate the amount of charge it must be able to move in order to supply this energy. Among the things to be considered are the energy needs and battery voltage. You may need to look ahead to interpret manufacturer's battery ratings in ampere-hours as energy in joules.

Glossary

electric potential

potential energy per unit charge

potential difference (or voltage)

change in potential energy of a charge moved from one point to another, divided by the charge; units of potential difference are joules per coulomb, known as volt

electron volt

the energy given to a fundamental charge accelerated through a potential difference of one volt

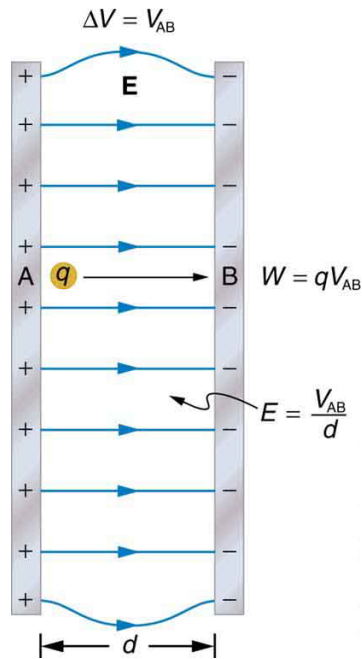
mechanical energy

sum of the kinetic energy and potential energy of a system; this sum is a constant

Electric Potential in a Uniform Electric Field

- Describe the relationship between voltage and electric field.
- Derive an expression for the electric potential and electric field.
- Calculate electric field strength given distance and voltage.

In the previous section, we explored the relationship between voltage and energy. In this section, we will explore the relationship between voltage and electric field. For example, a uniform electric field $\mathbf{E}$ is produced by placing a potential difference (or voltage) ΔV across two parallel metal plates, labeled A and B. (See [\[link\]](#).) Examining this will tell us what voltage is needed to produce a certain electric field strength; it will also reveal a more fundamental relationship between electric potential and electric field. From a physicist's point of view, either ΔV or $\mathbf{E}$ can be used to describe any charge distribution. ΔV is most closely tied to energy, whereas $\mathbf{E}$ is most closely related to force. ΔV is a **scalar** quantity and has no direction, while $\mathbf{E}$ is a **vector** quantity, having both magnitude and direction. (Note that the magnitude of the electric field strength, a scalar quantity, is represented by E below.) The relationship between ΔV and $\mathbf{E}$ is revealed by calculating the work done by the force in moving a charge from point A to point B. But, as noted in [Electric Potential Energy: Potential Difference](#), this is complex for arbitrary charge distributions, requiring calculus. We therefore look at a uniform electric field as an interesting special case.



The relationship between V and E for parallel conducting plates is $E = V/d$. (Note that $\Delta V = V_{AB}$ in magnitude. For a charge that is moved from plate A at higher potential to plate B at lower potential, a minus sign needs to be included as follows:
 $-\Delta V = V_A - V_B = V_{AB}$
 . See the text for details.)

The work done by the electric field in [\[link\]](#) to move a positive charge q from A, the positive plate, higher potential, to B, the negative plate, lower potential, is

Equation:

$$W = -\Delta PE = -q\Delta V.$$

The potential difference between points A and B is

Equation:

$$-\Delta V = -(V_B - V_A) = V_A - V_B = V_{AB}.$$

Entering this into the expression for work yields

Equation:

$$W = qV_{AB}.$$

Work is $W = Fd \cos \theta$; here $\cos \theta = 1$, since the path is parallel to the field, and so $W = Fd$. Since $F = qE$, we see that $W = qEd$. Substituting this expression for work into the previous equation gives

Equation:

$$qEd = qV_{AB}.$$

The charge cancels, and so the voltage between points A and B is seen to be

Equation:

$$\left. \begin{array}{l} V_{AB} = Ed \\ E = \frac{V_{AB}}{d} \end{array} \right\} (\text{uniform } E - \text{field only}),$$

where d is the distance from A to B, or the distance between the plates in [\[link\]](#). Note that the above equation implies the units for electric field are volts per meter. We already know the units for electric field are newtons per coulomb; thus the following relation among units is valid:

Equation:

$$1 \text{ N/C} = 1 \text{ V/m}.$$

Note:

Voltage between Points A and B

Equation:

$$\left. \begin{array}{l} V_{AB} = Ed \\ E = \frac{V_{AB}}{d} \end{array} \right\} (\text{uniform } E - \text{field only}),$$

where d is the distance from A to B, or the distance between the plates.

Example:**What Is the Highest Voltage Possible between Two Plates?**

Dry air will support a maximum electric field strength of about $3.0 \times 10^6 \text{ V/m}$. Above that value, the field creates enough ionization in the air to make the air a conductor. This allows a discharge or spark that reduces the field. What, then, is the maximum voltage between two parallel conducting plates separated by 2.5 cm of dry air?

Strategy

We are given the maximum electric field E between the plates and the distance d between them. The equation $V_{AB} = Ed$ can thus be used to calculate the maximum voltage.

Solution

The potential difference or voltage between the plates is

Equation:

$$V_{AB} = Ed.$$

Entering the given values for E and d gives

Equation:

$$V_{AB} = (3.0 \times 10^6 \text{ V/m})(0.025 \text{ m}) = 7.5 \times 10^4 \text{ V}$$

or

Equation:

$$V_{AB} = 75 \text{ kV}.$$

(The answer is quoted to only two digits, since the maximum field strength is approximate.)

Discussion

One of the implications of this result is that it takes about 75 kV to make a spark jump across a 2.5 cm (1 in.) gap, or 150 kV for a 5 cm spark. This limits the voltages that can exist between conductors, perhaps on a power transmission line. A smaller voltage will cause a spark if there are points on the surface, since points create greater fields than smooth surfaces. Humid air breaks down at a lower field strength, meaning that a smaller voltage will make a spark jump through humid air. The largest voltages can be built up, say with static electricity, on dry days.



A spark chamber is used to trace the paths of high-energy particles. Ionization created by the particles as they pass through the gas between the plates allows a spark to jump. The sparks are

perpendicular to the plates, following electric field lines between them. The potential difference between adjacent plates is not high enough to cause sparks without the ionization produced by particles from accelerator experiments (or cosmic rays).
(credit: Daderot, Wikimedia Commons)

Example:**Field and Force inside an Electron Gun**

(a) An electron gun has parallel plates separated by 4.00 cm and gives electrons 25.0 keV of energy. What is the electric field strength between the plates? (b) What force would this field exert on a piece of plastic with a $0.500\ \mu\text{C}$ charge that gets between the plates?

Strategy

Since the voltage and plate separation are given, the electric field strength can be calculated directly from the expression $E = \frac{V_{AB}}{d}$. Once the electric field strength is known, the force on a charge is found using $\mathbf{F} = q \mathbf{E}$. Since the electric field is in only one direction, we can write this equation in terms of the magnitudes, $F = q E$.

Solution for (a)

The expression for the magnitude of the electric field between two uniform metal plates is

Equation:

$$E = \frac{V_{AB}}{d}.$$

Since the electron is a single charge and is given 25.0 keV of energy, the potential difference must be 25.0 kV. Entering this value for V_{AB} and the plate separation of 0.0400 m, we obtain

Equation:

$$E = \frac{25.0 \text{ kV}}{0.0400 \text{ m}} = 6.25 \times 10^5 \text{ V/m}.$$

Solution for (b)

The magnitude of the force on a charge in an electric field is obtained from the equation

Equation:

$$F = qE.$$

Substituting known values gives

Equation:

$$F = (0.500 \times 10^{-6} \text{ C})(6.25 \times 10^5 \text{ V/m}) = 0.313 \text{ N}.$$

Discussion

Note that the units are newtons, since $1 \text{ V/m} = 1 \text{ N/C}$. The force on the charge is the same no matter where the charge is located between the plates. This is because the electric field is uniform between the plates.

In more general situations, regardless of whether the electric field is uniform, it points in the direction of decreasing potential, because the force on a positive charge is in the direction of $\mathbf{E}$ and also in the direction of lower potential V . Furthermore, the magnitude of $\mathbf{E}$ equals the rate of decrease of V with distance. The faster V decreases over distance, the

greater the electric field. In equation form, the general relationship between voltage and electric field is

Equation:

$$E = - \frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV , takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Note:

Relationship between Voltage and Electric Field

In equation form, the general relationship between voltage and electric field is

Equation:

$$E = - \frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV , takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential. The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

For continually changing potentials, ΔV and Δs become infinitesimals and differential calculus must be employed to determine the electric field.

Section Summary

- The voltage between points A and B is

Equation:

$$\left. \begin{aligned} V_{AB} &= Ed \\ E &= \frac{V_{AB}}{d} \end{aligned} \right\} (\text{uniform } E - \text{field only}),$$

where d is the distance from A to B, or the distance between the plates.

- In equation form, the general relationship between voltage and electric field is

Equation:

$$E = - \frac{\Delta V}{\Delta s},$$

where Δs is the distance over which the change in potential, ΔV , takes place. The minus sign tells us that $\mathbf{E}$ points in the direction of decreasing potential.) The electric field is said to be the *gradient* (as in grade or slope) of the electric potential.

Conceptual Questions

Exercise:

Problem:

Discuss how potential difference and electric field strength are related. Give an example.

Exercise:

Problem:

What is the strength of the electric field in a region where the electric potential is constant?

Exercise:

Problem:

Will a negative charge, initially at rest, move toward higher or lower potential? Explain why.

Problems & Exercises

Exercise:

Problem:

Show that units of V/m and N/C for electric field strength are indeed equivalent.

Exercise:

Problem:

What is the strength of the electric field between two parallel conducting plates separated by 1.00 cm and having a potential difference (voltage) between them of 1.50×10^4 V?

Exercise:

Problem:

The electric field strength between two parallel conducting plates separated by 4.00 cm is 7.50×10^4 V/m. (a) What is the potential difference between the plates? (b) The plate with the lowest potential is taken to be at zero volts. What is the potential 1.00 cm from that plate (and 3.00 cm from the other)?

Solution:

(a) 3.00 kV

(b) 750 V

Exercise:

Problem:

How far apart are two conducting plates that have an electric field strength of 4.50×10^3 V/m between them, if their potential difference is 15.0 kV?

Exercise:

Problem:

(a) Will the electric field strength between two parallel conducting plates exceed the breakdown strength for air ($3.0 \times 10^6 \text{ V/m}$) if the plates are separated by 2.00 mm and a potential difference of $5.0 \times 10^3 \text{ V}$ is applied? (b) How close together can the plates be with this applied voltage?

Solution:

(a) No. The electric field strength between the plates is $2.5 \times 10^6 \text{ V/m}$, which is lower than the breakdown strength for air ($3.0 \times 10^6 \text{ V/m}$).

(b) 1.7 mm

Exercise:**Problem:**

The voltage across a membrane forming a cell wall is 80.0 mV and the membrane is 9.00 nm thick. What is the electric field strength? (The value is surprisingly large, but correct. Membranes are discussed in [Capacitors and Dielectrics](#) and [Nerve Conduction—Electrocardiograms](#).) You may assume a uniform electric field.

Exercise:**Problem:**

Membrane walls of living cells have surprisingly large electric fields across them due to separation of ions. (Membranes are discussed in some detail in [Nerve Conduction—Electrocardiograms](#).) What is the voltage across an 8.00 nm-thick membrane if the electric field strength across it is 5.50 MV/m? You may assume a uniform electric field.

Solution:

44.0 mV

Exercise:**Problem:**

Two parallel conducting plates are separated by 10.0 cm, and one of them is taken to be at zero volts. (a) What is the electric field strength between them, if the potential 8.00 cm from the zero volt plate (and 2.00 cm from the other) is 450 V? (b) What is the voltage between the plates?

Exercise:**Problem:**

Find the maximum potential difference between two parallel conducting plates separated by 0.500 cm of air, given the maximum sustainable electric field strength in air to be $3.0 \times 10^6 \text{ V/m}$.

Solution:

15 kV

Exercise:**Problem:**

A doubly charged ion is accelerated to an energy of 32.0 keV by the electric field between two parallel conducting plates separated by 2.00 cm. What is the electric field strength between the plates?

Exercise:**Problem:**

An electron is to be accelerated in a uniform electric field having a strength of $2.00 \times 10^6 \text{ V/m}$. (a) What energy in keV is given to the electron if it is accelerated through 0.400 m? (b) Over what distance would it have to be accelerated to increase its energy by 50.0 GeV?

Solution:

(a) 800 KeV

(b) 25.0 km

Glossary

scalar

physical quantity with magnitude but no direction

vector

physical quantity with both magnitude and direction

Electrical Potential Due to a Point Charge

- Explain point charges and express the equation for electric potential of a point charge.
- Distinguish between electric potential and electric field.
- Determine the electric potential of a point charge given charge and distance.

Point charges, such as electrons, are among the fundamental building blocks of matter. Furthermore, spherical charge distributions (like on a metal sphere) create external electric fields exactly like a point charge. The electric potential due to a point charge is, thus, a case we need to consider. Using calculus to find the work needed to move a test charge q from a large distance away to a distance of r from a point charge Q , and noting the connection between work and potential ($W = -q\Delta V$), it can be shown that the *electric potential V of a point charge* is

Equation:

$$V = \frac{kQ}{r} \text{ (Point Charge),}$$

where k is a constant equal to $9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$.

Note:

Electric Potential V of a Point Charge

The electric potential V of a point charge is given by

Equation:

$$V = \frac{kQ}{r} \text{ (Point Charge).}$$

The potential at infinity is chosen to be zero. Thus V for a point charge decreases with distance, whereas $\mathbf{E}$ for a point charge decreases with distance squared:

Equation:

$$E = \frac{F}{q} = \frac{kQ}{r^2}.$$

Recall that the electric potential V is a scalar and has no direction, whereas the electric field $\mathbf{E}$ is a vector. To find the voltage due to a combination of point charges, you add the individual voltages as numbers. To find the total electric field, you must add the individual fields as *vectors*, taking magnitude and direction into account. This is consistent with the fact that V is closely associated with energy, a scalar, whereas $\mathbf{E}$ is closely associated with force, a vector.

Example:

What Voltage Is Produced by a Small Charge on a Metal Sphere?

Charges in static electricity are typically in the nanocoulomb (nC) to microcoulomb (μC) range. What is the voltage 5.00 cm away from the center of a 1-cm diameter metal sphere that has a -3.00 nC static charge?

Strategy

As we have discussed in [Electric Charge and Electric Field](#), charge on a metal sphere spreads out uniformly and produces a field like that of a point charge located at its center. Thus we can find the voltage using the equation $V = kQ/r$.

Solution

Entering known values into the expression for the potential of a point charge, we obtain

Equation:

$$\begin{aligned}
 V &= k \frac{Q}{r} \\
 &= (8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2) \left(\frac{-3.00 \times 10^{-9} \text{ C}}{5.00 \times 10^{-2} \text{ m}} \right) \\
 &= -539 \text{ V}.
 \end{aligned}$$

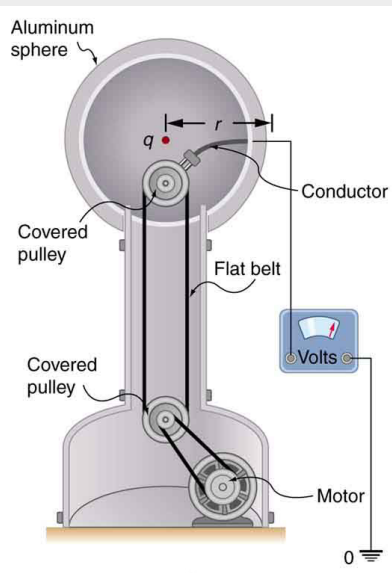
Discussion

The negative value for voltage means a positive charge would be attracted from a larger distance, since the potential is lower (more negative) than at larger distances. Conversely, a negative charge would be repelled, as expected.

Example:

What Is the Excess Charge on a Van de Graaff Generator

A demonstration Van de Graaff generator has a 25.0 cm diameter metal sphere that produces a voltage of 100 kV near its surface. (See [\[link\]](#).) What excess charge resides on the sphere? (Assume that each numerical value here is shown with three significant figures.)



The voltage of this demonstration Van de Graaff generator is measured

between the charged sphere and ground. Earth's potential is taken to be zero as a reference. The potential of the charged conducting sphere is the same as that of an equal point charge at its center.

Strategy

The potential on the surface will be the same as that of a point charge at the center of the sphere, 12.5 cm away. (The radius of the sphere is 12.5 cm.)

We can thus determine the excess charge using the equation

Equation:

$$V = \frac{kQ}{r}.$$

Solution

Solving for Q and entering known values gives

Equation:

$$\begin{aligned} Q &= \frac{rV}{k} \\ &= \frac{(0.125 \text{ m})(100 \times 10^3 \text{ V})}{8.99 \times 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2} \\ &= 1.39 \times 10^{-6} \text{ C} = 1.39 \text{ } \mu\text{C}. \end{aligned}$$

Discussion

This is a relatively small charge, but it produces a rather large voltage. We have another indication here that it is difficult to store isolated charges.

The voltages in both of these examples could be measured with a meter that compares the measured potential with ground potential. Ground potential is often taken to be zero (instead of taking the potential at infinity to be zero). It is the potential difference between two points that is of importance, and very often there is a tacit assumption that some reference point, such as Earth or a very distant point, is at zero potential. As noted in [Electric Potential Energy: Potential Difference](#), this is analogous to taking sea level as $h = 0$ when considering gravitational potential energy, $PE_g = mgh$.

Section Summary

- Electric potential of a point charge is $V = kQ/r$.
- Electric potential is a scalar, and electric field is a vector. Addition of voltages as numbers gives the voltage due to a combination of point charges, whereas addition of individual fields as vectors gives the total electric field.

Conceptual Questions

Exercise:

Problem:

In what region of space is the potential due to a uniformly charged sphere the same as that of a point charge? In what region does it differ from that of a point charge?

Exercise:

Problem:

Can the potential of a non-uniformly charged sphere be the same as that of a point charge? Explain.

Problems & Exercises

Exercise:

Problem:

A 0.500 cm diameter plastic sphere, used in a static electricity demonstration, has a uniformly distributed 40.0 pC charge on its surface. What is the potential near its surface?

Solution:

144 V

Exercise:**Problem:**

What is the potential 0.530×10^{-10} m from a proton (the average distance between the proton and electron in a hydrogen atom)?

Exercise:**Problem:**

(a) A sphere has a surface uniformly charged with 1.00 C. At what distance from its center is the potential 5.00 MV? (b) What does your answer imply about the practical aspect of isolating such a large charge?

Solution:

(a) 1.80 km

(b) A charge of 1 C is a very large amount of charge; a sphere of radius 1.80 km is not practical.

Exercise:**Problem:**

How far from a 1.00 μC point charge will the potential be 100 V? At what distance will it be 2.00×10^2 V?

Exercise:

Problem:

What are the sign and magnitude of a point charge that produces a potential of -2.00 V at a distance of 1.00 mm ?

Solution:

$$-2.22 \times 10^{-13}\text{ C}$$

Exercise:**Problem:**

If the potential due to a point charge is $5.00 \times 10^2\text{ V}$ at a distance of 15.0 m , what are the sign and magnitude of the charge?

Exercise:**Problem:**

In nuclear fission, a nucleus splits roughly in half. (a) What is the potential $2.00 \times 10^{-14}\text{ m}$ from a fragment that has 46 protons in it? (b) What is the potential energy in MeV of a similarly charged fragment at this distance?

Solution:

(a) $3.31 \times 10^6\text{ V}$

(b) 152 MeV

Exercise:**Problem:**

A research Van de Graaff generator has a 2.00-m -diameter metal sphere with a charge of 5.00 mC on it. (a) What is the potential near its surface? (b) At what distance from its center is the potential 1.00 MV ? (c) An oxygen atom with three missing electrons is released near the Van de Graaff generator. What is its energy in MeV at this distance?

Exercise:**Problem:**

An electrostatic paint sprayer has a 0.200-m-diameter metal sphere at a potential of 25.0 kV that repels paint droplets onto a grounded object.

(a) What charge is on the sphere? (b) What charge must a 0.100-mg drop of paint have to arrive at the object with a speed of 10.0 m/s?

Solution:

(a) $2.78 \times 10^{-7} \text{ C}$

(b) $2.00 \times 10^{-10} \text{ C}$

Exercise:**Problem:**

In one of the classic nuclear physics experiments at the beginning of the 20th century, an alpha particle was accelerated toward a gold nucleus, and its path was substantially deflected by the Coulomb interaction. If the energy of the doubly charged alpha nucleus was 5.00 MeV, how close to the gold nucleus (79 protons) could it come before being deflected?

Exercise:**Problem:**

(a) What is the potential between two points situated 10 cm and 20 cm from a $3.0 \mu\text{C}$ point charge? (b) To what location should the point at 20 cm be moved to increase this potential difference by a factor of two?

Exercise:**Problem: Unreasonable Results**

(a) What is the final speed of an electron accelerated from rest through a voltage of 25.0 MV by a negatively charged Van de Graaff terminal?

(b) What is unreasonable about this result?

(c) Which assumptions are responsible?

Solution:

(a) $2.96 \times 10^9 \text{ m/s}$

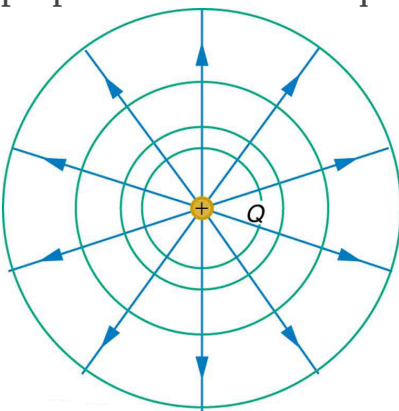
(b) This velocity is far too great. It is faster than the speed of light.

(c) The assumption that the speed of the electron is far less than that of light and that the problem does not require a relativistic treatment produces an answer greater than the speed of light.

Equipotential Lines

- Explain equipotential lines and equipotential surfaces.
- Describe the action of grounding an electrical appliance.
- Compare electric field and equipotential lines.

We can represent electric potentials (voltages) pictorially, just as we drew pictures to illustrate electric fields. Of course, the two are related. Consider [\[link\]](#), which shows an isolated positive point charge and its electric field lines. Electric field lines radiate out from a positive charge and terminate on negative charges. While we use blue arrows to represent the magnitude and direction of the electric field, we use green lines to represent places where the electric potential is constant. These are called **equipotential lines** in two dimensions, or *equipotential surfaces* in three dimensions. The term *equipotential* is also used as a noun, referring to an equipotential line or surface. The potential for a point charge is the same anywhere on an imaginary sphere of radius r surrounding the charge. This is true since the potential for a point charge is given by $V = kQ/r$ and, thus, has the same value at any point that is a given distance r from the charge. An equipotential sphere is a circle in the two-dimensional view of [\[link\]](#). Since the electric field lines point radially away from the charge, they are perpendicular to the equipotential lines.



An isolated point charge Q with its electric field lines in blue and equipotential lines

in green. The potential is the same along each equipotential line, meaning that no work is required to move a charge anywhere along one of those lines. Work is needed to move a charge from one equipotential line to another. Equipotential lines are perpendicular to electric field lines in every case.

It is important to note that *equipotential lines are always perpendicular to electric field lines*. No work is required to move a charge along an equipotential, since $\Delta V = 0$. Thus the work is

Equation:

$$W = -\Delta \text{PE} = -q\Delta V = 0.$$

Work is zero if force is perpendicular to motion. Force is in the same direction as $\mathbf{E}$, so that motion along an equipotential must be perpendicular to $\mathbf{E}$. More precisely, work is related to the electric field by

Equation:

$$W = Fd \cos \theta = qEd \cos \theta = 0.$$

Note that in the above equation, E and F symbolize the magnitudes of the electric field strength and force, respectively. Neither q nor $\mathbf{E}$ nor d is zero, and so $\cos \theta$ must be 0, meaning θ must be 90° . In other words, motion along an equipotential is perpendicular to $\mathbf{E}$.

One of the rules for static electric fields and conductors is that the electric field must be perpendicular to the surface of any conductor. This implies that a *conductor is an equipotential surface in static situations*. There can be no voltage difference across the surface of a conductor, or charges will flow. One of the uses of this fact is that a conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called **grounding**. Grounding can be a useful safety tool. For example, grounding the metal case of an electrical appliance ensures that it is at zero volts relative to the earth.

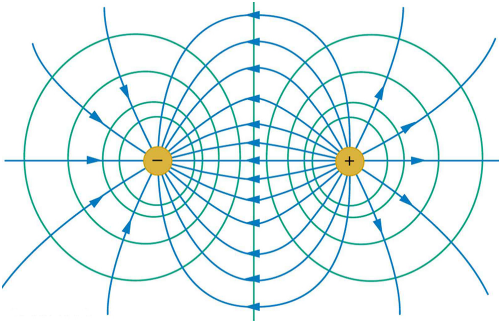
Note:

Grounding

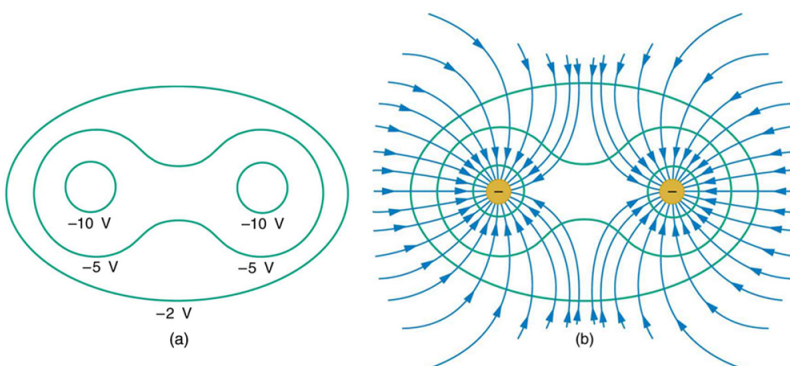
A conductor can be fixed at zero volts by connecting it to the earth with a good conductor—a process called grounding.

Because a conductor is an equipotential, it can replace any equipotential surface. For example, in [\[link\]](#) a charged spherical conductor can replace the point charge, and the electric field and potential surfaces outside of it will be unchanged, confirming the contention that a spherical charge distribution is equivalent to a point charge at its center.

[\[link\]](#) shows the electric field and equipotential lines for two equal and opposite charges. Given the electric field lines, the equipotential lines can be drawn simply by making them perpendicular to the electric field lines. Conversely, given the equipotential lines, as in [\[link\]\(a\)](#), the electric field lines can be drawn by making them perpendicular to the equipotentials, as in [\[link\]\(b\)](#).



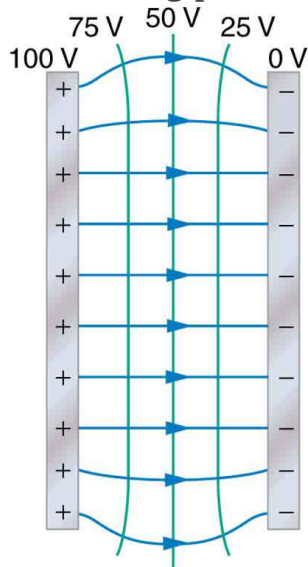
The electric field lines and equipotential lines for two equal but opposite charges. The equipotential lines can be drawn by making them perpendicular to the electric field lines, if those are known. Note that the potential is greatest (most positive) near the positive charge and least (most negative) near the negative charge.



(a) These equipotential lines might be measured with a voltmeter in a laboratory experiment. (b) The corresponding electric field lines are found by drawing them

perpendicular to the equipotentials. Note that these fields are consistent with two equal negative charges.

One of the most important cases is that of the familiar parallel conducting plates shown in [\[link\]](#). Between the plates, the equipotentials are evenly spaced and parallel. The same field could be maintained by placing conducting plates at the equipotential lines at the potentials shown.



The electric field and equipotential lines between two metal plates.

An important application of electric fields and equipotential lines involves the heart. The heart relies on electrical signals to maintain its rhythm. The movement of electrical signals causes the chambers of the heart to contract and relax. When a person has a heart attack, the movement of these electrical signals may be disturbed. An artificial pacemaker and a

defibrillator can be used to initiate the rhythm of electrical signals. The equipotential lines around the heart, the thoracic region, and the axis of the heart are useful ways of monitoring the structure and functions of the heart. An electrocardiogram (ECG) measures the small electric signals being generated during the activity of the heart. More about the relationship between electric fields and the heart is discussed in [Energy Stored in Capacitors](#).

Note:

PhET Explorations: Charges and Fields

Move point charges around on the playing field and then view the electric field, voltages, equipotential lines, and more. It's colorful, it's dynamic, it's free.

https://phet.colorado.edu/sims/html/charges-and-fields/latest/charges-and-fields_en.html

Section Summary

- An equipotential line is a line along which the electric potential is constant.
- An equipotential surface is a three-dimensional version of equipotential lines.
- Equipotential lines are always perpendicular to electric field lines.
- The process by which a conductor can be fixed at zero volts by connecting it to the earth with a good conductor is called grounding.

Conceptual Questions

Exercise:

Problem:

What is an equipotential line? What is an equipotential surface?

Exercise:

Problem:

Explain in your own words why equipotential lines and surfaces must be perpendicular to electric field lines.

Exercise:

Problem: Can different equipotential lines cross? Explain.

Problems & Exercises

Exercise:

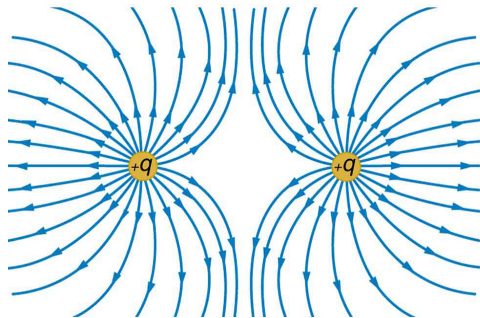
Problem:

(a) Sketch the equipotential lines near a point charge $+q$. Indicate the direction of increasing potential. (b) Do the same for a point charge $-3q$.

Exercise:

Problem:

Sketch the equipotential lines for the two equal positive charges shown in [\[link\]](#). Indicate the direction of increasing potential.



The electric field near two equal positive charges is directed away from each of the charges.

Exercise:

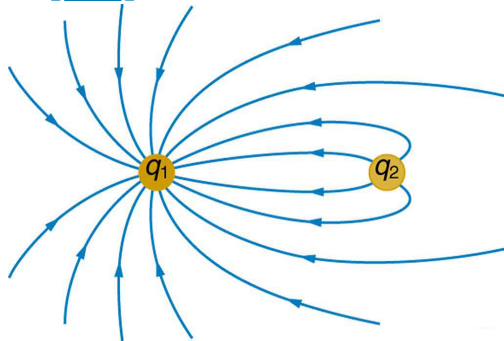
Problem:

[\[link\]](#) shows the electric field lines near two charges q_1 and q_2 , the first having a magnitude four times that of the second. Sketch the equipotential lines for these two charges, and indicate the direction of increasing potential.

Exercise:

Problem:

Sketch the equipotential lines a long distance from the charges shown in [\[link\]](#). Indicate the direction of increasing potential.



The electric field near
two charges.

Exercise:

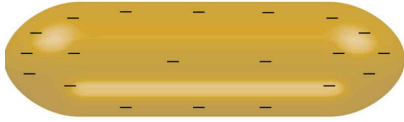
Problem:

Sketch the equipotential lines in the vicinity of two opposite charges, where the negative charge is three times as great in magnitude as the positive. See [\[link\]](#) for a similar situation. Indicate the direction of increasing potential.

Exercise:

Problem:

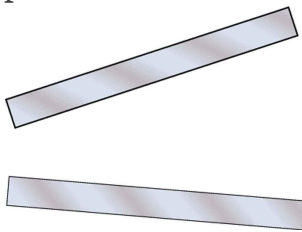
Sketch the equipotential lines in the vicinity of the negatively charged conductor in [\[link\]](#). How will these equipotentials look a long distance from the object?



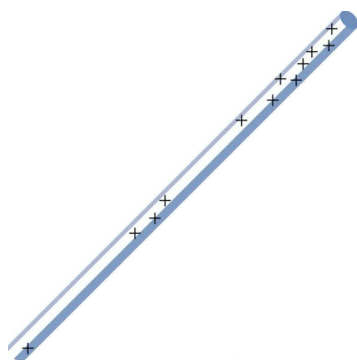
A negatively charged conductor.

Exercise:**Problem:**

Sketch the equipotential lines surrounding the two conducting plates shown in [\[link\]](#), given the top plate is positive and the bottom plate has an equal amount of negative charge. Be certain to indicate the distribution of charge on the plates. Is the field strongest where the plates are closest? Why should it be?

**Exercise:****Problem:**

(a) Sketch the electric field lines in the vicinity of the charged insulator in [\[link\]](#). Note its non-uniform charge distribution. (b) Sketch equipotential lines surrounding the insulator. Indicate the direction of increasing potential.



A charged insulating rod such as might be used in a classroom demonstration.

Exercise:

Problem:

The naturally occurring charge on the ground on a fine day out in the open country is -1.00 nC/m^2 . (a) What is the electric field relative to ground at a height of 3.00 m? (b) Calculate the electric potential at this height. (c) Sketch electric field and equipotential lines for this scenario.

Exercise:

Problem:

The lesser electric ray (*Narcine bancroftii*) maintains an incredible charge on its head and a charge equal in magnitude but opposite in sign on its tail ([\[link\]](#)). (a) Sketch the equipotential lines surrounding the ray. (b) Sketch the equipotentials when the ray is near a ship with a conducting surface. (c) How could this charge distribution be of use to the ray?



Lesser electric ray (*Narcine bancroftii*) (credit: National Oceanic and Atmospheric Administration, NOAA's Fisheries Collection).

Glossary

equipotential line

a line along which the electric potential is constant

grounding

fixing a conductor at zero volts by connecting it to the earth or ground

Capacitors and Dielectrics

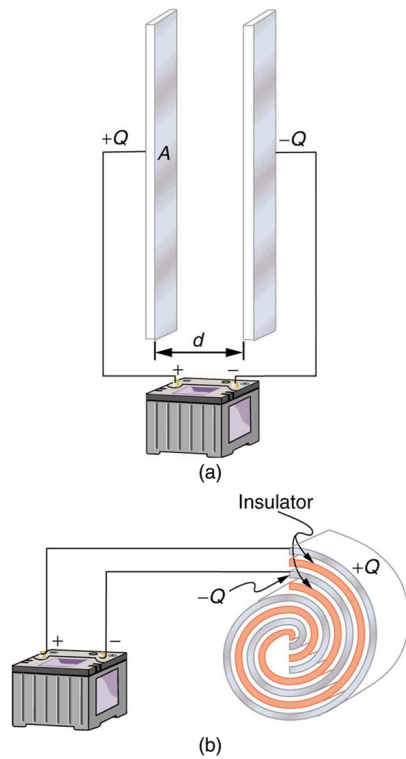
- Describe the action of a capacitor and define capacitance.
- Explain parallel plate capacitors and their capacitances.
- Discuss the process of increasing the capacitance of a dielectric.
- Determine capacitance given charge and voltage.

A **capacitor** is a device used to store electric charge. Capacitors have applications ranging from filtering static out of radio reception to energy storage in heart defibrillators. Typically, commercial capacitors have two conducting parts close to one another, but not touching, such as those in [\[link\]](#). (Most of the time an insulator is used between the two plates to provide separation—see the discussion on dielectrics below.) When battery terminals are connected to an initially uncharged capacitor, equal amounts of positive and negative charge, $+Q$ and $-Q$, are separated into its two plates. The capacitor remains neutral overall, but we refer to it as storing a charge Q in this circumstance.

Note:

Capacitor

A capacitor is a device used to store electric charge.



Both capacitors shown here were initially uncharged before being connected to a battery. They now have separated charges of $+Q$ and $-Q$ on their two halves. (a) A parallel plate capacitor. (b) A rolled capacitor with an insulating material between its two conducting sheets.

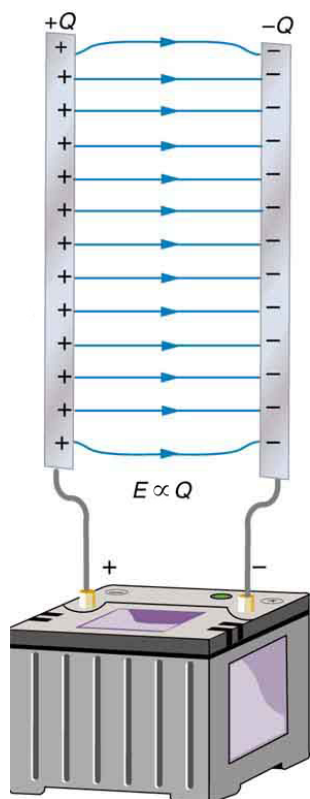
The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor’s physical characteristics, such as its size.

Note:

The Amount of Charge Q a Capacitor Can Store

The amount of charge Q a *capacitor* can store depends on two major factors—the voltage applied and the capacitor’s physical characteristics, such as its size.

A system composed of two identical, parallel conducting plates separated by a distance, as in [\[link\]](#), is called a **parallel plate capacitor**. It is easy to see the relationship between the voltage and the stored charge for a parallel plate capacitor, as shown in [\[link\]](#). Each electric field line starts on an individual positive charge and ends on a negative one, so that there will be more field lines if there is more charge. (Drawing a single field line per charge is a convenience, only. We can draw many field lines for each charge, but the total number is proportional to the number of charges.) The electric field strength is, thus, directly proportional to Q .



Electric field lines in this parallel plate capacitor, as always, start on positive charges and end on negative charges. Since the electric field strength is proportional to the density of field lines, it is also proportional to the amount

of charge on
the capacitor.

The field is proportional to the charge:

Equation:

$$E \propto Q,$$

where the symbol $\propto$ means “proportional to.” From the discussion in [Electric Potential in a Uniform Electric Field](#), we know that the voltage across parallel plates is $V = Ed$. Thus,

Equation:

$$V \propto E.$$

It follows, then, that $V \propto Q$, and conversely,

Equation:

$$Q \propto V.$$

This is true in general: The greater the voltage applied to any capacitor, the greater the charge stored in it.

Different capacitors will store different amounts of charge for the same applied voltage, depending on their physical characteristics. We define their **capacitance** C to be such that the charge Q stored in a capacitor is proportional to C . The charge stored in a capacitor is given by

Equation:

$$Q = CV.$$

This equation expresses the two major factors affecting the amount of charge stored. Those factors are the physical characteristics of the capacitor,

C , and the voltage, V . Rearranging the equation, we see that *capacitance C is the amount of charge stored per volt*, or

Equation:

$$C = \frac{Q}{V}.$$

Note:

Capacitance

Capacitance C is the amount of charge stored per volt, or

Equation:

$$C = \frac{Q}{V}.$$

The unit of capacitance is the farad (F), named for Michael Faraday (1791–1867), an English scientist who contributed to the fields of electromagnetism and electrochemistry. Since capacitance is charge per unit voltage, we see that a farad is a coulomb per volt, or

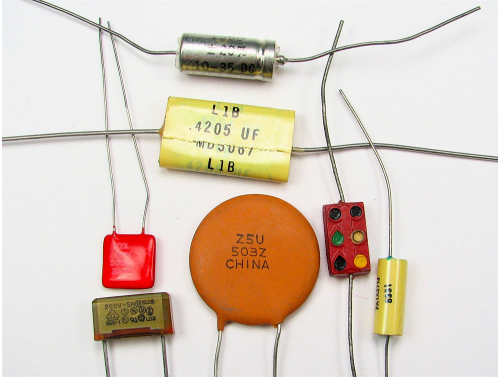
Equation:

$$1 \text{ F} = \frac{1 \text{ C}}{1 \text{ V}}.$$

A 1-farad capacitor would be able to store 1 coulomb (a very large amount of charge) with the application of only 1 volt. One farad is, thus, a very large capacitance. Typical capacitors range from fractions of a picofarad ($1 \text{ pF} = 10^{-12} \text{ F}$) to millifarads ($1 \text{ mF} = 10^{-3} \text{ F}$).

[\[link\]](#) shows some common capacitors. Capacitors are primarily made of ceramic, glass, or plastic, depending upon purpose and size. Insulating

materials, called dielectrics, are commonly used in their construction, as discussed below.

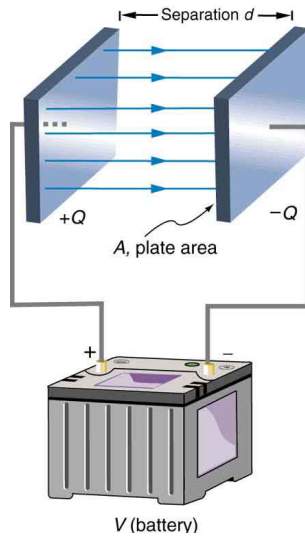


Some typical capacitors.

Size and value of
capacitance are not
necessarily related.
(credit: Windell Oskay)

Parallel Plate Capacitor

The parallel plate capacitor shown in [\[link\]](#) has two identical conducting plates, each having a surface area A , separated by a distance d (with no material between the plates). When a voltage V is applied to the capacitor, it stores a charge Q , as shown. We can see how its capacitance depends on A and d by considering the characteristics of the Coulomb force. We know that like charges repel, unlike charges attract, and the force between charges decreases with distance. So it seems quite reasonable that the bigger the plates are, the more charge they can store—because the charges can spread out more. Thus C should be greater for larger A . Similarly, the closer the plates are together, the greater the attraction of the opposite charges on them. So C should be greater for smaller d .



Parallel plate
capacitor
with plates
separated by
a distance d .
Each plate
has an area A

It can be shown that for a parallel plate capacitor there are only two factors (A and d) that affect its capacitance C . The capacitance of a parallel plate capacitor in equation form is given by

Equation:

$$C = \epsilon_0 \frac{A}{d}.$$

Note:

Capacitance of a Parallel Plate Capacitor

Equation:

$$C = \epsilon_0 \frac{A}{d}$$

A is the area of one plate in square meters, and d is the distance between the plates in meters. The constant ϵ_0 is the permittivity of free space; its numerical value in SI units is $\epsilon_0 = 8.85 \times 10^{-12} \text{ F/m}$. The units of F/m are equivalent to $\text{C}^2/\text{N} \cdot \text{m}^2$. The small numerical value of ϵ_0 is related to the large size of the farad. A parallel plate capacitor must have a large area to have a capacitance approaching a farad. (Note that the above equation is valid when the parallel plates are separated by air or free space. When another material is placed between the plates, the equation is modified, as discussed below.)

Example:

Capacitance and Charge Stored in a Parallel Plate Capacitor

(a) What is the capacitance of a parallel plate capacitor with metal plates, each of area 1.00 m^2 , separated by 1.00 mm ? (b) What charge is stored in this capacitor if a voltage of $3.00 \times 10^3 \text{ V}$ is applied to it?

Strategy

Finding the capacitance C is a straightforward application of the equation $C = \epsilon_0 A/d$. Once C is found, the charge stored can be found using the equation $Q = CV$.

Solution for (a)

Entering the given values into the equation for the capacitance of a parallel plate capacitor yields

Equation:

$$\begin{aligned} C &= \epsilon_0 \frac{A}{d} = \left(8.85 \times 10^{-12} \frac{\text{F}}{\text{m}} \right) \frac{1.00 \text{ m}^2}{1.00 \times 10^{-3} \text{ m}} \\ &= 8.85 \times 10^{-9} \text{ F} = 8.85 \text{ nF}. \end{aligned}$$

Discussion for (a)

This small value for the capacitance indicates how difficult it is to make a device with a large capacitance. Special techniques help, such as using very large area thin foils placed close together.

Solution for (b)

The charge stored in any capacitor is given by the equation $Q = CV$. Entering the known values into this equation gives

Equation:

$$\begin{aligned} Q &= CV = (8.85 \times 10^{-9} \text{ F})(3.00 \times 10^3 \text{ V}) \\ &= 26.6 \text{ } \mu\text{C}. \end{aligned}$$

Discussion for (b)

This charge is only slightly greater than those found in typical static electricity. Since air breaks down at about $3.00 \times 10^6 \text{ V/m}$, more charge cannot be stored on this capacitor by increasing the voltage.

Another interesting biological example dealing with electric potential is found in the cell's plasma membrane. The membrane sets a cell off from its surroundings and also allows ions to selectively pass in and out of the cell. There is a potential difference across the membrane of about -70 mV . This is due to the mainly negatively charged ions in the cell and the predominance of positively charged sodium (Na^+) ions outside. Things change when a nerve cell is stimulated. Na^+ ions are allowed to pass through the membrane into the cell, producing a positive membrane potential—the nerve signal. The cell membrane is about 7 to 10 nm thick. An approximate value of the electric field across it is given by

Equation:

$$E = \frac{V}{d} = \frac{-70 \times 10^{-3} \text{ V}}{8 \times 10^{-9} \text{ m}} = -9 \times 10^6 \text{ V/m}.$$

This electric field is enough to cause a breakdown in air.

Dielectric

The previous example highlights the difficulty of storing a large amount of charge in capacitors. If d is made smaller to produce a larger capacitance, then the maximum voltage must be reduced proportionally to avoid breakdown (since $E = V/d$). An important solution to this difficulty is to put an insulating material, called a **dielectric**, between the plates of a capacitor and allow d to be as small as possible. Not only does the smaller d make the capacitance greater, but many insulators can withstand greater electric fields than air before breaking down.

There is another benefit to using a dielectric in a capacitor. Depending on the material used, the capacitance is greater than that given by the equation $C = \epsilon_0 \frac{A}{d}$ by a factor κ , called the *dielectric constant*. A parallel plate capacitor with a dielectric between its plates has a capacitance given by **Equation:**

$$C = \kappa \epsilon_0 \frac{A}{d} \text{ (parallel plate capacitor with dielectric).}$$

Values of the dielectric constant κ for various materials are given in [\[link\]](#). Note that κ for vacuum is exactly 1, and so the above equation is valid in that case, too. If a dielectric is used, perhaps by placing Teflon between the plates of the capacitor in [\[link\]](#), then the capacitance is greater by the factor κ , which for Teflon is 2.1.

Note:**Take-Home Experiment: Building a Capacitor**

How large a capacitor can you make using a chewing gum wrapper? The plates will be the aluminum foil, and the separation (dielectric) in between will be the paper.

Material	Dielectric constant κ	Dielectric strength (V/m)
Vacuum	1.00000	—
Air	1.00059	3×10^6
Bakelite	4.9	24×10^6
Fused quartz	3.78	8×10^6
Neoprene rubber	6.7	12×10^6
Nylon	3.4	14×10^6
Paper	3.7	16×10^6
Polystyrene	2.56	24×10^6
Pyrex glass	5.6	14×10^6
Silicon oil	2.5	15×10^6
Strontium titanate	233	8×10^6
Teflon	2.1	60×10^6
Water	80	—

Dielectric Constants and Dielectric Strengths for Various Materials at 20°C

Note also that the dielectric constant for air is very close to 1, so that air-filled capacitors act much like those with vacuum between their plates *except* that the air can become conductive if the electric field strength

becomes too great. (Recall that $E = V/d$ for a parallel plate capacitor.) Also shown in [\[link\]](#) are maximum electric field strengths in V/m, called **dielectric strengths**, for several materials. These are the fields above which the material begins to break down and conduct. The dielectric strength imposes a limit on the voltage that can be applied for a given plate separation. For instance, in [\[link\]](#), the separation is 1.00 mm, and so the voltage limit for air is

Equation:

$$\begin{aligned} V &= E \cdot d \\ &= (3 \times 10^6 \text{ V/m})(1.00 \times 10^{-3} \text{ m}) \\ &= 3000 \text{ V.} \end{aligned}$$

However, the limit for a 1.00 mm separation filled with Teflon is 60,000 V, since the dielectric strength of Teflon is $60 \times 10^6 \text{ V/m}$. So the same capacitor filled with Teflon has a greater capacitance and can be subjected to a much greater voltage. Using the capacitance we calculated in the above example for the air-filled parallel plate capacitor, we find that the Teflon-filled capacitor can store a maximum charge of

Equation:

$$\begin{aligned} Q &= CV \\ &= \kappa C_{\text{air}} V \\ &= (2.1)(8.85 \text{ nF})(6.0 \times 10^4 \text{ V}) \\ &= 1.1 \text{ mC.} \end{aligned}$$

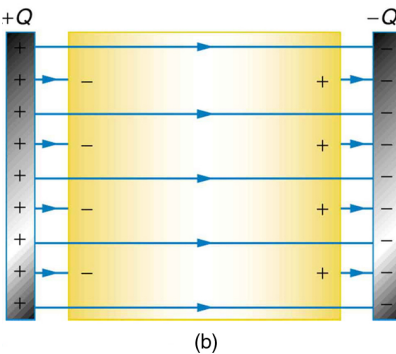
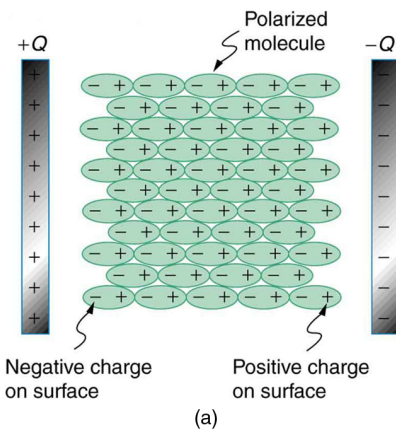
This is 42 times the charge of the same air-filled capacitor.

Note:

Dielectric Strength

The maximum electric field strength above which an insulating material begins to break down and conduct is called its dielectric strength.

Microscopically, how does a dielectric increase capacitance? Polarization of the insulator is responsible. The more easily it is polarized, the greater its dielectric constant κ . Water, for example, is a **polar molecule** because one end of the molecule has a slight positive charge and the other end has a slight negative charge. The polarity of water causes it to have a relatively large dielectric constant of 80. The effect of polarization can be best explained in terms of the characteristics of the Coulomb force. [\[link\]](#) shows the separation of charge schematically in the molecules of a dielectric material placed between the charged plates of a capacitor. The Coulomb force between the closest ends of the molecules and the charge on the plates is attractive and very strong, since they are very close together. This attracts more charge onto the plates than if the space were empty and the opposite charges were a distance d away.



(a) The molecules in the insulating material between

the plates of a capacitor are polarized by the charged plates. This produces a layer of opposite charge on the surface of the dielectric that attracts more charge onto the plate, increasing its capacitance. (b)

The dielectric reduces the electric field strength inside the capacitor, resulting in a smaller voltage between the plates for the same charge. The capacitor stores the same charge for a smaller voltage, implying that it has a larger capacitance because of the dielectric.

Another way to understand how a dielectric increases capacitance is to consider its effect on the electric field inside the capacitor. [\[link\]](#)(b) shows the electric field lines with a dielectric in place. Since the field lines end on charges in the dielectric, there are fewer of them going from one side of the capacitor to the other. So the electric field strength is less than if there were

a vacuum between the plates, even though the same charge is on the plates. The voltage between the plates is $V = Ed$, so it too is reduced by the dielectric. Thus there is a smaller voltage V for the same charge Q ; since $C = Q/V$, the capacitance C is greater.

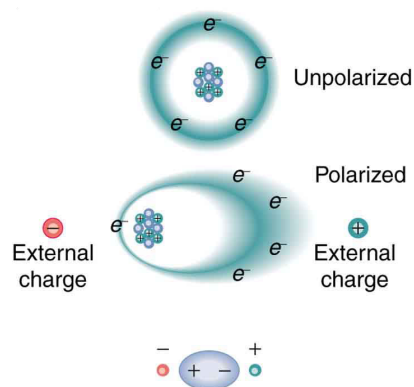
The dielectric constant is generally defined to be $\kappa = E_0/E$, or the ratio of the electric field in a vacuum to that in the dielectric material, and is intimately related to the polarizability of the material.

Note:

Things Great and Small

The Submicroscopic Origin of Polarization

Polarization is a separation of charge within an atom or molecule. As has been noted, the planetary model of the atom pictures it as having a positive nucleus orbited by negative electrons, analogous to the planets orbiting the Sun. Although this model is not completely accurate, it is very helpful in explaining a vast range of phenomena and will be refined elsewhere, such as in [Atomic Physics](#). The submicroscopic origin of polarization can be modeled as shown in [\[link\]](#).



Large-scale view of polarized atom

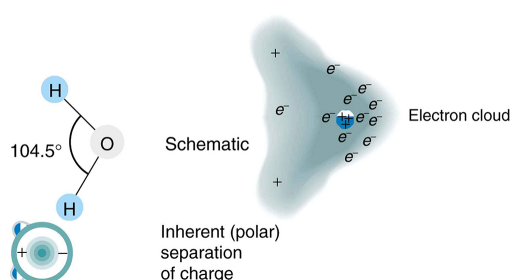
Artist's conception
of a polarized atom.
The orbits of

electrons around
the nucleus are
shifted slightly by
the external charges
(shown
exaggerated). The
resulting separation
of charge within
the atom means
that it is polarized.
Note that the unlike
charge is now
closer to the
external charges,
causing the
polarization.

We will find in [Atomic Physics](#) that the orbits of electrons are more properly viewed as electron clouds with the density of the cloud related to the probability of finding an electron in that location (as opposed to the definite locations and paths of planets in their orbits around the Sun). This cloud is shifted by the Coulomb force so that the atom on average has a separation of charge. Although the atom remains neutral, it can now be the source of a Coulomb force, since a charge brought near the atom will be closer to one type of charge than the other.

Some molecules, such as those of water, have an inherent separation of charge and are thus called polar molecules. [\[link\]](#) illustrates the separation of charge in a water molecule, which has two hydrogen atoms and one oxygen atom (H_2O). The water molecule is not symmetric—the hydrogen atoms are repelled to one side, giving the molecule a boomerang shape. The electrons in a water molecule are more concentrated around the more highly charged oxygen nucleus than around the hydrogen nuclei. This makes the oxygen end of the molecule slightly negative and leaves the hydrogen ends slightly positive. The inherent separation of charge in polar molecules

makes it easier to align them with external fields and charges. Polar molecules therefore exhibit greater polarization effects and have greater dielectric constants. Those who study chemistry will find that the polar nature of water has many effects. For example, water molecules gather ions much more effectively because they have an electric field and a separation of charge to attract charges of both signs. Also, as brought out in the previous chapter, polar water provides a shield or screening of the electric fields in the highly charged molecules of interest in biological systems.



Artist's conception of a water molecule. There is an inherent separation of charge, and so water is a polar molecule. Electrons in the molecule are attracted to the oxygen nucleus and leave an excess of positive charge near the two hydrogen nuclei. (Note that the schematic on the right is a rough illustration of the distribution of electrons in the water molecule. It does not show the actual numbers of protons and electrons involved in the structure.)

Note:**PhET Explorations: Capacitor Lab**

Explore how a capacitor works! Change the size of the plates and add a dielectric to see the effect on capacitance. Change the voltage and see charges built up on the plates. Observe the electric field in the capacitor. Measure the voltage and the electric field.

[Capacitor Lab](#)

Section Summary

- A capacitor is a device used to store charge.
- The amount of charge Q a capacitor can store depends on two major factors—the voltage applied and the capacitor's physical characteristics, such as its size.
- The capacitance C is the amount of charge stored per volt, or

Equation:

$$C = \frac{Q}{V}.$$

- The capacitance of a parallel plate capacitor is $C = \epsilon_0 \frac{A}{d}$, when the plates are separated by air or free space. ϵ_0 is called the permittivity of free space.
- A parallel plate capacitor with a dielectric between its plates has a capacitance given by

Equation:

$$C = \kappa \epsilon_0 \frac{A}{d},$$

where κ is the dielectric constant of the material.

- The maximum electric field strength above which an insulating material begins to break down and conduct is called dielectric strength.

Conceptual Questions

Exercise:

Problem:

Does the capacitance of a device depend on the applied voltage? What about the charge stored in it?

Exercise:

Problem:

Use the characteristics of the Coulomb force to explain why capacitance should be proportional to the plate area of a capacitor. Similarly, explain why capacitance should be inversely proportional to the separation between plates.

Exercise:

Problem:

Give the reason why a dielectric material increases capacitance compared with what it would be with air between the plates of a capacitor. What is the independent reason that a dielectric material also allows a greater voltage to be applied to a capacitor? (The dielectric thus increases C and permits a greater V .)

Exercise:

Problem:

How does the polar character of water molecules help to explain water's relatively large dielectric constant? ([link](#))

Exercise:**Problem:**

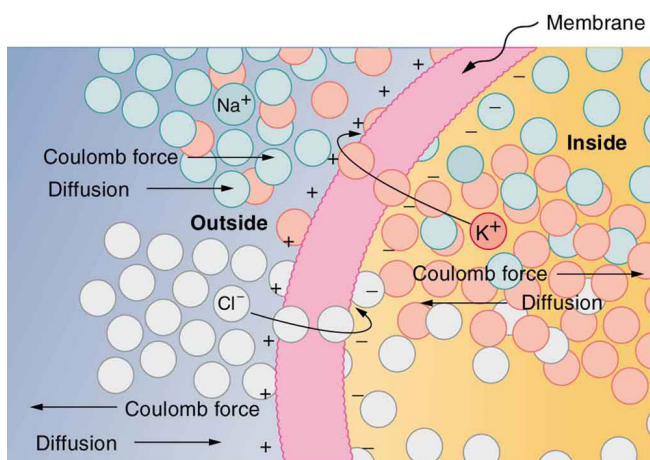
Sparks will occur between the plates of an air-filled capacitor at lower voltage when the air is humid than when dry. Explain why, considering the polar character of water molecules.

Exercise:**Problem:**

Water has a large dielectric constant, but it is rarely used in capacitors. Explain why.

Exercise:**Problem:**

Membranes in living cells, including those in humans, are characterized by a separation of charge across the membrane. Effectively, the membranes are thus charged capacitors with important functions related to the potential difference across the membrane. Is energy required to separate these charges in living membranes and, if so, is its source the metabolization of food energy or some other source?



The semipermeable membrane of a cell has different concentrations of ions inside and out. Diffusion moves the K^+ (potassium) and Cl^- (chloride) ions in the directions shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane. The membrane is normally impermeable to Na^+ (sodium ions).

Problems & Exercises

Exercise:

Problem:

What charge is stored in a $180\ \mu\text{F}$ capacitor when $120\ \text{V}$ is applied to it?

Solution:

$21.6\ \text{mC}$

Exercise:

Problem:

Find the charge stored when $5.50\ \text{V}$ is applied to an $8.00\ \text{pF}$ capacitor.

Exercise:

Problem: What charge is stored in the capacitor in [\[link\]](#)?

Solution:

80.0 mC

Exercise:

Problem:

Calculate the voltage applied to a $2.00\ \mu\text{F}$ capacitor when it holds $3.10\ \mu\text{C}$ of charge.

Exercise:

Problem:

What voltage must be applied to an $8.00\ \text{nF}$ capacitor to store $0.160\ \text{mC}$ of charge?

Solution:

20.0 kV

Exercise:

Problem:

What capacitance is needed to store $3.00\ \mu\text{C}$ of charge at a voltage of $120\ \text{V}$?

Exercise:

Problem:

What is the capacitance of a large Van de Graaff generator's terminal, given that it stores $8.00\ \text{mC}$ of charge at a voltage of $12.0\ \text{MV}$?

Solution:

667 pF

Exercise:

Problem:

Find the capacitance of a parallel plate capacitor having plates of area 5.00 m^2 that are separated by 0.100 mm of Teflon.

Exercise:**Problem:**

(a) What is the capacitance of a parallel plate capacitor having plates of area 1.50 m^2 that are separated by 0.0200 mm of neoprene rubber? (b) What charge does it hold when 9.00 V is applied to it?

Solution:

(a) $4.4 \text{ } \mu\text{F}$

(b) $4.0 \times 10^{-5} \text{ C}$

Exercise:**Problem: Integrated Concepts**

A prankster applies 450 V to an $80.0 \text{ } \mu\text{F}$ capacitor and then tosses it to an unsuspecting victim. The victim's finger is burned by the discharge of the capacitor through 0.200 g of flesh. What is the temperature increase of the flesh? Is it reasonable to assume no phase change?

Exercise:**Problem: Unreasonable Results**

(a) A certain parallel plate capacitor has plates of area 4.00 m^2 , separated by 0.0100 mm of nylon, and stores 0.170 C of charge. What is the applied voltage? (b) What is unreasonable about this result? (c) Which assumptions are responsible or inconsistent?

Solution:

(a) 14.2 kV

(b) The voltage is unreasonably large, more than 100 times the breakdown voltage of nylon.

(c) The assumed charge is unreasonably large and cannot be stored in a capacitor of these dimensions.

Glossary

capacitor

a device that stores electric charge

capacitance

amount of charge stored per unit volt

dielectric

an insulating material

dielectric strength

the maximum electric field above which an insulating material begins to break down and conduct

parallel plate capacitor

two identical conducting plates separated by a distance

polar molecule

a molecule with inherent separation of charge

Capacitors in Series and Parallel

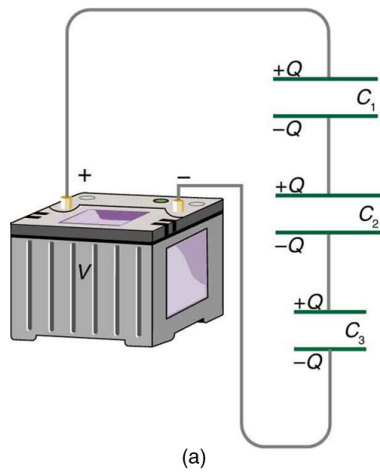
- Derive expressions for total capacitance in series and in parallel.
- Identify series and parallel parts in the combination of connection of capacitors.
- Calculate the effective capacitance in series and parallel given individual capacitances.

Several capacitors may be connected together in a variety of applications. Multiple connections of capacitors act like a single equivalent capacitor. The total capacitance of this equivalent single capacitor depends both on the individual capacitors and how they are connected. There are two simple and common types of connections, called *series* and *parallel*, for which we can easily calculate the total capacitance. Certain more complicated connections can also be related to combinations of series and parallel.

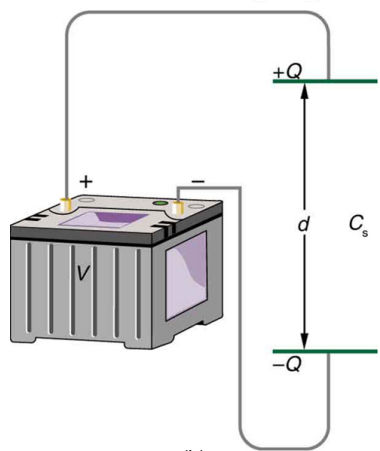
Capacitance in Series

[\[link\]](#)(a) shows a series connection of three capacitors with a voltage applied. As for any capacitor, the capacitance of the combination is related to charge and voltage by $C = \frac{Q}{V}$.

Note in [\[link\]](#) that opposite charges of magnitude Q flow to either side of the originally uncharged combination of capacitors when the voltage V is applied. Conservation of charge requires that equal-magnitude charges be created on the plates of the individual capacitors, since charge is only being separated in these originally neutral devices. The end result is that the combination resembles a single capacitor with an effective plate separation greater than that of the individual capacitors alone. (See [\[link\]](#)(b).) Larger plate separation means smaller capacitance. It is a general feature of series connections of capacitors that the total capacitance is less than any of the individual capacitances.



(a)



(b)

(a) Capacitors connected in series. The magnitude of the charge on each plate is Q . (b) An equivalent capacitor has a larger plate separation d .

Series connections produce a total capacitance that is less than that of

any of the individual capacitors.

We can find an expression for the total capacitance by considering the voltage across the individual capacitors shown in [\[link\]](#). Solving $C = \frac{Q}{V}$ for V gives $V = \frac{Q}{C}$. The voltages across the individual capacitors are thus $V_1 = \frac{Q}{C_1}$, $V_2 = \frac{Q}{C_2}$, and $V_3 = \frac{Q}{C_3}$. The total voltage is the sum of the individual voltages:

Equation:

$$V = V_1 + V_2 + V_3.$$

Now, calling the total capacitance C_S for series capacitance, consider that

Equation:

$$V = \frac{Q}{C_S} = V_1 + V_2 + V_3.$$

Entering the expressions for V_1 , V_2 , and V_3 , we get

Equation:

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3}.$$

Canceling the Q s, we obtain the equation for the total capacitance in series C_S to be

Equation:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots,$$

where “...” indicates that the expression is valid for any number of capacitors connected in series. An expression of this form always results in a total capacitance C_S that is less than any of the individual capacitances $C_1, C_2, \dots$, as the next example illustrates.

Note:

Total Capacitance in Series, C_S

Total capacitance in series: $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$

Example:**What Is the Series Capacitance?**

Find the total capacitance for three capacitors connected in series, given their individual capacitances are 1.000, 5.000, and 8.000 μF .

Strategy

With the given information, the total capacitance can be found using the equation for capacitance in series.

Solution

Entering the given capacitances into the expression for $\frac{1}{C_S}$ gives

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}.$$

Equation:

$$\frac{1}{C_S} = \frac{1}{1.000 \mu\text{F}} + \frac{1}{5.000 \mu\text{F}} + \frac{1}{8.000 \mu\text{F}} = \frac{1.325}{\mu\text{F}}$$

Inverting to find C_S yields $C_S = \frac{\mu\text{F}}{1.325} = 0.755 \mu\text{F}$.

Discussion

The total series capacitance C_S is less than the smallest individual capacitance, as promised. In series connections of capacitors, the sum is less than the parts. In fact, it is less than any individual. Note that it is sometimes possible, and more convenient, to solve an equation like the above by finding the least common denominator, which in this case (showing only whole-number calculations) is 40. Thus,

Equation:

$$\frac{1}{C_s} = \frac{40}{40 \mu\text{F}} + \frac{8}{40 \mu\text{F}} + \frac{5}{40 \mu\text{F}} = \frac{53}{40 \mu\text{F}},$$

so that

Equation:

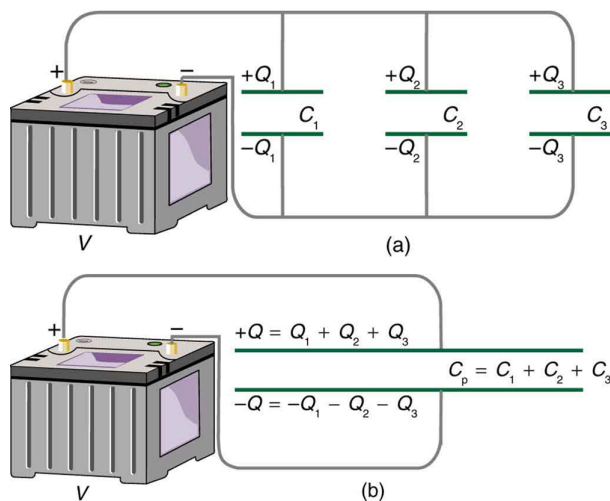
$$C_s = \frac{40 \mu\text{F}}{53} = 0.755 \mu\text{F}.$$

Capacitors in Parallel

[\[link\]](#)(a) shows a parallel connection of three capacitors with a voltage applied. Here the total capacitance is easier to find than in the series case. To find the equivalent total capacitance C_p , we first note that the voltage across each capacitor is V , the same as that of the source, since they are connected directly to it through a conductor. (Conductors are equipotentials, and so the voltage across the capacitors is the same as that across the voltage source.) Thus the capacitors have the same charges on them as they would have if connected individually to the voltage source. The total charge Q is the sum of the individual charges:

Equation:

$$Q = Q_1 + Q_2 + Q_3.$$



(a) Capacitors in parallel. Each is connected directly to the voltage source just as if it were all alone, and so the total capacitance in parallel is just the sum of the individual capacitances. (b) The equivalent capacitor has a larger plate area and can therefore hold more charge than the individual capacitors.

Using the relationship $Q = CV$, we see that the total charge is $Q = C_p V$, and the individual charges are $Q_1 = C_1 V$, $Q_2 = C_2 V$, and $Q_3 = C_3 V$. Entering these into the previous equation gives

Equation:

$$C_p V = C_1 V + C_2 V + C_3 V.$$

Canceling V from the equation, we obtain the equation for the total capacitance in parallel C_p :

Equation:

$$C_p = C_1 + C_2 + C_3 + \dots$$

Total capacitance in parallel is simply the sum of the individual capacitances. (Again the “...” indicates the expression is valid for any number of capacitors connected in parallel.) So, for example, if the capacitors in the example above were connected in parallel, their capacitance would be

Equation:

$$C_p = 1.000 \mu\text{F} + 5.000 \mu\text{F} + 8.000 \mu\text{F} = 14.000 \mu\text{F}.$$

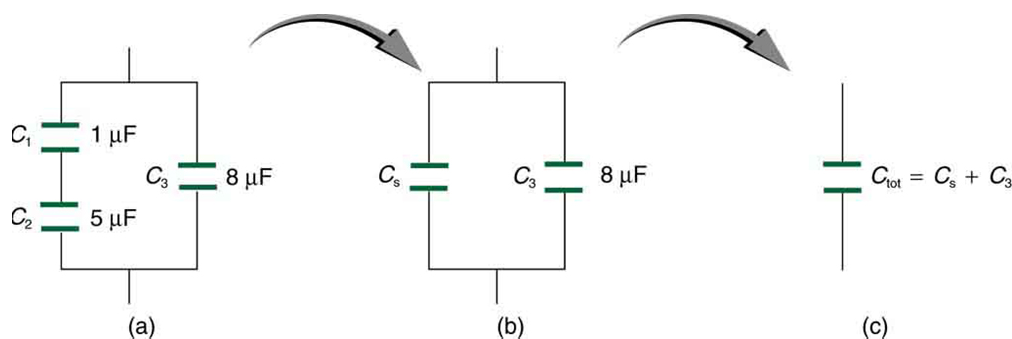
The equivalent capacitor for a parallel connection has an effectively larger plate area and, thus, a larger capacitance, as illustrated in [\[link\]](#)(b).

Note:

Total Capacitance in Parallel, C_p

Total capacitance in parallel $C_p = C_1 + C_2 + C_3 + \dots$

More complicated connections of capacitors can sometimes be combinations of series and parallel. (See [\[link\]](#).) To find the total capacitance of such combinations, we identify series and parallel parts, compute their capacitances, and then find the total.



(a) This circuit contains both series and parallel connections of capacitors. See [\[link\]](#) for the calculation of the overall capacitance of the circuit. (b) C_1 and C_2 are in series; their equivalent capacitance C_S is less than either of them. (c) Note that C_S is in parallel with C_3 . The total capacitance is, thus, the sum of C_S and C_3 .

Example:**A Mixture of Series and Parallel Capacitance**

Find the total capacitance of the combination of capacitors shown in [\[link\]](#). Assume the capacitances in [\[link\]](#) are known to three decimal places ($C_1 = 1.000 \mu\text{F}$, $C_2 = 5.000 \mu\text{F}$, and $C_3 = 8.000 \mu\text{F}$), and round your answer to three decimal places.

Strategy

To find the total capacitance, we first identify which capacitors are in series and which are in parallel. Capacitors C_1 and C_2 are in series. Their combination, labeled C_S in the figure, is in parallel with C_3 .

Solution

Since C_1 and C_2 are in series, their total capacitance is given by $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$. Entering their values into the equation gives

Equation:

$$\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} = \frac{1}{1.000 \mu\text{F}} + \frac{1}{5.000 \mu\text{F}} = \frac{1.200}{\mu\text{F}}.$$

Inverting gives

Equation:

$$C_S = 0.833 \mu\text{F}.$$

This equivalent series capacitance is in parallel with the third capacitor; thus, the total is the sum

Equation:

$$\begin{aligned}C_{\text{tot}} &= C_S + C_S \\&= 0.833 \mu\text{F} + 8.000 \mu\text{F} \\&= 8.833 \mu\text{F}.\end{aligned}$$

Discussion

This technique of analyzing the combinations of capacitors piece by piece until a total is obtained can be applied to larger combinations of capacitors.

Section Summary

- Total capacitance in series $\frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \dots$
- Total capacitance in parallel $C_p = C_1 + C_2 + C_3 + \dots$
- If a circuit contains a combination of capacitors in series and parallel, identify series and parallel parts, compute their capacitances, and then find the total.

Conceptual Questions

Exercise:

Problem:

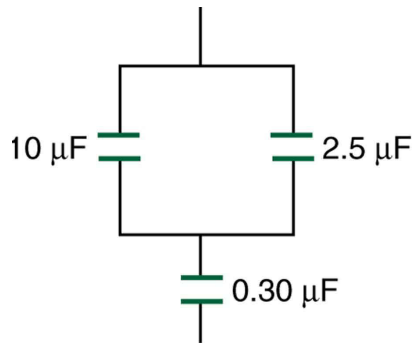
If you wish to store a large amount of energy in a capacitor bank, would you connect capacitors in series or parallel? Explain.

Problems & Exercises

Exercise:

Problem:

Find the total capacitance of the combination of capacitors in [\[link\]](#).



A combination of series and parallel connections of capacitors.

Solution:

$0.293\ \mu\text{F}$

Exercise:

Problem:

Suppose you want a capacitor bank with a total capacitance of $0.750\ \text{F}$ and you possess numerous $1.50\ \text{mF}$ capacitors. What is the smallest number you could hook together to achieve your goal, and how would you connect them?

Exercise:

Problem:

What total capacitances can you make by connecting a $5.00\ \mu\text{F}$ and an $8.00\ \mu\text{F}$ capacitor together?

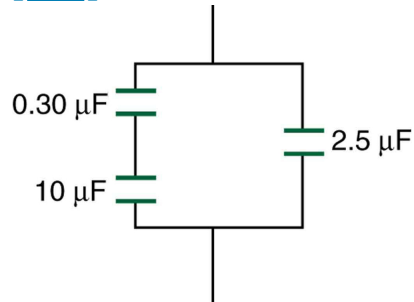
Solution:

$3.08\ \mu\text{F}$ in series combination, $13.0\ \mu\text{F}$ in parallel combination

Exercise:

Problem:

Find the total capacitance of the combination of capacitors shown in [\[link\]](#).



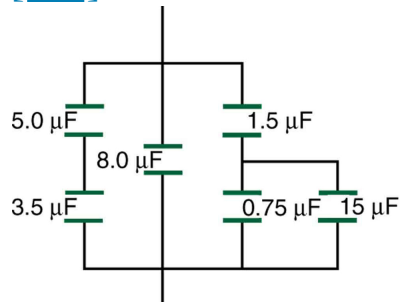
A combination of
series and parallel
connections of
capacitors.

Solution:

$$2.79\ \mu\text{F}$$

Exercise:**Problem:**

Find the total capacitance of the combination of capacitors shown in [\[link\]](#).



A combination of
series and parallel

connections of
capacitors.

Exercise:

Problem: Unreasonable Results

(a) An $8.00\ \mu\text{F}$ capacitor is connected in parallel to another capacitor, producing a total capacitance of $5.00\ \mu\text{F}$. What is the capacitance of the second capacitor? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $-3.00\ \mu\text{F}$

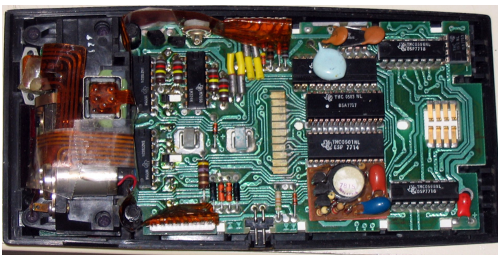
(b) You cannot have a negative value of capacitance.

(c) The assumption that the capacitors were hooked up in parallel, rather than in series, was incorrect. A parallel connection always produces a greater capacitance, while here a smaller capacitance was assumed. This could happen only if the capacitors are connected in series.

Energy Stored in Capacitors

- List some uses of capacitors.
- Express in equation form the energy stored in a capacitor.
- Explain the function of a defibrillator.

Most of us have seen dramatizations in which medical personnel use a **defibrillator** to pass an electric current through a patient's heart to get it to beat normally. (Review [\[link\]](#).) Often realistic in detail, the person applying the shock directs another person to “make it 400 joules this time.” The energy delivered by the defibrillator is stored in a capacitor and can be adjusted to fit the situation. SI units of joules are often employed. Less dramatic is the use of capacitors in microelectronics, such as certain handheld calculators, to supply energy when batteries are charged. (See [\[link\]](#).) Capacitors are also used to supply energy for flash lamps on cameras.



Energy stored in the large capacitor is used to preserve the memory of an electronic calculator when its batteries are charged. (credit: Kucharek, Wikimedia Commons)

Energy stored in a capacitor is electrical potential energy, and it is thus related to the charge Q and voltage V on the capacitor. We must be careful when applying the equation for electrical potential energy $\Delta PE = q\Delta V$ to a capacitor. Remember that ΔPE is the potential energy of a charge q going through a voltage ΔV . But the capacitor starts with zero voltage and gradually comes up to its full voltage as it is charged. The first charge placed on a capacitor experiences a change in voltage $\Delta V = 0$, since the capacitor has zero voltage when uncharged. The final charge placed on a capacitor experiences $\Delta V = V$, since the capacitor now has its full voltage V on it. The average voltage on the capacitor during the charging process is $V/2$, and so the average voltage experienced by the full charge q is $V/2$. Thus the energy stored in a capacitor, E_{cap} , is

Equation:

$$E_{\text{cap}} = \frac{QV}{2},$$

where Q is the charge on a capacitor with a voltage V applied. (Note that the energy is not QV , but $QV/2$.) Charge and voltage are related to the capacitance C of a capacitor by $Q = CV$, and so the expression for E_{cap} can be algebraically manipulated into three equivalent expressions:

Equation:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge and V the voltage on a capacitor C . The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

Note:

Energy Stored in Capacitors

The energy stored in a capacitor can be expressed in three ways:

Equation:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules for a charge in coulombs, voltage in volts, and capacitance in farads.

In a defibrillator, the delivery of a large charge in a short burst to a set of paddles across a person's chest can be a lifesaver. The person's heart attack might have arisen from the onset of fast, irregular beating of the heart—cardiac or ventricular fibrillation. The application of a large shock of electrical energy can terminate the arrhythmia and allow the body's pacemaker to resume normal patterns. Today it is common for ambulances to carry a defibrillator, which also uses an electrocardiogram to analyze the patient's heartbeat pattern. Automated external defibrillators (AED) are found in many public places ([\[link\]](#)). These are designed to be used by lay persons. The device automatically diagnoses the patient's heart condition and then applies the shock with appropriate energy and waveform. CPR is recommended in many cases before use of an AED.



Automated external
defibrillators are found in
many public places.
These portable units
provide verbal
instructions for use in the
important first few
minutes for a person
suffering a cardiac attack.
(credit: Owain Davies,
Wikimedia Commons)

Example:**Capacitance in a Heart Defibrillator**

A heart defibrillator delivers 4.00×10^2 J of energy by discharging a capacitor initially at 1.00×10^4 V. What is its capacitance?

Strategy

We are given E_{cap} and V , and we are asked to find the capacitance C . Of the three expressions in the equation for E_{cap} , the most convenient relationship is

Equation:

$$E_{\text{cap}} = \frac{CV^2}{2}.$$

Solution

Solving this expression for C and entering the given values yields

Equation:

$$\begin{aligned} C &= \frac{2E_{\text{cap}}}{V^2} = \frac{2(4.00 \times 10^2 \text{ J})}{(1.00 \times 10^4 \text{ V})^2} = 8.00 \times 10^{-6} \text{ F} \\ &= 8.00 \text{ } \mu\text{F}. \end{aligned}$$

Discussion

This is a fairly large, but manageable, capacitance at $1.00 \times 10^4 \text{ V}$.

Section Summary

- Capacitors are used in a variety of devices, including defibrillators, microelectronics such as calculators, and flash lamps, to supply energy.
- The energy stored in a capacitor can be expressed in three ways:

Equation:

$$E_{\text{cap}} = \frac{QV}{2} = \frac{CV^2}{2} = \frac{Q^2}{2C},$$

where Q is the charge, V is the voltage, and C is the capacitance of the capacitor. The energy is in joules when the charge is in coulombs, voltage is in volts, and capacitance is in farads.

Conceptual Questions

Exercise:

Problem:

How does the energy contained in a charged capacitor change when a dielectric is inserted, assuming the capacitor is isolated and its charge is constant? Does this imply that work was done?

Exercise:

Problem:

What happens to the energy stored in a capacitor connected to a battery when a dielectric is inserted? Was work done in the process?

Problems & Exercises

Exercise:**Problem:**

(a) What is the energy stored in the $10.0\ \mu\text{F}$ capacitor of a heart defibrillator charged to $9.00 \times 10^3\ \text{V}$? (b) Find the amount of stored charge.

Solution:

(a) 405 J

(b) 90.0 mC

Exercise:**Problem:**

In open heart surgery, a much smaller amount of energy will defibrillate the heart. (a) What voltage is applied to the $8.00\ \mu\text{F}$ capacitor of a heart defibrillator that stores 40.0 J of energy? (b) Find the amount of stored charge.

Solution:

(a) 3.16 kV

(b) 25.3 mC

Exercise:**Problem:**

A $165\ \mu\text{F}$ capacitor is used in conjunction with a motor. How much energy is stored in it when 119 V is applied?

Exercise:

Problem:

Suppose you have a 9.00 V battery, a 2.00 μF capacitor, and a 7.40 μF capacitor. (a) Find the charge and energy stored if the capacitors are connected to the battery in series. (b) Do the same for a parallel connection.

Solution:

(a) $1.42 \times 10^{-5} \text{ C}$, $6.38 \times 10^{-5} \text{ J}$

(b) $8.46 \times 10^{-5} \text{ C}$, $3.81 \times 10^{-4} \text{ J}$

Exercise:**Problem:**

A nervous physicist worries that the two metal shelves of his wood frame bookcase might obtain a high voltage if charged by static electricity, perhaps produced by friction. (a) What is the capacitance of the empty shelves if they have area $1.00 \times 10^2 \text{ m}^2$ and are 0.200 m apart? (b) What is the voltage between them if opposite charges of magnitude 2.00 nC are placed on them? (c) To show that this voltage poses a small hazard, calculate the energy stored.

Solution:

(a) $4.43 \times 10^{-12} \text{ F}$

(b) 452 V

(c) $4.52 \times 10^{-7} \text{ J}$

Exercise:

Problem:

Show that for a given dielectric material the maximum energy a parallel plate capacitor can store is directly proportional to the volume of dielectric (Volume = $A \cdot d$). Note that the applied voltage is limited by the dielectric strength.

Exercise:**Problem: Construct Your Own Problem**

Consider a heart defibrillator similar to that discussed in [\[link\]](#). Construct a problem in which you examine the charge stored in the capacitor of a defibrillator as a function of stored energy. Among the things to be considered are the applied voltage and whether it should vary with energy to be delivered, the range of energies involved, and the capacitance of the defibrillator. You may also wish to consider the much smaller energy needed for defibrillation during open-heart surgery as a variation on this problem.

Exercise:**Problem: Unreasonable Results**

(a) On a particular day, it takes 9.60×10^3 J of electric energy to start a truck's engine. Calculate the capacitance of a capacitor that could store that amount of energy at 12.0 V. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Solution:

(a) 133 F

(b) Such a capacitor would be too large to carry with a truck. The size of the capacitor would be enormous.

(c) It is unreasonable to assume that a capacitor can store the amount of energy needed.

Glossary

defibrillator

a machine used to provide an electrical shock to a heart attack victim's heart in order to restore the heart's normal rhythmic pattern

Introduction to Electric Current, Resistance, and Ohm's Law

class="introduction"

Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailem power station located along the Krishna River in India, by the movement of charge—that is, by electric current.
(credit: Chintohere, Wikimedia Commons)



The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling its load over a mountain pass, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve *electric current, the movement of charge*. Humankind has indeed harnessed electricity, the basis of technology, to improve our quality of life. Whereas the previous two chapters concentrated on static electricity and the fundamental force underlying its behavior, the next few chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into nature—in particular, the fact that all magnetism results from electric current.

Current

- Define electric current, ampere, and drift velocity
- Describe the direction of charge flow in conventional current.
- Use drift velocity to calculate current and vice versa.

Electric Current

Electric current is defined to be the rate at which charge flows. A large current, such as that used to start a truck engine, moves a large amount of charge in a small time, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge over a long period of time. In equation form, **electric current** I is defined to be

Equation:

$$I = \frac{\Delta Q}{\Delta t},$$

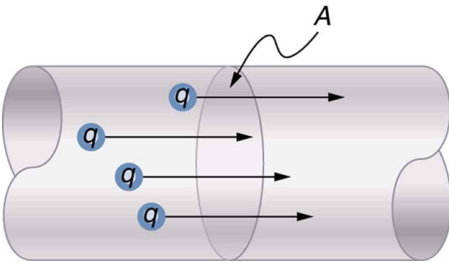
where ΔQ is the amount of charge passing through a given area in time Δt . (As in previous chapters, initial time is often taken to be zero, in which case $\Delta t = t$.) (See [\[link\]](#).) The SI unit for current is the **ampere** (A), named for the French physicist André-Marie Ampère (1775–1836). Since $I = \Delta Q / \Delta t$, we see that an ampere is one coulomb per second:

Equation:

$$1 \text{ A} = 1 \text{ C/s}$$

Not only are fuses and circuit breakers rated in amperes (or amps), so are many electrical appliances.

Current = flow of charge



The rate of flow of charge is current. An ampere is the flow of one coulomb through an area in one second.

Example:

Calculating Currents: Current in a Truck Battery and a Handheld Calculator

(a) What is the current involved when a truck battery sets in motion 720 C of charge in 4.00 s while starting an engine? (b) How long does it take 1.00 C of charge to flow through a handheld calculator if a 0.300-mA current is flowing?

Strategy

We can use the definition of current in the equation $I = \Delta Q / \Delta t$ to find the current in part (a), since charge and time are given. In part (b), we rearrange the definition of current and use the given values of charge and current to find the time required.

Solution for (a)

Entering the given values for charge and time into the definition of current gives

Equation:

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} = \frac{720 \text{ C}}{4.00 \text{ s}} = 180 \text{ C/s} \\ &= 180 \text{ A.} \end{aligned}$$

Discussion for (a)

This large value for current illustrates the fact that a large charge is moved in a small amount of time. The currents in these “starter motors” are fairly large because large frictional forces need to be overcome when setting something in motion.

Solution for (b)

Solving the relationship $I = \Delta Q / \Delta t$ for time Δt , and entering the known values for charge and current gives

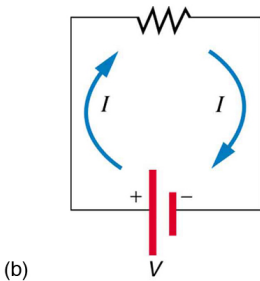
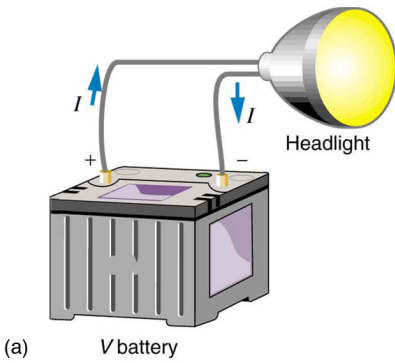
Equation:

$$\begin{aligned}\Delta t &= \frac{\Delta Q}{I} = \frac{1.00 \text{ C}}{0.300 \times 10^{-3} \text{ C/s}} \\ &= 3.33 \times 10^3 \text{ s.}\end{aligned}$$

Discussion for (b)

This time is slightly less than an hour. The small current used by the handheld calculator takes a much longer time to move a smaller charge than the large current of the truck starter. So why can we operate our calculators only seconds after turning them on? It's because calculators require very little energy. Such small current and energy demands allow handheld calculators to operate from solar cells or to get many hours of use out of small batteries. Remember, calculators do not have moving parts in the same way that a truck engine has with cylinders and pistons, so the technology requires smaller currents.

[\[link\]](#) shows a simple circuit and the standard schematic representation of a battery, conducting path, and load (a resistor). Schematics are very useful in visualizing the main features of a circuit. A single schematic can represent a wide variety of situations. The schematic in [\[link\]](#) (b), for example, can represent anything from a truck battery connected to a headlight lighting the street in front of the truck to a small battery connected to a penlight lighting a keyhole in a door. Such schematics are useful because the analysis is the same for a wide variety of situations. We need to understand a few schematics to apply the concepts and analysis to many more situations.



(a) A simple electric circuit. A closed path for current to flow through is supplied by conducting wires connecting a load to the terminals of a battery. (b) In this schematic, the battery is represented by the two parallel red lines, conducting wires are shown as straight lines, and the zigzag represents the load. The schematic represents a wide

variety of similar
circuits.

Note that the direction of current flow in [\[link\]](#) is from positive to negative. *The direction of conventional current is the direction that positive charge would flow.* Depending on the situation, positive charges, negative charges, or both may move. In metal wires, for example, current is carried by electrons—that is, negative charges move. In ionic solutions, such as salt water, both positive and negative charges move. This is also true in nerve cells. A Van de Graaff generator used for nuclear research can produce a current of pure positive charges, such as protons. [\[link\]](#) illustrates the movement of charged particles that compose a current. The fact that conventional current is taken to be in the direction that positive charge would flow can be traced back to American politician and scientist Benjamin Franklin in the 1700s. He named the type of charge associated with electrons negative, long before they were known to carry current in so many situations. Franklin, in fact, was totally unaware of the small-scale structure of electricity.

It is important to realize that there is an electric field in conductors responsible for producing the current, as illustrated in [\[link\]](#). Unlike static electricity, where a conductor in equilibrium cannot have an electric field in it, conductors carrying a current have an electric field and are not in static equilibrium. An electric field is needed to supply energy to move the charges.

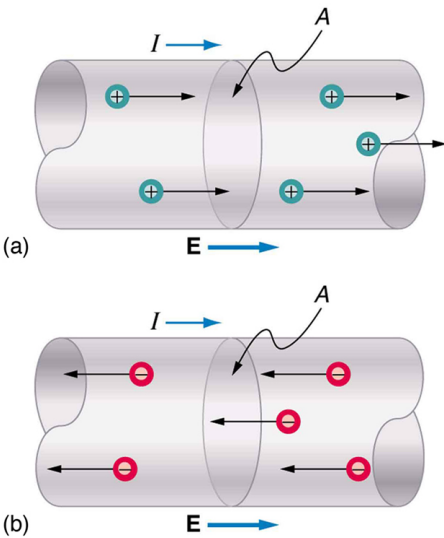
Note:

Making Connections: Take-Home Investigation—Electric Current Illustration

Find a straw and little peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you pop one pea in at one end, a different pea should pop out the other end. This demonstration is an analogy for an electric current. Identify what compares

to the electrons and what compares to the supply of energy. What other analogies can you find for an electric current?

Note that the flow of peas is based on the peas physically bumping into each other; electrons flow due to mutually repulsive electrostatic forces.



Current I is the rate at which charge moves through an area A , such as the cross-section of a wire.

Conventional current is defined to move in the direction of the electric field. (a)

Positive charges move in the direction of the electric field and the same direction as conventional current.

(b) Negative charges move in the direction opposite to the electric field. Conventional

current is in the direction opposite to the movement of negative charge. The flow of electrons is sometimes referred to as electronic flow.

Example:**Calculating the Number of Electrons that Move through a Calculator**

If the 0.300-mA current through the calculator mentioned in the [\[link\]](#) example is carried by electrons, how many electrons per second pass through it?

Strategy

The current calculated in the previous example was defined for the flow of positive charge. For electrons, the magnitude is the same, but the sign is opposite, $I_{\text{electrons}} = -0.300 \times 10^{-3} \text{ C/s}$. Since each electron (e^-) has a charge of $-1.60 \times 10^{-19} \text{ C}$, we can convert the current in coulombs per second to electrons per second.

Solution

Starting with the definition of current, we have

Equation:

$$I_{\text{electrons}} = \frac{\Delta Q_{\text{electrons}}}{\Delta t} = \frac{-0.300 \times 10^{-3} \text{ C}}{\text{s}}.$$

We divide this by the charge per electron, so that

Equation:

$$\begin{aligned} \frac{e^-}{\text{s}} &= \frac{-0.300 \times 10^{-3} \text{ C}}{\text{s}} \times \frac{1 e^-}{-1.60 \times 10^{-19} \text{ C}} \\ &= 1.88 \times 10^{15} \frac{e^-}{\text{s}}. \end{aligned}$$

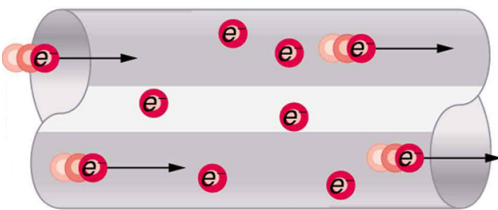
Discussion

There are so many charged particles moving, even in small currents, that individual charges are not noticed, just as individual water molecules are not noticed in water flow. Even more amazing is that they do not always keep moving forward like soldiers in a parade. Rather they are like a crowd of people with movement in different directions but a general trend to move forward. There are lots of collisions with atoms in the metal wire and, of course, with other electrons.

Drift Velocity

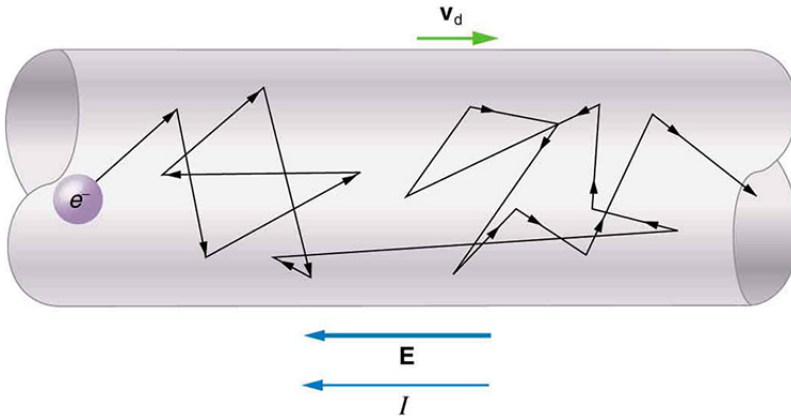
Electrical signals are known to move very rapidly. Telephone conversations carried by currents in wires cover large distances without noticeable delays. Lights come on as soon as a switch is flicked. Most electrical signals carried by currents travel at speeds on the order of 10^8 m/s, a significant fraction of the speed of light. Interestingly, the individual charges that make up the current move *much* more slowly on average, typically drifting at speeds on the order of 10^{-4} m/s. How do we reconcile these two speeds, and what does it tell us about standard conductors?

The high speed of electrical signals results from the fact that the force between charges acts rapidly at a distance. Thus, when a free charge is forced into a wire, as in [\[link\]](#), the incoming charge pushes other charges ahead of it, which in turn push on charges farther down the line. The density of charge in a system cannot easily be increased, and so the signal is passed on rapidly. The resulting electrical shock wave moves through the system at nearly the speed of light. To be precise, this rapidly moving signal or shock wave is a rapidly propagating change in electric field.



When charged particles are forced into this volume of a conductor, an equal number are quickly forced to leave. The repulsion between like charges makes it difficult to increase the number of charges in a volume. Thus, as one charge enters, another leaves almost immediately, carrying the signal rapidly forward.

Good conductors have large numbers of free charges in them. In metals, the free charges are free electrons. [\[link\]](#) shows how free electrons move through an ordinary conductor. The distance that an individual electron can move between collisions with atoms or other electrons is quite small. The electron paths thus appear nearly random, like the motion of atoms in a gas. But there is an electric field in the conductor that causes the electrons to drift in the direction shown (opposite to the field, since they are negative). The **drift velocity** v_d is the average velocity of the free charges. Drift velocity is quite small, since there are so many free charges. If we have an estimate of the density of free electrons in a conductor, we can calculate the drift velocity for a given current. The larger the density, the lower the velocity required for a given current.



Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of the free charges is called the drift velocity, v_d , and it is in the direction opposite to the electric field for electrons. The collisions normally transfer energy to the conductor, requiring a constant supply of energy to maintain a steady current.

Note:

Conduction of Electricity and Heat

Good electrical conductors are often good heat conductors, too. This is because large numbers of free electrons can carry electrical current and can transport thermal energy.

The free-electron collisions transfer energy to the atoms of the conductor. The electric field does work in moving the electrons through a distance, but that work does not increase the kinetic energy (nor speed, therefore) of the electrons. The work is transferred to the conductor's atoms, possibly

increasing temperature. Thus a continuous power input is required to keep a current flowing. An exception, of course, is found in superconductors, for reasons we shall explore in a later chapter. Superconductors can have a steady current without a continual supply of energy—a great energy savings. In contrast, the supply of energy can be useful, such as in a lightbulb filament. The supply of energy is necessary to increase the temperature of the tungsten filament, so that the filament glows.

Note:

Making Connections: Take-Home Investigation—Filament Observations

Find a lightbulb with a filament. Look carefully at the filament and describe its structure. To what points is the filament connected?

We can obtain an expression for the relationship between current and drift velocity by considering the number of free charges in a segment of wire, as illustrated in [\[link\]](#). The number of free charges per unit volume is given the symbol n and depends on the material. The shaded segment has a volume Ax , so that the number of free charges in it is nAx . The charge ΔQ in this segment is thus $qnAx$, where q is the amount of charge on each carrier. (Recall that for electrons, q is -1.60×10^{-19} C.) Current is charge moved per unit time; thus, if all the original charges move out of this segment in time Δt , the current is

Equation:

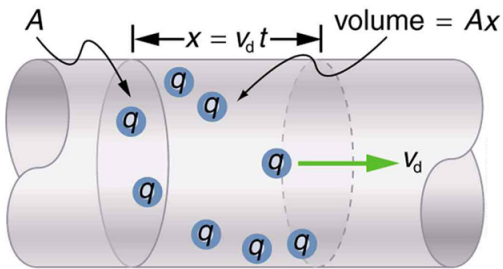
$$I = \frac{\Delta Q}{\Delta t} = \frac{qnAx}{\Delta t}.$$

Note that $x/\Delta t$ is the magnitude of the drift velocity, v_d , since the charges move an average distance x in a time Δt . Rearranging terms gives

Equation:

$$I = nqAv_d,$$

where I is the current through a wire of cross-sectional area A made of a material with a free charge density n . The carriers of the current each have charge q and move with a drift velocity of magnitude v_d .



All the charges in the shaded volume of this wire move out in a time t , having a drift velocity of magnitude $v_d = x/t$. See text for further discussion.

Note that simple drift velocity is not the entire story. The speed of an electron is much greater than its drift velocity. In addition, not all of the electrons in a conductor can move freely, and those that do might move somewhat faster or slower than the drift velocity. So what do we mean by free electrons? Atoms in a metallic conductor are packed in the form of a lattice structure. Some electrons are far enough away from the atomic nuclei that they do not experience the attraction of the nuclei as much as the inner electrons do. These are the free electrons. They are not bound to a single atom but can instead move freely among the atoms in a “sea” of electrons. These free electrons respond by accelerating when an electric field is applied. Of course as they move they collide with the atoms in the lattice and other electrons, generating thermal energy, and the conductor gets warmer. In an insulator, the organization of the atoms and the structure do not allow for such free electrons.

Example:**Calculating Drift Velocity in a Common Wire**

Calculate the drift velocity of electrons in a 12-gauge copper wire (which has a diameter of 2.053 mm) carrying a 20.0-A current, given that there is one free electron per copper atom. (Household wiring often contains 12-gauge copper wire, and the maximum current allowed in such wire is usually 20 A.) The density of copper is $8.80 \times 10^3 \text{ kg/m}^3$.

Strategy

We can calculate the drift velocity using the equation $I = nqAv_d$. The current $I = 20.0 \text{ A}$ is given, and $q = -1.60 \times 10^{-19} \text{ C}$ is the charge of an electron. We can calculate the area of a cross-section of the wire using the formula $A = \pi r^2$, where r is one-half the given diameter, 2.053 mm. We are given the density of copper, $8.80 \times 10^3 \text{ kg/m}^3$, and the periodic table shows that the atomic mass of copper is 63.54 g/mol. We can use these two quantities along with Avogadro's number, $6.02 \times 10^{23} \text{ atoms/mol}$, to determine n , the number of free electrons per cubic meter.

Solution

First, calculate the density of free electrons in copper. There is one free electron per copper atom. Therefore, is the same as the number of copper atoms per m^3 . We can now find n as follows:

Equation:

$$\begin{aligned} n &= \frac{1 e^-}{\text{atom}} \times \frac{6.02 \times 10^{23} \text{ atoms}}{\text{mol}} \times \frac{1 \text{ mol}}{63.54 \text{ g}} \times \frac{1000 \text{ g}}{\text{kg}} \times \frac{8.80 \times 10^3 \text{ kg}}{1 \text{ m}^3} \\ &= 8.342 \times 10^{28} e^-/\text{m}^3. \end{aligned}$$

The cross-sectional area of the wire is

Equation:

$$\begin{aligned} A &= \pi r^2 \\ &= \pi \left(\frac{2.053 \times 10^{-3} \text{ m}}{2} \right)^2 \\ &= 3.310 \times 10^{-6} \text{ m}^2. \end{aligned}$$

Rearranging $I = nqAv_d$ to isolate drift velocity gives

Equation:

$$\begin{aligned}
 v_d &= \frac{I}{nqA} \\
 &= \frac{20.0 \text{ A}}{(8.342 \times 10^{28} / \text{m}^3)(-1.60 \times 10^{-19} \text{ C})(3.310 \times 10^{-6} \text{ m}^2)} \\
 &= -4.53 \times 10^{-4} \text{ m/s}.
 \end{aligned}$$

Discussion

The minus sign indicates that the negative charges are moving in the direction opposite to conventional current. The small value for drift velocity (on the order of 10^{-4} m/s) confirms that the signal moves on the order of 10^{12} times faster (about 10^8 m/s) than the charges that carry it.

Section Summary

- Electric current I is the rate at which charge flows, given by

Equation:

$$I = \frac{\Delta Q}{\Delta t},$$

where ΔQ is the amount of charge passing through an area in time Δt .

- The direction of conventional current is taken as the direction in which positive charge moves.
- The SI unit for current is the ampere (A), where $1 \text{ A} = 1 \text{ C/s}$.
- Current is the flow of free charges, such as electrons and ions.
- Drift velocity v_d is the average speed at which these charges move.
- Current I is proportional to drift velocity v_d , as expressed in the relationship $I = nqAv_d$. Here, I is the current through a wire of cross-sectional area A . The wire's material has a free-charge density n , and each carrier has charge q and a drift velocity v_d .
- Electrical signals travel at speeds about 10^{12} times greater than the drift velocity of free electrons.

Conceptual Questions

Exercise:

Problem:

Can a wire carry a current and still be neutral—that is, have a total charge of zero? Explain.

Exercise:**Problem:**

Car batteries are rated in ampere-hours ($A \cdot h$). To what physical quantity do ampere-hours correspond (voltage, charge, . . .), and what relationship do ampere-hours have to energy content?

Exercise:**Problem:**

If two different wires having identical cross-sectional areas carry the same current, will the drift velocity be higher or lower in the better conductor? Explain in terms of the equation $v_d = \frac{I}{nqA}$, by considering how the density of charge carriers n relates to whether or not a material is a good conductor.

Exercise:**Problem:**

Why are two conducting paths from a voltage source to an electrical device needed to operate the device?

Exercise:**Problem:**

In cars, one battery terminal is connected to the metal body. How does this allow a single wire to supply current to electrical devices rather than two wires?

Exercise:

Problem:

Why isn't a bird sitting on a high-voltage power line electrocuted? Contrast this with the situation in which a large bird hits two wires simultaneously with its wings.

Problems & Exercises**Exercise:****Problem:**

What is the current in milliamperes produced by the solar cells of a pocket calculator through which 4.00 C of charge passes in 4.00 h?

Solution:

0.278 mA

Exercise:**Problem:**

A total of 600 C of charge passes through a flashlight in 0.500 h. What is the average current?

Exercise:**Problem:**

What is the current when a typical static charge of $0.250\ \mu\text{C}$ moves from your finger to a metal doorknob in $1.00\ \mu\text{s}$?

Solution:

0.250 A

Exercise:

Problem:

Find the current when 2.00 nC jumps between your comb and hair over a 0.500 - μ s time interval.

Exercise:**Problem:**

A large lightning bolt had a 20,000-A current and moved 30.0 C of charge. What was its duration?

Solution:

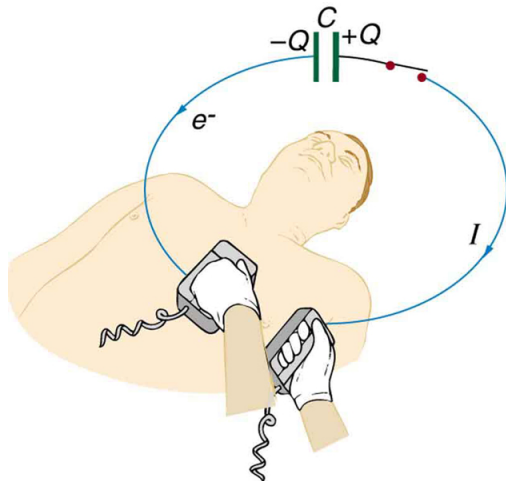
1.50ms

Exercise:**Problem:**

The 200-A current through a spark plug moves 0.300 mC of charge. How long does the spark last?

Exercise:**Problem:**

(a) A defibrillator sends a 6.00-A current through the chest of a patient by applying a 10,000-V potential as in the figure below. What is the resistance of the path? (b) The defibrillator paddles make contact with the patient through a conducting gel that greatly reduces the path resistance. Discuss the difficulties that would ensue if a larger voltage were used to produce the same current through the patient, but with the path having perhaps 50 times the resistance. (Hint: The current must be about the same, so a higher voltage would imply greater power. Use this equation for power: $P = I^2 R$.)



The capacitor in a defibrillation unit drives a current through the heart of a patient.

Solution:

(a) $1.67\text{k}\Omega$

(b) If a 50 times larger resistance existed, keeping the current about the same, the power would be increased by a factor of about 50 (based on the equation $P = I^2 R$), causing much more energy to be transferred to the skin, which could cause serious burns. The gel used reduces the resistance, and therefore reduces the power transferred to the skin.

Exercise:

Problem:

During open-heart surgery, a defibrillator can be used to bring a patient out of cardiac arrest. The resistance of the path is $500\ \Omega$ and a 10.0-mA current is needed. What voltage should be applied?

Exercise:

Problem:

(a) A defibrillator passes 12.0 A of current through the torso of a person for 0.0100 s. How much charge moves? (b) How many electrons pass through the wires connected to the patient? (See figure two problems earlier.)

Solution:

(a) 0.120 C

(b) 7.50×10^{17} electrons

Exercise:**Problem:**

A clock battery wears out after moving 10,000 C of charge through the clock at a rate of 0.500 mA. (a) How long did the clock run? (b) How many electrons per second flowed?

Exercise:**Problem:**

The batteries of a submerged non-nuclear submarine supply 1000 A at full speed ahead. How long does it take to move Avogadro's number (6.02×10^{23}) of electrons at this rate?

Solution:

96.3 s

Exercise:

Problem:

Electron guns are used in X-ray tubes. The electrons are accelerated through a relatively large voltage and directed onto a metal target, producing X-rays. (a) How many electrons per second strike the target if the current is 0.500 mA? (b) What charge strikes the target in 0.750 s?

Exercise:**Problem:**

A large cyclotron directs a beam of He^{++} nuclei onto a target with a beam current of 0.250 mA. (a) How many He^{++} nuclei per second is this? (b) How long does it take for 1.00 C to strike the target? (c) How long before 1.00 mol of He^{++} nuclei strike the target?

Solution:

(a) $7.81 \times 10^{14} \text{ He}^{++} \text{ nuclei/s}$

(b) $4.00 \times 10^3 \text{ s}$

(c) $7.71 \times 10^8 \text{ s}$

Exercise:**Problem:**

Repeat the above example on [\[link\]](#), but for a wire made of silver and given there is one free electron per silver atom.

Exercise:**Problem:**

Using the results of the above example on [\[link\]](#), find the drift velocity in a copper wire of twice the diameter and carrying 20.0 A.

Solution:

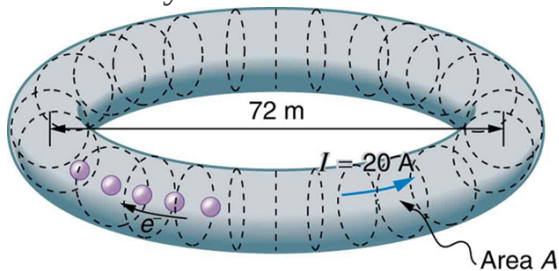
$$-1.13 \times 10^{-4} \text{ m/s}$$

Exercise:**Problem:**

A 14-gauge copper wire has a diameter of 1.628 mm. What magnitude current flows when the drift velocity is 1.00 mm/s? (See above example on [\[link\]](#) for useful information.)

Exercise:**Problem:**

SPEAR, a storage ring about 72.0 m in diameter at the Stanford Linear Accelerator (closed in 2009), has a 20.0-A circulating beam of electrons that are moving at nearly the speed of light. (See [\[link\]](#).) How many electrons are in the beam?



Electrons circulating in the storage ring called SPEAR constitute a 20.0-A current. Because they travel close to the speed of light, each electron completes many orbits in each second.

Solution:

$$9.42 \times 10^{13} \text{ electrons}$$

Glossary

electric current

the rate at which charge flows, $I = \Delta Q / \Delta t$

ampere

(amp) the SI unit for current; $1 \text{ A} = 1 \text{ C/s}$

drift velocity

the average velocity at which free charges flow in response to an electric field

Ohm's Law: Resistance and Simple Circuits

- Explain the origin of Ohm's law.
- Calculate voltages, currents, or resistances with Ohm's law.
- Explain what an ohmic material is.
- Describe a simple circuit.

What drives current? We can think of various devices—such as batteries, generators, wall outlets, and so on—which are necessary to maintain a current. All such devices create a potential difference and are loosely referred to as voltage sources. When a voltage source is connected to a conductor, it applies a potential difference V that creates an electric field. The electric field in turn exerts force on charges, causing current.

Ohm's Law

The current that flows through most substances is directly proportional to the voltage V applied to it. The German physicist Georg Simon Ohm (1787–1854) was the first to demonstrate experimentally that the current in a metal wire is *directly proportional to the voltage applied*:

Equation:

$$I \propto V.$$

This important relationship is known as **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage the cause and current the effect. This is an empirical law like that for friction—an experimentally observed phenomenon. Such a linear relationship doesn't always occur.

Resistance and Simple Circuits

If voltage drives current, what impedes it? The electric property that impedes current (crudely similar to friction and air resistance) is called **resistance** R . Collisions of moving charges with atoms and molecules in a substance transfer energy to the substance and limit current. Resistance is defined as inversely proportional to current, or

Equation:

$$I \propto \frac{1}{R}.$$

Thus, for example, current is cut in half if resistance doubles. Combining the relationships of current to voltage and current to resistance gives

Equation:

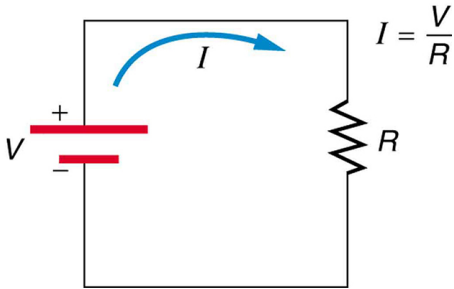
$$I = \frac{V}{R}.$$

This relationship is also called Ohm's law. Ohm's law in this form really defines resistance for certain materials. Ohm's law (like Hooke's law) is not universally valid. The many substances for which Ohm's law holds are called **ohmic**. These include good conductors like copper and aluminum, and some poor conductors under certain circumstances. Ohmic materials have a resistance R that is independent of voltage V and current I . An object that has simple resistance is called a *resistor*, even if its resistance is small. The unit for resistance is an **ohm** and is given the symbol Ω (upper case Greek omega). Rearranging $I = V/R$ gives $R = V/I$, and so the units of resistance are 1 ohm = 1 volt per ampere:

Equation:

$$1 \Omega = 1 \frac{V}{A}.$$

[\[link\]](#) shows the schematic for a simple circuit. A **simple circuit** has a single voltage source and a single resistor. The wires connecting the voltage source to the resistor can be assumed to have negligible resistance, or their resistance can be included in R .



A simple electric circuit in which a closed path for current to flow is supplied by conductors (usually metal wires) connecting a load to the terminals of a battery, represented by the red parallel lines.

The zigzag symbol represents the single resistor and includes any resistance in the connections to the voltage source.

Example:**Calculating Resistance: An Automobile Headlight**

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?

Strategy

We can rearrange Ohm's law as stated by $I = V/R$ and use it to find the resistance.

Solution

Rearranging $I = V/R$ and substituting known values gives

Equation:

$$R = \frac{V}{I} = \frac{12.0 \text{ V}}{2.50 \text{ A}} = 4.80 \Omega.$$

Discussion

This is a relatively small resistance, but it is larger than the cold resistance of the headlight. As we shall see in [Resistance and Resistivity](#), resistance usually increases with temperature, and so the bulb has a lower resistance when it is first switched on and will draw considerably more current during its brief warm-up period.

Resistances range over many orders of magnitude. Some ceramic insulators, such as those used to support power lines, have resistances of $10^{12} \Omega$ or more. A dry person may have a hand-to-foot resistance of $10^5 \Omega$, whereas the resistance of the human heart is about $10^3 \Omega$. A meter-long piece of large-diameter copper wire may have a resistance of $10^{-5} \Omega$, and superconductors have no resistance at all (they are non-ohmic). Resistance is related to the shape of an object and the material of which it is composed, as will be seen in [Resistance and Resistivity](#).

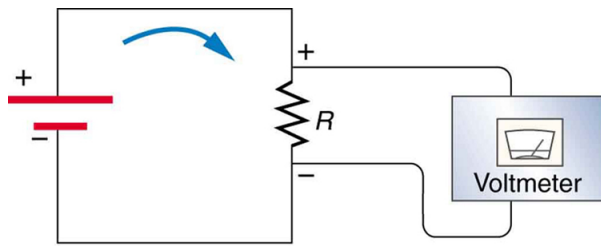
Additional insight is gained by solving $I = V/R$ for V , yielding

Equation:

$$V = IR.$$

This expression for V can be interpreted as the *voltage drop across a resistor produced by the flow of current I* . The phrase *IR drop* is often used for this voltage. For instance, the headlight in [\[link\]](#) has an IR drop of 12.0 V. If voltage is measured at various points in a circuit, it will be seen to increase at the voltage source and decrease at the resistor. Voltage is similar to fluid pressure. The voltage source is like a pump, creating a pressure difference, causing current—the flow of charge. The resistor is like a pipe that reduces pressure and limits flow because of its resistance. Conservation of energy has important consequences here. The voltage source supplies

energy (causing an electric field and a current), and the resistor converts it to another form (such as thermal energy). In a simple circuit (one with a single simple resistor), the voltage supplied by the source equals the voltage drop across the resistor, since $PE = q\Delta V$, and the same q flows through each. Thus the energy supplied by the voltage source and the energy converted by the resistor are equal. (See [\[link\]](#).)



$$V = IR = 18 \text{ V}$$

The voltage drop across a resistor in a simple circuit equals the voltage output of the battery.

Note:**Making Connections: Conservation of Energy**

In a simple electrical circuit, the sole resistor converts energy supplied by the source into another form. Conservation of energy is evidenced here by the fact that all of the energy supplied by the source is converted to another form by the resistor alone. We will find that conservation of energy has other important applications in circuits and is a powerful tool in circuit analysis.

Note:

PhET Explorations: Ohm's Law

See how the equation form of Ohm's law relates to a simple circuit. Adjust the voltage and resistance, and see the current change according to Ohm's law. The sizes of the symbols in the equation change to match the circuit diagram.

https://phet.colorado.edu/sims/html/ohms-law/latest/ohms-law_en.html

Section Summary

- A simple circuit *is* one in which there is a single voltage source and a single resistance.
- One statement of Ohm's law gives the relationship between current I , voltage V , and resistance R in a simple circuit to be $I = \frac{V}{R}$.
- Resistance has units of ohms (Ω), related to volts and amperes by $1 \Omega = 1 \text{ V/A}$.
- There is a voltage or IR drop across a resistor, caused by the current flowing through it, given by $V = IR$.

Conceptual Questions

Exercise:

Problem:

The IR drop across a resistor means that there is a change in potential or voltage across the resistor. Is there any change in current as it passes through a resistor? Explain.

Exercise:

Problem:

How is the IR drop in a resistor similar to the pressure drop in a fluid flowing through a pipe?

Problems & Exercises

Exercise:**Problem:**

What current flows through the bulb of a 3.00-V flashlight when its hot resistance is $3.60\ \Omega$?

Solution:

0.833 A

Exercise:**Problem:**

Calculate the effective resistance of a pocket calculator that has a 1.35-V battery and through which 0.200 mA flows.

Exercise:**Problem:**

What is the effective resistance of a car's starter motor when 150 A flows through it as the car battery applies 11.0 V to the motor?

Solution:

$7.33 \times 10^{-2}\ \Omega$

Exercise:**Problem:**

How many volts are supplied to operate an indicator light on a DVD player that has a resistance of $140\ \Omega$, given that 25.0 mA passes through it?

Exercise:

Problem:

(a) Find the voltage drop in an extension cord having a $0.0600\text{-}\Omega$ resistance and through which 5.00 A is flowing. (b) A cheaper cord utilizes thinner wire and has a resistance of $0.300\text{ }\Omega$. What is the voltage drop in it when 5.00 A flows? (c) Why is the voltage to whatever appliance is being used reduced by this amount? What is the effect on the appliance?

Solution:

(a) 0.300 V

(b) 1.50 V

(c) The voltage supplied to whatever appliance is being used is reduced because the total voltage drop from the wall to the final output of the appliance is fixed. Thus, if the voltage drop across the extension cord is large, the voltage drop across the appliance is significantly decreased, so the power output by the appliance can be significantly decreased, reducing the ability of the appliance to work properly.

Exercise:**Problem:**

A power transmission line is hung from metal towers with glass insulators having a resistance of $1.00 \times 10^9\text{ }\Omega$. What current flows through the insulator if the voltage is 200 kV ? (Some high-voltage lines are DC.)

Glossary**Ohm's law**

an empirical relation stating that the current I is proportional to the potential difference V , $\propto V$; it is often written as $I = V/R$, where R is the resistance

resistance

the electric property that impedes current; for ohmic materials, it is the ratio of voltage to current, $R = V/I$

ohm

the unit of resistance, given by $1\Omega = 1 \text{ V/A}$

ohmic

a type of a material for which Ohm's law is valid

simple circuit

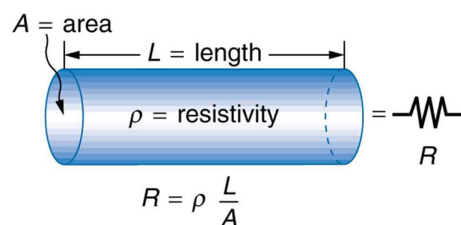
a circuit with a single voltage source and a single resistor

Resistance and Resistivity

- Explain the concept of resistivity.
- Use resistivity to calculate the resistance of specified configurations of material.
- Use the thermal coefficient of resistivity to calculate the change of resistance with temperature.

Material and Shape Dependence of Resistance

The resistance of an object depends on its shape and the material of which it is composed. The cylindrical resistor in [\[link\]](#) is easy to analyze, and, by so doing, we can gain insight into the resistance of more complicated shapes. As you might expect, the cylinder's electric resistance R is directly proportional to its length L , similar to the resistance of a pipe to fluid flow. The longer the cylinder, the more collisions charges will make with its atoms. The greater the diameter of the cylinder, the more current it can carry (again similar to the flow of fluid through a pipe). In fact, R is inversely proportional to the cylinder's cross-sectional area A .



A uniform cylinder of length L and cross-sectional area A . Its resistance to the flow of current is similar to the resistance posed by a pipe to fluid flow.

The longer the cylinder, the greater its

resistance. The larger its cross-sectional area A , the smaller its resistance.

For a given shape, the resistance depends on the material of which the object is composed. Different materials offer different resistance to the flow of charge. We define the **resistivity** ρ of a substance so that the **resistance** R of an object is directly proportional to ρ . Resistivity ρ is an *intrinsic* property of a material, independent of its shape or size. The resistance R of a uniform cylinder of length L , of cross-sectional area A , and made of a material with resistivity ρ , is

Equation:

$$R = \frac{\rho L}{A}.$$

[\[link\]](#) gives representative values of ρ . The materials listed in the table are separated into categories of conductors, semiconductors, and insulators, based on broad groupings of resistivities. Conductors have the smallest resistivities, and insulators have the largest; semiconductors have intermediate resistivities. Conductors have varying but large free charge densities, whereas most charges in insulators are bound to atoms and are not free to move. Semiconductors are intermediate, having far fewer free charges than conductors, but having properties that make the number of free charges depend strongly on the type and amount of impurities in the semiconductor. These unique properties of semiconductors are put to use in modern electronics, as will be explored in later chapters.

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
<i>Conductors</i>	
Silver	1.59×10^{-8}
Copper	1.72×10^{-8}
Gold	2.44×10^{-8}
Aluminum	2.65×10^{-8}
Tungsten	5.6×10^{-8}
Iron	9.71×10^{-8}
Platinum	10.6×10^{-8}
Steel	20×10^{-8}
Lead	22×10^{-8}

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Manganin (Cu, Mn, Ni alloy)	44×10^{-8}
Constantan (Cu, Ni alloy)	49×10^{-8}
Mercury	96×10^{-8}
Nichrome (Ni, Fe, Cr alloy)	100×10^{-8}
<i>Semiconductors</i> [footnote] Values depend strongly on amounts and types of impurities	
Carbon (pure)	3.5×10^{-5}
Carbon	$(3.5 - 60) \times 10^{-5}$
Germanium (pure)	600×10^{-3}
Germanium	$(1 - 600) \times 10^{-3}$

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Silicon (pure)	2300
Silicon	0.1–2300
<i>Insulators</i>	
Amber	5×10^{14}
Glass	$10^9 - 10^{14}$
Lucite	$>10^{13}$
Mica	$10^{11} - 10^{15}$
Quartz (fused)	75×10^{16}
Rubber (hard)	$10^{13} - 10^{16}$
Sulfur	10^{15}

Material	Resistivity ρ ($\Omega \cdot \text{m}$)
Teflon	$>10^{13}$
Wood	$10^8 - 10^{11}$

Resistivities ρ of Various materials at 20°C

Example:

Calculating Resistor Diameter: A Headlight Filament

A car headlight filament is made of tungsten and has a cold resistance of 0.350Ω . If the filament is a cylinder 4.00 cm long (it may be coiled to save space), what is its diameter?

Strategy

We can rearrange the equation $R = \frac{\rho L}{A}$ to find the cross-sectional area A of the filament from the given information. Then its diameter can be found by assuming it has a circular cross-section.

Solution

The cross-sectional area, found by rearranging the expression for the resistance of a cylinder given in $R = \frac{\rho L}{A}$, is

Equation:

$$A = \frac{\rho L}{R}.$$

Substituting the given values, and taking ρ from [\[link\]](#), yields

Equation:

$$\begin{aligned} A &= \frac{(5.6 \times 10^{-8} \Omega \cdot \text{m})(4.00 \times 10^{-2} \text{ m})}{0.350 \Omega} \\ &= 6.40 \times 10^{-9} \text{ m}^2. \end{aligned}$$

The area of a circle is related to its diameter D by

Equation:

$$A = \frac{\pi D^2}{4}.$$

Solving for the diameter D , and substituting the value found for A , gives

Equation:

$$\begin{aligned} D &= 2\left(\frac{A}{\pi}\right)^{\frac{1}{2}} = 2\left(\frac{6.40 \times 10^{-9} \text{ m}^2}{3.14}\right)^{\frac{1}{2}} \\ &= 9.0 \times 10^{-5} \text{ m}. \end{aligned}$$

Discussion

The diameter is just under a tenth of a millimeter. It is quoted to only two digits, because ρ is known to only two digits.

Temperature Variation of Resistance

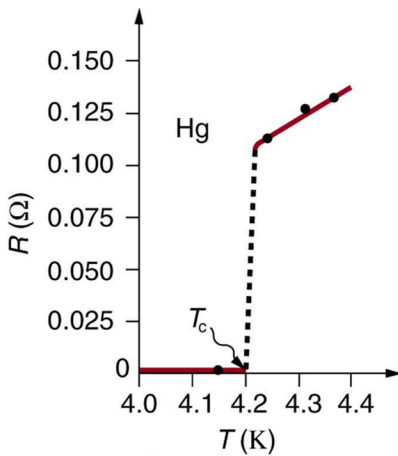
The resistivity of all materials depends on temperature. Some even become superconductors (zero resistivity) at very low temperatures. (See [\[link\]](#).) Conversely, the resistivity of conductors increases with increasing temperature. Since the atoms vibrate more rapidly and over larger distances at higher temperatures, the electrons moving through a metal make more collisions, effectively making the resistivity higher. Over relatively small temperature changes (about 100°C or less), resistivity ρ varies with temperature change ΔT as expressed in the following equation

Equation:

$$\rho = \rho_0(1 + \alpha\Delta T),$$

where ρ_0 is the original resistivity and α is the **temperature coefficient of resistivity**. (See the values of α in [\[link\]](#) below.) For larger temperature changes, α may vary or a nonlinear equation may be needed to find ρ . Note

that α is positive for metals, meaning their resistivity increases with temperature. Some alloys have been developed specifically to have a small temperature dependence. Manganin (which is made of copper, manganese and nickel), for example, has α close to zero (to three digits on the scale in [\[link\]](#)), and so its resistivity varies only slightly with temperature. This is useful for making a temperature-independent resistance standard, for example.



The resistance of a sample of mercury is zero at very low temperatures—it is a superconductor up to about 4.2 K. Above that critical temperature, its resistance makes a sudden jump and then increases nearly linearly with temperature.

Material	Coefficient $\alpha(1/^{\circ}\text{C})$ [footnote] Values at 20°C.
<i>Conductors</i>	
Silver	3.8×10^{-3}
Copper	3.9×10^{-3}
Gold	3.4×10^{-3}
Aluminum	3.9×10^{-3}
Tungsten	4.5×10^{-3}
Iron	5.0×10^{-3}
Platinum	3.93×10^{-3}
Lead	3.9×10^{-3}
Manganin (Cu, Mn, Ni alloy)	0.000×10^{-3}

Material	Coefficient $\alpha(1/^{\circ}\text{C})$ [footnote] Values at 20°C.
Constantan (Cu, Ni alloy)	0.002×10^{-3}
Mercury	0.89×10^{-3}
Nichrome (Ni, Fe, Cr alloy)	0.4×10^{-3}
<i>Semiconductors</i>	
Carbon (pure)	-0.5×10^{-3}
Germanium (pure)	-50×10^{-3}
Silicon (pure)	-70×10^{-3}

Temperature Coefficients of Resistivity α

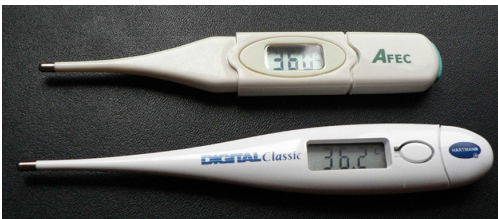
Note also that α is negative for the semiconductors listed in [\[link\]](#), meaning that their resistivity decreases with increasing temperature. They become better conductors at higher temperature, because increased thermal agitation increases the number of free charges available to carry current. This property of decreasing ρ with temperature is also related to the type and amount of impurities present in the semiconductors.

The resistance of an object also depends on temperature, since R_0 is directly proportional to ρ . For a cylinder we know $R = \rho L/A$, and so, if L and A do not change greatly with temperature, R will have the same temperature dependence as ρ . (Examination of the coefficients of linear expansion shows them to be about two orders of magnitude less than typical temperature coefficients of resistivity, and so the effect of temperature on L and A is about two orders of magnitude less than on ρ .) Thus,

Equation:

$$R = R_0(1 + \alpha\Delta T)$$

is the temperature dependence of the resistance of an object, where R_0 is the original resistance and R is the resistance after a temperature change ΔT . Numerous thermometers are based on the effect of temperature on resistance. (See [\[link\]](#).) One of the most common is the thermistor, a semiconductor crystal with a strong temperature dependence, the resistance of which is measured to obtain its temperature. The device is small, so that it quickly comes into thermal equilibrium with the part of a person it touches.



These familiar
thermometers are based
on the automated
measurement of a
thermistor's temperature-
dependent resistance.
(credit: Biol, Wikimedia
Commons)

Example:**Calculating Resistance: Hot-Filament Resistance**

Although caution must be used in applying $\rho = \rho_0(1 + \alpha\Delta T)$ and $R = R_0(1 + \alpha\Delta T)$ for temperature changes greater than 100°C, for tungsten the equations work reasonably well for very large temperature changes. What, then, is the resistance of the tungsten filament in the previous example if its temperature is increased from room temperature (20°C) to a typical operating temperature of 2850°C?

Strategy

This is a straightforward application of $R = R_0(1 + \alpha\Delta T)$, since the original resistance of the filament was given to be $R_0 = 0.350 \, \Omega$, and the temperature change is $\Delta T = 2830^\circ\text{C}$.

Solution

The hot resistance R is obtained by entering known values into the above equation:

Equation:

$$\begin{aligned} R &= R_0(1 + \alpha\Delta T) \\ &= (0.350 \, \Omega)[1 + (4.5 \times 10^{-3}/^\circ\text{C})(2830^\circ\text{C})] \\ &= 4.8 \, \Omega. \end{aligned}$$

Discussion

This value is consistent with the headlight resistance example in [Ohm's Law: Resistance and Simple Circuits](#).

Note:**PhET Explorations: Resistance in a Wire**

Learn about the physics of resistance in a wire. Change its resistivity, length, and area to see how they affect the wire's resistance. The sizes of the symbols in the equation change along with the diagram of a wire.

Section Summary

- The resistance R of a cylinder of length L and cross-sectional area A is $R = \frac{\rho L}{A}$, where ρ is the resistivity of the material.
- Values of ρ in [\[link\]](#) show that materials fall into three groups—*conductors, semiconductors, and insulators*.
- Temperature affects resistivity; for relatively small temperature changes ΔT , resistivity is $\rho = \rho_0(1 + \alpha\Delta T)$, where ρ_0 is the original resistivity and α is the temperature coefficient of resistivity.
- [\[link\]](#) gives values for α , the temperature coefficient of resistivity.
- The resistance R of an object also varies with temperature: $R = R_0(1 + \alpha\Delta T)$, where R_0 is the original resistance, and R is the resistance after the temperature change.

Conceptual Questions

Exercise:

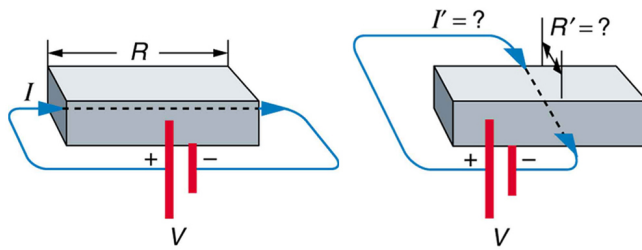
Problem:

In which of the three semiconducting materials listed in [\[link\]](#) do impurities supply free charges? (Hint: Examine the range of resistivity for each and determine whether the pure semiconductor has the higher or lower conductivity.)

Exercise:

Problem:

Does the resistance of an object depend on the path current takes through it? Consider, for example, a rectangular bar—is its resistance the same along its length as across its width? (See [\[link\]](#).)



Does current taking two different paths through the same object encounter different resistance?

Exercise:

Problem:

If aluminum and copper wires of the same length have the same resistance, which has the larger diameter? Why?

Exercise:

Problem:

Explain why $R = R_0(1 + \alpha\Delta T)$ for the temperature variation of the resistance R of an object is not as accurate as $\rho = \rho_0(1 + \alpha\Delta T)$, which gives the temperature variation of resistivity ρ .

Problems & Exercises

Exercise:

Problem:

What is the resistance of a 20.0-m-long piece of 12-gauge copper wire having a 2.053-mm diameter?

Solution:

0.104 Ω

Exercise:**Problem:**

The diameter of 0-gauge copper wire is 8.252 mm. Find the resistance of a 1.00-km length of such wire used for power transmission.

Exercise:**Problem:**

If the 0.100-mm diameter tungsten filament in a light bulb is to have a resistance of $0.200\ \Omega$ at 20.0°C , how long should it be?

Solution:

$$2.8 \times 10^{-2}\ \text{m}$$

Exercise:**Problem:**

Find the ratio of the diameter of aluminum to copper wire, if they have the same resistance per unit length (as they might in household wiring).

Exercise:**Problem:**

What current flows through a 2.54-cm-diameter rod of pure silicon that is 20.0 cm long, when $1.00 \times 10^3\ \text{V}$ is applied to it? (Such a rod may be used to make nuclear-particle detectors, for example.)

Solution:

$$1.10 \times 10^{-3}\ \text{A}$$

Exercise:

Problem:

(a) To what temperature must you raise a copper wire, originally at 20.0°C , to double its resistance, neglecting any changes in dimensions? (b) Does this happen in household wiring under ordinary circumstances?

Exercise:**Problem:**

A resistor made of Nichrome wire is used in an application where its resistance cannot change more than 1.00% from its value at 20.0°C . Over what temperature range can it be used?

Solution:

-5°C to 45°C

Exercise:**Problem:**

Of what material is a resistor made if its resistance is 40.0% greater at 100°C than at 20.0°C ?

Exercise:**Problem:**

An electronic device designed to operate at any temperature in the range from -10.0°C to 55.0°C contains pure carbon resistors. By what factor does their resistance increase over this range?

Solution:

1.03

Exercise:

Problem:

(a) Of what material is a wire made, if it is 25.0 m long with a 0.100 mm diameter and has a resistance of $77.7\ \Omega$ at 20.0°C ? (b) What is its resistance at 150°C ?

Exercise:**Problem:**

Assuming a constant temperature coefficient of resistivity, what is the maximum percent decrease in the resistance of a constantan wire starting at 20.0°C ?

Solution:

0.06%

Exercise:**Problem:**

A wire is drawn through a die, stretching it to four times its original length. By what factor does its resistance increase?

Exercise:**Problem:**

A copper wire has a resistance of $0.500\ \Omega$ at 20.0°C , and an iron wire has a resistance of $0.525\ \Omega$ at the same temperature. At what temperature are their resistances equal?

Solution:

-17°C

Exercise:

Problem:

(a) Digital medical thermometers determine temperature by measuring the resistance of a semiconductor device called a thermistor (which has $\alpha = -0.0600/^{\circ}\text{C}$) when it is at the same temperature as the patient. What is a patient's temperature if the thermistor's resistance at that temperature is 82.0% of its value at 37.0°C (normal body temperature)? (b) The negative value for α may not be maintained for very low temperatures. Discuss why and whether this is the case here. (Hint: Resistance can't become negative.)

Exercise:**Problem: Integrated Concepts**

(a) Redo [\[link\]](#) taking into account the thermal expansion of the tungsten filament. You may assume a thermal expansion coefficient of $12 \times 10^{-6}/^{\circ}\text{C}$. (b) By what percentage does your answer differ from that in the example?

Solution:

(a) $4.7\ \Omega$ (total)

(b) 3.0% decrease

Exercise:**Problem: Unreasonable Results**

(a) To what temperature must you raise a resistor made of constantan to double its resistance, assuming a constant temperature coefficient of resistivity? (b) To cut it in half? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable, or which premises are inconsistent?

Glossary

resistivity

an intrinsic property of a material, independent of its shape or size,
directly proportional to the resistance, denoted by ρ

temperature coefficient of resistivity

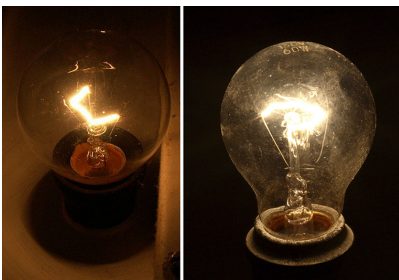
an empirical quantity, denoted by α , which describes the change in
resistance or resistivity of a material with temperature

Electric Power and Energy

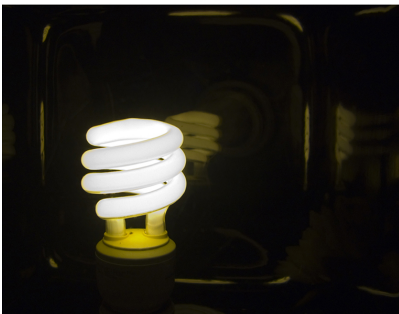
- Calculate the power dissipated by a resistor and power supplied by a power supply.
- Calculate the cost of electricity under various circumstances.

Power in Electric Circuits

Power is associated by many people with electricity. Knowing that power is the rate of energy use or energy conversion, what is the expression for **electric power**? Power transmission lines might come to mind. We also think of lightbulbs in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb. (See [\[link\]](#)(a).) Since both operate on the same voltage, the 60-W bulb must draw more current to have a greater power rating. Thus the 60-W bulb's resistance must be lower than that of a 25-W bulb. If we increase voltage, we also increase power. For example, when a 25-W bulb that is designed to operate on 120 V is connected to 240 V, it briefly glows very brightly and then burns out. Precisely how are voltage, current, and resistance related to electric power?



(a)



(b)

(a) Which of these lightbulbs, the 25-W bulb (upper left) or the 60-W bulb (upper right), has the higher resistance? Which draws more current? Which uses the most energy? Can you tell from the color that the 25-W filament is cooler? Is the brighter bulb a different color and if so why?

(credits: Dickbauch, Wikimedia Commons; Greg Westfall, Flickr) (b)

This compact fluorescent light (CFL) puts out the same intensity of light as the 60-W bulb, but at 1/4 to 1/10 the input power. (credit: dbgg1979, Flickr)

Electric energy depends on both the voltage involved and the charge moved. This is expressed most simply as $PE = qV$, where q is the charge moved and V is the voltage (or more precisely, the potential difference the

charge moves through). Power is the rate at which energy is moved, and so electric power is

Equation:

$$P = \frac{PE}{t} = \frac{qV}{t}.$$

Recognizing that current is $I = q/t$ (note that $\Delta t = t$ here), the expression for power becomes

Equation:

$$P = IV.$$

Electric power (P) is simply the product of current times voltage. Power has familiar units of watts. Since the SI unit for potential energy (PE) is the joule, power has units of joules per second, or watts. Thus, $1 \text{ A} \cdot \text{V} = 1 \text{ W}$. For example, cars often have one or more auxiliary power outlets with which you can charge a cell phone or other electronic devices. These outlets may be rated at 20 A, so that the circuit can deliver a maximum power $P = IV = (20 \text{ A})(12 \text{ V}) = 240 \text{ W}$. In some applications, electric power may be expressed as volt-amperes or even kilovolt-amperes ($1 \text{ kA} \cdot \text{V} = 1 \text{ kW}$).

To see the relationship of power to resistance, we combine Ohm's law with $P = IV$. Substituting $I = V/R$ gives $P = (V/R)V = V^2/R$. Similarly, substituting $V = IR$ gives $P = I(IR) = I^2R$. Three expressions for electric power are listed together here for convenience:

Equation:

$$P = IV$$

Equation:

$$P = \frac{V^2}{R}$$

Equation:

$$P = I^2 R.$$

Note that the first equation is always valid, whereas the other two can be used only for resistors. In a simple circuit, with one voltage source and a single resistor, the power supplied by the voltage source and that dissipated by the resistor are identical. (In more complicated circuits, P can be the power dissipated by a single device and not the total power in the circuit.)

Different insights can be gained from the three different expressions for electric power. For example, $P = V^2/R$ implies that the lower the resistance connected to a given voltage source, the greater the power delivered. Furthermore, since voltage is squared in $P = V^2/R$, the effect of applying a higher voltage is perhaps greater than expected. Thus, when the voltage is doubled to a 25-W bulb, its power nearly quadruples to about 100 W, burning it out. If the bulb's resistance remained constant, its power would be exactly 100 W, but at the higher temperature its resistance is higher, too.

Example:**Calculating Power Dissipation and Current: Hot and Cold Power**

(a) Consider the examples given in [Ohm's Law: Resistance and Simple Circuits](#) and [Resistance and Resistivity](#). Then find the power dissipated by the car headlight in these examples, both when it is hot and when it is cold.
(b) What current does it draw when cold?

Strategy for (a)

For the hot headlight, we know voltage and current, so we can use $P = IV$ to find the power. For the cold headlight, we know the voltage and resistance, so we can use $P = V^2/R$ to find the power.

Solution for (a)

Entering the known values of current and voltage for the hot headlight, we obtain

Equation:

$$P = IV = (2.50 \text{ A})(12.0 \text{ V}) = 30.0 \text{ W}.$$

The cold resistance was $0.350\ \Omega$, and so the power it uses when first switched on is

Equation:

$$P = \frac{V^2}{R} = \frac{(12.0\ \text{V})^2}{0.350\ \Omega} = 411\ \text{W}.$$

Discussion for (a)

The 30 W dissipated by the hot headlight is typical. But the 411 W when cold is surprisingly higher. The initial power quickly decreases as the bulb's temperature increases and its resistance increases.

Strategy and Solution for (b)

The current when the bulb is cold can be found several different ways. We rearrange one of the power equations, $P = I^2 R$, and enter known values, obtaining

Equation:

$$I = \sqrt{\frac{P}{R}} = \sqrt{\frac{411\ \text{W}}{0.350\ \Omega}} = 34.3\ \text{A}.$$

Discussion for (b)

The cold current is remarkably higher than the steady-state value of 2.50 A, but the current will quickly decline to that value as the bulb's temperature increases. Most fuses and circuit breakers (used to limit the current in a circuit) are designed to tolerate very high currents briefly as a device comes on. In some cases, such as with electric motors, the current remains high for several seconds, necessitating special "slow blow" fuses.

The Cost of Electricity

The more electric appliances you use and the longer they are left on, the higher your electric bill. This familiar fact is based on the relationship between energy and power. You pay for the energy used. Since $P = E/t$, we see that

Equation:

$$E = Pt$$

is the energy used by a device using power P for a time interval t . For example, the more lightbulbs burning, the greater P used; the longer they are on, the greater t is. The energy unit on electric bills is the kilowatt-hour ($\text{kW} \cdot \text{h}$), consistent with the relationship $E = Pt$. It is easy to estimate the cost of operating electric appliances if you have some idea of their power consumption rate in watts or kilowatts, the time they are on in hours, and the cost per kilowatt-hour for your electric utility. Kilowatt-hours, like all other specialized energy units such as food calories, can be converted to joules. You can prove to yourself that $1 \text{ kW} \cdot \text{h} = 3.6 \times 10^6 \text{ J}$.

The electrical energy (E) used can be reduced either by reducing the time of use or by reducing the power consumption of that appliance or fixture. This will not only reduce the cost, but it will also result in a reduced impact on the environment. Improvements to lighting are some of the fastest ways to reduce the electrical energy used in a home or business. About 20% of a home's use of energy goes to lighting, while the number for commercial establishments is closer to 40%. Fluorescent lights are about four times more efficient than incandescent lights—this is true for both the long tubes and the compact fluorescent lights (CFL). (See [\[link\]](#)(b).) Thus, a 60-W incandescent bulb can be replaced by a 15-W CFL, which has the same brightness and color. CFLs have a bent tube inside a globe or a spiral-shaped tube, all connected to a standard screw-in base that fits standard incandescent light sockets. (Original problems with color, flicker, shape, and high initial investment for CFLs have been addressed in recent years.) The heat transfer from these CFLs is less, and they last up to 10 times longer. The significance of an investment in such bulbs is addressed in the next example. New white LED lights (which are clusters of small LED bulbs) are even more efficient (twice that of CFLs) and last 5 times longer than CFLs. However, their cost is still high.

Note:

Making Connections: Energy, Power, and Time

The relationship $E = Pt$ is one that you will find useful in many different contexts. The energy your body uses in exercise is related to the power level and duration of your activity, for example. The amount of heating by a power source is related to the power level and time it is applied. Even the radiation dose of an X-ray image is related to the power and time of exposure.

Example:

Calculating the Cost Effectiveness of Compact Fluorescent Lights (CFL)

If the cost of electricity in your area is 12 cents per kWh, what is the total cost (capital plus operation) of using a 60-W incandescent bulb for 1000 hours (the lifetime of that bulb) if the bulb cost 25 cents? (b) If we replace this bulb with a compact fluorescent light that provides the same light output, but at one-quarter the wattage, and which costs \$1.50 but lasts 10 times longer (10,000 hours), what will that total cost be?

Strategy

To find the operating cost, we first find the energy used in kilowatt-hours and then multiply by the cost per kilowatt-hour.

Solution for (a)

The energy used in kilowatt-hours is found by entering the power and time into the expression for energy:

Equation:

$$E = Pt = (60 \text{ W})(1000 \text{ h}) = 60,000 \text{ W} \cdot \text{h}.$$

In kilowatt-hours, this is

Equation:

$$E = 60.0 \text{ kW} \cdot \text{h}.$$

Now the electricity cost is

Equation:

$$\text{cost} = (60.0 \text{ kW} \cdot \text{h})(\$0.12/\text{kW} \cdot \text{h}) = \$7.20.$$

The total cost will be \$7.20 for 1000 hours (about one-half year at 5 hours per day).

Solution for (b)

Since the CFL uses only 15 W and not 60 W, the electricity cost will be $\$7.20/4 = \$1.80$. The CFL will last 10 times longer than the incandescent, so that the investment cost will be 1/10 of the bulb cost for that time period of use, or $0.1(\$1.50) = \$0.15$. Therefore, the total cost will be \$1.95 for 1000 hours.

Discussion

Therefore, it is much cheaper to use the CFLs, even though the initial investment is higher. The increased cost of labor that a business must include for replacing the incandescent bulbs more often has not been figured in here.

Note:

Making Connections: Take-Home Experiment—Electrical Energy Use Inventory

1) Make a list of the power ratings on a range of appliances in your home or room. Explain why something like a toaster has a higher rating than a digital clock. Estimate the energy consumed by these appliances in an average day (by estimating their time of use). Some appliances might only state the operating current. If the household voltage is 120 V, then use $P = IV$. 2) Check out the total wattage used in the rest rooms of your school's floor or building. (You might need to assume the long fluorescent lights in use are rated at 32 W.) Suppose that the building was closed all weekend and that these lights were left on from 6 p.m. Friday until 8 a.m. Monday. What would this oversight cost? How about for an entire year of weekends?

Section Summary

- Electric power P is the rate (in watts) that energy is supplied by a source or dissipated by a device.

- Three expressions for electrical power are
Equation:

$$P = IV,$$

Equation:

$$P = \frac{V^2}{R},$$

and

Equation:

$$P = I^2 R.$$

- The energy used by a device with a power P over a time t is $E = Pt$.

Conceptual Questions

Exercise:

Problem:

Why do incandescent lightbulbs grow dim late in their lives, particularly just before their filaments break?

Exercise:

Problem:

The power dissipated in a resistor is given by $P = V^2/R$, which means power decreases if resistance increases. Yet this power is also given by $P = I^2 R$, which means power increases if resistance increases. Explain why there is no contradiction here.

Problem Exercises

Exercise:

Problem:

What is the power of a 1.00×10^2 MV lightning bolt having a current of 2.00×10^4 A?

Solution:

$$2.00 \times 10^{12} \text{ W}$$

Exercise:**Problem:**

What power is supplied to the starter motor of a large truck that draws 250 A of current from a 24.0-V battery hookup?

Exercise:**Problem:**

A charge of 4.00 C of charge passes through a pocket calculator's solar cells in 4.00 h. What is the power output, given the calculator's voltage output is 3.00 V? (See [\[link\]](#).)



The strip of solar cells just above the keys of this calculator convert

light to electricity
to supply its energy
needs. (credit:
Evan-Amos,
Wikimedia
Commons)

Exercise:

Problem:

How many watts does a flashlight that has 6.00×10^2 C pass through it in 0.500 h use if its voltage is 3.00 V?

Exercise:

Problem:

Find the power dissipated in each of these extension cords: (a) an extension cord having a $0.0600\ \Omega$ resistance and through which 5.00 A is flowing; (b) a cheaper cord utilizing thinner wire and with a resistance of $0.300\ \Omega$.

Solution:

(a) 1.50 W

(b) 7.50 W

Exercise:

Problem:

Verify that the units of a volt-ampere are watts, as implied by the equation $P = IV$.

Exercise:

Problem:

Show that the units $1 \text{ V}^2/\Omega = 1 \text{ W}$, as implied by the equation $P = V^2/R$.

Solution:

$$\frac{V^2}{\Omega} = \frac{V^2}{V/A} = AV = \left(\frac{C}{s}\right)\left(\frac{J}{C}\right) = \frac{J}{s} = 1 \text{ W}$$

Exercise:**Problem:**

Show that the units $1 \text{ A}^2 \cdot \Omega = 1 \text{ W}$, as implied by the equation $P = I^2 R$.

Exercise:**Problem:**

Verify the energy unit equivalence that $1 \text{ kW} \cdot \text{h} = 3.60 \times 10^6 \text{ J}$.

Solution:

$$1 \text{ kW} \cdot \text{h} = \left(\frac{1 \times 10^3 \text{ J}}{1 \text{ s}}\right)(1 \text{ h})\left(\frac{3600 \text{ s}}{1 \text{ h}}\right) = 3.60 \times 10^6 \text{ J}$$

Exercise:**Problem:**

Electrons in an X-ray tube are accelerated through $1.00 \times 10^2 \text{ kV}$ and directed toward a target to produce X-rays. Calculate the power of the electron beam in this tube if it has a current of 15.0 mA .

Exercise:

Problem:

An electric water heater consumes 5.00 kW for 2.00 h per day. What is the cost of running it for one year if electricity costs 12.0 cents/kW · h? See [\[link\]](#).



On-demand electric hot water heater. Heat is supplied to water only when needed.
(credit: aviddavid, Flickr)

Solution:

\$438/y

Exercise:**Problem:**

With a 1200-W toaster, how much electrical energy is needed to make a slice of toast (cooking time = 1 minute)? At 9.0 cents/kW · h, how much does this cost?

Exercise:

Problem:

What would be the maximum cost of a CFL such that the total cost (investment plus operating) would be the same for both CFL and incandescent 60-W bulbs? Assume the cost of the incandescent bulb is 25 cents and that electricity costs 10 cents/kWh. Calculate the cost for 1000 hours, as in the cost effectiveness of CFL example.

Solution:

\$6.25

Exercise:**Problem:**

Some makes of older cars have 6.00-V electrical systems. (a) What is the hot resistance of a 30.0-W headlight in such a car? (b) What current flows through it?

Exercise:**Problem:**

Alkaline batteries have the advantage of putting out constant voltage until very nearly the end of their life. How long will an alkaline battery rated at 1.00 A · h and 1.58 V keep a 1.00-W flashlight bulb burning?

Solution:

1.58 h

Exercise:**Problem:**

A cauterizer, used to stop bleeding in surgery, puts out 2.00 mA at 15.0 kV. (a) What is its power output? (b) What is the resistance of the path?

Exercise:

Problem:

The average television is said to be on 6 hours per day. Estimate the yearly cost of electricity to operate 100 million TVs, assuming their power consumption averages 150 W and the cost of electricity averages 12.0 cents/kW · h.

Solution:

\$3.94 billion/year

Exercise:**Problem:**

An old lightbulb draws only 50.0 W, rather than its original 60.0 W, due to evaporative thinning of its filament. By what factor is its diameter reduced, assuming uniform thinning along its length? Neglect any effects caused by temperature differences.

Exercise:**Problem:**

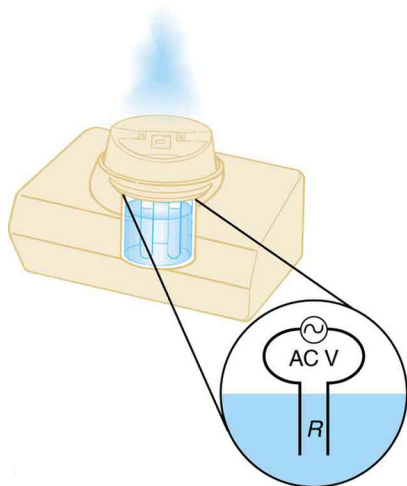
00-gauge copper wire has a diameter of 9.266 mm. Calculate the power loss in a kilometer of such wire when it carries 1.00×10^2 A.

Solution:

25.5 W

Exercise:**Problem: Integrated Concepts**

Cold vaporizers pass a current through water, evaporating it with only a small increase in temperature. One such home device is rated at 3.50 A and utilizes 120 V AC with 95.0% efficiency. (a) What is the vaporization rate in grams per minute? (b) How much water must you put into the vaporizer for 8.00 h of overnight operation? (See [\[link\]](#).)



This cold vaporizer
passes current
directly through
water, vaporizing it
directly with
relatively little
temperature
increase.

Exercise:

Problem: Integrated Concepts

(a) What energy is dissipated by a lightning bolt having a 20,000-A current, a voltage of 1.00×10^2 MV, and a length of 1.00 ms? (b) What mass of tree sap could be raised from 18.0°C to its boiling point and then evaporated by this energy, assuming sap has the same thermal characteristics as water?

Solution:

(a) 2.00×10^9 J

(b) 769 kg

Exercise:**Problem: Integrated Concepts**

What current must be produced by a 12.0-V battery-operated bottle warmer in order to heat 75.0 g of glass, 250 g of baby formula, and 3.00×10^2 g of aluminum from 20.0°C to 90.0°C in 5.00 min?

Exercise:**Problem: Integrated Concepts**

How much time is needed for a surgical cauterizer to raise the temperature of 1.00 g of tissue from 37.0°C to 100°C and then boil away 0.500 g of water, if it puts out 2.00 mA at 15.0 kV? Ignore heat transfer to the surroundings.

Solution:

45.0 s

Exercise:**Problem: Integrated Concepts**

Hydroelectric generators (see [\[link\]](#)) at Hoover Dam produce a maximum current of 8.00×10^3 A at 250 kV. (a) What is the power output? (b) The water that powers the generators enters and leaves the system at low speed (thus its kinetic energy does not change) but loses 160 m in altitude. How many cubic meters per second are needed, assuming 85.0% efficiency?



Hydroelectric generators
at the Hoover dam.
(credit: Jon Sullivan)

Exercise:

Problem: Integrated Concepts

(a) Assuming 95.0% efficiency for the conversion of electrical power by the motor, what current must the 12.0-V batteries of a 750-kg electric car be able to supply: (a) To accelerate from rest to 25.0 m/s in 1.00 min? (b) To climb a 2.00×10^2 -m-high hill in 2.00 min at a constant 25.0-m/s speed while exerting 5.00×10^2 N of force to overcome air resistance and friction? (c) To travel at a constant 25.0-m/s speed, exerting a 5.00×10^2 N force to overcome air resistance and friction? See [\[link\]](#).



This REVAi, an electric

car, gets recharged on a street in London. (credit: Frank Hebbert)

Solution:

(a) 343 A

(b) 2.17×10^3 A

(c) 1.10×10^3 A

Exercise:

Problem: Integrated Concepts

A light-rail commuter train draws 630 A of 650-V DC electricity when accelerating. (a) What is its power consumption rate in kilowatts? (b) How long does it take to reach 20.0 m/s starting from rest if its loaded mass is 5.30×10^4 kg, assuming 95.0% efficiency and constant power? (c) Find its average acceleration. (d) Discuss how the acceleration you found for the light-rail train compares to what might be typical for an automobile.

Exercise:

Problem: Integrated Concepts

(a) An aluminum power transmission line has a resistance of $0.0580 \Omega/\text{km}$. What is its mass per kilometer? (b) What is the mass per kilometer of a copper line having the same resistance? A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

Solution:

(a) $1.23 \times 10^3 \text{ kg}$

(b) $2.64 \times 10^3 \text{ kg}$

Exercise:

Problem: Integrated Concepts

(a) An immersion heater utilizing 120 V can raise the temperature of a 1.00×10^2 -g aluminum cup containing 350 g of water from 20.0°C to 95.0°C in 2.00 min. Find its resistance, assuming it is constant during the process. (b) A lower resistance would shorten the heating time. Discuss the practical limits to speeding the heating by lowering the resistance.

Exercise:

Problem: Integrated Concepts

(a) What is the cost of heating a hot tub containing 1500 kg of water from 10.0°C to 40.0°C, assuming 75.0% efficiency to account for heat transfer to the surroundings? The cost of electricity is 9 cents/kW · h. (b) What current was used by the 220-V AC electric heater, if this took 4.00 h?

Exercise:

Problem: Unreasonable Results

(a) What current is needed to transmit 1.00×10^2 MW of power at 480 V? (b) What power is dissipated by the transmission lines if they have a $1.00 - \Omega$ resistance? (c) What is unreasonable about this result? (d) Which assumptions are unreasonable, or which premises are inconsistent?

Solution:

(a) $2.08 \times 10^5 \text{ A}$

(b) 4.33×10^4 MW

(c) The transmission lines dissipate more power than they are supposed to transmit.

(d) A voltage of 480 V is unreasonably low for a transmission voltage. Long-distance transmission lines are kept at much higher voltages (often hundreds of kilovolts) to reduce power losses.

Exercise:

Problem: Unreasonable Results

(a) What current is needed to transmit 1.00×10^2 MW of power at 10.0 kV? (b) Find the resistance of 1.00 km of wire that would cause a 0.0100% power loss. (c) What is the diameter of a 1.00-km-long copper wire having this resistance? (d) What is unreasonable about these results? (e) Which assumptions are unreasonable, or which premises are inconsistent?

Exercise:

Problem: Construct Your Own Problem

Consider an electric immersion heater used to heat a cup of water to make tea. Construct a problem in which you calculate the needed resistance of the heater so that it increases the temperature of the water and cup in a reasonable amount of time. Also calculate the cost of the electrical energy used in your process. Among the things to be considered are the voltage used, the masses and heat capacities involved, heat losses, and the time over which the heating takes place. Your instructor may wish for you to consider a thermal safety switch (perhaps bimetallic) that will halt the process before damaging temperatures are reached in the immersion unit.

Glossary

electric power

the rate at which electrical energy is supplied by a source or dissipated by a device; it is the product of current times voltage

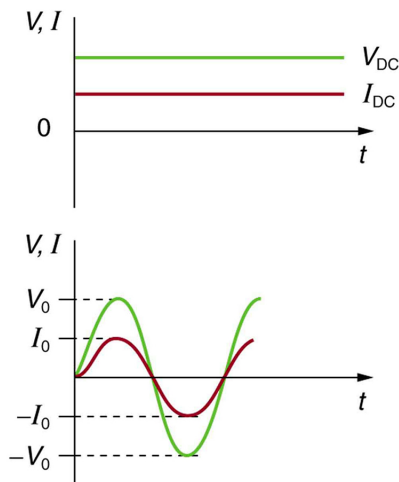
Alternating Current versus Direct Current

- Explain the differences and similarities between AC and DC current.
- Calculate rms voltage, current, and average power.
- Explain why AC current is used for power transmission.

Alternating Current

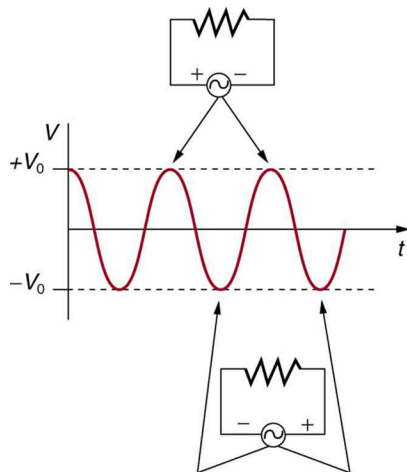
Most of the examples dealt with so far, and particularly those utilizing batteries, have constant voltage sources. Once the current is established, it is thus also a constant. **Direct current** (DC) is the flow of electric charge in only one direction. It is the steady state of a constant-voltage circuit. Most well-known applications, however, use a time-varying voltage source.

Alternating current (AC) is the flow of electric charge that periodically reverses direction. If the source varies periodically, particularly sinusoidally, the circuit is known as an alternating current circuit. Examples include the commercial and residential power that serves so many of our needs. [\[link\]](#) shows graphs of voltage and current versus time for typical DC and AC power. The AC voltages and frequencies commonly used in homes and businesses vary around the world.



(a) DC voltage and current are constant in time, once the

current is established. (b) A graph of voltage and current versus time for 60-Hz AC power. The voltage and current are sinusoidal and are in phase for a simple resistance circuit. The frequencies and peak voltages of AC sources differ greatly.



The potential difference V between the terminals of an AC voltage source fluctuates as

shown. The
mathematical
expression for V is
given by
 $V = V_0 \sin 2\pi ft$.

[\[link\]](#) shows a schematic of a simple circuit with an AC voltage source. The voltage between the terminals fluctuates as shown, with the **AC voltage** given by

Equation:

$$V = V_0 \sin 2\pi ft,$$

where V is the voltage at time t , V_0 is the peak voltage, and f is the frequency in hertz. For this simple resistance circuit, $I = V/R$, and so the **AC current** is

Equation:

$$I = I_0 \sin 2\pi ft,$$

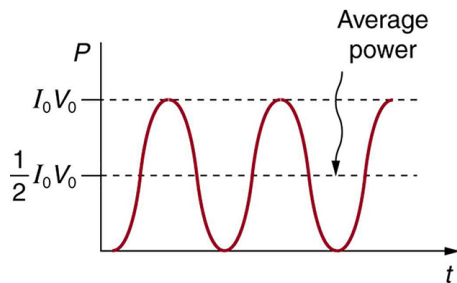
where I is the current at time t , and $I_0 = V_0/R$ is the peak current. For this example, the voltage and current are said to be in phase, as seen in [\[link\]](#)(b).

Current in the resistor alternates back and forth just like the driving voltage, since $I = V/R$. If the resistor is a fluorescent light bulb, for example, it brightens and dims 120 times per second as the current repeatedly goes through zero. A 120-Hz flicker is too rapid for your eyes to detect, but if you wave your hand back and forth between your face and a fluorescent light, you will see a stroboscopic effect evidencing AC. The fact that the light output fluctuates means that the power is fluctuating. The power supplied is $P = IV$. Using the expressions for I and V above, we see that the time dependence of power is $P = I_0 V_0 \sin^2 2\pi ft$, as shown in [\[link\]](#).

Note:**Making Connections: Take-Home Experiment—AC/DC Lights**

Wave your hand back and forth between your face and a fluorescent light bulb. Do you observe the same thing with the headlights on your car?

Explain what you observe. *Warning: Do not look directly at very bright light.*



AC power as a function of time. Since the voltage and current are in phase here, their product is non-negative and fluctuates between zero and $I_0 V_0$. Average power is $(1/2) I_0 V_0$.

We are most often concerned with average power rather than its fluctuations—that 60-W light bulb in your desk lamp has an average power consumption of 60 W, for example. As illustrated in [\[link\]](#), the average power P_{ave} is

Equation:

$$P_{\text{ave}} = \frac{1}{2} I_0 V_0.$$

This is evident from the graph, since the areas above and below the $(1/2)I_0V_0$ line are equal, but it can also be proven using trigonometric identities. Similarly, we define an average or **rms current** I_{rms} and average or **rms voltage** V_{rms} to be, respectively,

Equation:

$$I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$$

and

Equation:

$$V_{\text{rms}} = \frac{V_0}{\sqrt{2}}.$$

where rms stands for root mean square, a particular kind of average. In general, to obtain a root mean square, the particular quantity is squared, its mean (or average) is found, and the square root is taken. This is useful for AC, since the average value is zero. Now,

Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

which gives

Equation:

$$P_{\text{ave}} = \frac{I_0}{\sqrt{2}} \cdot \frac{V_0}{\sqrt{2}} = \frac{1}{2} I_0 V_0,$$

as stated above. It is standard practice to quote I_{rms} , V_{rms} , and P_{ave} rather than the peak values. For example, most household electricity is 120 V AC, which means that V_{rms} is 120 V. The common 10-A circuit breaker will interrupt a sustained I_{rms} greater than 10 A. Your 1.0-kW microwave oven

consumes $P_{\text{ave}} = 1.0 \text{ kW}$, and so on. You can think of these rms and average values as the equivalent DC values for a simple resistive circuit.

To summarize, when dealing with AC, Ohm's law and the equations for power are completely analogous to those for DC, but rms and average values are used for AC. Thus, for AC, Ohm's law is written

Equation:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{R}.$$

The various expressions for AC power P_{ave} are

Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}},$$

Equation:

$$P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R},$$

and

Equation:

$$P_{\text{ave}} = I_{\text{rms}}^2 R.$$

Example:

Peak Voltage and Power for AC

(a) What is the value of the peak voltage for 120-V AC power? (b) What is the peak power consumption rate of a 60.0-W AC light bulb?

Strategy

We are told that V_{rms} is 120 V and P_{ave} is 60.0 W. We can use $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ to find the peak voltage, and we can manipulate the definition of power to

find the peak power from the given average power.

Solution for (a)

Solving the equation $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$ for the peak voltage V_0 and substituting the known value for V_{rms} gives

Equation:

$$V_0 = \sqrt{2}V_{\text{rms}} = 1.414(120 \text{ V}) = 170 \text{ V}.$$

Discussion for (a)

This means that the AC voltage swings from 170 V to -170 V and back 60 times every second. An equivalent DC voltage is a constant 120 V.

Solution for (b)

Peak power is peak current times peak voltage. Thus,

Equation:

$$P_0 = I_0 V_0 = 2 \left(\frac{1}{2} I_0 V_0 \right) = 2P_{\text{ave}}.$$

We know the average power is 60.0 W, and so

Equation:

$$P_0 = 2(60.0 \text{ W}) = 120 \text{ W}.$$

Discussion

So the power swings from zero to 120 W one hundred twenty times per second (twice each cycle), and the power averages 60 W.

Why Use AC for Power Distribution?

Most large power-distribution systems are AC. Moreover, the power is transmitted at much higher voltages than the 120-V AC (240 V in most parts of the world) we use in homes and on the job. Economies of scale make it cheaper to build a few very large electric power-generation plants than to build numerous small ones. This necessitates sending power long distances, and it is obviously important that energy losses en route be

minimized. High voltages can be transmitted with much smaller power losses than low voltages, as we shall see. (See [\[link\]](#).) For safety reasons, the voltage at the user is reduced to familiar values. The crucial factor is that it is much easier to increase and decrease AC voltages than DC, so AC is used in most large power distribution systems.



Power is distributed over large distances at high voltage to reduce power loss in the transmission lines. The voltages generated at the power plant are stepped up by passive devices called transformers (see [Transformers](#)) to 330,000 volts (or more in some places worldwide). At the point of use, the transformers reduce the voltage transmitted for safe residential and commercial use.

(Credit: GeorgHH, Wikimedia Commons)

Example:**Power Losses Are Less for High-Voltage Transmission**

(a) What current is needed to transmit 100 MW of power at 200 kV? (b) What is the power dissipated by the transmission lines if they have a resistance of $1.00\ \Omega$? (c) What percentage of the power is lost in the transmission lines?

Strategy

We are given $P_{\text{ave}} = 100\text{ MW}$, $V_{\text{rms}} = 200\text{ kV}$, and the resistance of the lines is $R = 1.00\ \Omega$. Using these givens, we can find the current flowing (from $P = IV$) and then the power dissipated in the lines ($P = I^2 R$), and we take the ratio to the total power transmitted.

Solution

To find the current, we rearrange the relationship $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$ and substitute known values. This gives

Equation:

$$I_{\text{rms}} = \frac{P_{\text{ave}}}{V_{\text{rms}}} = \frac{100 \times 10^6\text{ W}}{200 \times 10^3\text{ V}} = 500\text{ A}.$$

Solution

Knowing the current and given the resistance of the lines, the power dissipated in them is found from $P_{\text{ave}} = I_{\text{rms}}^2 R$. Substituting the known values gives

Equation:

$$P_{\text{ave}} = I_{\text{rms}}^2 R = (500\text{ A})^2 (1.00\ \Omega) = 250\text{ kW}.$$

Solution

The percent loss is the ratio of this lost power to the total or input power, multiplied by 100:

Equation:

$$\% \text{ loss} = \frac{250 \text{ kW}}{100 \text{ MW}} \times 100 = 0.250 \%$$

Discussion

One-fourth of a percent is an acceptable loss. Note that if 100 MW of power had been transmitted at 25 kV, then a current of 4000 A would have been needed. This would result in a power loss in the lines of 16.0 MW, or 16.0% rather than 0.250%. The lower the voltage, the more current is needed, and the greater the power loss in the fixed-resistance transmission lines. Of course, lower-resistance lines can be built, but this requires larger and more expensive wires. If superconducting lines could be economically produced, there would be no loss in the transmission lines at all. But, as we shall see in a later chapter, there is a limit to current in superconductors, too. In short, high voltages are more economical for transmitting power, and AC voltage is much easier to raise and lower, so that AC is used in most large-scale power distribution systems.

It is widely recognized that high voltages pose greater hazards than low voltages. But, in fact, some high voltages, such as those associated with common static electricity, can be harmless. So it is not voltage alone that determines a hazard. It is not so widely recognized that AC shocks are often more harmful than similar DC shocks. Thomas Edison thought that AC shocks were more harmful and set up a DC power-distribution system in New York City in the late 1800s. There were bitter fights, in particular between Edison and George Westinghouse and Nikola Tesla, who were advocating the use of AC in early power-distribution systems. AC has prevailed largely due to transformers and lower power losses with high-voltage transmission.

Note:**PhET Explorations: Generator**

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

Generato

r

Section Summary

- Direct current (DC) is the flow of electric current in only one direction. It refers to systems where the source voltage is constant.
- The voltage source of an alternating current (AC) system puts out $V = V_0 \sin 2\pi ft$, where V is the voltage at time t , V_0 is the peak voltage, and f is the frequency in hertz.
- In a simple circuit, $I = V/R$ and AC current is $I = I_0 \sin 2\pi ft$, where I is the current at time t , and $I_0 = V_0/R$ is the peak current.
- The average AC power is $P_{\text{ave}} = \frac{1}{2} I_0 V_0$.
- Average (rms) current I_{rms} and average (rms) voltage V_{rms} are $I_{\text{rms}} = \frac{I_0}{\sqrt{2}}$ and $V_{\text{rms}} = \frac{V_0}{\sqrt{2}}$, where rms stands for root mean square.
- Thus, $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$.
- Ohm's law for AC is $I_{\text{rms}} = \frac{V_{\text{rms}}}{R}$.
- Expressions for the average power of an AC circuit are $P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}}$, $P_{\text{ave}} = \frac{V_{\text{rms}}^2}{R}$, and $P_{\text{ave}} = I_{\text{rms}}^2 R$, analogous to the expressions for DC circuits.

Conceptual Questions

Exercise:

Problem:

Give an example of a use of AC power other than in the household. Similarly, give an example of a use of DC power other than that supplied by batteries.

Exercise:**Problem:**

Why do voltage, current, and power go through zero 120 times per second for 60-Hz AC electricity?

Exercise:**Problem:**

You are riding in a train, gazing into the distance through its window. As close objects streak by, you notice that the nearby fluorescent lights make *dashed* streaks. Explain.

Problem Exercises**Exercise:****Problem:**

(a) What is the hot resistance of a 25-W light bulb that runs on 120-V AC? (b) If the bulb's operating temperature is 2700°C , what is its resistance at 2600°C ?

Exercise:**Problem:**

Certain heavy industrial equipment uses AC power that has a peak voltage of 679 V. What is the rms voltage?

Solution:

480 V

Exercise:**Problem:**

A certain circuit breaker trips when the rms current is 15.0 A. What is the corresponding peak current?

Exercise:**Problem:**

Military aircraft use 400-Hz AC power, because it is possible to design lighter-weight equipment at this higher frequency. What is the time for one complete cycle of this power?

Solution:

2.50 ms

Exercise:**Problem:**

A North American tourist takes his 25.0-W, 120-V AC razor to Europe, finds a special adapter, and plugs it into 240 V AC. Assuming constant resistance, what power does the razor consume as it is ruined?

Exercise:**Problem:**

In this problem, you will verify statements made at the end of the power losses for [\[link\]](#). (a) What current is needed to transmit 100 MW of power at a voltage of 25.0 kV? (b) Find the power loss in a 1.00 - Ω transmission line. (c) What percent loss does this represent?

Solution:

(a) 4.00 kA

(b) 16.0 MW

(c) 16.0%

Exercise:

Problem:

A small office-building air conditioner operates on 408-V AC and consumes 50.0 kW. (a) What is its effective resistance? (b) What is the cost of running the air conditioner during a hot summer month when it is on 8.00 h per day for 30 days and electricity costs 9.00 cents/kW · h?

Exercise:**Problem:**

What is the peak power consumption of a 120-V AC microwave oven that draws 10.0 A?

Solution:

2.40 kW

Exercise:**Problem:**

What is the peak current through a 500-W room heater that operates on 120-V AC power?

Exercise:**Problem:**

Two different electrical devices have the same power consumption, but one is meant to be operated on 120-V AC and the other on 240-V AC. (a) What is the ratio of their resistances? (b) What is the ratio of their currents? (c) Assuming its resistance is unaffected, by what factor will the power increase if a 120-V AC device is connected to 240-V AC?

Solution:

(a) 4.0

(b) 0.50

(c) 4.0

Exercise:

Problem:

Nichrome wire is used in some radiative heaters. (a) Find the resistance needed if the average power output is to be 1.00 kW utilizing 120-V AC. (b) What length of Nichrome wire, having a cross-sectional area of 5.00mm^2 , is needed if the operating temperature is 500°C ? (c) What power will it draw when first switched on?

Exercise:

Problem:

Find the time after $t = 0$ when the instantaneous voltage of 60-Hz AC first reaches the following values: (a) $V_0/2$ (b) V_0 (c) 0.

Solution:

(a) 1.39 ms

(b) 4.17 ms

(c) 8.33 ms

Exercise:

Problem:

(a) At what two times in the first period following $t = 0$ does the instantaneous voltage in 60-Hz AC equal V_{rms} ? (b) $-V_{\text{rms}}$?

Glossary

direct current

(DC) the flow of electric charge in only one direction

alternating current

(AC) the flow of electric charge that periodically reverses direction

AC voltage

voltage that fluctuates sinusoidally with time, expressed as $V = V_0 \sin 2\pi ft$, where V is the voltage at time t , V_0 is the peak voltage, and f is the frequency in hertz

AC current

current that fluctuates sinusoidally with time, expressed as $I = I_0 \sin 2\pi ft$, where I is the current at time t , I_0 is the peak current, and f is the frequency in hertz

rms current

the root mean square of the current, $I_{\text{rms}} = I_0/\sqrt{2}$, where I_0 is the peak current, in an AC system

rms voltage

the root mean square of the voltage, $V_{\text{rms}} = V_0/\sqrt{2}$, where V_0 is the peak voltage, in an AC system

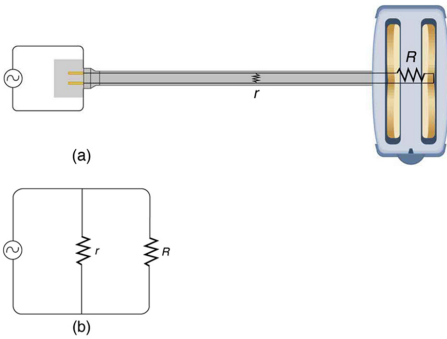
Electric Hazards and the Human Body

- Define thermal hazard, shock hazard, and short circuit.
- Explain what effects various levels of current have on the human body.

There are two known hazards of electricity—thermal and shock. A **thermal hazard** is one where excessive electric power causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. This section considers these hazards and the various factors affecting them in a quantitative manner. [Electrical Safety: Systems and Devices](#) will consider systems and devices for preventing electrical hazards.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted to thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the **short circuit**, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in [\[link\]](#). Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact. Such an undesired contact with a high voltage is called a *short*. Since the resistance of the short, r , is very small, the power dissipated in the short, $P = V^2/r$, is very large. For example, if V is 120 V and r is 0.100 Ω , then the power is 144 kW, *much* greater than that used by a typical household appliance. Thermal energy delivered at this rate will very quickly raise the temperature of surrounding materials, melting or perhaps igniting them.

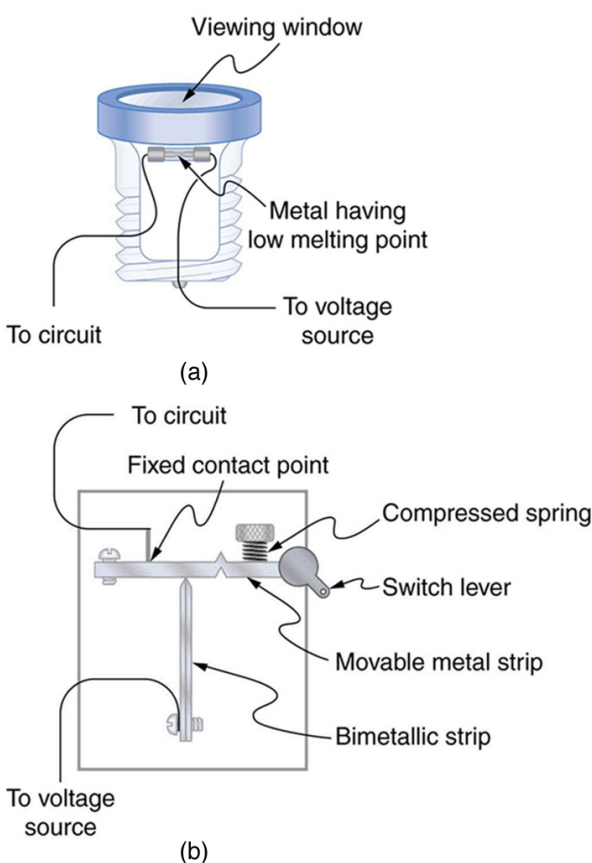


A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r . Since $P = V^2/r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

One particularly insidious aspect of a short circuit is that its resistance may actually be decreased due to the increase in temperature. This can happen if the short creates ionization. These charged atoms and molecules are free to move and, thus, lower the resistance r . Since $P = V^2/r$, the power dissipated in the short rises, possibly causing more ionization, more power, and so on. High voltages, such as the 480-V AC used in some industrial applications, lend themselves to this hazard, because higher voltages create higher initial power production in a short.

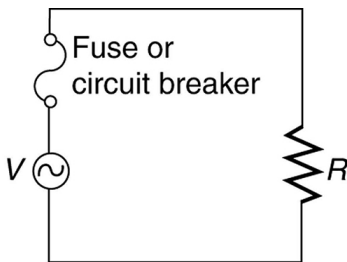
Another serious, but less dramatic, thermal hazard occurs when wires supplying power to a user are overloaded with too great a current. As

discussed in the previous section, the power dissipated in the supply wires is $P = I^2 R_w$, where R_w is the resistance of the wires and I the current flowing through them. If either I or R_w is too large, the wires overheat. For example, a worn appliance cord (with some of its braided wires broken) may have $R_w = 2.00 \, \Omega$ rather than the $0.100 \, \Omega$ it should be. If $10.0 \, \text{A}$ of current passes through the cord, then $P = I^2 R_w = 200 \, \text{W}$ is dissipated in the cord—much more than is safe. Similarly, if a wire with a $0.100 \, \Omega$ resistance is meant to carry a few amps, but is instead carrying $100 \, \text{A}$, it will severely overheat. The power dissipated in the wire will in that case be $P = 1000 \, \text{W}$. Fuses and circuit breakers are used to limit excessive currents. (See [\[link\]](#) and [\[link\]](#).) Each device opens the circuit automatically when a sustained current exceeds safe limits.



(a) A fuse has a metal strip with a low melting point that, when overheated by an excessive

current, permanently breaks the connection of a circuit to a voltage source. (b) A circuit breaker is an automatic but restorable electric switch. The one shown here has a bimetallic strip that bends to the right and into the notch if overheated. The spring then forces the metal strip downward, breaking the electrical connection at the points.



Schematic of a circuit with a fuse or circuit breaker in it.

Fuses and circuit breakers act like automatic switches that open when sustained current exceeds desired limits.

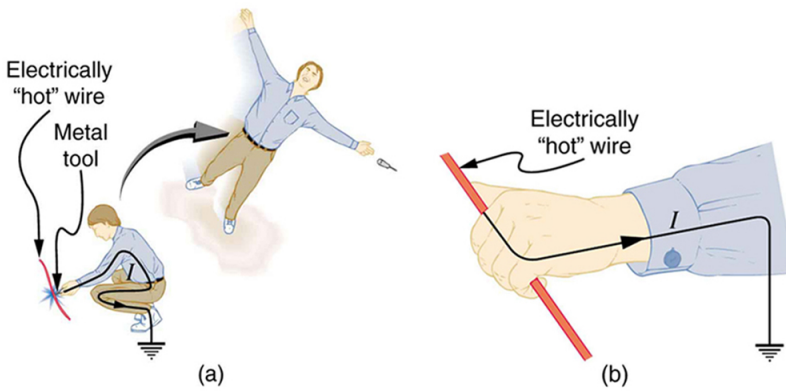
Fuses and circuit breakers for typical household voltages and currents are relatively simple to produce, but those for large voltages and currents experience special problems. For example, when a circuit breaker tries to interrupt the flow of high-voltage electricity, a spark can jump across its points that ionizes the air in the gap and allows the current to continue flowing. Large circuit breakers found in power-distribution systems employ insulating gas and even use jets of gas to blow out such sparks. Here AC is safer than DC, since AC current goes through zero 120 times per second, giving a quick opportunity to extinguish these arcs.

Shock Hazards

Electrical currents through people produce tremendously varied effects. An electrical current can be used to block back pain. The possibility of using electrical current to stimulate muscle action in paralyzed limbs, perhaps allowing paraplegics to walk, is under study. TV dramatizations in which electrical shocks are used to bring a heart attack victim out of ventricular fibrillation (a massively irregular, often fatal, beating of the heart) are more than common. Yet most electrical shock fatalities occur because a current put the heart into fibrillation. A pacemaker uses electrical shocks to stimulate the heart to beat properly. Some fatal shocks do not produce burns, but warts can be safely burned off with electric current (though freezing using liquid nitrogen is now more common). Of course, there are consistent explanations for these disparate effects. The major factors upon which the effects of electrical shock depend are

1. The amount of current I
2. The path taken by the current
3. The duration of the shock
4. The frequency f of the current ($f = 0$ for DC)

[\[link\]](#) gives the effects of electrical shocks as a function of current for a typical accidental shock. The effects are for a shock that passes through the trunk of the body, has a duration of 1 s, and is caused by 60-Hz power.



An electric current can cause muscular contractions with varying effects. (a) The victim is “thrown” backward by involuntary muscle contractions that extend the legs and torso. (b) The victim can’t let go of the wire that is stimulating all the muscles in the hand. Those that close the fingers are stronger than those that open them.

Current (mA)	Effect
1	Threshold of sensation
5	Maximum harmless current
10–20	Onset of sustained muscular contraction; cannot let go for duration of shock; contraction of chest muscles may stop breathing during shock

Current (mA)	Effect
50	Onset of pain
100–300+	Ventricular fibrillation possible; often fatal
300	Onset of burns depending on concentration of current
6000 (6 A)	Onset of sustained ventricular contraction and respiratory paralysis; both cease when shock ends; heartbeat may return to normal; used to defibrillate the heart

Effects of Electrical Shock as a Function of Current^{[footnote](#)}

For an average male shocked through trunk of body for 1 s by 60-Hz AC. Values for females are 60–80% of those listed.

Our bodies are relatively good conductors due to the water in our bodies. Given that larger currents will flow through sections with lower resistance (to be further discussed in the next chapter), electric currents preferentially flow through paths in the human body that have a minimum resistance in a direct path to earth. The earth is a natural electron sink. Wearing insulating shoes, a requirement in many professions, prohibits a pathway for electrons by providing a large resistance in that path. Whenever working with high-power tools (drills), or in risky situations, ensure that you do not provide a pathway for current flow (especially through the heart).

Very small currents pass harmlessly and unfelt through the body. This happens to you regularly without your knowledge. The threshold of sensation is only 1 mA and, although unpleasant, shocks are apparently harmless for currents less than 5 mA. A great number of safety rules take the 5-mA value for the maximum allowed shock. At 10 to 20 mA and above, the current can stimulate sustained muscular contractions much as regular nerve impulses do. People sometimes say they were knocked across the room by a shock, but what really happened was that certain muscles

contracted, propelling them in a manner not of their own choosing. (See [\[link\]](#)(a).) More frightening, and potentially more dangerous, is the “can’t let go” effect illustrated in [\[link\]](#)(b). The muscles that close the fingers are stronger than those that open them, so the hand closes involuntarily on the wire shocking it. This can prolong the shock indefinitely. It can also be a danger to a person trying to rescue the victim, because the rescuer’s hand may close about the victim’s wrist. Usually the best way to help the victim is to give the fist a hard knock/blow/jar with an insulator or to throw an insulator at the fist. Modern electric fences, used in animal enclosures, are now pulsed on and off to allow people who touch them to get free, rendering them less lethal than in the past.

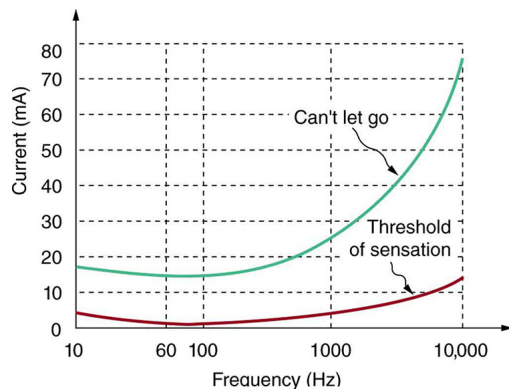
Greater currents may affect the heart. Its electrical patterns can be disrupted, so that it beats irregularly and ineffectively in a condition called “ventricular fibrillation.” This condition often lingers after the shock and is fatal due to a lack of blood circulation. The threshold for ventricular fibrillation is between 100 and 300 mA. At about 300 mA and above, the shock can cause burns, depending on the concentration of current—the more concentrated, the greater the likelihood of burns.

Very large currents cause the heart and diaphragm to contract for the duration of the shock. Both the heart and breathing stop. Interestingly, both often return to normal following the shock. The electrical patterns on the heart are completely erased in a manner that the heart can start afresh with normal beating, as opposed to the permanent disruption caused by smaller currents that can put the heart into ventricular fibrillation. The latter is something like scribbling on a blackboard, whereas the former completely erases it. TV dramatizations of electric shock used to bring a heart attack victim out of ventricular fibrillation also show large paddles. These are used to spread out current passed through the victim to reduce the likelihood of burns.

Current is the major factor determining shock severity (given that other conditions such as path, duration, and frequency are fixed, such as in the table and preceding discussion). A larger voltage is more hazardous, but since $I = V/R$, the severity of the shock depends on the combination of voltage and resistance. For example, a person with dry skin has a resistance

of about $200\text{ k}\Omega$. If he comes into contact with 120-V AC, a current $I = (120\text{ V})/(200\text{ k}\Omega) = 0.6\text{ mA}$ passes harmlessly through him. The same person soaking wet may have a resistance of $10.0\text{ k}\Omega$ and the same 120 V will produce a current of 12 mA—above the “can’t let go” threshold and potentially dangerous.

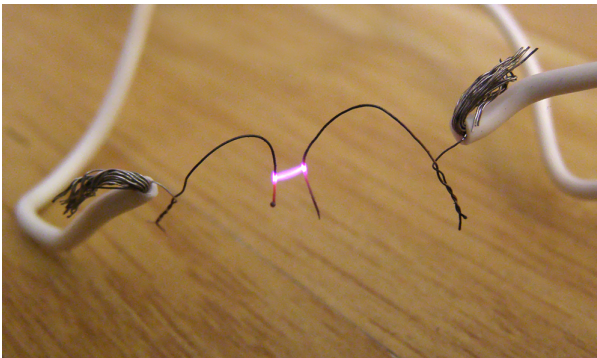
Most of the body’s resistance is in its dry skin. When wet, salts go into ion form, lowering the resistance significantly. The interior of the body has a much lower resistance than dry skin because of all the ionic solutions and fluids it contains. If skin resistance is bypassed, such as by an intravenous infusion, a catheter, or exposed pacemaker leads, a person is rendered **microshock sensitive**. In this condition, currents about 1/1000 those listed in [\[link\]](#) produce similar effects. During open-heart surgery, currents as small as $20\text{ }\mu\text{A}$ can be used to still the heart. Stringent electrical safety requirements in hospitals, particularly in surgery and intensive care, are related to the doubly disadvantaged microshock-sensitive patient. The break in the skin has reduced his resistance, and so the same voltage causes a greater current, and a much smaller current has a greater effect.



Graph of average values
for the threshold of
sensation and the “can’t
let go” current as a
function of frequency.
The lower the value, the

more sensitive the body is
at that frequency.

Factors other than current that affect the severity of a shock are its path, duration, and AC frequency. Path has obvious consequences. For example, the heart is unaffected by an electric shock through the brain, such as may be used to treat manic depression. And it is a general truth that the longer the duration of a shock, the greater its effects. [\[link\]](#) presents a graph that illustrates the effects of frequency on a shock. The curves show the minimum current for two different effects, as a function of frequency. The lower the current needed, the more sensitive the body is at that frequency. Ironically, the body is most sensitive to frequencies near the 50- or 60-Hz frequencies in common use. The body is slightly less sensitive for DC ($f = 0$), mildly confirming Edison's claims that AC presents a greater hazard. At higher and higher frequencies, the body becomes progressively less sensitive to any effects that involve nerves. This is related to the maximum rates at which nerves can fire or be stimulated. At very high frequencies, electrical current travels only on the surface of a person. Thus a wart can be burned off with very high frequency current without causing the heart to stop. (Do not try this at home with 60-Hz AC!) Some of the spectacular demonstrations of electricity, in which high-voltage arcs are passed through the air and over people's bodies, employ high frequencies and low currents. (See [\[link\]](#).) Electrical safety devices and techniques are discussed in detail in [Electrical Safety: Systems and Devices](#).



Is this electric arc dangerous?

The answer depends on the AC frequency and the power involved. (credit: Khimich Alex, Wikimedia Commons)

Section Summary

- The two types of electric hazards are thermal (excessive power) and shock (current through a person).
- Shock severity is determined by current, path, duration, and AC frequency.
- [\[link\]](#) lists shock hazards as a function of current.
- [\[link\]](#) graphs the threshold current for two hazards as a function of frequency.

Conceptual Questions

Exercise:

Problem:

Using an ohmmeter, a student measures the resistance between various points on his body. He finds that the resistance between two points on the same finger is about the same as the resistance between two points on opposite hands—both are several hundred thousand ohms. Furthermore, the resistance decreases when more skin is brought into contact with the probes of the ohmmeter. Finally, there is a dramatic drop in resistance (to a few thousand ohms) when the skin is wet. Explain these observations and their implications regarding skin and internal resistance of the human body.

Exercise:

Problem: What are the two major hazards of electricity?

Exercise:

Problem: Why isn't a short circuit a shock hazard?

Exercise:

Problem:

What determines the severity of a shock? Can you say that a certain voltage is hazardous without further information?

Exercise:

Problem:

An electrified needle is used to burn off warts, with the circuit being completed by having the patient sit on a large butt plate. Why is this plate large?

Exercise:

Problem:

Some surgery is performed with high-voltage electricity passing from a metal scalpel through the tissue being cut. Considering the nature of electric fields at the surface of conductors, why would you expect most of the current to flow from the sharp edge of the scalpel? Do you think high- or low-frequency AC is used?

Exercise:

Problem:

Some devices often used in bathrooms, such as hairdryers, often have safety messages saying "Do not use when the bathtub or basin is full of water." Why is this so?

Exercise:

Problem:

We are often advised to not flick electric switches with wet hands, dry your hand first. We are also advised to never throw water on an electric fire. Why is this so?

Exercise:**Problem:**

Before working on a power transmission line, linemen will touch the line with the back of the hand as a final check that the voltage is zero. Why the back of the hand?

Exercise:**Problem:**

Why is the resistance of wet skin so much smaller than dry, and why do blood and other bodily fluids have low resistances?

Exercise:**Problem:**

Could a person on intravenous infusion (an IV) be microshock sensitive?

Exercise:**Problem:**

In view of the small currents that cause shock hazards and the larger currents that circuit breakers and fuses interrupt, how do they play a role in preventing shock hazards?

Problem Exercises**Exercise:****Problem:**

(a) How much power is dissipated in a short circuit of 240-V AC through a resistance of $0.250\ \Omega$? (b) What current flows?

Solution:

(a) 230 kW

(b) 960 A

Exercise:

Problem:

What voltage is involved in a 1.44-kW short circuit through a $0.100\text{ }\Omega$ resistance?

Exercise:

Problem:

Find the current through a person and identify the likely effect on her if she touches a 120-V AC source: (a) if she is standing on a rubber mat and offers a total resistance of $300\text{ k}\Omega$; (b) if she is standing barefoot on wet grass and has a resistance of only $4000\text{ k}\Omega$.

Solution:

(a) 0.400 mA, no effect

(b) 26.7 mA, muscular contraction for duration of the shock (can't let go)

Exercise:

Problem:

While taking a bath, a person touches the metal case of a radio. The path through the person to the drainpipe and ground has a resistance of $4000\text{ }\Omega$. What is the smallest voltage on the case of the radio that could cause ventricular fibrillation?

Exercise:

Problem:

Foolishly trying to fish a burning piece of bread from a toaster with a metal butter knife, a man comes into contact with 120-V AC. He does not even feel it since, luckily, he is wearing rubber-soled shoes. What is the minimum resistance of the path the current follows through the person?

Solution:

$$1.20 \times 10^5 \, \Omega$$

Exercise:**Problem:**

(a) During surgery, a current as small as $20.0 \, \mu\text{A}$ applied directly to the heart may cause ventricular fibrillation. If the resistance of the exposed heart is $300 \, \Omega$, what is the smallest voltage that poses this danger? (b) Does your answer imply that special electrical safety precautions are needed?

Exercise:**Problem:**

(a) What is the resistance of a 220-V AC short circuit that generates a peak power of 96.8 kW? (b) What would the average power be if the voltage was 120 V AC?

Solution:

(a) $1.00 \, \Omega$

(b) 14.4 kW

Exercise:

Problem:

A heart defibrillator passes 10.0 A through a patient's torso for 5.00 ms in an attempt to restore normal beating. (a) How much charge passed? (b) What voltage was applied if 500 J of energy was dissipated? (c) What was the path's resistance? (d) Find the temperature increase caused in the 8.00 kg of affected tissue.

Exercise:**Problem: Integrated Concepts**

A short circuit in a 120-V appliance cord has a $0.500\text{-}\Omega$ resistance. Calculate the temperature rise of the 2.00 g of surrounding materials, assuming their specific heat capacity is $0.200\text{ cal/g}\cdot^{\circ}\text{C}$ and that it takes 0.0500 s for a circuit breaker to interrupt the current. Is this likely to be damaging?

Solution:

Temperature increases 860°C . It is very likely to be damaging.

Exercise:**Problem: Construct Your Own Problem**

Consider a person working in an environment where electric currents might pass through her body. Construct a problem in which you calculate the resistance of insulation needed to protect the person from harm. Among the things to be considered are the voltage to which the person might be exposed, likely body resistance (dry, wet, ...), and acceptable currents (safe but sensed, safe and unfelt, ...).

Glossary

thermal hazard

a hazard in which electric current causes undesired thermal effects

shock hazard

when electric current passes through a person

short circuit

also known as a “short,” a low-resistance path between terminals of a voltage source

microshock sensitive

a condition in which a person’s skin resistance is bypassed, possibly by a medical procedure, rendering the person vulnerable to electrical shock at currents about 1/1000 the normally required level

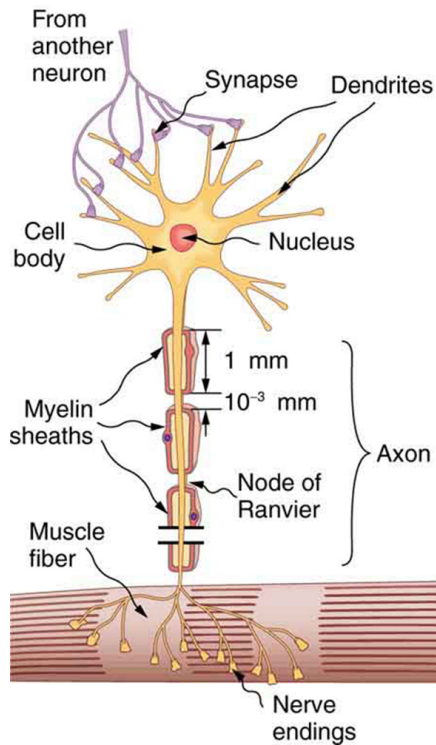
Nerve Conduction–Electrocardiograms

- Explain the process by which electric signals are transmitted along a neuron.
- Explain the effects myelin sheaths have on signal propagation.
- Explain what the features of an ECG signal indicate.

Nerve Conduction

Electric currents in the vastly complex system of billions of nerves in our body allow us to sense the world, control parts of our body, and think. These are representative of the three major functions of nerves. First, nerves carry messages from our sensory organs and others to the central nervous system, consisting of the brain and spinal cord. Second, nerves carry messages from the central nervous system to muscles and other organs. Third, nerves transmit and process signals within the central nervous system. The sheer number of nerve cells and the incredibly greater number of connections between them makes this system the subtle wonder that it is. **Nerve conduction** is a general term for electrical signals carried by nerve cells. It is one aspect of **bioelectricity**, or electrical effects in and created by biological systems.

Nerve cells, properly called *neurons*, look different from other cells—they have tendrils, some of them many centimeters long, connecting them with other cells. (See [\[link\]](#).) Signals arrive at the cell body across *synapses* or through *dendrites*, stimulating the neuron to generate its own signal, sent along its long *axon* to other nerve or muscle cells. Signals may arrive from many other locations and be transmitted to yet others, conditioning the synapses by use, giving the system its complexity and its ability to learn.

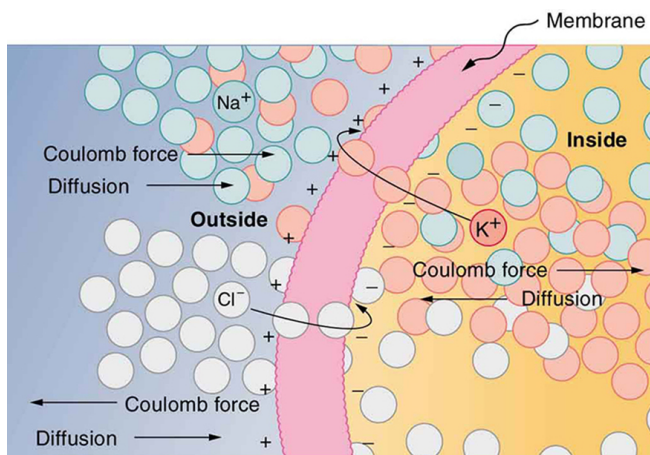


A neuron with its dendrites and long axon. Signals in the form of electric currents reach the cell body through dendrites and across synapses, stimulating the neuron to generate its own signal sent down the axon. The number of interconnections can be far greater than shown here.

The method by which these electric currents are generated and transmitted is more complex than the simple movement of free charges in a conductor,

but it can be understood with principles already discussed in this text. The most important of these are the Coulomb force and diffusion.

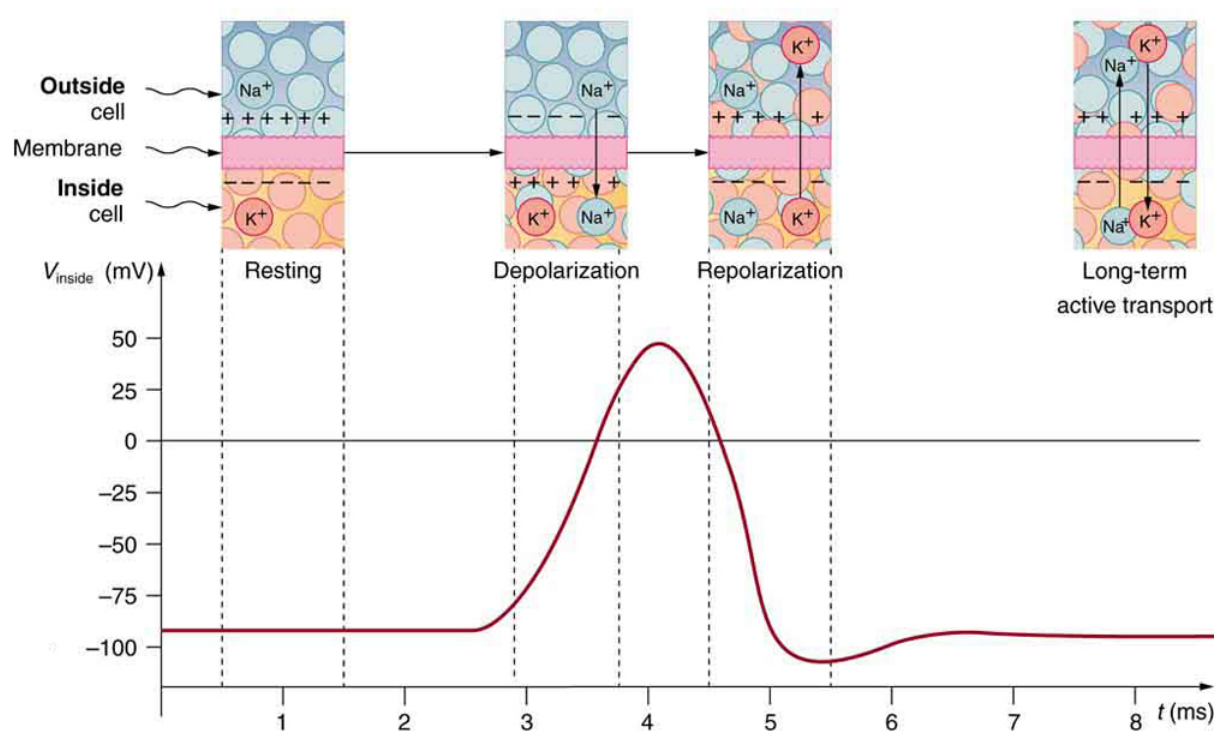
[\[link\]](#) illustrates how a voltage (potential difference) is created across the cell membrane of a neuron in its resting state. This thin membrane separates electrically neutral fluids having differing concentrations of ions, the most important varieties being Na^+ , K^+ , and Cl^- (these are sodium, potassium, and chlorine ions with single plus or minus charges as indicated). As discussed in [Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes](#), free ions will diffuse from a region of high concentration to one of low concentration. But the cell membrane is **semipermeable**, meaning that some ions may cross it while others cannot. In its resting state, the cell membrane is permeable to K^+ and Cl^- , and impermeable to Na^+ . Diffusion of K^+ and Cl^- thus creates the layers of positive and negative charge on the outside and inside of the membrane. The Coulomb force prevents the ions from diffusing across in their entirety. Once the charge layer has built up, the repulsion of like charges prevents more from moving across, and the attraction of unlike charges prevents more from leaving either side. The result is two layers of charge right on the membrane, with diffusion being balanced by the Coulomb force. A tiny fraction of the charges move across and the fluids remain neutral (other ions are present), while a separation of charge and a voltage have been created across the membrane.



The semipermeable membrane of a

cell has different concentrations of ions inside and out. Diffusion moves the Na^+ and K^+ ions in the direction shown, until the Coulomb force halts further transfer. This results in a layer of positive charge on the outside, a layer of negative charge on the inside, and thus a voltage across the cell membrane.

The membrane is normally impermeable to K^+ ions.



An action potential is the pulse of voltage inside a nerve cell graphed here. It is caused by movements of ions across the cell membrane as shown. Depolarization occurs when a stimulus makes the membrane permeable to Na^+ ions. Repolarization follows as the membrane

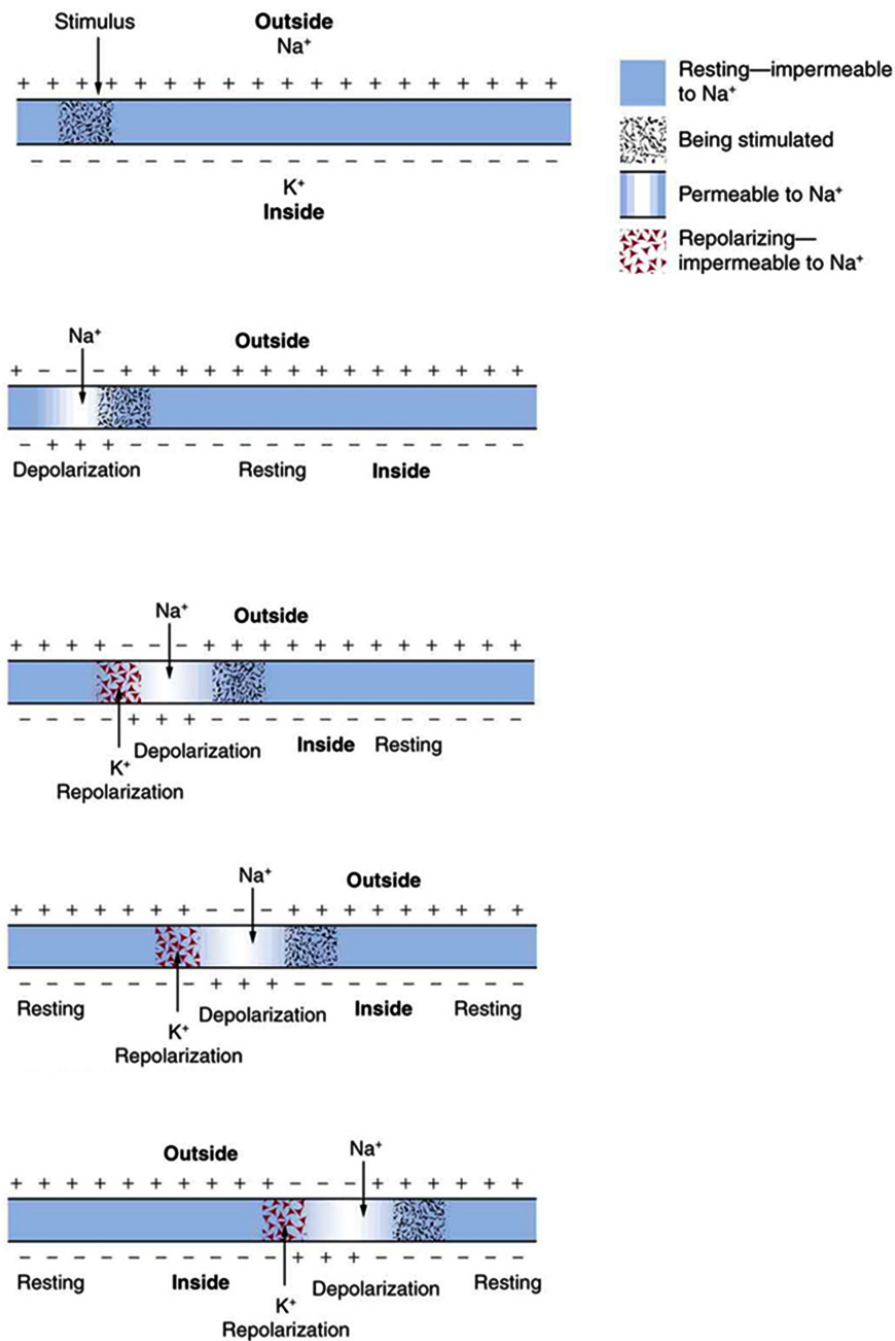
again becomes impermeable to K^+ and Na^+ moves from high to low concentration. In the long term, active transport slowly maintains the concentration differences, but the cell may fire hundreds of times in rapid succession without seriously depleting them.

The separation of charge creates a potential difference of 70 to 90 mV across the cell membrane. While this is a small voltage, the resulting electric field (10^6 V/m) across the only 8-nm-thick membrane is immense (on the order of 11 MV/m!) and has fundamental effects on its structure and permeability. Now, if the exterior of a neuron is taken to be at 0 V, then the interior has a *resting potential* of about -90 mV. Such voltages are created across the membranes of almost all types of animal cells but are largest in nerve and muscle cells. In fact, fully 25% of the energy used by cells goes toward creating and maintaining these potentials.

Electric currents along the cell membrane are created by any stimulus that changes the membrane's permeability. The membrane thus temporarily becomes permeable to Na^+ , which then rushes in, driven both by diffusion and the Coulomb force. This inrush of Na^+ first neutralizes the inside membrane, or *depolarizes* it, and then makes it slightly positive. The depolarization causes the membrane to again become impermeable to Na^+ , and the movement of K^+ quickly returns the cell to its resting potential, or *repolarizes* it. This sequence of events results in a voltage pulse, called the *action potential*. (See [\[link\]](#).) Only small fractions of the ions move, so that the cell can fire many hundreds of times without depleting the excess concentrations of Na^+ and K^+ . Eventually, the cell must replenish these ions to maintain the concentration differences that create bioelectricity. This sodium-potassium pump is an example of *active transport*, wherein cell energy is used to move ions across membranes against diffusion gradients and the Coulomb force.

The action potential is a voltage pulse at one location on a cell membrane. How does it get transmitted along the cell membrane, and in particular down an axon, as a nerve impulse? The answer is that the changing voltage and electric fields affect the permeability of the adjacent cell membrane, so

that the same process takes place there. The adjacent membrane depolarizes, affecting the membrane further down, and so on, as illustrated in [\[link\]](#). Thus the action potential stimulated at one location triggers a *nerve impulse* that moves slowly (about 1 m/s) along the cell membrane.



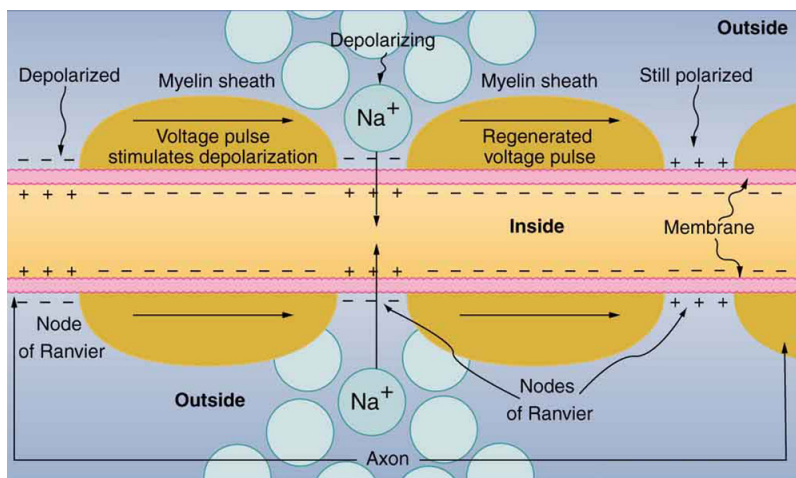
A nerve impulse is the propagation of an action potential along a cell membrane. A stimulus causes an action potential at one location, which changes the permeability of the adjacent membrane, causing an action potential there. This in turn affects the membrane further down, so that the action potential moves slowly (in electrical terms) along the cell membrane. Although the impulse is due to and going across the membrane, it is equivalent to a wave of charge moving along the outside and inside of the membrane.

Some axons, like that in [\[link\]](#), are sheathed with *myelin*, consisting of fat-containing cells. [\[link\]](#) shows an enlarged view of an axon having myelin sheaths characteristically separated by unmyelinated gaps (called nodes of Ranvier). This arrangement gives the axon a number of interesting properties. Since myelin is an insulator, it prevents signals from jumping between adjacent nerves (cross talk). Additionally, the myelinated regions transmit electrical signals at a very high speed, as an ordinary conductor or resistor would. There is no action potential in the myelinated regions, so that no cell energy is used in them. There is an signal loss in the myelin, but the signal is regenerated in the gaps, where the voltage pulse triggers the action potential at full voltage. So a myelinated axon transmits a nerve impulse faster, with less energy consumption, and is better protected from cross talk than an unmyelinated one. Not all axons are myelinated, so that cross talk and slow signal transmission are a characteristic of the normal operation of these axons, another variable in the nervous system.

The degeneration or destruction of the myelin sheaths that surround the nerve fibers impairs signal transmission and can lead to numerous neurological effects. One of the most prominent of these diseases comes from the body's own immune system attacking the myelin in the central nervous system—multiple sclerosis. MS symptoms include fatigue, vision problems, weakness of arms and legs, loss of balance, and tingling or

numbness in one's extremities (neuropathy). It is more apt to strike younger adults, especially females. Causes might come from infection, environmental or geographic affects, or genetics. At the moment there is no known cure for MS.

Most animal cells can fire or create their own action potential. Muscle cells contract when they fire and are often induced to do so by a nerve impulse. In fact, nerve and muscle cells are physiologically similar, and there are even hybrid cells, such as in the heart, that have characteristics of both nerves and muscles. Some animals, like the infamous electric eel (see [\[link\]](#)), use muscles ganged so that their voltages add in order to create a shock great enough to stun prey.



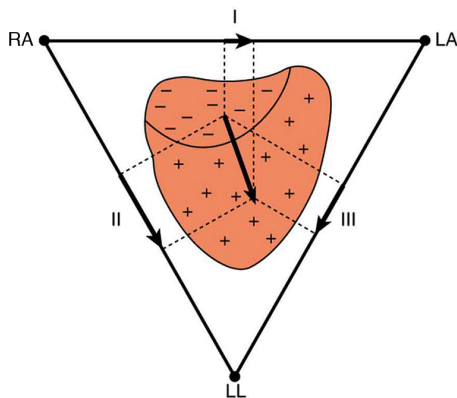
Propagation of a nerve impulse down a myelinated axon, from left to right. The signal travels very fast and without energy input in the myelinated regions, but it loses voltage. It is regenerated in the gaps. The signal moves faster than in unmyelinated axons and is insulated from signals in other nerves, limiting cross talk.



An electric eel flexes its muscles to create a voltage that stuns prey.
(credit: chrisbb, Flickr)

Electrocardiograms

Just as nerve impulses are transmitted by depolarization and repolarization of adjacent membrane, the depolarization that causes muscle contraction can also stimulate adjacent muscle cells to depolarize (fire) and contract. Thus, a depolarization wave can be sent across the heart, coordinating its rhythmic contractions and enabling it to perform its vital function of propelling blood through the circulatory system. [\[link\]](#) is a simplified graphic of a depolarization wave spreading across the heart from the *sinoarterial (SA) node*, the heart's natural pacemaker.



The outer surface of the heart changes from positive to negative during depolarization.

This wave of depolarization is spreading from the top of the heart and is represented by a vector pointing in the direction of the wave.

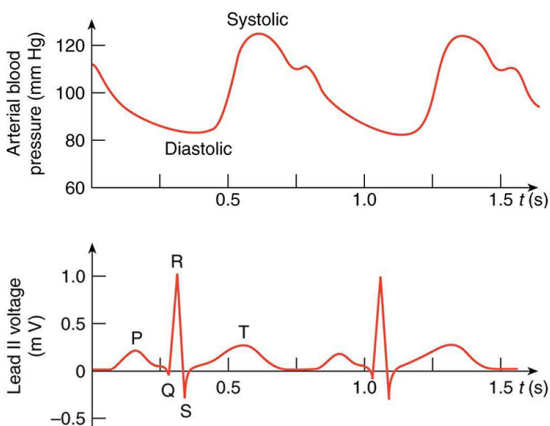
This vector is a voltage (potential difference) vector.

Three electrodes, labeled RA, LA, and LL, are placed on the patient. Each pair (called leads I, II, and III) measures a component of the depolarization vector and is graphed in an ECG.

An **electrocardiogram (ECG)** is a record of the voltages created by the wave of depolarization and subsequent repolarization in the heart. Voltages between pairs of electrodes placed on the chest are vector components of the voltage wave on the heart. Standard ECGs have 12 or more electrodes, but only three are shown in [\[link\]](#) for clarity. Decades ago, three-electrode ECGs were performed by placing electrodes on the left and right arms and the left leg. The voltage between the right arm and the left leg is called the *lead II potential* and is the most often graphed. We shall examine the lead II potential as an indicator of heart-muscle function and see that it is coordinated with arterial blood pressure as well.

Heart function and its four-chamber action are explored in [Viscosity and Laminar Flow; Poiseuille's Law](#). Basically, the right and left atria receive blood from the body and lungs, respectively, and pump the blood into the ventricles. The right and left ventricles, in turn, pump blood through the lungs and the rest of the body, respectively. Depolarization of the heart muscle causes it to contract. After contraction it is repolarized to ready it for the next beat. The ECG measures components of depolarization and repolarization of the heart muscle and can yield significant information on the functioning and malfunctioning of the heart.

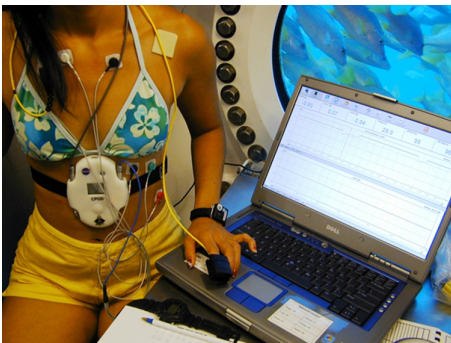
[\[link\]](#) shows an ECG of the lead II potential and a graph of the corresponding arterial blood pressure. The major features are labeled P, Q, R, S, and T. The *P wave* is generated by the depolarization and contraction of the atria as they pump blood into the ventricles. The *QRS complex* is created by the depolarization of the ventricles as they pump blood to the lungs and body. Since the shape of the heart and the path of the depolarization wave are not simple, the QRS complex has this typical shape and time span. The lead II QRS signal also masks the repolarization of the atria, which occur at the same time. Finally, the *T wave* is generated by the repolarization of the ventricles and is followed by the next P wave in the next heartbeat. Arterial blood pressure varies with each part of the heartbeat, with systolic (maximum) pressure occurring closely after the QRS complex, which signals contraction of the ventricles.



A lead II ECG with

corresponding arterial blood pressure. The QRS complex is created by the depolarization and contraction of the ventricles and is followed shortly by the maximum or systolic blood pressure. See text for further description.

Taken together, the 12 leads of a state-of-the-art ECG can yield a wealth of information about the heart. For example, regions of damaged heart tissue, called infarcts, reflect electrical waves and are apparent in one or more lead potentials. Subtle changes due to slight or gradual damage to the heart are most readily detected by comparing a recent ECG to an older one. This is particularly the case since individual heart shape, size, and orientation can cause variations in ECGs from one individual to another. ECG technology has advanced to the point where a portable ECG monitor with a liquid crystal instant display and a printer can be carried to patients' homes or used in emergency vehicles. See [\[link\]](#).



This NASA scientist and NEEMO 5 aquanaut's heart rate and other vital signs

are being recorded by
a portable device
while living in an
underwater habitat.
(credit: NASA, Life
Sciences Data Archive
at Johnson Space
Center, Houston,
Texas)

Note:

PhET Explorations: Neuron

Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

https://phet.colorado.edu/sims/html/neuron/latest/neuron_en.html

Section Summary

- Electric potentials in neurons and other cells are created by ionic concentration differences across semipermeable membranes.
- Stimuli change the permeability and create action potentials that propagate along neurons.
- Myelin sheaths speed this process and reduce the needed energy input.
- This process in the heart can be measured with an electrocardiogram (ECG).

Conceptual Questions

Exercise:

Problem:

Note that in [\[link\]](#), both the concentration gradient and the Coulomb force tend to move K^+ ions into the cell. What prevents this?

Exercise:**Problem:**

Define depolarization, repolarization, and the action potential.

Exercise:**Problem:**

Explain the properties of myelinated nerves in terms of the insulating properties of myelin.

Problems & Exercises**Exercise:****Problem: Integrated Concepts**

Use the ECG in [\[link\]](#) to determine the heart rate in beats per minute assuming a constant time between beats.

Solution:

80 beats/minute

Exercise:**Problem: Integrated Concepts**

(a) Referring to [\[link\]](#), find the time systolic pressure lags behind the middle of the QRS complex. (b) Discuss the reasons for the time lag.

Glossary

nerve conduction

the transport of electrical signals by nerve cells

bioelectricity

electrical effects in and created by biological systems

semipermeable

property of a membrane that allows only certain types of ions to cross it

electrocardiogram (ECG)

usually abbreviated ECG, a record of voltages created by depolarization and repolarization, especially in the heart

Introduction to Circuits and DC Instruments

class="introduction"

Electric
circuits in
a
computer
allow
large
amounts
of data to
be
quickly
and
accuratel
y
analyzed..
(credit:
Airman
1st Class
Mike
Meares,
United
States Air
Force)



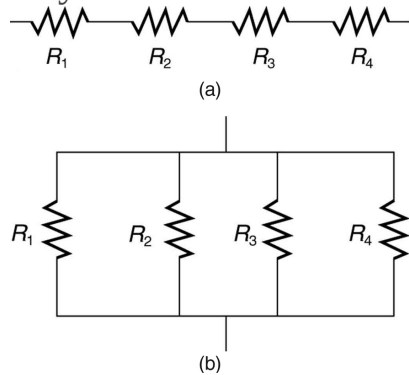
Electric circuits are commonplace. Some are simple, such as those in flashlights. Others, such as those used in supercomputers, are extremely complex.

This collection of modules takes the topic of electric circuits a step beyond simple circuits. When the circuit is purely resistive, everything in this module applies to both DC and AC. Matters become more complex when capacitance is involved. We do consider what happens when capacitors are connected to DC voltage sources, but the interaction of capacitors and other nonresistive devices with AC is left for a later chapter. Finally, a number of important DC instruments, such as meters that measure voltage and current, are covered in this chapter.

Resistors in Series and Parallel

- Draw a circuit with resistors in parallel and in series.
- Calculate the voltage drop of a current across a resistor using Ohm's law.
- Contrast the way total resistance is calculated for resistors in series and in parallel.
- Explain why total resistance of a parallel circuit is less than the smallest resistance of any of the resistors in that circuit.
- Calculate total resistance of a circuit that contains a mixture of resistors connected in series and in parallel.

Most circuits have more than one component, called a **resistor** that limits the flow of charge in the circuit. A measure of this limit on charge flow is called **resistance**. The simplest combinations of resistors are the series and parallel connections illustrated in [\[link\]](#). The total resistance of a combination of resistors depends on both their individual values and how they are connected.

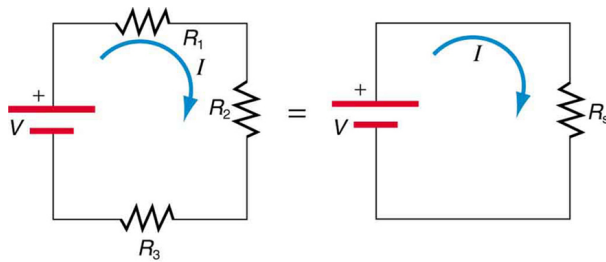


(a) A series connection of resistors. (b) A parallel connection of resistors.

Resistors in Series

When are resistors in **series**? Resistors are in series whenever the flow of charge, called the **current**, must flow through devices sequentially. For example, if current flows through a person holding a screwdriver and into the Earth, then R_1 in [\[link\]](#)(a) could be the resistance of the screwdriver's shaft, R_2 the resistance of its handle, R_3 the person's body resistance, and R_4 the resistance of her shoes.

[\[link\]](#) shows resistors in series connected to a **voltage** source. It seems reasonable that the total resistance is the sum of the individual resistances, considering that the current has to pass through each resistor in sequence. (This fact would be an advantage to a person wishing to avoid an electrical shock, who could reduce the current by wearing high-resistance rubber-soled shoes. It could be a disadvantage if one of the resistances were a faulty high-resistance cord to an appliance that would reduce the operating current.)



Three resistors connected in series to a battery (left) and the equivalent single or series resistance (right).

To verify that resistances in series do indeed add, let us consider the loss of electrical power, called a **voltage drop**, in each resistor in [\[link\]](#).

According to **Ohm's law**, the voltage drop, V , across a resistor when a current flows through it is calculated using the equation $V = IR$, where I equals the current in amps (A) and R is the resistance in ohms (Ω). Another

way to think of this is that V is the voltage necessary to make a current I flow through a resistance R .

So the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the source; that is,

Equation:

$$V = V_1 + V_2 + V_3.$$

This equation is based on the conservation of energy and conservation of charge. Electrical potential energy can be described by the equation $PE = qV$, where q is the electric charge and V is the voltage. Thus the energy supplied by the source is qV , while that dissipated by the resistors is

Equation:

$$qV_1 + qV_2 + qV_3.$$

Note:

Connections: Conservation Laws

The derivations of the expressions for series and parallel resistance are based on the laws of conservation of energy and conservation of charge, which state that total charge and total energy are constant in any process. These two laws are directly involved in all electrical phenomena and will be invoked repeatedly to explain both specific effects and the general behavior of electricity.

These energies must be equal, because there is no other source and no other destination for energy in the circuit. Thus, $qV = qV_1 + qV_2 + qV_3$. The charge q cancels, yielding $V = V_1 + V_2 + V_3$, as stated. (Note that the same amount of charge passes through the battery and each resistor in a given amount of time, since there is no capacitance to store charge, there is no place for charge to leak, and charge is conserved.)

Now substituting the values for the individual voltages gives

Equation:

$$V = IR_1 + IR_2 + IR_3 = I(R_1 + R_2 + R_3).$$

Note that for the equivalent single series resistance R_s , we have

Equation:

$$V = IR_s.$$

This implies that the total or equivalent series resistance R_s of three resistors is $R_s = R_1 + R_2 + R_3$.

This logic is valid in general for any number of resistors in series; thus, the total resistance R_s of a series connection is

Equation:

$$R_s = R_1 + R_2 + R_3 + \dots,$$

as proposed. Since all of the current must pass through each resistor, it experiences the resistance of each, and resistances in series simply add up.

Example:

Calculating Resistance, Current, Voltage Drop, and Power

Dissipation: Analysis of a Series Circuit

Suppose the voltage output of the battery in [\[link\]](#) is 12.0 V, and the resistances are $R_1 = 1.00 \, \Omega$, $R_2 = 6.00 \, \Omega$, and $R_3 = 13.0 \, \Omega$. (a) What is the total resistance? (b) Find the current. (c) Calculate the voltage drop in each resistor, and show these add to equal the voltage output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance is simply the sum of the individual resistances, as given by this equation:

Equation:

$$\begin{aligned}R_s &= R_1 + R_2 + R_3 \\&= 1.00\ \Omega + 6.00\ \Omega + 13.0\ \Omega \\&= 20.0\ \Omega.\end{aligned}$$

Strategy and Solution for (b)

The current is found using Ohm's law, $V = IR$. Entering the value of the applied voltage and the total resistance yields the current for the circuit:

Equation:

$$I = \frac{V}{R_s} = \frac{12.0\ \text{V}}{20.0\ \Omega} = 0.600\ \text{A}.$$

Strategy and Solution for (c)

The voltage—or IR drop—in a resistor is given by Ohm's law. Entering the current and the value of the first resistance yields

Equation:

$$V_1 = IR_1 = (0.600\ \text{A})(1.0\ \Omega) = 0.600\ \text{V}.$$

Similarly,

Equation:

$$V_2 = IR_2 = (0.600\ \text{A})(6.0\ \Omega) = 3.60\ \text{V}$$

and

Equation:

$$V_3 = IR_3 = (0.600\ \text{A})(13.0\ \Omega) = 7.80\ \text{V}.$$

Discussion for (c)

The three IR drops add to 12.0 V, as predicted:

Equation:

$$V_1 + V_2 + V_3 = (0.600 + 3.60 + 7.80)\ \text{V} = 12.0\ \text{V}.$$

Strategy and Solution for (d)

The easiest way to calculate power in watts (W) dissipated by a resistor in a DC circuit is to use **Joule's law**, $P = IV$, where P is electric power. In this case, each resistor has the same full current flowing through it. By substituting Ohm's law $V = IR$ into Joule's law, we get the power dissipated by the first resistor as

Equation:

$$P_1 = I^2 R_1 = (0.600 \text{ A})^2 (1.00 \Omega) = 0.360 \text{ W}.$$

Similarly,

Equation:

$$P_2 = I^2 R_2 = (0.600 \text{ A})^2 (6.00 \Omega) = 2.16 \text{ W}$$

and

Equation:

$$P_3 = I^2 R_3 = (0.600 \text{ A})^2 (13.0 \Omega) = 4.68 \text{ W}.$$

Discussion for (d)

Power can also be calculated using either $P = IV$ or $P = \frac{V^2}{R}$, where V is the voltage drop across the resistor (not the full voltage of the source). The same values will be obtained.

Strategy and Solution for (e)

The easiest way to calculate power output of the source is to use $P = IV$, where V is the source voltage. This gives

Equation:

$$P = (0.600 \text{ A})(12.0 \text{ V}) = 7.20 \text{ W}.$$

Discussion for (e)

Note, coincidentally, that the total power dissipated by the resistors is also 7.20 W, the same as the power put out by the source. That is,

Equation:

$$P_1 + P_2 + P_3 = (0.360 + 2.16 + 4.68) \text{ W} = 7.20 \text{ W}.$$

Power is energy per unit time (watts), and so conservation of energy requires the power output of the source to be equal to the total power dissipated by the resistors.

Note:

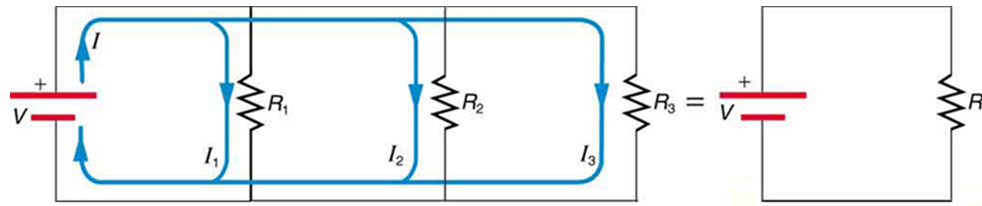
Major Features of Resistors in Series

1. Series resistances add: $R_s = R_1 + R_2 + R_3 + \dots$
2. The same current flows through each resistor in series.
3. Individual resistors in series do not get the total source voltage, but divide it.

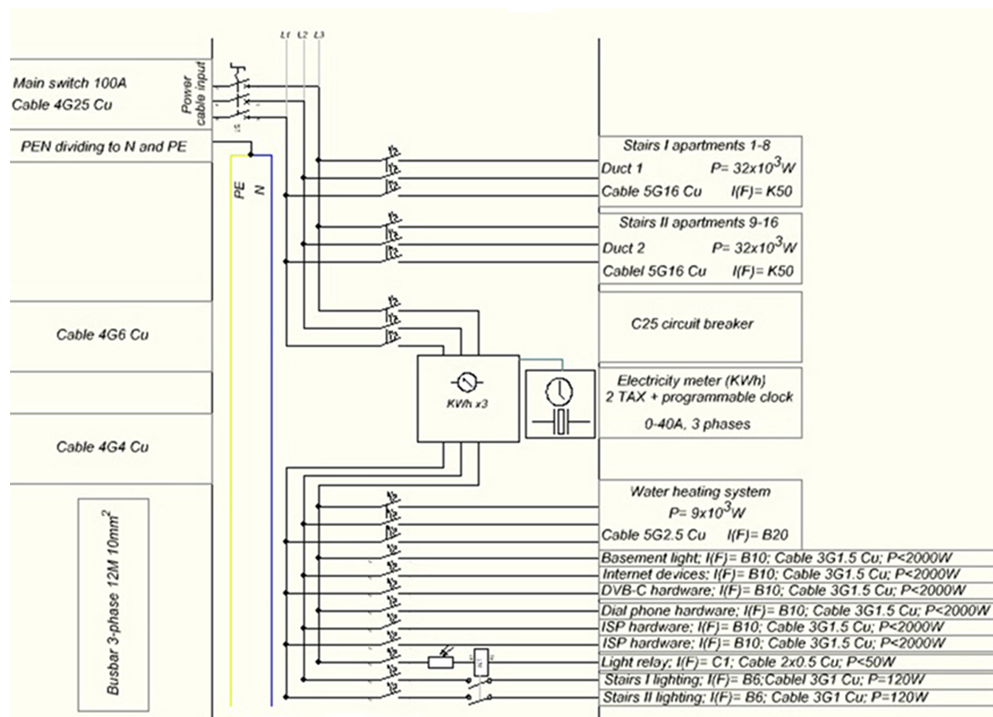
Resistors in Parallel

[\[link\]](#) shows resistors in **parallel**, wired to a voltage source. Resistors are in parallel when each resistor is connected directly to the voltage source by connecting wires having negligible resistance. Each resistor thus has the full voltage of the source applied to it.

Each resistor draws the same current it would if it alone were connected to the voltage source (provided the voltage source is not overloaded). For example, an automobile's headlights, radio, and so on, are wired in parallel, so that they utilize the full voltage of the source and can operate completely independently. The same is true in your house, or any building. (See [\[link\]](#) (b).)



(a)



(b)

(a) Three resistors connected in parallel to a battery and the equivalent single or parallel resistance. (b) Electrical power setup in a house. (credit: Dmitry G, Wikimedia Commons)

To find an expression for the equivalent parallel resistance R_p , let us consider the currents that flow and how they are related to resistance. Since each resistor in the circuit has the full voltage, the currents flowing through the individual resistors are $I_1 = \frac{V}{R_1}$, $I_2 = \frac{V}{R_2}$, and $I_3 = \frac{V}{R_3}$. Conservation of charge implies that the total current I produced by the source is the sum of these currents:

Equation:

$$I = I_1 + I_2 + I_3.$$

Substituting the expressions for the individual currents gives

Equation:

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right).$$

Note that Ohm's law for the equivalent single resistance gives

Equation:

$$I = \frac{V}{R_p} = V \left(\frac{1}{R_p} \right).$$

The terms inside the parentheses in the last two equations must be equal. Generalizing to any number of resistors, the total resistance R_p of a parallel connection is related to the individual resistances by

Equation:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

This relationship results in a total resistance R_p that is less than the smallest of the individual resistances. (This is seen in the next example.) When resistors are connected in parallel, more current flows from the source than would flow for any of them individually, and so the total resistance is lower.

Example:

Calculating Resistance, Current, Power Dissipation, and Power Output: Analysis of a Parallel Circuit

Let the voltage output of the battery and resistances in the parallel connection in [\[link\]](#) be the same as the previously considered series

connection: $V = 12.0 \text{ V}$, $R_1 = 1.00 \text{ } \Omega$, $R_2 = 6.00 \text{ } \Omega$, and $R_3 = 13.0 \text{ } \Omega$.

(a) What is the total resistance? (b) Find the total current. (c) Calculate the currents in each resistor, and show these add to equal the total current output of the source. (d) Calculate the power dissipated by each resistor. (e) Find the power output of the source, and show that it equals the total power dissipated by the resistors.

Strategy and Solution for (a)

The total resistance for a parallel combination of resistors is found using the equation below. Entering known values gives

Equation:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{1.00 \text{ } \Omega} + \frac{1}{6.00 \text{ } \Omega} + \frac{1}{13.0 \text{ } \Omega}.$$

Thus,

Equation:

$$\frac{1}{R_p} = \frac{1.00}{\Omega} + \frac{0.1667}{\Omega} + \frac{0.07692}{\Omega} = \frac{1.2436}{\Omega}.$$

(Note that in these calculations, each intermediate answer is shown with an extra digit.)

We must invert this to find the total resistance R_p . This yields

Equation:

$$R_p = \frac{1}{1.2436} \Omega = 0.8041 \text{ } \Omega.$$

The total resistance with the correct number of significant digits is $R_p = 0.804 \text{ } \Omega$.

Discussion for (a)

R_p is, as predicted, less than the smallest individual resistance.

Strategy and Solution for (b)

The total current can be found from Ohm's law, substituting R_p for the total resistance. This gives

Equation:

$$I = \frac{V}{R_p} = \frac{12.0 \text{ V}}{0.8041 \Omega} = 14.92 \text{ A.}$$

Discussion for (b)

Current I for each device is much larger than for the same devices connected in series (see the previous example). A circuit with parallel connections has a smaller total resistance than the resistors connected in series.

Strategy and Solution for (c)

The individual currents are easily calculated from Ohm's law, since each resistor gets the full voltage. Thus,

Equation:

$$I_1 = \frac{V}{R_1} = \frac{12.0 \text{ V}}{1.00 \Omega} = 12.0 \text{ A.}$$

Similarly,

Equation:

$$I_2 = \frac{V}{R_2} = \frac{12.0 \text{ V}}{6.00 \Omega} = 2.00 \text{ A}$$

and

Equation:

$$I_3 = \frac{V}{R_3} = \frac{12.0 \text{ V}}{13.0 \Omega} = 0.92 \text{ A.}$$

Discussion for (c)

The total current is the sum of the individual currents:

Equation:

$$I_1 + I_2 + I_3 = 14.92 \text{ A.}$$

This is consistent with conservation of charge.

Strategy and Solution for (d)

The power dissipated by each resistor can be found using any of the equations relating power to current, voltage, and resistance, since all three

are known. Let us use $P = \frac{V^2}{R}$, since each resistor gets full voltage. Thus,
Equation:

$$P_1 = \frac{V^2}{R_1} = \frac{(12.0 \text{ V})^2}{1.00 \, \Omega} = 144 \text{ W}.$$

Similarly,
Equation:

$$P_2 = \frac{V^2}{R_2} = \frac{(12.0 \text{ V})^2}{6.00 \, \Omega} = 24.0 \text{ W}$$

and
Equation:

$$P_3 = \frac{V^2}{R_3} = \frac{(12.0 \text{ V})^2}{13.0 \, \Omega} = 11.1 \text{ W}.$$

Discussion for (d)

The power dissipated by each resistor is considerably higher in parallel than when connected in series to the same voltage source.

Strategy and Solution for (e)

The total power can also be calculated in several ways. Choosing $P = IV$, and entering the total current, yields

Equation:

$$P = IV = (14.92 \text{ A})(12.0 \text{ V}) = 179 \text{ W}.$$

Discussion for (e)

Total power dissipated by the resistors is also 179 W:

Equation:

$$P_1 + P_2 + P_3 = 144 \text{ W} + 24.0 \text{ W} + 11.1 \text{ W} = 179 \text{ W}.$$

This is consistent with the law of conservation of energy.

Overall Discussion

Note that both the currents and powers in parallel connections are greater than for the same devices in series.

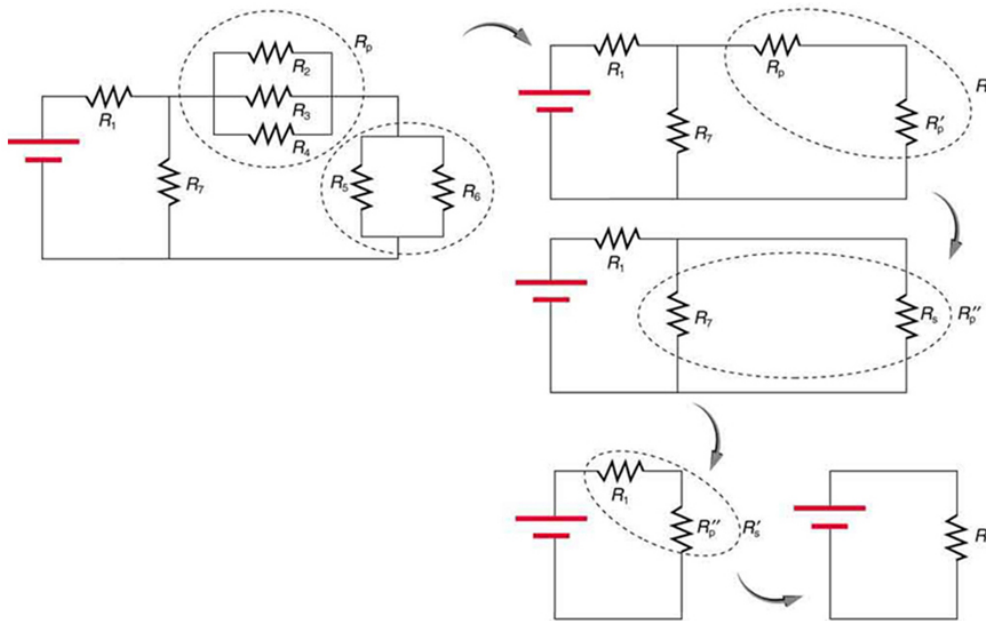
Note:**Major Features of Resistors in Parallel**

1. Parallel resistance is found from $\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$, and it is smaller than any individual resistance in the combination.
2. Each resistor in parallel has the same full voltage of the source applied to it. (Power distribution systems most often use parallel connections to supply the myriad devices served with the same voltage and to allow them to operate independently.)
3. Parallel resistors do not each get the total current; they divide it.

Combinations of Series and Parallel

More complex connections of resistors are sometimes just combinations of series and parallel. These are commonly encountered, especially when wire resistance is considered. In that case, wire resistance is in series with other resistances that are in parallel.

Combinations of series and parallel can be reduced to a single equivalent resistance using the technique illustrated in [\[link\]](#). Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.



This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

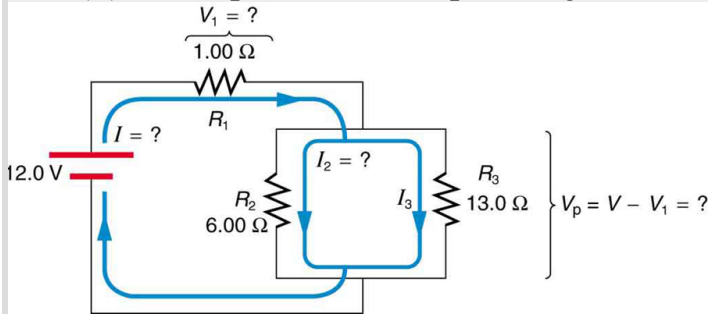
The simplest combination of series and parallel resistance, shown in [\[link\]](#), is also the most instructive, since it is found in many applications. For example, R_1 could be the resistance of wires from a car battery to its electrical devices, which are in parallel. R_2 and R_3 could be the starter motor and a passenger compartment light. We have previously assumed that wire resistance is negligible, but, when it is not, it has important effects, as the next example indicates.

Example:

Calculating Resistance, IR Drop, Current, and Power Dissipation: Combining Series and Parallel Circuits

[\[link\]](#) shows the resistors from the previous two examples wired in a different way—a combination of series and parallel. We can consider R_1 to be the resistance of wires leading to R_2 and R_3 . (a) Find the total

resistance. (b) What is the IR drop in R_1 ? (c) Find the current I_2 through R_2 . (d) What power is dissipated by R_2 ?



These three resistors are connected to a voltage source so that R_2 and R_3 are in parallel with one another and that combination is in series with R_1 .

Strategy and Solution for (a)

To find the total resistance, we note that R_2 and R_3 are in parallel and their combination R_p is in series with R_1 . Thus the total (equivalent) resistance of this combination is

Equation:

$$R_{\text{tot}} = R_1 + R_p.$$

First, we find R_p using the equation for resistors in parallel and entering known values:

Equation:

$$\frac{1}{R_p} = \frac{1}{R_2} + \frac{1}{R_3} = \frac{1}{6.00 \, \Omega} + \frac{1}{13.0 \, \Omega} = \frac{0.2436}{\Omega}.$$

Inverting gives

Equation:

$$R_p = \frac{1}{0.2436} \, \Omega = 4.11 \, \Omega.$$

So the total resistance is

Equation:

$$R_{\text{tot}} = R_1 + R_p = 1.00 \, \Omega + 4.11 \, \Omega = 5.11 \, \Omega.$$

Discussion for (a)

The total resistance of this combination is intermediate between the pure series and pure parallel values ($20.0 \, \Omega$ and $0.804 \, \Omega$, respectively) found for the same resistors in the two previous examples.

Strategy and Solution for (b)

To find the IR drop in R_1 , we note that the full current I flows through R_1 . Thus its IR drop is

Equation:

$$V_1 = IR_1.$$

We must find I before we can calculate V_1 . The total current I is found using Ohm's law for the circuit. That is,

Equation:

$$I = \frac{V}{R_{\text{tot}}} = \frac{12.0 \, \text{V}}{5.11 \, \Omega} = 2.35 \, \text{A}.$$

Entering this into the expression above, we get

Equation:

$$V_1 = IR_1 = (2.35 \, \text{A})(1.00 \, \Omega) = 2.35 \, \text{V}.$$

Discussion for (b)

The voltage applied to R_2 and R_3 is less than the total voltage by an amount V_1 . When wire resistance is large, it can significantly affect the operation of the devices represented by R_2 and R_3 .

Strategy and Solution for (c)

To find the current through R_2 , we must first find the voltage applied to it. We call this voltage V_p , because it is applied to a parallel combination of resistors. The voltage applied to both R_2 and R_3 is reduced by the amount V_1 , and so it is

Equation:

$$V_p = V - V_1 = 12.0 \text{ V} - 2.35 \text{ V} = 9.65 \text{ V}.$$

Now the current I_2 through resistance R_2 is found using Ohm's law:

Equation:

$$I_2 = \frac{V_p}{R_2} = \frac{9.65 \text{ V}}{6.00 \Omega} = 1.61 \text{ A}.$$

Discussion for (c)

The current is less than the 2.00 A that flowed through R_2 when it was connected in parallel to the battery in the previous parallel circuit example.

Strategy and Solution for (d)

The power dissipated by R_2 is given by

Equation:

$$P_2 = (I_2)^2 R_2 = (1.61 \text{ A})^2 (6.00 \Omega) = 15.5 \text{ W}.$$

Discussion for (d)

The power is less than the 24.0 W this resistor dissipated when connected in parallel to the 12.0-V source.

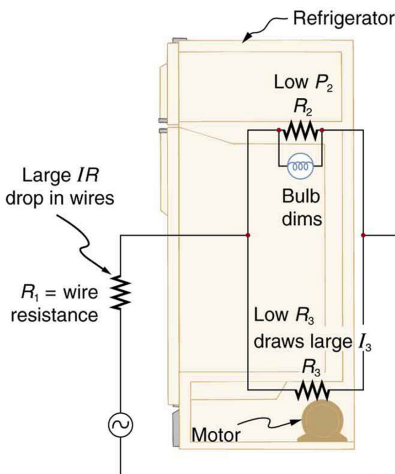
Practical Implications

One implication of this last example is that resistance in wires reduces the current and power delivered to a resistor. If wire resistance is relatively large, as in a worn (or a very long) extension cord, then this loss can be significant. If a large current is drawn, the IR drop in the wires can also be significant.

For example, when you are rummaging in the refrigerator and the motor comes on, the refrigerator light dims momentarily. Similarly, you can see the passenger compartment light dim when you start the engine of your car (although this may be due to resistance inside the battery itself).

What is happening in these high-current situations is illustrated in [\[link\]](#). The device represented by R_3 has a very low resistance, and so when it is

switched on, a large current flows. This increased current causes a larger IR drop in the wires represented by R_1 , reducing the voltage across the light bulb (which is R_2), which then dims noticeably.



Why do lights dim when a large appliance is switched on? The answer is that the large current the appliance motor draws causes a significant IR drop in the wires and reduces the voltage across the light.

Exercise:
Check Your Understanding

Problem:

Can any arbitrary combination of resistors be broken down into series and parallel combinations? See if you can draw a circuit diagram of resistors that cannot be broken down into combinations of series and parallel.

Solution:

No, there are many ways to connect resistors that are not combinations of series and parallel, including loops and junctions. In such cases Kirchhoff's rules, to be introduced in [Kirchhoff's Rules](#), will allow you to analyze the circuit.

Note:**Problem-Solving Strategies for Series and Parallel Resistors**

1. Draw a clear circuit diagram, labeling all resistors and voltage sources. This step includes a list of the knowns for the problem, since they are labeled in your circuit diagram.
2. Identify exactly what needs to be determined in the problem (identify the unknowns). A written list is useful.
3. Determine whether resistors are in series, parallel, or a combination of both series and parallel. Examine the circuit diagram to make this assessment. Resistors are in series if the same current must pass sequentially through them.
4. Use the appropriate list of major features for series or parallel connections to solve for the unknowns. There is one list for series and another for parallel. If your problem has a combination of series and parallel, reduce it in steps by considering individual groups of series or parallel connections, as done in this module and the examples. Special note: When finding R_p , the reciprocal must be taken with care.
5. Check to see whether the answers are reasonable and consistent. Units and numerical results must be reasonable. Total series resistance

should be greater, whereas total parallel resistance should be smaller, for example. Power should be greater for the same devices in parallel compared with series, and so on.

Section Summary

- The total resistance of an electrical circuit with resistors wired in a series is the sum of the individual resistances:
 $R_s = R_1 + R_2 + R_3 + \dots$
- Each resistor in a series circuit has the same amount of current flowing through it.
- The voltage drop, or power dissipation, across each individual resistor in a series is different, and their combined total adds up to the power source input.
- The total resistance of an electrical circuit with resistors wired in parallel is less than the lowest resistance of any of the components and can be determined using the formula:

Equation:

$$\frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \dots$$

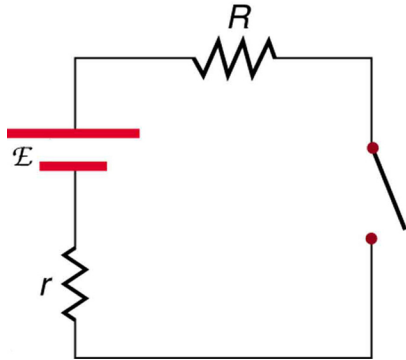
- Each resistor in a parallel circuit has the same full voltage of the source applied to it.
- The current flowing through each resistor in a parallel circuit is different, depending on the resistance.
- If a more complex connection of resistors is a combination of series and parallel, it can be reduced to a single equivalent resistance by identifying its various parts as series or parallel, reducing each to its equivalent, and continuing until a single resistance is eventually reached.

Conceptual Questions

Exercise:

Problem:

A switch has a variable resistance that is nearly zero when closed and extremely large when open, and it is placed in series with the device it controls. Explain the effect the switch in [\[link\]](#) has on current when open and when closed.



A switch is ordinarily in series with a resistance and voltage source. Ideally, the switch has nearly zero resistance when closed but has an extremely large resistance when open. (Note that in this diagram, the script E represents the voltage (or electromotive force) of the battery.)

Exercise:

Problem: What is the voltage across the open switch in [\[link\]](#)?

Exercise:

Problem:

There is a voltage across an open switch, such as in [\[link\]](#). Why, then, is the power dissipated by the open switch small?

Exercise:

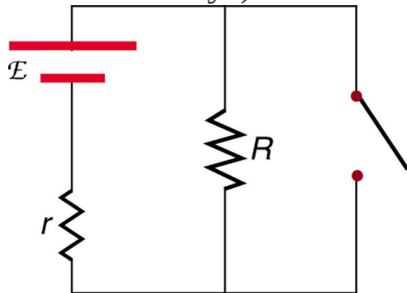
Problem:

Why is the power dissipated by a closed switch, such as in [\[link\]](#), small?

Exercise:

Problem:

A student in a physics lab mistakenly wired a light bulb, battery, and switch as shown in [\[link\]](#). Explain why the bulb is on when the switch is open, and off when the switch is closed. (Do not try this—it is hard on the battery!)



A wiring mistake put this switch in parallel with the device represented by R . (Note that in this diagram, the script E represents the voltage (or

electromotive
force) of the
battery.)

Exercise:

Problem:

Knowing that the severity of a shock depends on the magnitude of the current through your body, would you prefer to be in series or parallel with a resistance, such as the heating element of a toaster, if shocked by it? Explain.

Exercise:

Problem:

Would your headlights dim when you start your car's engine if the wires in your automobile were superconductors? (Do not neglect the battery's internal resistance.) Explain.

Exercise:

Problem:

Some strings of holiday lights are wired in series to save wiring costs. An old version utilized bulbs that break the electrical connection, like an open switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 40 identical bulbs, what is the normal operating voltage of each? Newer versions use bulbs that short circuit, like a closed switch, when they burn out. If one such bulb burns out, what happens to the others? If such a string operates on 120 V and has 39 remaining identical bulbs, what is then the operating voltage of each?

Exercise:

Problem:

If two household lightbulbs rated 60 W and 100 W are connected in series to household power, which will be brighter? Explain.

Exercise:**Problem:**

Suppose you are doing a physics lab that asks you to put a resistor into a circuit, but all the resistors supplied have a larger resistance than the requested value. How would you connect the available resistances to attempt to get the smaller value asked for?

Exercise:**Problem:**

Before World War II, some radios got power through a “resistance cord” that had a significant resistance. Such a resistance cord reduces the voltage to a desired level for the radio’s tubes and the like, and it saves the expense of a transformer. Explain why resistance cords become warm and waste energy when the radio is on.

Exercise:**Problem:**

Some light bulbs have three power settings (not including zero), obtained from multiple filaments that are individually switched and wired in parallel. What is the minimum number of filaments needed for three power settings?

Problem Exercises

Note: Data taken from figures can be assumed to be accurate to three significant digits.

Exercise:

Problem:

- (a) What is the resistance of ten $275\text{-}\Omega$ resistors connected in series?
(b) In parallel?
-

Solution:

- (a) $2.75\text{ k}\Omega$
(b) $27.5\text{ }\Omega$

Exercise:**Problem:**

- (a) What is the resistance of a $1.00 \times 10^2\text{-}\Omega$, a $2.50\text{-k}\Omega$, and a $4.00\text{-k}\Omega$ resistor connected in series? (b) In parallel?

Exercise:**Problem:**

What are the largest and smallest resistances you can obtain by connecting a $36.0\text{-}\Omega$, a $50.0\text{-}\Omega$, and a $700\text{-}\Omega$ resistor together?

Solution:

- (a) $786\text{ }\Omega$
(b) $20.3\text{ }\Omega$

Exercise:**Problem:**

An 1800-W toaster, a 1400-W electric frying pan, and a 75-W lamp are plugged into the same outlet in a 15-A , 120-V circuit. (The three devices are in parallel when plugged into the same socket.). (a) What current is drawn by each device? (b) Will this combination blow the 15-A fuse?

Exercise:

Problem:

Your car's 30.0-W headlight and 2.40-kW starter are ordinarily connected in parallel in a 12.0-V system. What power would one headlight and the starter consume if connected in series to a 12.0-V battery? (Neglect any other resistance in the circuit and any change in resistance in the two devices.)

Solution:

29.6 W

Exercise:**Problem:**

(a) Given a 48.0-V battery and 24.0- Ω and 96.0- Ω resistors, find the current and power for each when connected in series. (b) Repeat when the resistances are in parallel.

Exercise:**Problem:**

Referring to the example combining series and parallel circuits and [\[link\]](#), calculate I_3 in the following two different ways: (a) from the known values of I and I_2 ; (b) using Ohm's law for R_3 . In both parts explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).

Solution:

(a) 0.74 A

(b) 0.742 A

Exercise:

Problem:

Referring to [\[link\]](#): (a) Calculate P_3 and note how it compares with P_3 found in the first two example problems in this module. (b) Find the total power supplied by the source and compare it with the sum of the powers dissipated by the resistors.

Exercise:**Problem:**

Refer to [\[link\]](#) and the discussion of lights dimming when a heavy appliance comes on. (a) Given the voltage source is 120 V, the wire resistance is $0.400\ \Omega$, and the bulb is nominally 75.0 W, what power will the bulb dissipate if a total of 15.0 A passes through the wires when the motor comes on? Assume negligible change in bulb resistance. (b) What power is consumed by the motor?

Solution:

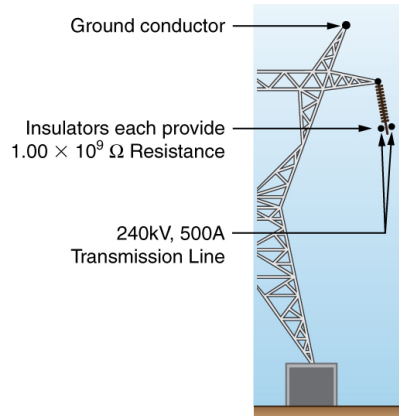
(a) 60.8 W

(b) 3.18 kW

Exercise:**Problem:**

A 240-kV power transmission line carrying $5.00 \times 10^2\ \text{A}$ is hung from grounded metal towers by ceramic insulators, each having a $1.00 \times 10^9\ \Omega$ resistance. [\[link\]](#). (a) What is the resistance to ground of 100 of these insulators? (b) Calculate the power dissipated by 100 of them. (c) What fraction of the power carried by the line is this?

Explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).



High-voltage (240-kV) transmission line carrying $5.00 \times 10^2 \text{ A}$ is hung from a grounded metal transmission tower. The row of ceramic insulators provide $1.00 \times 10^9 \Omega$ of resistance each.

Exercise:

Problem:

Show that if two resistors R_1 and R_2 are combined and one is much greater than the other ($R_1 \gg R_2$): (a) Their series resistance is very nearly equal to the greater resistance R_1 . (b) Their parallel resistance is very nearly equal to smaller resistance R_2 .

Solution:

$$R_s = R_1 + R_2$$

(a) $\Rightarrow R_s \approx R_1 (R_1 \gg R_2)$

$$(b) \frac{1}{R_p} = \frac{1}{R_1} + \frac{1}{R_2} = \frac{R_1 + R_2}{R_1 R_2},$$

so that

$$R_p = \frac{R_1 R_2}{R_1 + R_2} \approx \frac{R_1 R_2}{R_1} = R_2 (R_1 \gg R_2).$$

Exercise:

Problem: Unreasonable Results

Two resistors, one having a resistance of $145 \, \Omega$, are connected in parallel to produce a total resistance of $150 \, \Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Exercise:

Problem: Unreasonable Results

Two resistors, one having a resistance of $900 \, \text{k}\Omega$, are connected in series to produce a total resistance of $0.500 \, \text{M}\Omega$. (a) What is the value of the second resistance? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $-400 \, \text{k}\Omega$

(b) Resistance cannot be negative.

(c) Series resistance is said to be less than one of the resistors, but it must be greater than any of the resistors.

Glossary

series

a sequence of resistors or other components wired into a circuit one after the other

resistor

a component that provides resistance to the current flowing through an electrical circuit

resistance

causing a loss of electrical power in a circuit

Ohm's law

the relationship between current, voltage, and resistance within an electrical circuit: $V = IR$

voltage

the electrical potential energy per unit charge; electric pressure created by a power source, such as a battery

voltage drop

the loss of electrical power as a current travels through a resistor, wire or other component

current

the flow of charge through an electric circuit past a given point of measurement

Joule's law

the relationship between potential electrical power, voltage, and resistance in an electrical circuit, given by: $P_e = IV$

parallel

the wiring of resistors or other components in an electrical circuit such that each component receives an equal voltage from the power source; often pictured in a ladder-shaped diagram, with each component on a rung of the ladder

Electromotive Force: Terminal Voltage

- Compare and contrast the voltage and the electromagnetic force of an electric power source.
- Describe what happens to the terminal voltage, current, and power delivered to a load as internal resistance of the voltage source increases (due to aging of batteries, for example).
- Explain why it is beneficial to use more than one voltage source connected in parallel.

When you forget to turn off your car lights, they slowly dim as the battery runs down. Why don't they simply blink off when the battery's energy is gone? Their gradual dimming implies that battery output voltage decreases as the battery is depleted.

Furthermore, if you connect an excessive number of 12-V lights in parallel to a car battery, they will be dim even when the battery is fresh and even if the wires to the lights have very low resistance. This implies that the battery's output voltage is reduced by the overload.

The reason for the decrease in output voltage for depleted or overloaded batteries is that all voltage sources have two fundamental parts—a source of electrical energy and an **internal resistance**. Let us examine both.

Electromotive Force

You can think of many different types of voltage sources. Batteries themselves come in many varieties. There are many types of mechanical/electrical generators, driven by many different energy sources, ranging from nuclear to wind. Solar cells create voltages directly from light, while thermoelectric devices create voltage from temperature differences.

A few voltage sources are shown in [\[link\]](#). All such devices create a **potential difference** and can supply current if connected to a resistance. On the small scale, the potential difference creates an electric field that exerts force on charges, causing current. We thus use the name **electromotive force**, abbreviated emf.

Emf is not a force at all; it is a special type of potential difference. To be precise, the electromotive force (emf) is the potential difference of a source when no current is flowing. Units of emf are volts.



A variety of voltage sources (clockwise from top left): the Brazos Wind Farm in Fluvanna, Texas (credit: Leaflet, Wikimedia Commons); the Krasnoyarsk Dam in Russia (credit: Alex Polezhaev); a solar farm (credit: U.S. Department of Energy); and a group of nickel metal hydride batteries (credit: Tiaa Monto). The voltage output of each depends on its construction and load, and equals emf only if there is no load.

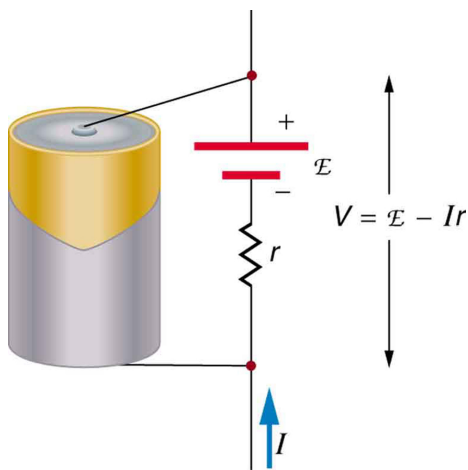
Electromotive force is directly related to the source of potential difference, such as the particular combination of chemicals in a battery. However, emf

differs from the voltage output of the device when current flows. The voltage across the terminals of a battery, for example, is less than the emf when the battery supplies current, and it declines further as the battery is depleted or loaded down. However, if the device's output voltage can be measured without drawing current, then output voltage will equal emf (even for a very depleted battery).

Internal Resistance

As noted before, a 12-V truck battery is physically larger, contains more charge and energy, and can deliver a larger current than a 12-V motorcycle battery. Both are lead-acid batteries with identical emf, but, because of its size, the truck battery has a smaller internal resistance r . Internal resistance is the inherent resistance to the flow of current within the source itself.

[\[link\]](#) is a schematic representation of the two fundamental parts of any voltage source. The emf (represented by a script $\mathcal{E}$ in the figure) and internal resistance r are in series. The smaller the internal resistance for a given emf, the more current and the more power the source can supply.



Any voltage source (in this case, a carbon-zinc dry cell) has an emf related to its source of

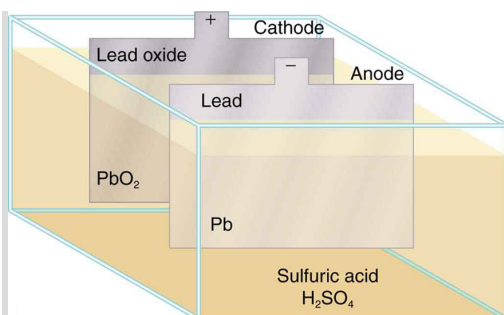
potential difference,
and an internal
resistance r related to
its construction. (Note
that the script E stands
for emf.). Also shown
are the output
terminals across which
the terminal voltage V
is measured. Since
 $V = \text{emf} - Ir$,
terminal voltage equals
emf only if there is no
current flowing.

The internal resistance r can behave in complex ways. As noted, r increases as a battery is depleted. But internal resistance may also depend on the magnitude and direction of the current through a voltage source, its temperature, and even its history. The internal resistance of rechargeable nickel-cadmium cells, for example, depends on how many times and how deeply they have been depleted.

Note:

Things Great and Small: The Submicroscopic Origin of Battery Potential
Various types of batteries are available, with emfs determined by the combination of chemicals involved. We can view this as a molecular reaction (what much of chemistry is about) that separates charge.

The lead-acid battery used in cars and other vehicles is one of the most common types. A single cell (one of six) of this battery is seen in [\[link\]](#). The cathode (positive) terminal of the cell is connected to a lead oxide plate, while the anode (negative) terminal is connected to a lead plate. Both plates are immersed in sulfuric acid, the electrolyte for the system.

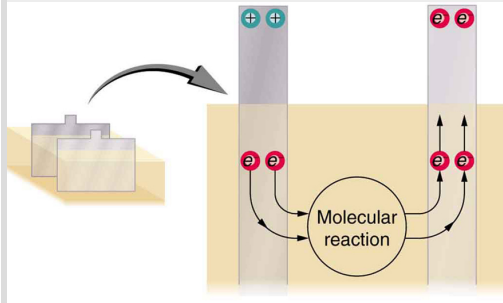


Artist's conception of a lead-acid cell. Chemical reactions in a lead-acid cell separate charge, sending negative charge to the anode, which is connected to the lead plates. The lead oxide plates are connected to the positive or cathode terminal of the cell. Sulfuric acid conducts the charge as well as participating in the chemical reaction.

The details of the chemical reaction are left to the reader to pursue in a chemistry text, but their results at the molecular level help explain the potential created by the battery. [\[link\]](#) shows the result of a single chemical reaction. Two electrons are placed on the anode, making it negative, provided that the cathode supplied two electrons. This leaves the cathode positively charged, because it has lost two electrons. In short, a separation of charge has been driven by a chemical reaction.

Note that the reaction will not take place unless there is a complete circuit to allow two electrons to be supplied to the cathode. Under many circumstances, these electrons come from the anode, flow through a resistance, and return to the cathode. Note also that since the chemical

reactions involve substances with resistance, it is not possible to create the emf without an internal resistance.



Artist's conception of two electrons being forced onto the anode of a cell and two electrons being removed from the cathode of the cell. The chemical reaction in a lead-acid battery places two electrons on the anode and removes two from the cathode. It requires a closed circuit to proceed, since the two electrons must be supplied to the cathode.

Why are the chemicals able to produce a unique potential difference? Quantum mechanical descriptions of molecules, which take into account the types of atoms and numbers of electrons in them, are able to predict the energy states they can have and the energies of reactions between them.

In the case of a lead-acid battery, an energy of 2 eV is given to each electron sent to the anode. Voltage is defined as the electrical potential

energy divided by charge: $V = \frac{P_E}{q}$. An electron volt is the energy given to a single electron by a voltage of 1 V. So the voltage here is 2 V, since 2 eV is given to each electron. It is the energy produced in each molecular reaction that produces the voltage. A different reaction produces a different energy and, hence, a different voltage.

Terminal Voltage

The voltage output of a device is measured across its terminals and, thus, is called its **terminal voltage** V . Terminal voltage is given by

Equation:

$$V = \text{emf} - Ir,$$

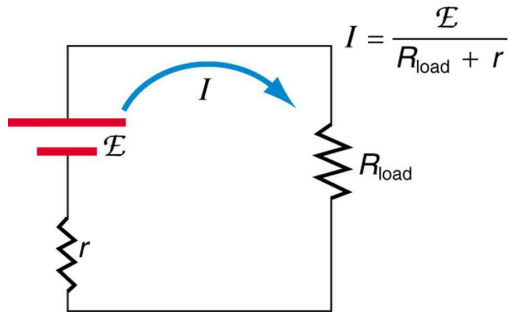
where r is the internal resistance and I is the current flowing at the time of the measurement.

I is positive if current flows away from the positive terminal, as shown in [\[link\]](#). You can see that the larger the current, the smaller the terminal voltage. And it is likewise true that the larger the internal resistance, the smaller the terminal voltage.

Suppose a load resistance R_{load} is connected to a voltage source, as in [\[link\]](#). Since the resistances are in series, the total resistance in the circuit is $R_{\text{load}} + r$. Thus the current is given by Ohm's law to be

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r}.$$



Schematic of a voltage source and its load R_{load} .

Since the internal resistance r is in series with the load, it can significantly affect the terminal voltage and current delivered to the load. (Note that the script E stands for emf.)

We see from this expression that the smaller the internal resistance r , the greater the current the voltage source supplies to its load R_{load} . As batteries are depleted, r increases. If r becomes a significant fraction of the load resistance, then the current is significantly reduced, as the following example illustrates.

Example:

Calculating Terminal Voltage, Power Dissipation, Current, and Resistance: Terminal Voltage and Load

A certain battery has a 12.0-V emf and an internal resistance of 0.100 Ω .

(a) Calculate its terminal voltage when connected to a 10.0- Ω load. (b) What is the terminal voltage when connected to a 0.500- Ω load? (c) What power does the 0.500- Ω load dissipate? (d) If the internal resistance grows

to $0.500\ \Omega$, find the current, terminal voltage, and power dissipated by a $0.500\text{-}\Omega$ load.

Strategy

The analysis above gave an expression for current when internal resistance is taken into account. Once the current is found, the terminal voltage can be calculated using the equation $V = \text{emf} - Ir$. Once current is found, the power dissipated by a resistor can also be found.

Solution for (a)

Entering the given values for the emf, load resistance, and internal resistance into the expression above yields

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\ \text{V}}{10.1\ \Omega} = 1.188\ \text{A}.$$

Enter the known values into the equation $V = \text{emf} - Ir$ to get the terminal voltage:

Equation:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\ \text{V} - (1.188\ \text{A})(0.100\ \Omega) \\ &= 11.9\ \text{V}. \end{aligned}$$

Discussion for (a)

The terminal voltage here is only slightly lower than the emf, implying that $10.0\ \Omega$ is a light load for this particular battery.

Solution for (b)

Similarly, with $R_{\text{load}} = 0.500\ \Omega$, the current is

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\ \text{V}}{0.600\ \Omega} = 20.0\ \text{A}.$$

The terminal voltage is now

Equation:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\ \text{V} - (20.0\ \text{A})(0.100\ \Omega) \\ &= 10.0\ \text{V}. \end{aligned}$$

Discussion for (b)

This terminal voltage exhibits a more significant reduction compared with emf, implying $0.500\ \Omega$ is a heavy load for this battery.

Solution for (c)

The power dissipated by the $0.500\text{ - }\Omega$ load can be found using the formula $P = I^2 R$. Entering the known values gives

Equation:

$$P_{\text{load}} = I^2 R_{\text{load}} = (20.0\text{ A})^2 (0.500\ \Omega) = 2.00 \times 10^2\text{ W}.$$

Discussion for (c)

Note that this power can also be obtained using the expressions $\frac{V^2}{R}$ or IV , where V is the terminal voltage (10.0 V in this case).

Solution for (d)

Here the internal resistance has increased, perhaps due to the depletion of the battery, to the point where it is as great as the load resistance. As before, we first find the current by entering the known values into the expression, yielding

Equation:

$$I = \frac{\text{emf}}{R_{\text{load}} + r} = \frac{12.0\text{ V}}{1.00\ \Omega} = 12.0\text{ A}.$$

Now the terminal voltage is

Equation:

$$\begin{aligned} V &= \text{emf} - Ir = 12.0\text{ V} - (12.0\text{ A})(0.500\ \Omega) \\ &= 6.00\text{ V}, \end{aligned}$$

and the power dissipated by the load is

Equation:

$$P_{\text{load}} = I^2 R_{\text{load}} = (12.0\text{ A})^2 (0.500\ \Omega) = 72.0\text{ W}.$$

Discussion for (d)

We see that the increased internal resistance has significantly decreased terminal voltage, current, and power delivered to a load.

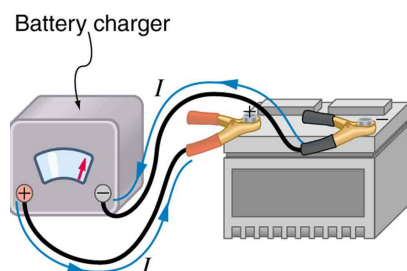
Battery testers, such as those in [\[link\]](#), use small load resistors to intentionally draw current to determine whether the terminal voltage drops below an acceptable level. They really test the internal resistance of the battery. If internal resistance is high, the battery is weak, as evidenced by its low terminal voltage.



These two battery testers measure terminal voltage under a load to determine the condition of a battery. The large device is being used by a U.S. Navy electronics technician to test large batteries aboard the aircraft carrier USS *Nimitz* and has a small resistance that can dissipate large amounts of power. (credit: U.S. Navy photo by Photographer's Mate Airman Jason A. Johnston) The small device is used on small batteries and has a digital display to indicate the acceptability of their terminal voltage. (credit: Keith Williamson)

Some batteries can be recharged by passing a current through them in the direction opposite to the current they supply to a resistance. This is done routinely in cars and batteries for small electrical appliances and electronic devices, and is represented pictorially in [\[link\]](#). The voltage output of the battery charger must be greater than the emf of the battery to reverse current

through it. This will cause the terminal voltage of the battery to be greater than the emf, since $V = \text{emf} - Ir$, and I is now negative.



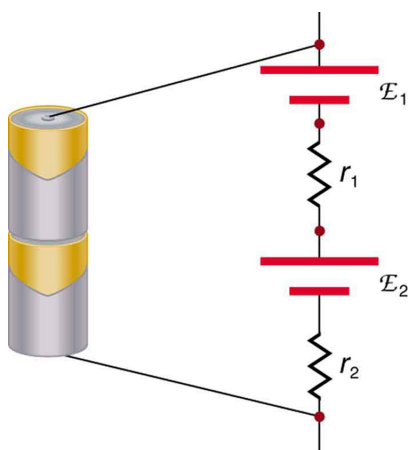
A car battery charger reverses the normal direction of current through a battery, reversing its chemical reaction and replenishing its chemical potential.

Multiple Voltage Sources

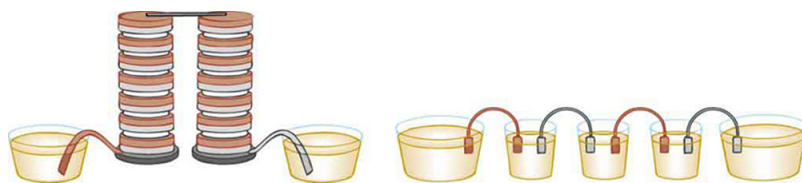
There are two voltage sources when a battery charger is used. Voltage sources connected in series are relatively simple. When voltage sources are in series, their internal resistances add and their emfs add algebraically. (See [\[link\]](#).) Series connections of voltage sources are common—for example, in flashlights, toys, and other appliances. Usually, the cells are in series in order to produce a larger total emf.

But if the cells oppose one another, such as when one is put into an appliance backward, the total emf is less, since it is the algebraic sum of the individual emfs.

A battery is a multiple connection of voltaic cells, as shown in [\[link\]](#). The disadvantage of series connections of cells is that their internal resistances add. One of the authors once owned a 1957 MGA that had two 6-V batteries in series, rather than a single 12-V battery. This arrangement produced a large internal resistance that caused him many problems in starting the engine.

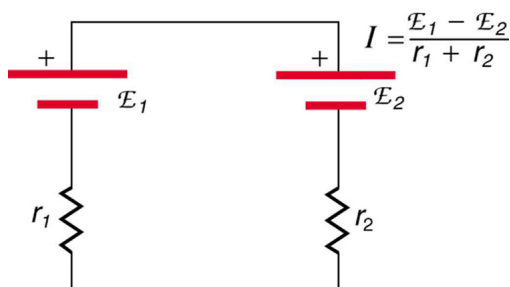


A series connection
of two voltage
sources. The emfs
(each labeled with
a script E) and
internal resistances
add, giving a total
emf of
 $\text{emf}_1 + \text{emf}_2$ and a
total internal
resistance of
 $r_1 + r_2$.



Batteries are multiple connections of individual cells, as shown in this modern rendition of an old print. Single cells, such as AA or C cells, are commonly called batteries, although this is technically incorrect.

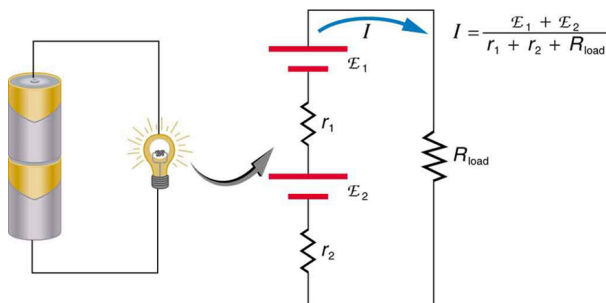
If the *series* connection of two voltage sources is made into a complete circuit with the emfs in opposition, then a current of magnitude $I = \frac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ flows. See [\[link\]](#), for example, which shows a circuit exactly analogous to the battery charger discussed above. If two voltage sources in series with emfs in the same sense are connected to a load R_{load} , as in [\[link\]](#), then $I = \frac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$ flows.



These two voltage sources are connected in series with their emfs in opposition. Current flows in the direction of the greater emf and is limited

to $I = \frac{(\text{emf}_1 - \text{emf}_2)}{r_1 + r_2}$ by the sum of the internal resistances. (Note that each emf is represented by script E in the figure.)

A battery charger connected to a battery is an example of such a connection. The charger must have a larger emf than the battery to reverse current through it.



This schematic represents a flashlight with two cells (voltage sources) and a single bulb (load resistance) in series.

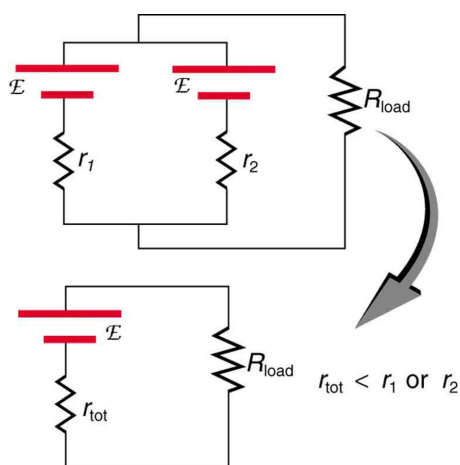
The current that flows is $I = \frac{(\text{emf}_1 + \text{emf}_2)}{r_1 + r_2 + R_{\text{load}}}$. (Note that each emf is represented by script E in the figure.)

Note:**Take-Home Experiment: Flashlight Batteries**

Find a flashlight that uses several batteries and find new and old batteries. Based on the discussions in this module, predict the brightness of the flashlight when different combinations of batteries are used. Do your predictions match what you observe? Now place new batteries in the flashlight and leave the flashlight switched on for several hours. Is the flashlight still quite bright? Do the same with the old batteries. Is the flashlight as bright when left on for the same length of time with old and new batteries? What does this say for the case when you are limited in the number of available new batteries?

[\[link\]](#) shows two voltage sources with identical emfs in parallel and connected to a load resistance. In this simple case, the total emf is the same as the individual emfs. But the total internal resistance is reduced, since the internal resistances are in parallel. The parallel connection thus can produce a larger current.

Here, $I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$ flows through the load, and r_{tot} is less than those of the individual batteries. For example, some diesel-powered cars use two 12-V batteries in parallel; they produce a total emf of 12 V but can deliver the larger current needed to start a diesel engine.



Two voltage sources
with identical emfs
(each labeled by script
E) connected in
parallel produce the
same emf but have a
smaller total internal
resistance than the
individual sources.
Parallel combinations
are often used to
deliver more current.
Here $I = \frac{\text{emf}}{(r_{\text{tot}} + R_{\text{load}})}$
flows through the
load.

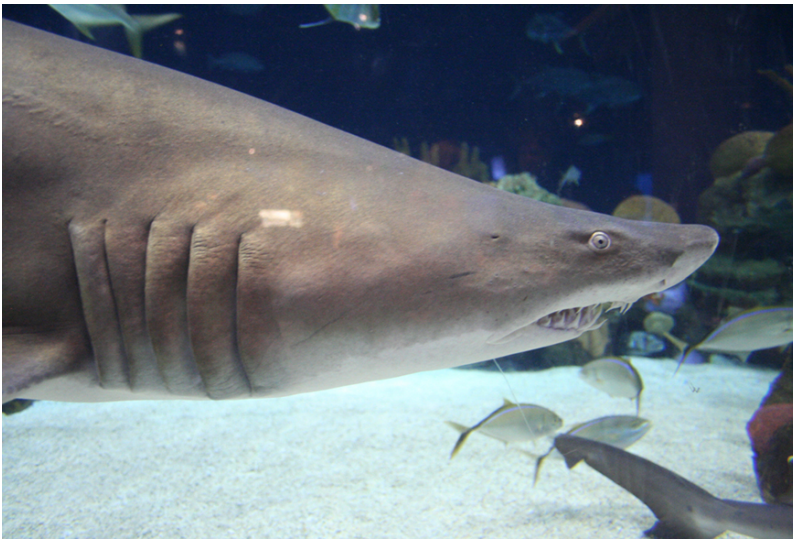
Animals as Electrical Detectors

A number of animals both produce and detect electrical signals. Fish, sharks, platypuses, and echidnas (spiny anteaters) all detect electric fields generated by nerve activity in prey. Electric eels produce their own emf through biological cells (electric organs) called electroplaques, which are arranged in both series and parallel as a set of batteries.

Electroplaques are flat, disk-like cells; those of the electric eel have a voltage of 0.15 V across each one. These cells are usually located toward the head or tail of the animal, although in the case of the electric eel, they are found along the entire body. The electroplaques in the South American eel are arranged in 140 rows, with each row stretching horizontally along the body and containing 5,000 electroplaques. This can yield an emf of approximately 600 V, and a current of 1 A—deadly.

The mechanism for detection of external electric fields is similar to that for producing nerve signals in the cell through depolarization and

repolarization—the movement of ions across the cell membrane. Within the fish, weak electric fields in the water produce a current in a gel-filled canal that runs from the skin to sensing cells, producing a nerve signal. The Australian platypus, one of the very few mammals that lay eggs, can detect fields of $30 \frac{\text{mV}}{\text{m}}$, while sharks have been found to be able to sense a field in their snouts as small as $100 \frac{\text{mV}}{\text{m}}$ ([link](#)). Electric eels use their own electric fields produced by the electroplaques to stun their prey or enemies.



Sand tiger sharks (*Carcharias taurus*), like this one at the Minnesota Zoo, use electroreceptors in their snouts to locate prey. (credit: Jim Winstead, Flickr)

Solar Cell Arrays

Another example dealing with multiple voltage sources is that of combinations of solar cells—wired in both series and parallel combinations to yield a desired voltage and current. Photovoltaic generation (PV), the conversion of sunlight directly into electricity, is based upon the

photoelectric effect, in which photons hitting the surface of a solar cell create an electric current in the cell.

Most solar cells are made from pure silicon—either as single-crystal silicon, or as a thin film of silicon deposited upon a glass or metal backing. Most single cells have a voltage output of about 0.5 V, while the current output is a function of the amount of sunlight upon the cell (the incident solar radiation—the insolation). Under bright noon sunlight, a current of about 100 mA/cm^2 of cell surface area is produced by typical single-crystal cells.

Individual solar cells are connected electrically in modules to meet electrical-energy needs. They can be wired together in series or in parallel—connected like the batteries discussed earlier. A solar-cell array or module usually consists of between 36 and 72 cells, with a power output of 50 W to 140 W.

The output of the solar cells is direct current. For most uses in a home, AC is required, so a device called an inverter must be used to convert the DC to AC. Any extra output can then be passed on to the outside electrical grid for sale to the utility.

Note:

Take-Home Experiment: Virtual Solar Cells

One can assemble a “virtual” solar cell array by using playing cards, or business or index cards, to represent a solar cell. Combinations of these cards in series and/or parallel can model the required array output. Assume each card has an output of 0.5 V and a current (under bright light) of 2 A. Using your cards, how would you arrange them to produce an output of 6 A at 3 V (18 W)?

Suppose you were told that you needed only 18 W (but no required voltage). Would you need more cards to make this arrangement?

Section Summary

- All voltage sources have two fundamental parts—a source of electrical energy that has a characteristic electromotive force (emf), and an internal resistance r .
- The emf is the potential difference of a source when no current is flowing.
- The numerical value of the emf depends on the source of potential difference.
- The internal resistance r of a voltage source affects the output voltage when a current flows.
- The voltage output of a device is called its terminal voltage V and is given by $V = \text{emf} - Ir$, where I is the electric current and is positive when flowing away from the positive terminal of the voltage source.
- When multiple voltage sources are in series, their internal resistances add and their emfs add algebraically.
- Solar cells can be wired in series or parallel to provide increased voltage or current, respectively.

Conceptual Questions

Exercise:

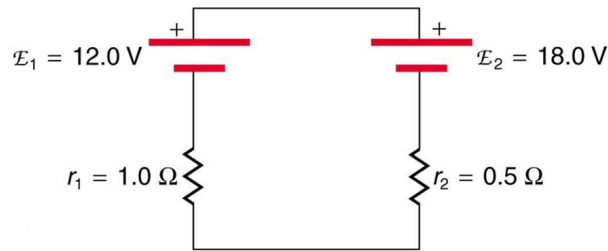
Problem:

Is every emf a potential difference? Is every potential difference an emf? Explain.

Exercise:

Problem:

Explain which battery is doing the charging and which is being charged in [\[link\]](#).



Exercise:

Problem:

Given a battery, an assortment of resistors, and a variety of voltage and current measuring devices, describe how you would determine the internal resistance of the battery.

Exercise:

Problem:

Two different 12-V automobile batteries on a store shelf are rated at 600 and 850 “cold cranking amps.” Which has the smallest internal resistance?

Exercise:

Problem:

What are the advantages and disadvantages of connecting batteries in series? In parallel?

Exercise:

Problem:

Semitractor trucks use four large 12-V batteries. The starter system requires 24 V, while normal operation of the truck’s other electrical components utilizes 12 V. How could the four batteries be connected to produce 24 V? To produce 12 V? Why is 24 V better than 12 V for starting the truck’s engine (a very heavy load)?

Problem Exercises

Exercise:**Problem:**

Standard automobile batteries have six lead-acid cells in series, creating a total emf of 12.0 V. What is the emf of an individual lead-acid cell?

Solution:

2.00 V

Exercise:**Problem:**

Carbon-zinc dry cells (sometimes referred to as non-alkaline cells) have an emf of 1.54 V, and they are produced as single cells or in various combinations to form other voltages. (a) How many 1.54-V cells are needed to make the common 9-V battery used in many small electronic devices? (b) What is the actual emf of the approximately 9-V battery? (c) Discuss how internal resistance in the series connection of cells will affect the terminal voltage of this approximately 9-V battery.

Exercise:**Problem:**

What is the output voltage of a 3.0000-V lithium cell in a digital wristwatch that draws 0.300 mA, if the cell's internal resistance is $2.00\ \Omega$?

Solution:

2.9994 V

Exercise:

Problem:

(a) What is the terminal voltage of a large 1.54-V carbon-zinc dry cell used in a physics lab to supply 2.00 A to a circuit, if the cell's internal resistance is $0.100\ \Omega$? (b) How much electrical power does the cell produce? (c) What power goes to its load?

Exercise:**Problem:**

What is the internal resistance of an automobile battery that has an emf of 12.0 V and a terminal voltage of 15.0 V while a current of 8.00 A is charging it?

Solution:

$0.375\ \Omega$

Exercise:**Problem:**

(a) Find the terminal voltage of a 12.0-V motorcycle battery having a $0.600\text{-}\Omega$ internal resistance, if it is being charged by a current of 10.0 A. (b) What is the output voltage of the battery charger?

Exercise:**Problem:**

A car battery with a 12-V emf and an internal resistance of $0.050\ \Omega$ is being charged with a current of 60 A. Note that in this process the battery is being charged. (a) What is the potential difference across its terminals? (b) At what rate is thermal energy being dissipated in the battery? (c) At what rate is electric energy being converted to chemical energy? (d) What are the answers to (a) and (b) when the battery is used to supply 60 A to the starter motor?

Exercise:

Problem:

The hot resistance of a flashlight bulb is $2.30\ \Omega$, and it is run by a 1.58-V alkaline cell having a $0.100\text{-}\Omega$ internal resistance. (a) What current flows? (b) Calculate the power supplied to the bulb using $I^2 R_{\text{bulb}}$. (c) Is this power the same as calculated using $\frac{V^2}{R_{\text{bulb}}}$?

Solution:

(a) 0.658 A

(b) 0.997 W

(c) 0.997 W; yes

Exercise:**Problem:**

The label on a portable radio recommends the use of rechargeable nickel-cadmium cells (nicads), although they have a 1.25-V emf while alkaline cells have a 1.58-V emf. The radio has a $3.20\text{-}\Omega$ resistance. (a) Draw a circuit diagram of the radio and its batteries. Now, calculate the power delivered to the radio. (b) When using Nicad cells each having an internal resistance of $0.0400\ \Omega$. (c) When using alkaline cells each having an internal resistance of $0.200\ \Omega$. (d) Does this difference seem significant, considering that the radio's effective resistance is lowered when its volume is turned up?

Exercise:

Problem:

An automobile starter motor has an equivalent resistance of $0.0500\ \Omega$ and is supplied by a 12.0-V battery with a $0.0100\text{-}\Omega$ internal resistance. (a) What is the current to the motor? (b) What voltage is applied to it? (c) What power is supplied to the motor? (d) Repeat these calculations for when the battery connections are corroded and add $0.0900\ \Omega$ to the circuit. (Significant problems are caused by even small amounts of unwanted resistance in low-voltage, high-current applications.)

Solution:

- (a) 200 A
- (b) 10.0 V
- (c) 2.00 kW
- (d) $0.1000\ \Omega$; 80.0 A, 4.0 V, 320 W

Exercise:**Problem:**

A child's electronic toy is supplied by three 1.58-V alkaline cells having internal resistances of $0.0200\ \Omega$ in series with a 1.53-V carbon-zinc dry cell having a $0.100\text{-}\Omega$ internal resistance. The load resistance is $10.0\ \Omega$. (a) Draw a circuit diagram of the toy and its batteries. (b) What current flows? (c) How much power is supplied to the load? (d) What is the internal resistance of the dry cell if it goes bad, resulting in only 0.500 W being supplied to the load?

Exercise:

Problem:

(a) What is the internal resistance of a voltage source if its terminal voltage drops by 2.00 V when the current supplied increases by 5.00 A? (b) Can the emf of the voltage source be found with the information supplied?

Solution:

(a) $0.400\ \Omega$

(b) No, there is only one independent equation, so only r can be found.

Exercise:**Problem:**

A person with body resistance between his hands of $10.0\ \text{k}\Omega$ accidentally grasps the terminals of a 20.0-kV power supply. (Do NOT do this!) (a) Draw a circuit diagram to represent the situation. (b) If the internal resistance of the power supply is $2000\ \Omega$, what is the current through his body? (c) What is the power dissipated in his body? (d) If the power supply is to be made safe by increasing its internal resistance, what should the internal resistance be for the maximum current in this situation to be 1.00 mA or less? (e) Will this modification compromise the effectiveness of the power supply for driving low-resistance devices? Explain your reasoning.

Exercise:**Problem:**

Electric fish generate current with biological cells called electroplaques, which are physiological emf devices. The electroplaques in the South American eel are arranged in 140 rows, each row stretching horizontally along the body and each containing 5000 electroplaques. Each electroplaque has an emf of 0.15 V and internal resistance of $0.25\ \Omega$. If the water surrounding the fish has resistance of $800\ \Omega$, how much current can the eel produce in water from near its head to near its tail?

Exercise:**Problem: Integrated Concepts**

A 12.0-V emf automobile battery has a terminal voltage of 16.0 V when being charged by a current of 10.0 A. (a) What is the battery's internal resistance? (b) What power is dissipated inside the battery? (c) At what rate (in $^{\circ}\text{C}/\text{min}$) will its temperature increase if its mass is 20.0 kg and it has a specific heat of $0.300 \text{ kcal}/\text{kg} \cdot ^{\circ}\text{C}$, assuming no heat escapes?

Exercise:**Problem: Unreasonable Results**

A 1.58-V alkaline cell with a $0.200\text{-}\Omega$ internal resistance is supplying 8.50 A to a load. (a) What is its terminal voltage? (b) What is the value of the load resistance? (c) What is unreasonable about these results? (d) Which assumptions are unreasonable or inconsistent?

Solution:

(a) -0.120 V

(b) $-1.41 \times 10^{-2} \Omega$

(c) Negative terminal voltage; negative load resistance.

(d) The assumption that such a cell could provide 8.50 A is inconsistent with its internal resistance.

Exercise:**Problem: Unreasonable Results**

(a) What is the internal resistance of a 1.54-V dry cell that supplies 1.00 W of power to a $15.0\text{-}\Omega$ bulb? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Glossary

electromotive force (emf)

the potential difference of a source of electricity when no current is flowing; measured in volts

internal resistance

the amount of resistance within the voltage source

potential difference

the difference in electric potential between two points in an electric circuit, measured in volts

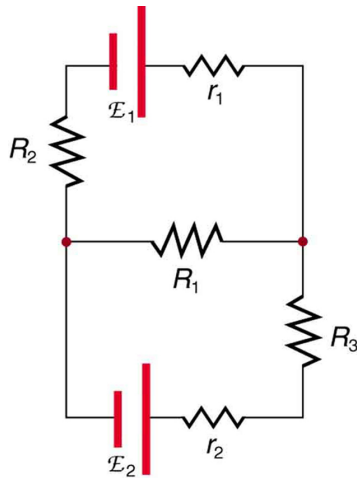
terminal voltage

the voltage measured across the terminals of a source of potential difference

Kirchhoff's Rules

- Analyze a complex circuit using Kirchhoff's rules, using the conventions for determining the correct signs of various terms.

Many complex circuits, such as the one in [\[link\]](#), cannot be analyzed with the series-parallel techniques developed in [Resistors in Series and Parallel](#) and [Electromotive Force: Terminal Voltage](#). There are, however, two circuit analysis rules that can be used to analyze any circuit, simple or complex. These rules are special cases of the laws of conservation of charge and conservation of energy. The rules are known as **Kirchhoff's rules**, after their inventor Gustav Kirchhoff (1824–1887).



This circuit cannot be reduced to a combination of series and parallel connections. Kirchhoff's rules, special applications of the laws of conservation of charge and energy, can be

used to analyze
it. (Note: The
script E in the
figure represents
electromotive
force, emf.)

Note:

Kirchhoff's Rules

- Kirchhoff's first rule—the junction rule. The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule. The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.

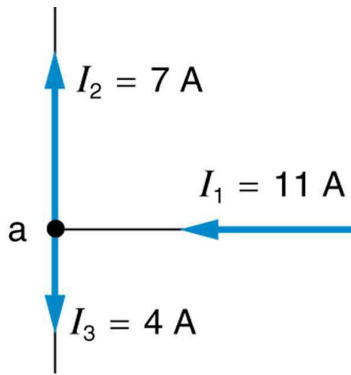
Explanations of the two rules will now be given, followed by problem-solving hints for applying Kirchhoff's rules, and a worked example that uses them.

Kirchhoff's First Rule

Kirchhoff's first rule (the **junction rule**) is an application of the conservation of charge to a junction; it is illustrated in [\[link\]](#). Current is the flow of charge, and charge is conserved; thus, whatever charge flows into the junction must flow out. Kirchhoff's first rule requires that $I_1 = I_2 + I_3$ (see figure). Equations like this can and will be used to analyze circuits and to solve circuit problems.

Note:**Making Connections: Conservation Laws**

Kirchhoff's rules for circuit analysis are applications of **conservation laws** to circuits. The first rule is the application of conservation of charge, while the second rule is the application of conservation of energy. Conservation laws, even used in a specific application, such as circuit analysis, are so basic as to form the foundation of that application.



$$I_1 = I_2 + I_3$$

The junction rule. The diagram shows an example of Kirchhoff's first rule where the sum of the currents into a junction equals the sum of the currents out of a junction. In this case, the current going into the junction splits and comes out as

two currents, so
that

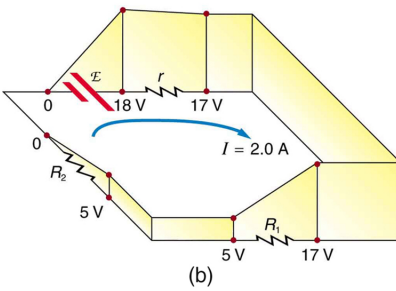
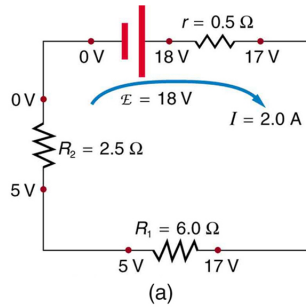
$$I_1 = I_2 + I_3.$$

Here I_1 must be
11 A, since I_2 is
7 A and I_3 is 4
A.

Kirchhoff's Second Rule

Kirchhoff's second rule (the **loop rule**) is an application of conservation of energy. The loop rule is stated in terms of potential, V , rather than potential energy, but the two are related since $PE_{\text{elec}} = qV$. Recall that **emf** is the potential difference of a source when no current is flowing. In a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. [\[link\]](#) illustrates the changes in potential in a simple series circuit loop.

Kirchhoff's second rule requires $\text{emf} - Ir - IR_1 - IR_2 = 0$. Rearranged, this is $\text{emf} = Ir + IR_1 + IR_2$, which means the emf equals the sum of the IR (voltage) drops in the loop.



The loop rule. An example of Kirchhoff's second rule where the sum of the changes in potential around a closed loop must be zero. (a) In this standard schematic of a simple series circuit, the emf supplies 18 V, which is reduced to zero by the resistances, with 1 V across the internal resistance, and 12 V and 5 V across the two load resistances, for a total of 18 V. (b) This perspective view represents the

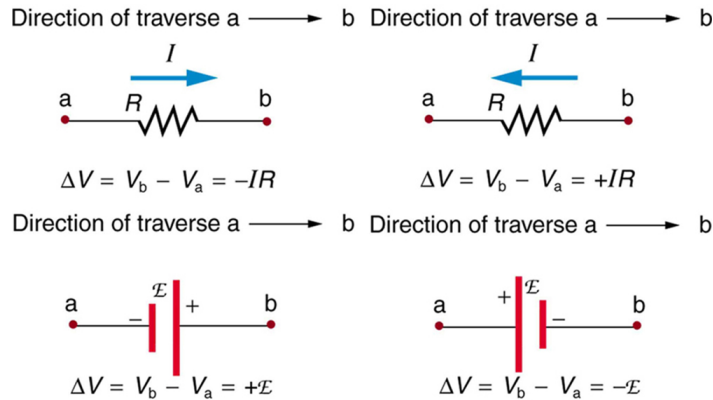
potential as something like a roller coaster, where charge is raised in potential by the emf and lowered by the resistances. (Note that the script E stands for emf.)

Applying Kirchhoff's Rules

By applying Kirchhoff's rules, we generate equations that allow us to find the unknowns in circuits. The unknowns may be currents, emfs, or resistances. Each time a rule is applied, an equation is produced. If there are as many independent equations as unknowns, then the problem can be solved. There are two decisions you must make when applying Kirchhoff's rules. These decisions determine the signs of various quantities in the equations you obtain from applying the rules.

1. When applying Kirchhoff's first rule, the junction rule, you must label the current in each branch and decide in what direction it is going. For example, in [\[link\]](#), [\[link\]](#), and [\[link\]](#), currents are labeled I_1 , I_2 , I_3 , and I , and arrows indicate their directions. There is no risk here, for if you choose the wrong direction, the current will be of the correct magnitude but negative.
2. When applying Kirchhoff's second rule, the loop rule, you must identify a closed loop and decide in which direction to go around it, clockwise or counterclockwise. For example, in [\[link\]](#) the loop was traversed in the same direction as the current (clockwise). Again, there is no risk; going around the circuit in the opposite direction reverses the sign of every term in the equation, which is like multiplying both sides of the equation by -1 .

[\[link\]](#) and the following points will help you get the plus or minus signs right when applying the loop rule. Note that the resistors and emfs are traversed by going from a to b. In many circuits, it will be necessary to construct more than one loop. In traversing each loop, one needs to be consistent for the sign of the change in potential. (See [\[link\]](#).)

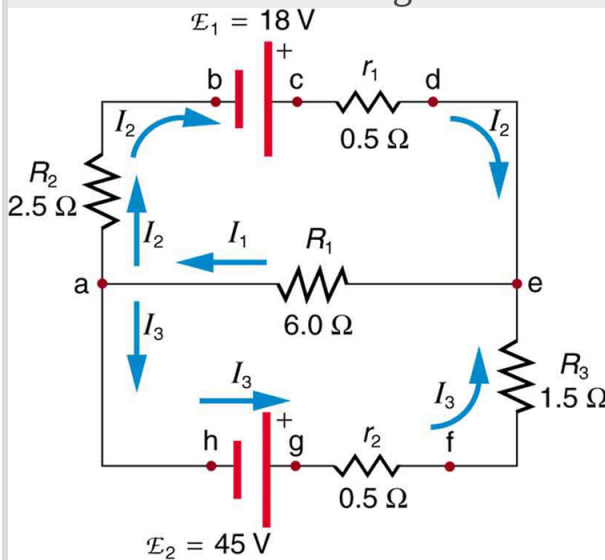


Each of these resistors and voltage sources is traversed from a to b. The potential changes are shown beneath each element and are explained in the text. (Note that the script E stands for emf.)

- When a resistor is traversed in the same direction as the current, the change in potential is $-IR$. (See [\[link\]](#).)
- When a resistor is traversed in the direction opposite to the current, the change in potential is $+IR$. (See [\[link\]](#).)
- When an emf is traversed from $-$ to $+$ (the same direction it moves positive charge), the change in potential is $+\text{emf}$. (See [\[link\]](#).)
- When an emf is traversed from $+$ to $-$ (opposite to the direction it moves positive charge), the change in potential is $-\text{emf}$. (See [\[link\]](#).)

Example:**Calculating Current: Using Kirchhoff's Rules**

Find the currents flowing in the circuit in [\[link\]](#).



This circuit is similar to that in [\[link\]](#), but the resistances and emfs are specified. (Each emf is denoted by script E.) The currents in each branch are labeled and assumed to move in the directions shown. This example uses Kirchhoff's rules to find the currents.

Strategy

This circuit is sufficiently complex that the currents cannot be found using Ohm's law and the series-parallel techniques—it is necessary to use Kirchhoff's rules. Currents have been labeled I_1 , I_2 , and I_3 in the figure and assumptions have been made about their directions. Locations on the diagram have been labeled with letters a through h. In the solution we will apply the junction and loop rules, seeking three independent equations to allow us to solve for the three unknown currents.

Solution

We begin by applying Kirchhoff's first or junction rule at point a. This gives

Equation:

$$I_1 = I_2 + I_3,$$

since I_1 flows into the junction, while I_2 and I_3 flow out. Applying the junction rule at e produces exactly the same equation, so that no new information is obtained. This is a single equation with three unknowns—three independent equations are needed, and so the loop rule must be applied.

Now we consider the loop abcdea. Going from a to b, we traverse R_2 in the same (assumed) direction of the current I_2 , and so the change in potential is $-I_2 R_2$. Then going from b to c, we go from $-$ to $+$, so that the change in potential is $+\text{emf}_1$. Traversing the internal resistance r_1 from c to d gives $-I_2 r_1$. Completing the loop by going from d to a again traverses a resistor in the same direction as its current, giving a change in potential of $-I_1 R_1$.

The loop rule states that the changes in potential sum to zero. Thus,

Equation:

$$-I_2 R_2 + \text{emf}_1 - I_2 r_1 - I_1 R_1 = -I_2(R_2 + r_1) + \text{emf}_1 - I_1 R_1 = 0.$$

Substituting values from the circuit diagram for the resistances and emf, and canceling the ampere unit gives

Equation:

$$-3I_2 + 18 - 6I_1 = 0.$$

Now applying the loop rule to aefgha (we could have chosen abcdefgha as well) similarly gives

Equation:

$$+ I_1 R_1 + I_3 R_3 + I_3 r_2 - \text{emf}_2 = +I_1 R_1 + I_3(R_3 + r_2) - \text{emf}_2 = 0.$$

Note that the signs are reversed compared with the other loop, because elements are traversed in the opposite direction. With values entered, this becomes

Equation:

$$+ 6I_1 + 2I_3 - 45 = 0.$$

These three equations are sufficient to solve for the three unknown currents. First, solve the second equation for I_2 :

Equation:

$$I_2 = 6 - 2I_1.$$

Now solve the third equation for I_3 :

Equation:

$$I_3 = 22.5 - 3I_1.$$

Substituting these two new equations into the first one allows us to find a value for I_1 :

Equation:

$$I_1 = I_2 + I_3 = (6 - 2I_1) + (22.5 - 3I_1) = 28.5 - 5I_1.$$

Combining terms gives

Equation:

$$6I_1 = 28.5, \text{ and}$$

Equation:

$$I_1 = 4.75 \text{ A.}$$

Substituting this value for I_1 back into the fourth equation gives

Equation:

$$I_2 = 6 - 2I_1 = 6 - 9.50$$

Equation:

$$I_2 = -3.50 \text{ A.}$$

The minus sign means I_2 flows in the direction opposite to that assumed in [\[link\]](#).

Finally, substituting the value for I_1 into the fifth equation gives

Equation:

$$I_3 = 22.5 - 3I_1 = 22.5 - 14.25$$

Equation:

$$I_3 = 8.25 \text{ A.}$$

Discussion

Just as a check, we note that indeed $I_1 = I_2 + I_3$. The results could also have been checked by entering all of the values into the equation for the abcdefgha loop.

Note:**Problem-Solving Strategies for Kirchhoff's Rules**

1. Make certain there is a clear circuit diagram on which you can label all known and unknown resistances, emfs, and currents. If a current is unknown, you must assign it a direction. This is necessary for determining the signs of potential changes. If you assign the direction incorrectly, the current will be found to have a negative value—no harm done.
2. Apply the junction rule to any junction in the circuit. Each time the junction rule is applied, you should get an equation with a current that does not appear in a previous application—if not, then the equation is redundant.
3. Apply the loop rule to as many loops as needed to solve for the unknowns in the problem. (There must be as many independent equations as unknowns.) To apply the loop rule, you must choose a direction to go around the loop. Then carefully and consistently determine the signs of the potential changes for each element using the four bulleted points discussed above in conjunction with [\[link\]](#).
4. Solve the simultaneous equations for the unknowns. This may involve many algebraic steps, requiring careful checking and rechecking.
5. Check to see whether the answers are reasonable and consistent. The numbers should be of the correct order of magnitude, neither

exceedingly large nor vanishingly small. The signs should be reasonable—for example, no resistance should be negative. Check to see that the values obtained satisfy the various equations obtained from applying the rules. The currents should satisfy the junction rule, for example.

The material in this section is correct in theory. We should be able to verify it by making measurements of current and voltage. In fact, some of the devices used to make such measurements are straightforward applications of the principles covered so far and are explored in the next modules. As we shall see, a very basic, even profound, fact results—making a measurement alters the quantity being measured.

Exercise:

Check Your Understanding

Problem:

Can Kirchhoff's rules be applied to simple series and parallel circuits or are they restricted for use in more complicated circuits that are not combinations of series and parallel?

Solution:

Kirchhoff's rules can be applied to any circuit since they are applications to circuits of two conservation laws. Conservation laws are the most broadly applicable principles in physics. It is usually mathematically simpler to use the rules for series and parallel in simpler circuits so we emphasize Kirchhoff's rules for use in more complicated situations. But the rules for series and parallel can be derived from Kirchhoff's rules. Moreover, Kirchhoff's rules can be expanded to devices other than resistors and emfs, such as capacitors, and are one of the basic analysis devices in circuit analysis.

Section Summary

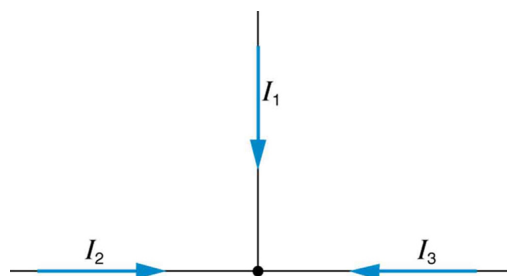
- Kirchhoff's rules can be used to analyze any circuit, simple or complex.
- Kirchhoff's first rule—the junction rule: The sum of all currents entering a junction must equal the sum of all currents leaving the junction.
- Kirchhoff's second rule—the loop rule: The algebraic sum of changes in potential around any closed circuit path (loop) must be zero.
- The two rules are based, respectively, on the laws of conservation of charge and energy.
- When calculating potential and current using Kirchhoff's rules, a set of conventions must be followed for determining the correct signs of various terms.
- The simpler series and parallel rules are special cases of Kirchhoff's rules.

Conceptual Questions

Exercise:

Problem:

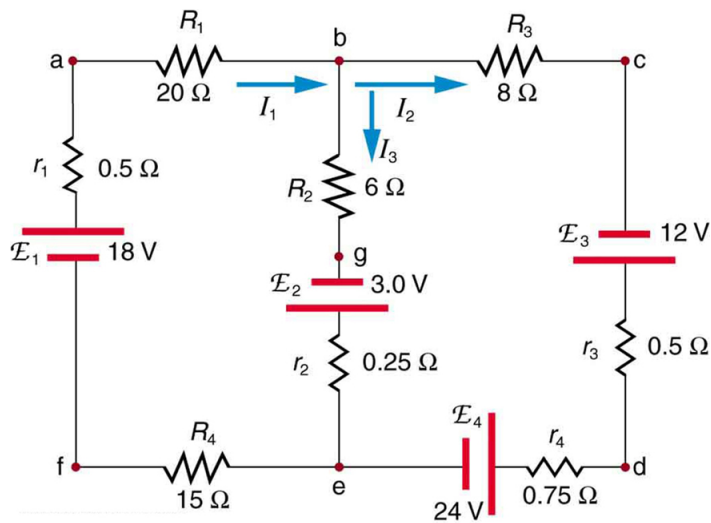
Can all of the currents going into the junction in [\[link\]](#) be positive? Explain.



Exercise:

Problem:

Apply the junction rule to junction b in [\[link\]](#). Is any new information gained by applying the junction rule at e? (In the figure, each emf is represented by script E.)

**Exercise:****Problem:**

(a) What is the potential difference going from point a to point b in [\[link\]](#)? (b) What is the potential difference going from c to b? (c) From e to g? (d) From e to d?

Exercise:

Problem: Apply the loop rule to loop afedcba in [\[link\]](#).

Exercise:

Problem: Apply the loop rule to loops abgefa and cbgedc in [\[link\]](#).

Problem Exercises

Exercise:

Problem: Apply the loop rule to loop abcdefgha in [\[link\]](#).

Solution:

Equation:

$$-I_2 R_2 + \text{emf}_1 - I_2 r_1 + I_3 R_3 + I_3 r_2 - \text{emf}_2 = 0$$

Exercise:

Problem: Apply the loop rule to loop aedcba in [\[link\]](#).

Exercise:

Problem:

Verify the second equation in [\[link\]](#) by substituting the values found for the currents I_1 and I_2 .

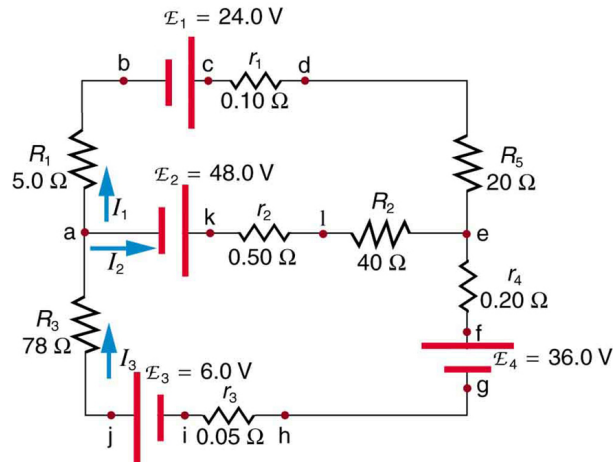
Exercise:

Problem:

Verify the third equation in [\[link\]](#) by substituting the values found for the currents I_1 and I_3 .

Exercise:

Problem: Apply the junction rule at point a in [\[link\]](#).



Solution:

Equation:

$$I_3 = I_1 + I_2$$

Exercise:

Problem: Apply the loop rule to loop abcdefghija in [\[link\]](#).

Exercise:

Problem: Apply the loop rule to loop akledcba in [\[link\]](#).

Solution:

Equation:

$$\text{emf}_2 - I_2 r_2 - I_2 R_2 + I_1 R_5 + I_1 r_1 - \text{emf}_1 + I_1 R_1 = 0$$

Exercise:

Problem:

Find the currents flowing in the circuit in [\[link\]](#). Explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).

Exercise:

Problem:

Solve [\[link\]](#), but use loop abcdefgha instead of loop akledcba. Explicitly show how you follow the steps in the [Problem-Solving Strategies for Series and Parallel Resistors](#).

Solution:

(a) $I_1 = 4.75 \text{ A}$

(b) $I_2 = -3.5 \text{ A}$

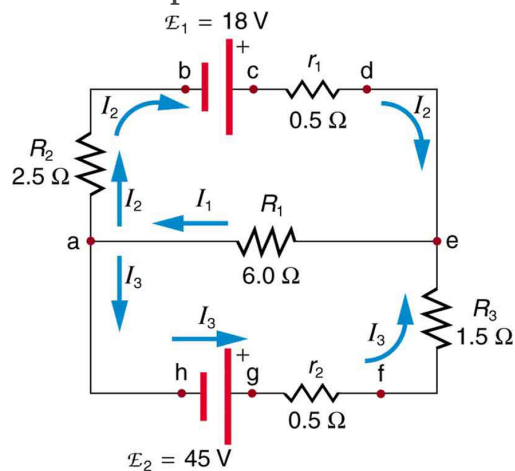
(c) $I_3 = 8.25 \text{ A}$

Exercise:

Problem: Find the currents flowing in the circuit in [\[link\]](#).

Exercise:**Problem: Unreasonable Results**

Consider the circuit in [\[link\]](#), and suppose that the emfs are unknown and the currents are given to be $I_1 = 5.00 \text{ A}$, $I_2 = 3.0 \text{ A}$, and $I_3 = -2.00 \text{ A}$. (a) Could you find the emfs? (b) What is wrong with the assumptions?



Solution:

- (a) No, you would get inconsistent equations to solve.
- (b) $I_1 \neq I_2 + I_3$. The assumed currents violate the junction rule.

Glossary**Kirchhoff's rules**

a set of two rules, based on conservation of charge and energy, governing current and changes in potential in an electric circuit

junction rule

Kirchhoff's first rule, which applies the conservation of charge to a junction; current is the flow of charge; thus, whatever charge flows into the junction must flow out; the rule can be stated $I_1 = I_2 + I_3$

loop rule

Kirchhoff's second rule, which states that in a closed loop, whatever energy is supplied by emf must be transferred into other forms by devices in the loop, since there are no other ways in which energy can be transferred into or out of the circuit. Thus, the emf equals the sum of the IR (voltage) drops in the loop and can be stated:
$$\text{emf} = Ir + IR_1 + IR_2$$

conservation laws

require that energy and charge be conserved in a system

DC Voltmeters and Ammeters

- Explain why a voltmeter must be connected in parallel with the circuit.
- Draw a diagram showing an ammeter correctly connected in a circuit.
- Describe how a galvanometer can be used as either a voltmeter or an ammeter.
- Find the resistance that must be placed in series with a galvanometer to allow it to be used as a voltmeter with a given reading.
- Explain why measuring the voltage or current in a circuit can never be exact.

Voltmeters measure voltage, whereas **ammeters** measure current. Some of the meters in automobile dashboards, digital cameras, cell phones, and tuner-amplifiers are voltmeters or ammeters. (See [\[link\]](#).) The internal construction of the simplest of these meters and how they are connected to the system they monitor give further insight into applications of series and parallel connections.

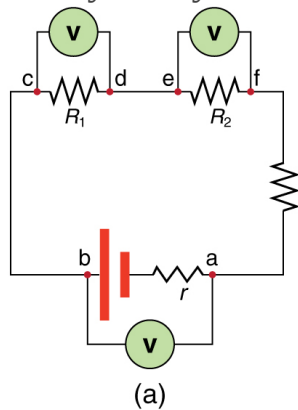


The fuel and temperature gauges (far right and far left, respectively) in this 1996 Volkswagen are voltmeters that register the voltage output of “sender” units, which are hopefully proportional to the amount of gasoline in the tank and the engine

temperature. (credit:
Christian Giersing)

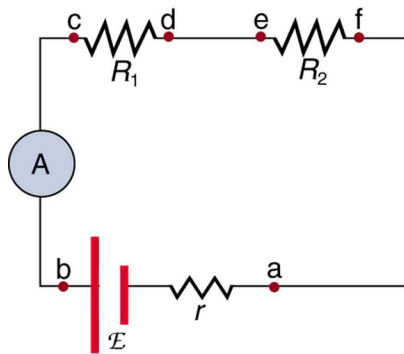
Voltmeters are connected in parallel with whatever device's voltage is to be measured. A parallel connection is used because objects in parallel experience the same potential difference. (See [\[link\]](#), where the voltmeter is represented by the symbol V.)

Ammeters are connected in series with whatever device's current is to be measured. A series connection is used because objects in series have the same current passing through them. (See [\[link\]](#), where the ammeter is represented by the symbol A.)



(a) To measure potential differences in this series circuit, the voltmeter (V) is placed in parallel with the

voltage source or either of the resistors. Note that terminal voltage is measured between points a and b. It is not possible to connect the voltmeter directly across the emf without including its internal resistance, r . (b) A digital voltmeter in use.
(credit: Messtechniker, Wikimedia Commons)



An ammeter (A) is placed in series to measure current. All of the current in this circuit flows through the meter.

The ammeter would have the same reading if located between points d and e or between points f and a as it does in the position shown. (Note that the script

capital E stands for
emf, and r stands
for the internal
resistance of the
source of potential
difference.)

Analog Meters: Galvanometers

Analog meters have a needle that swivels to point at numbers on a scale, as opposed to **digital meters**, which have numerical readouts similar to a hand-held calculator. The heart of most analog meters is a device called a **galvanometer**, denoted by G . Current flow through a galvanometer, I_G , produces a proportional needle deflection. (This deflection is due to the force of a magnetic field upon a current-carrying wire.)

The two crucial characteristics of a given galvanometer are its resistance and current sensitivity. **Current sensitivity** is the current that gives a **full-scale deflection** of the galvanometer's needle, the maximum current that the instrument can measure. For example, a galvanometer with a current sensitivity of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it, reads half-scale when $25\ \mu\text{A}$ flows through it, and so on.

If such a galvanometer has a $25\text{-}\Omega$ resistance, then a voltage of only $V = IR = (50\ \mu\text{A})(25\ \Omega) = 1.25\ \text{mV}$ produces a full-scale reading. By connecting resistors to this galvanometer in different ways, you can use it as either a voltmeter or ammeter that can measure a broad range of voltages or currents.

Galvanometer as Voltmeter

[\[link\]](#) shows how a galvanometer can be used as a voltmeter by connecting it in series with a large resistance, R . The value of the resistance R is

determined by the maximum voltage to be measured. Suppose you want 10 V to produce a full-scale deflection of a voltmeter containing a $25\text{-}\Omega$ galvanometer with a $50\text{-}\mu\text{A}$ sensitivity. Then 10 V applied to the meter must produce a current of $50\text{ }\mu\text{A}$. The total resistance must be

Equation:

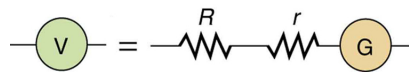
$$R_{\text{tot}} = R + r = \frac{V}{I} = \frac{10\text{ V}}{50\text{ }\mu\text{A}} = 200\text{ k}\Omega, \text{ or}$$

Equation:

$$R = R_{\text{tot}} - r = 200\text{ k}\Omega - 25\text{ }\Omega \approx 200\text{ k}\Omega.$$

(R is so large that the galvanometer resistance, r , is nearly negligible.) Note that 5 V applied to this voltmeter produces a half-scale deflection by producing a $25\text{-}\mu\text{A}$ current through the meter, and so the voltmeter's reading is proportional to voltage as desired.

This voltmeter would not be useful for voltages less than about half a volt, because the meter deflection would be small and difficult to read accurately. For other voltage ranges, other resistances are placed in series with the galvanometer. Many meters have a choice of scales. That choice involves switching an appropriate resistance into series with the galvanometer.



A large resistance
 R placed in series
 with a
 galvanometer G
 produces a
 voltmeter, the full-
 scale deflection of
 which depends on
 the choice of R .

The larger the voltage to be measured, the larger R must be. (Note that r represents the internal resistance of the galvanometer.)

Galvanometer as Ammeter

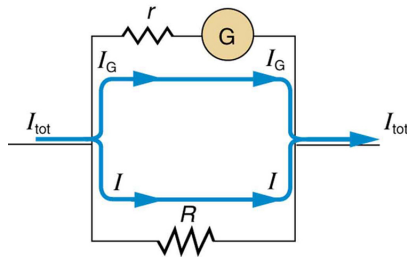
The same galvanometer can also be made into an ammeter by placing it in parallel with a small resistance R , often called the **shunt resistance**, as shown in [\[link\]](#). Since the shunt resistance is small, most of the current passes through it, allowing an ammeter to measure currents much greater than those producing a full-scale deflection of the galvanometer.

Suppose, for example, an ammeter is needed that gives a full-scale deflection for 1.0 A, and contains the same $25\text{-}\Omega$ galvanometer with its $50\text{-}\mu\text{A}$ sensitivity. Since R and r are in parallel, the voltage across them is the same.

These IR drops are $IR = I_G r$ so that $IR = \frac{I_G}{I} = \frac{R}{r}$. Solving for R , and noting that I_G is $50\text{ }\mu\text{A}$ and I is 0.999950 A , we have

Equation:

$$R = r \frac{I_G}{I} = (25\text{ }\Omega) \frac{50\text{ }\mu\text{A}}{0.999950\text{ A}} = 1.25 \times 10^{-3}\text{ }\Omega.$$



A small shunt resistance R placed in parallel with a galvanometer G produces an ammeter, the full-scale deflection of which depends on the choice of R .

The larger the current to be measured, the smaller R must be. Most of the current (I) flowing through the meter is shunted through R to protect the galvanometer.

(Note that r represents the internal resistance of the galvanometer.)

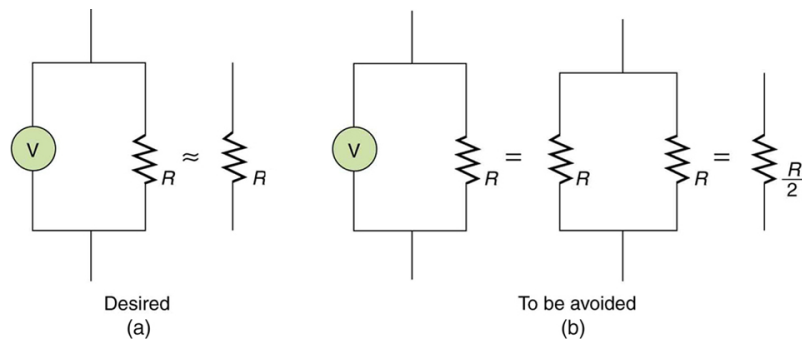
Ammeters may also have multiple scales for greater flexibility in application. The various scales are

achieved by
switching various
shunt resistances in
parallel with the
galvanometer—the
greater the
maximum current
to be measured, the
smaller the shunt
resistance must be.

Taking Measurements Alters the Circuit

When you use a voltmeter or ammeter, you are connecting another resistor to an existing circuit and, thus, altering the circuit. Ideally, voltmeters and ammeters do not appreciably affect the circuit, but it is instructive to examine the circumstances under which they do or do not interfere.

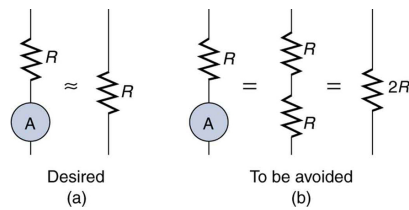
First, consider the voltmeter, which is always placed in parallel with the device being measured. Very little current flows through the voltmeter if its resistance is a few orders of magnitude greater than the device, and so the circuit is not appreciably affected. (See [\[link\]](#)(a).) (A large resistance in parallel with a small one has a combined resistance essentially equal to the small one.) If, however, the voltmeter's resistance is comparable to that of the device being measured, then the two in parallel have a smaller resistance, appreciably affecting the circuit. (See [\[link\]](#)(b).) The voltage across the device is not the same as when the voltmeter is out of the circuit.



(a) A voltmeter having a resistance much larger than the device ($R_{\text{Voltmeter}} \gg R$) with which it is in parallel produces a parallel resistance essentially the same as the device and does not appreciably affect the circuit being measured. (b) Here the voltmeter has the same resistance as the device ($R_{\text{Voltmeter}} \cong R$), so that the parallel resistance is half of what it is when the voltmeter is not connected. This is an example of a significant alteration of the circuit and is to be avoided.

An ammeter is placed in series in the branch of the circuit being measured, so that its resistance adds to that branch. Normally, the ammeter's resistance is very small compared with the resistances of the devices in the circuit, and so the extra resistance is negligible. (See [\[link\]](#)(a).) However, if very small load resistances are involved, or if the ammeter is not as low in resistance as it should be, then the total series resistance is significantly greater, and the current in the branch being measured is reduced. (See [\[link\]](#)(b).)

A practical problem can occur if the ammeter is connected incorrectly. If it was put in parallel with the resistor to measure the current in it, you could possibly damage the meter; the low resistance of the ammeter would allow most of the current in the circuit to go through the galvanometer, and this current would be larger since the effective resistance is smaller.



(a) An ammeter normally has such a small resistance that the total series resistance in the branch being measured is not appreciably increased. The circuit is essentially unaltered compared with when the ammeter is absent.

(b) Here the ammeter's resistance is the same as that of the branch, so that the total resistance is doubled and the current is half what it is without the ammeter. This significant alteration of the circuit is to be avoided.

One solution to the problem of voltmeters and ammeters interfering with the circuits being measured is to use galvanometers with greater sensitivity.

This allows construction of voltmeters with greater resistance and ammeters with smaller resistance than when less sensitive galvanometers are used.

There are practical limits to galvanometer sensitivity, but it is possible to get analog meters that make measurements accurate to a few percent. Note that the inaccuracy comes from altering the circuit, not from a fault in the meter.

Note:**Connections: Limits to Knowledge**

Making a measurement alters the system being measured in a manner that produces uncertainty in the measurement. For macroscopic systems, such as the circuits discussed in this module, the alteration can usually be made negligibly small, but it cannot be eliminated entirely. For submicroscopic systems, such as atoms, nuclei, and smaller particles, measurement alters the system in a manner that cannot be made arbitrarily small. This actually limits knowledge of the system—even limiting what nature can know about itself. We shall see profound implications of this when the Heisenberg uncertainty principle is discussed in the modules on quantum mechanics.

There is another measurement technique based on drawing no current at all and, hence, not altering the circuit at all. These are called null measurements and are the topic of [Null Measurements](#). Digital meters that employ solid-state electronics and null measurements can attain accuracies of one part in 10^6 .

Exercise:**Check Your Understanding****Problem:**

Digital meters are able to detect smaller currents than analog meters employing galvanometers. How does this explain their ability to measure voltage and current more accurately than analog meters?

Solution:

Since digital meters require less current than analog meters, they alter the circuit less than analog meters. Their resistance as a voltmeter can be far greater than an analog meter, and their resistance as an ammeter can be far less than an analog meter. Consult [\[link\]](#) and [\[link\]](#) and their discussion in the text.

Note:

PhET Explorations: Circuit Construction Kit (DC Only), Virtual Lab
Stimulate a neuron and monitor what happens. Pause, rewind, and move forward in time in order to observe the ions as they move across the neuron membrane.

[Circuit
Construction
Kit \(DC
Only\),
Virtual Lab](#)

Section Summary

- Voltmeters measure voltage, and ammeters measure current.
- A voltmeter is placed in parallel with the voltage source to receive full voltage and must have a large resistance to limit its effect on the circuit.
- An ammeter is placed in series to get the full current flowing through a branch and must have a small resistance to limit its effect on the circuit.

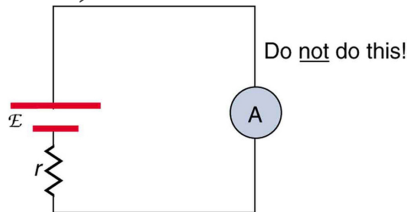
- Both can be based on the combination of a resistor and a galvanometer, a device that gives an analog reading of current.
- Standard voltmeters and ammeters alter the circuit being measured and are thus limited in accuracy.

Conceptual Questions

Exercise:

Problem:

Why should you not connect an ammeter directly across a voltage source as shown in [\[link\]](#)? (Note that script E in the figure stands for emf.)



Exercise:

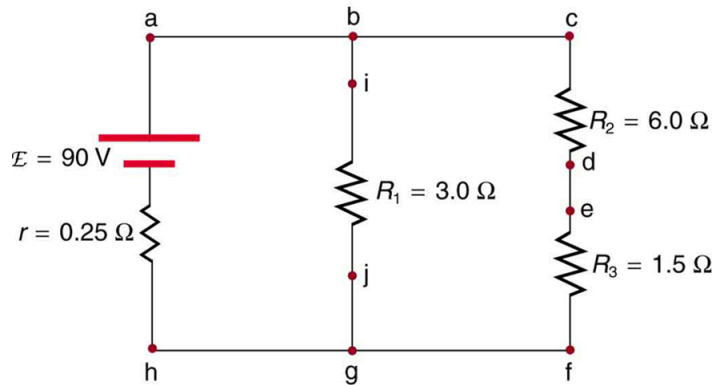
Problem:

Suppose you are using a multimeter (one designed to measure a range of voltages, currents, and resistances) to measure current in a circuit and you inadvertently leave it in a voltmeter mode. What effect will the meter have on the circuit? What would happen if you were measuring voltage but accidentally put the meter in the ammeter mode?

Exercise:

Problem:

Specify the points to which you could connect a voltmeter to measure the following potential differences in [\[link\]](#): (a) the potential difference of the voltage source; (b) the potential difference across R_1 ; (c) across R_2 ; (d) across R_3 ; (e) across R_2 and R_3 . Note that there may be more than one answer to each part.



Exercise:

Problem:

To measure currents in [\[link\]](#), you would replace a wire between two points with an ammeter. Specify the points between which you would place an ammeter to measure the following: (a) the total current; (b) the current flowing through R_1 ; (c) through R_2 ; (d) through R_3 . Note that there may be more than one answer to each part.

Problem Exercises

Exercise:

Problem:

What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $1.00\text{-M}\Omega$ resistance on its 30.0-V scale?

Solution:

$$30\ \mu\text{A}$$

Exercise:

Problem:

What is the sensitivity of the galvanometer (that is, what current gives a full-scale deflection) inside a voltmeter that has a $25.0\text{-k}\Omega$ resistance on its 100-V scale?

Exercise:**Problem:**

Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 0.100-V full-scale reading.

Solution:

$1.98\text{ k}\Omega$

Exercise:**Problem:**

Find the resistance that must be placed in series with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as a voltmeter with a 3000-V full-scale reading. Include a circuit diagram with your solution.

Exercise:**Problem:**

Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 10.0-A full-scale reading. Include a circuit diagram with your solution.

Solution:**Equation:**

$$1.25 \times 10^{-4} \Omega$$

Exercise:**Problem:**

Find the resistance that must be placed in parallel with a $25.0\text{-}\Omega$ galvanometer having a $50.0\text{-}\mu\text{A}$ sensitivity (the same as the one discussed in the text) to allow it to be used as an ammeter with a 300-mA full-scale reading.

Exercise:**Problem:**

Find the resistance that must be placed in series with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as a voltmeter with: (a) a 300-V full-scale reading, and (b) a 0.300-V full-scale reading.

Solution:

(a) $3.00\text{ M}\Omega$

(b) $2.99\text{ k}\Omega$

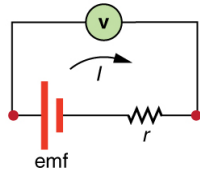
Exercise:**Problem:**

Find the resistance that must be placed in parallel with a $10.0\text{-}\Omega$ galvanometer having a $100\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter with: (a) a 20.0-A full-scale reading, and (b) a 100-mA full-scale reading.

Exercise:

Problem:

Suppose you measure the terminal voltage of a 1.585-V alkaline cell having an internal resistance of $0.100\ \Omega$ by placing a $1.00\text{-k}\Omega$ voltmeter across its terminals. (See [\[link\]](#).) (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.



Solution:

(a) 1.58 mA

(b) 1.5848 V (need four digits to see the difference)

(c) 0.99990 (need five digits to see the difference from unity)

Exercise:**Problem:**

Suppose you measure the terminal voltage of a 3.200-V lithium cell having an internal resistance of $5.00\ \Omega$ by placing a $1.00\text{-k}\Omega$ voltmeter across its terminals. (a) What current flows? (b) Find the terminal voltage. (c) To see how close the measured terminal voltage is to the emf, calculate their ratio.

Exercise:**Problem:**

A certain ammeter has a resistance of $5.00 \times 10^{-5}\ \Omega$ on its 3.00-A scale and contains a $10.0\text{-}\Omega$ galvanometer. What is the sensitivity of the galvanometer?

Solution:

15.0 μA

Exercise:

Problem:

A 1.00-M Ω voltmeter is placed in parallel with a 75.0-k Ω resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) What is the resistance of the combination? (c) If the voltage across the combination is kept the same as it was across the 75.0-k Ω resistor alone, what is the percent increase in current? (d) If the current through the combination is kept the same as it was through the 75.0-k Ω resistor alone, what is the percentage decrease in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

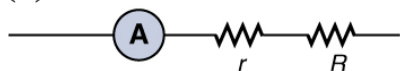
Exercise:

Problem:

A 0.0200- Ω ammeter is placed in series with a 10.00- Ω resistor in a circuit. (a) Draw a circuit diagram of the connection. (b) Calculate the resistance of the combination. (c) If the voltage is kept the same across the combination as it was through the 10.00- Ω resistor alone, what is the percent decrease in current? (d) If the current is kept the same through the combination as it was through the 10.00- Ω resistor alone, what is the percent increase in voltage? (e) Are the changes found in parts (c) and (d) significant? Discuss.

Solution:

(a)



(b) 10.02 Ω

(c) 0.9980, or a 2.0×10^{-1} percent decrease

(d) 1.002, or a 2.0×10^{-1} percent increase

(e) Not significant.

Exercise:

Problem: Unreasonable Results

Suppose you have a $40.0\text{-}\Omega$ galvanometer with a $25.0\text{-}\mu\text{A}$ sensitivity.

(a) What resistance would you put in series with it to allow it to be used as a voltmeter that has a full-scale deflection for 0.500 mV ? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Unreasonable Results

(a) What resistance would you put in parallel with a $40.0\text{-}\Omega$ galvanometer having a $25.0\text{-}\mu\text{A}$ sensitivity to allow it to be used as an ammeter that has a full-scale deflection for $10.0\text{-}\mu\text{A}$? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Solution:

(a) $-66.7\text{ }\Omega$

(b) You can't have negative resistance.

(c) It is unreasonable that I_G is greater than I_{tot} (see [\[link\]](#)). You cannot achieve a full-scale deflection using a current less than the sensitivity of the galvanometer.

Glossary

voltmeter

an instrument that measures voltage

ammeter

an instrument that measures current

analog meter

a measuring instrument that gives a readout in the form of a needle movement over a marked gauge

digital meter

a measuring instrument that gives a readout in a digital form

galvanometer

an analog measuring device, denoted by G , that measures current flow using a needle deflection caused by a magnetic field force acting upon a current-carrying wire

current sensitivity

the maximum current that a galvanometer can read

full-scale deflection

the maximum deflection of a galvanometer needle, also known as current sensitivity; a galvanometer with a full-scale deflection of $50\ \mu\text{A}$ has a maximum deflection of its needle when $50\ \mu\text{A}$ flows through it

shunt resistance

a small resistance R placed in parallel with a galvanometer G to produce an ammeter; the larger the current to be measured, the smaller R must be; most of the current flowing through the meter is shunted through R to protect the galvanometer

Null Measurements

- Explain why a null measurement device is more accurate than a standard voltmeter or ammeter.
- Demonstrate how a Wheatstone bridge can be used to accurately calculate the resistance in a circuit.

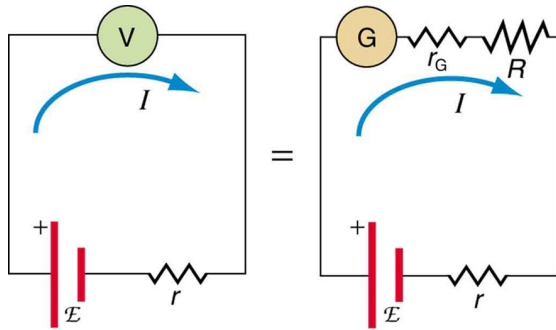
Standard measurements of voltage and current alter the circuit being measured, introducing uncertainties in the measurements. Voltmeters draw some extra current, whereas ammeters reduce current flow. **Null measurements** balance voltages so that there is no current flowing through the measuring device and, therefore, no alteration of the circuit being measured.

Null measurements are generally more accurate but are also more complex than the use of standard voltmeters and ammeters, and they still have limits to their precision. In this module, we shall consider a few specific types of null measurements, because they are common and interesting, and they further illuminate principles of electric circuits.

The Potentiometer

Suppose you wish to measure the emf of a battery. Consider what happens if you connect the battery directly to a standard voltmeter as shown in [\[link\]](#). (Once we note the problems with this measurement, we will examine a null measurement that improves accuracy.) As discussed before, the actual quantity measured is the terminal voltage V , which is related to the emf of the battery by $V = \text{emf} - Ir$, where I is the current that flows and r is the internal resistance of the battery.

The emf could be accurately calculated if r were very accurately known, but it is usually not. If the current I could be made zero, then $V = \text{emf}$, and so emf could be directly measured. However, standard voltmeters need a current to operate; thus, another technique is needed.



An analog voltmeter attached to a battery draws a small but nonzero current and measures a terminal voltage that differs from the emf of the battery. (Note that the script capital $\mathcal{E}$ symbolizes electromotive force, or emf.) Since the internal resistance of the battery is not known precisely, it is not possible to calculate the emf precisely.

A **potentiometer** is a null measurement device for measuring potentials (voltages). (See [\[link\]](#).) A voltage source is connected to a resistor R , say, a long wire, and passes a constant current through it. There is a steady drop in potential (an IR drop) along the wire, so that a variable potential can be obtained by making contact at varying locations along the wire.

[\[link\]](#)(b) shows an unknown emf_x (represented by script $\mathcal{E}_x$ in the figure) connected in series with a galvanometer. Note that emf_x opposes the other voltage source. The location of the contact point (see the arrow on the drawing) is adjusted until the galvanometer reads zero. When the galvanometer reads zero, $\text{emf}_x = IR_x$, where R_x is the resistance of the section of wire up to the contact point. Since no current flows through the

galvanometer, none flows through the unknown emf, and so emf_x is directly sensed.

Now, a very precisely known standard emf_s is substituted for emf_x , and the contact point is adjusted until the galvanometer again reads zero, so that $\text{emf}_s = IR_s$. In both cases, no current passes through the galvanometer, and so the current I through the long wire is the same. Upon taking the ratio $\frac{\text{emf}_x}{\text{emf}_s}$, I cancels, giving

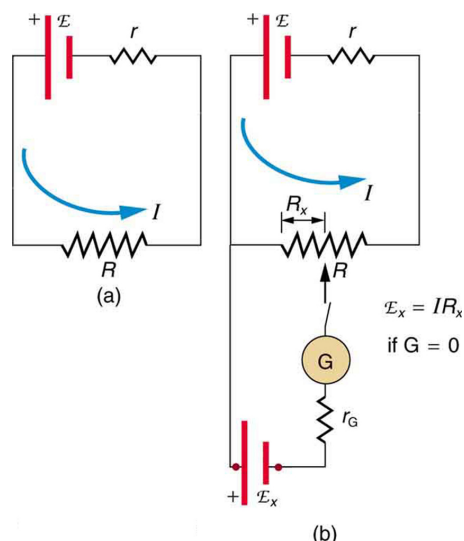
Equation:

$$\frac{\text{emf}_x}{\text{emf}_s} = \frac{IR_x}{IR_s} = \frac{R_x}{R_s}.$$

Solving for emf_x gives

Equation:

$$\text{emf}_x = \text{emf}_s \frac{R_x}{R_s}.$$



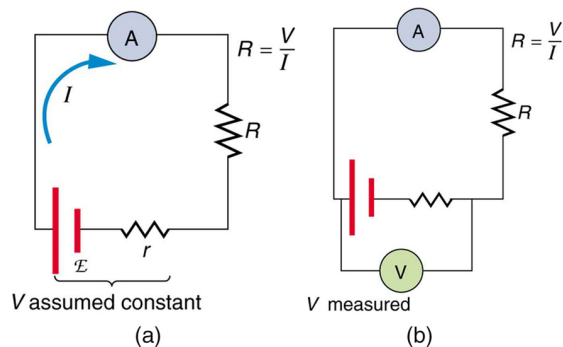
The potentiometer, a
null measurement

device. (a) A voltage source connected to a long wire resistor passes a constant current I through it. (b) An unknown emf (labeled script E_x in the figure) is connected as shown, and the point of contact along R is adjusted until the galvanometer reads zero. The segment of wire has a resistance R_x and script $E_x = IR_x$, where I is unaffected by the connection since no current flows through the galvanometer. The unknown emf is thus proportional to the resistance of the wire segment.

Because a long uniform wire is used for R , the ratio of resistances R_x/R_s is the same as the ratio of the lengths of wire that zero the galvanometer for each emf. The three quantities on the right-hand side of the equation are now known or measured, and emf_x can be calculated. The uncertainty in this calculation can be considerably smaller than when using a voltmeter directly, but it is not zero. There is always some uncertainty in the ratio of resistances R_x/R_s and in the standard emf_s . Furthermore, it is not possible to tell when the galvanometer reads exactly zero, which introduces error into both R_x and R_s , and may also affect the current I .

Resistance Measurements and the Wheatstone Bridge

There is a variety of so-called **ohmmeters** that purport to measure resistance. What the most common ohmmeters actually do is to apply a voltage to a resistance, measure the current, and calculate the resistance using Ohm's law. Their readout is this calculated resistance. Two configurations for ohmmeters using standard voltmeters and ammeters are shown in [\[link\]](#). Such configurations are limited in accuracy, because the meters alter both the voltage applied to the resistor and the current that flows through it.



Two methods for measuring resistance with standard meters. (a) Assuming a known voltage for the source, an ammeter measures current, and resistance is calculated as $R = \frac{V}{I}$. (b) Since the terminal voltage V varies with current, it is better to measure it. V is most accurately known when I is small, but I itself is most accurately known when it is large.

The **Wheatstone bridge** is a null measurement device for calculating resistance by balancing potential drops in a circuit. (See [\[link\]](#).) The device is called a bridge because the galvanometer forms a bridge between two branches. A variety of **bridge devices** are used to make null measurements in circuits.

Resistors R_1 and R_2 are precisely known, while the arrow through R_3 indicates that it is a variable resistance. The value of R_3 can be precisely read. With the unknown resistance R_x in the circuit, R_3 is adjusted until the galvanometer reads zero. The potential difference between points b and d is then zero, meaning that b and d are at the same potential. With no current running through the galvanometer, it has no effect on the rest of the circuit. So the branches abc and adc are in parallel, and each branch has the full voltage of the source. That is, the IR drops along abc and adc are the same. Since b and d are at the same potential, the IR drop along ad must equal the IR drop along ab. Thus,

Equation:

$$I_1 R_1 = I_2 R_3.$$

Again, since b and d are at the same potential, the IR drop along dc must equal the IR drop along bc. Thus,

Equation:

$$I_1 R_2 = I_2 R_x.$$

Taking the ratio of these last two expressions gives

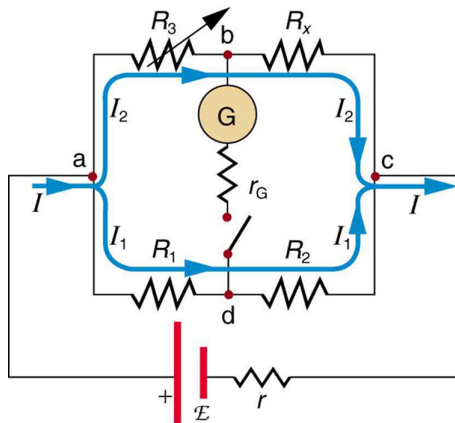
Equation:

$$\frac{I_1 R_1}{I_1 R_2} = \frac{I_2 R_3}{I_2 R_x}.$$

Canceling the currents and solving for R_x yields

Equation:

$$R_x = R_3 \frac{R_2}{R_1}.$$



The Wheatstone bridge is used to calculate unknown resistances. The variable resistance R_3 is adjusted until the galvanometer reads zero with the switch closed. This simplifies the circuit, allowing R_x to be calculated based on the IR drops as discussed in the text.

This equation is used to calculate the unknown resistance when current through the galvanometer is zero. This method can be very accurate (often to four significant digits), but it is limited by two factors. First, it is not possible to get the current through the galvanometer to be exactly zero.

Second, there are always uncertainties in R_1 , R_2 , and R_3 , which contribute to the uncertainty in R_x .

Exercise:

Check Your Understanding

Problem:

Identify other factors that might limit the accuracy of null measurements. Would the use of a digital device that is more sensitive than a galvanometer improve the accuracy of null measurements?

Solution:

One factor would be resistance in the wires and connections in a null measurement. These are impossible to make zero, and they can change over time. Another factor would be temperature variations in resistance, which can be reduced but not completely eliminated by choice of material. Digital devices sensitive to smaller currents than analog devices do improve the accuracy of null measurements because they allow you to get the current closer to zero.

Section Summary

- Null measurement techniques achieve greater accuracy by balancing a circuit so that no current flows through the measuring device.
- One such device, for determining voltage, is a potentiometer.
- Another null measurement device, for determining resistance, is the Wheatstone bridge.
- Other physical quantities can also be measured with null measurement techniques.

Conceptual questions

Exercise:

Problem:

Why can a null measurement be more accurate than one using standard voltmeters and ammeters? What factors limit the accuracy of null measurements?

Exercise:**Problem:**

If a potentiometer is used to measure cell emfs on the order of a few volts, why is it most accurate for the standard emf_s to be the same order of magnitude and the resistances to be in the range of a few ohms?

Problem Exercises**Exercise:****Problem:**

What is the emf_x of a cell being measured in a potentiometer, if the standard cell's emf is 12.0 V and the potentiometer balances for $R_x = 5.000\ \Omega$ and $R_s = 2.500\ \Omega$?

Solution:

24.0 V

Exercise:**Problem:**

Calculate the emf_x of a dry cell for which a potentiometer is balanced when $R_x = 1.200\ \Omega$, while an alkaline standard cell with an emf of 1.600 V requires $R_s = 1.247\ \Omega$ to balance the potentiometer.

Exercise:

Problem:

When an unknown resistance R_x is placed in a Wheatstone bridge, it is possible to balance the bridge by adjusting R_3 to be $2500\ \Omega$. What is R_x if $\frac{R_2}{R_1} = 0.625$?

Solution:

$1.56\ \text{k}\Omega$

Exercise:**Problem:**

To what value must you adjust R_3 to balance a Wheatstone bridge, if the unknown resistance R_x is $100\ \Omega$, R_1 is $50.0\ \Omega$, and R_2 is $175\ \Omega$?

Exercise:**Problem:**

(a) What is the unknown emf_x in a potentiometer that balances when R_x is $10.0\ \Omega$, and balances when R_s is $15.0\ \Omega$ for a standard 3.000-V emf ? (b) The same emf_x is placed in the same potentiometer, which now balances when R_s is $15.0\ \Omega$ for a standard emf of $3.100\ \text{V}$. At what resistance R_x will the potentiometer balance?

Solution:

(a) $2.00\ \text{V}$

(b) $9.68\ \Omega$

Exercise:**Problem:**

Suppose you want to measure resistances in the range from $10.0\ \Omega$ to $10.0\ \text{k}\Omega$ using a Wheatstone bridge that has $\frac{R_2}{R_1} = 2.000$. Over what range should R_3 be adjustable?

Solution:
Equation:

$$\text{Range} = 5.00 \, \Omega \text{ to } 5.00 \, \text{k}\Omega$$

Glossary

null measurements

methods of measuring current and voltage more accurately by balancing the circuit so that no current flows through the measurement device

potentiometer

a null measurement device for measuring potentials (voltages)

ohmmeter

an instrument that applies a voltage to a resistance, measures the current, calculates the resistance using Ohm's law, and provides a readout of this calculated resistance

bridge device

a device that forms a bridge between two branches of a circuit; some bridge devices are used to make null measurements in circuits

Wheatstone bridge

a null measurement device for calculating resistance by balancing potential drops in a circuit

DC Circuits Containing Resistors and Capacitors

- Explain the importance of the time constant, τ , and calculate the time constant for a given resistance and capacitance.
- Explain why batteries in a flashlight gradually lose power and the light dims over time.
- Describe what happens to a graph of the voltage across a capacitor over time as it charges.
- Explain how a timing circuit works and list some applications.
- Calculate the necessary speed of a strobe flash needed to “stop” the movement of an object over a particular length.

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and a number of other phenomena that involve charging and discharging capacitors are discussed in this module.

RC Circuits

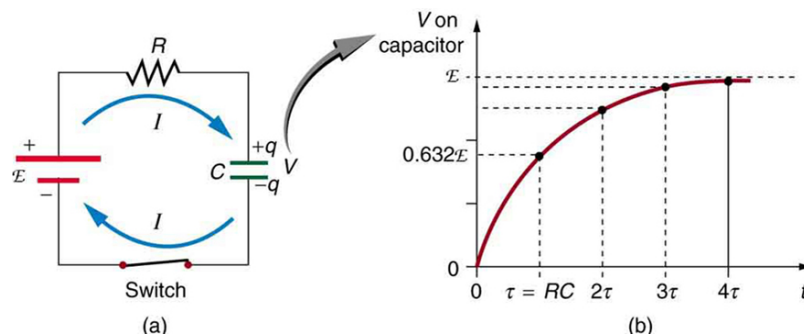
An **RC circuit** is one containing a **resistor** R and a **capacitor** C . The capacitor is an electrical component that stores electric charge.

[\[link\]](#) shows a simple RC circuit that employs a DC (direct current) voltage source. The capacitor is initially uncharged. As soon as the switch is closed, current flows to and from the initially uncharged capacitor. As charge increases on the capacitor plates, there is increasing opposition to the flow of charge by the repulsion of like charges on each plate.

In terms of voltage, this is because voltage across the capacitor is given by $V_c = Q/C$, where Q is the amount of charge stored on each plate and C is the **capacitance**. This voltage opposes the battery, growing from zero to the maximum emf when fully charged. The current thus decreases from its initial value of $I_0 = \frac{\text{emf}}{R}$ to zero as the voltage on the capacitor reaches the same value as the emf. When there is no current, there is no IR drop, and so the voltage on the capacitor must then equal the emf of the voltage source. This can also be explained with Kirchhoff’s second rule (the loop rule),

discussed in [Kirchhoff's Rules](#), which says that the algebraic sum of changes in potential around any closed loop must be zero.

The initial current is $I_0 = \frac{\text{emf}}{R}$, because all of the IR drop is in the resistance. Therefore, the smaller the resistance, the faster a given capacitor will be charged. Note that the internal resistance of the voltage source is included in R , as are the resistances of the capacitor and the connecting wires. In the flash camera scenario above, when the batteries powering the camera begin to wear out, their internal resistance rises, reducing the current and lengthening the time it takes to get ready for the next flash.



- (a) An RC circuit with an initially uncharged capacitor. Current flows in the direction shown (opposite of electron flow) as soon as the switch is closed. Mutual repulsion of like charges in the capacitor progressively slows the flow as the capacitor is charged, stopping the current when the capacitor is fully charged and $Q = C \cdot \text{emf}$.
- (b) A graph of voltage across the capacitor versus time, with the switch closing at time $t = 0$. (Note that in the two parts of the figure, the capital script E stands for emf, q stands for the charge stored on the capacitor, and τ is the RC time constant.)

Voltage on the capacitor is initially zero and rises rapidly at first, since the initial current is a maximum. [\[link\]](#)(b) shows a graph of capacitor voltage versus time (t) starting when the switch is closed at $t = 0$. The voltage approaches emf asymptotically, since the closer it gets to emf the less current flows. The equation for voltage versus time when charging a capacitor C through a resistor R , derived using calculus, is

Equation:

$$V = \text{emf}(1 - e^{-t/RC}) \text{ (charging),}$$

where V is the voltage across the capacitor, emf is equal to the emf of the DC voltage source, and the exponential $e = 2.718 \dots$ is the base of the natural logarithm. Note that the units of RC are seconds. We define

Equation:

$$\tau = RC,$$

where τ (the Greek letter tau) is called the time constant for an RC circuit. As noted before, a small resistance R allows the capacitor to charge faster. This is reasonable, since a larger current flows through a smaller resistance. It is also reasonable that the smaller the capacitor C , the less time needed to charge it. Both factors are contained in $\tau = RC$.

More quantitatively, consider what happens when $t = \tau = RC$. Then the voltage on the capacitor is

Equation:

$$V = \text{emf}(1 - e^{-1}) = \text{emf}(1 - 0.368) = 0.632 \cdot \text{emf}.$$

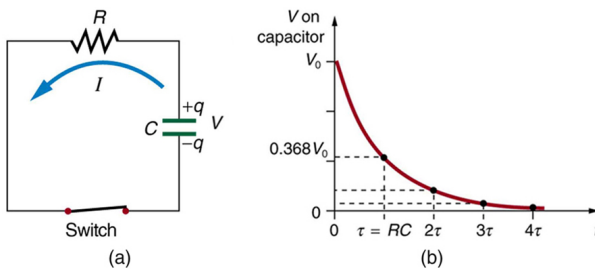
This means that in the time $\tau = RC$, the voltage rises to 0.632 of its final value. The voltage will rise 0.632 of the remainder in the next time τ . It is a characteristic of the exponential function that the final value is never reached, but 0.632 of the remainder to that value is achieved in every time, τ . In just a few multiples of the time constant τ , then, the final value is very nearly achieved, as the graph in [\[link\]](#)(b) illustrates.

Discharging a Capacitor

Discharging a capacitor through a resistor proceeds in a similar fashion, as [\[link\]](#) illustrates. Initially, the current is $I_0 = \frac{V_0}{R}$, driven by the initial voltage V_0 on the capacitor. As the voltage decreases, the current and hence the rate of discharge decreases, implying another exponential formula for V . Using calculus, the voltage V on a capacitor C being discharged through a resistor R is found to be

Equation:

$$V = V_0 e^{-t/RC} \text{ (discharging).}$$



The graph in [\[link\]](#)(b) is an example of this exponential decay. Again, the time constant is $\tau = RC$. A small resistance R allows the capacitor to discharge in a small time, since the current is larger. Similarly, a small capacitance requires less time to discharge, since less charge is stored. In the first time interval $\tau = RC$ after the switch is closed, the voltage falls to 0.368 of its initial value, since $V = V_0 \cdot e^{-1} = 0.368V_0$.

During each successive time τ , the voltage falls to 0.368 of its preceding value. In a few multiples of τ , the voltage becomes very close to zero, as indicated by the graph in [\[link\]](#)(b).

Now we can explain why the flash camera in our scenario takes so much longer to charge than discharge; the resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower. (You may have noticed this.)

The flash discharge is through a low-resistance ionized gas in the flash tube and proceeds very rapidly. Flash photographs, such as in [\[link\]](#), can capture a brief instant of a rapid motion because the flash can be less than a microsecond in duration. Such flashes can be made extremely intense.

During World War II, nighttime reconnaissance photographs were made from the air with a single flash illuminating more than a square kilometer of enemy territory. The brevity of the flash eliminated blurring due to the surveillance aircraft's motion. Today, an important use of intense flash lamps is to pump energy into a laser. The short intense flash can rapidly energize a laser and allow it to reemit the energy in another form.



This stop-motion photograph of a rufous hummingbird (*Selasphorus rufus*) feeding on a flower was obtained with an extremely brief and intense flash of light powered by the discharge of a capacitor through a gas.
(credit: Dean E. Biggins, U.S. Fish and Wildlife Service)

Example:

Integrated Concept Problem: Calculating Capacitor Size—Strobe Lights

High-speed flash photography was pioneered by Doc Edgerton in the 1930s, while he was a professor of electrical engineering at MIT. You might have seen examples of his work in the amazing shots of hummingbirds in motion, a drop of milk splattering on a table, or a bullet penetrating an apple (see [\[link\]](#)). To stop the motion and capture these pictures, one needs a high-intensity, very short pulsed flash, as mentioned earlier in this module.

Suppose one wished to capture the picture of a bullet (moving at 5.0×10^2 m/s) that was passing through an apple. The duration of the flash is related to the RC time constant, τ . What size capacitor would one

need in the RC circuit to succeed, if the resistance of the flash tube was $10.0\ \Omega$? Assume the apple is a sphere with a diameter of $8.0 \times 10^{-2}\ \text{m}$.

Strategy

We begin by identifying the physical principles involved. This example deals with the strobe light, as discussed above. [\[link\]](#) shows the circuit for this probe. The characteristic time τ of the strobe is given as $\tau = RC$.

Solution

We wish to find C , but we don't know τ . We want the flash to be on only while the bullet traverses the apple. So we need to use the kinematic equations that describe the relationship between distance x , velocity v , and time t :

Equation:

$$x = vt \text{ or } t = \frac{x}{v}.$$

The bullet's velocity is given as $5.0 \times 10^2\ \text{m/s}$, and the distance x is $8.0 \times 10^{-2}\ \text{m}$. The traverse time, then, is

Equation:

$$t = \frac{x}{v} = \frac{8.0 \times 10^{-2}\ \text{m}}{5.0 \times 10^2\ \text{m/s}} = 1.6 \times 10^{-4}\ \text{s}.$$

We set this value for the crossing time t equal to τ . Therefore,

Equation:

$$C = \frac{t}{R} = \frac{1.6 \times 10^{-4}\ \text{s}}{10.0\ \Omega} = 16\ \mu\text{F}.$$

(Note: Capacitance C is typically measured in farads, F , defined as Coulombs per volt. From the equation, we see that C can also be stated in units of seconds per ohm.)

Discussion

The flash interval of $160\ \mu\text{s}$ (the traverse time of the bullet) is relatively easy to obtain today. Strobe lights have opened up new worlds from science to entertainment. The information from the picture of the apple and bullet was used in the Warren Commission Report on the assassination of

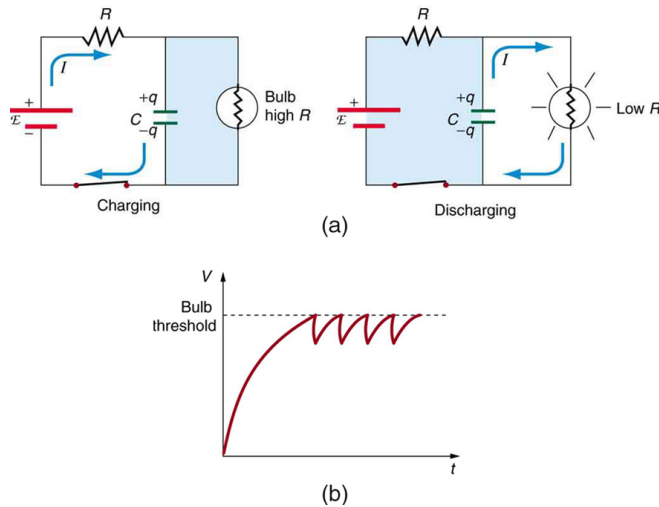
President John F. Kennedy in 1963 to confirm that only one bullet was fired.

RC Circuits for Timing

RC circuits are commonly used for timing purposes. A mundane example of this is found in the ubiquitous intermittent wiper systems of modern cars. The time between wipes is varied by adjusting the resistance in an RC circuit. Another example of an RC circuit is found in novelty jewelry, Halloween costumes, and various toys that have battery-powered flashing lights. (See [\[link\]](#) for a timing circuit.)

A more crucial use of RC circuits for timing purposes is in the artificial pacemaker, used to control heart rate. The heart rate is normally controlled by electrical signals generated by the sino-atrial (SA) node, which is on the wall of the right atrium chamber. This causes the muscles to contract and pump blood. Sometimes the heart rhythm is abnormal and the heartbeat is too high or too low.

The artificial pacemaker is inserted near the heart to provide electrical signals to the heart when needed with the appropriate time constant. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during exercise to meet the body's increased needs for blood and oxygen.



- (a) The lamp in this RC circuit ordinarily has a very high resistance, so that the battery charges the capacitor as if the lamp were not there. When the voltage reaches a threshold value, a current flows through the lamp that dramatically reduces its resistance, and the capacitor discharges through the lamp as if the battery and charging resistor were not there. Once discharged, the process starts again, with the flash period determined by the RC constant τ .
- (b) A graph of voltage versus time for this circuit.

Example:

Calculating Time: RC Circuit in a Heart Defibrillator

A heart defibrillator is used to resuscitate an accident victim by discharging a capacitor through the trunk of her body. A simplified version of the

circuit is seen in [\[link\]](#). (a) What is the time constant if an $8.00\text{-}\mu\text{F}$ capacitor is used and the path resistance through her body is $1.00 \times 10^3 \Omega$? (b) If the initial voltage is 10.0 kV , how long does it take to decline to $5.00 \times 10^2 \text{ V}$?

Strategy

Since the resistance and capacitance are given, it is straightforward to multiply them to give the time constant asked for in part (a). To find the time for the voltage to decline to $5.00 \times 10^2 \text{ V}$, we repeatedly multiply the initial voltage by 0.368 until a voltage less than or equal to $5.00 \times 10^2 \text{ V}$ is obtained. Each multiplication corresponds to a time of τ seconds.

Solution for (a)

The time constant τ is given by the equation $\tau = RC$. Entering the given values for resistance and capacitance (and remembering that units for a farad can be expressed as s/Ω) gives

Equation:

$$\tau = RC = (1.00 \times 10^3 \Omega)(8.00 \mu\text{F}) = 8.00 \text{ ms}.$$

Solution for (b)

In the first 8.00 ms , the voltage (10.0 kV) declines to 0.368 of its initial value. That is:

Equation:

$$V = 0.368V_0 = 3.680 \times 10^3 \text{ V at } t = 8.00 \text{ ms}.$$

(Notice that we carry an extra digit for each intermediate calculation.) After another 8.00 ms , we multiply by 0.368 again, and the voltage is

Equation:

$$\begin{aligned} V' &= 0.368V \\ &= (0.368)(3.680 \times 10^3 \text{ V}) \\ &= 1.354 \times 10^3 \text{ V at } t = 16.0 \text{ ms}. \end{aligned}$$

Similarly, after another 8.00 ms , the voltage is

Equation:

$$\begin{aligned} V'' &= 0.368V' = (0.368)(1.354 \times 10^3 \text{ V}) \\ &= 498 \text{ V at } t = 24.0 \text{ ms.} \end{aligned}$$

Discussion

So after only 24.0 ms, the voltage is down to 498 V, or 4.98% of its original value. Such brief times are useful in heart defibrillation, because the brief but intense current causes a brief but effective contraction of the heart. The actual circuit in a heart defibrillator is slightly more complex than the one in [\[link\]](#), to compensate for magnetic and AC effects that will be covered in [Magnetism](#).

Exercise:

Check Your Understanding

Problem: When is the potential difference across a capacitor an emf?

Solution:

Only when the current being drawn from or put into the capacitor is zero. Capacitors, like batteries, have internal resistance, so their output voltage is not an emf unless current is zero. This is difficult to measure in practice so we refer to a capacitor's voltage rather than its emf. But the source of potential difference in a capacitor is fundamental and it is an emf.

Note:

PhET Explorations: Circuit Construction Kit (DC only)

An electronics kit in your computer! Build circuits with resistors, light bulbs, batteries, and switches. Take measurements with the realistic ammeter and voltmeter. View the circuit as a schematic diagram, or switch to a life-like view.

<https://archive.cnx.org/specials/f23ce496-c9d1-11e5-bdc8-bb04dc1eecb6/circuit-construction-kit-dc-only/#sim-cck>

Section Summary

- An RC circuit is one that has both a resistor and a capacitor.
- The time constant τ for an RC circuit is $\tau = RC$.
- When an initially uncharged ($V_0 = 0$ at $t = 0$) capacitor in series with a resistor is charged by a DC voltage source, the voltage rises, asymptotically approaching the emf of the voltage source; as a function of time,

Equation:

$$V = \text{emf}(1 - e^{-t/RC}) \text{(charging)}.$$

- Within the span of each time constant τ , the voltage rises by 0.632 of the remaining value, approaching the final voltage asymptotically.
- If a capacitor with an initial voltage V_0 is discharged through a resistor starting at $t = 0$, then its voltage decreases exponentially as given by

Equation:

$$V = V_0 e^{-t/RC} \text{(discharging)}.$$

- In each time constant τ , the voltage falls by 0.368 of its remaining initial value, approaching zero asymptotically.

Conceptual questions

Exercise:

Problem:

Regarding the units involved in the relationship $\tau = RC$, verify that the units of resistance times capacitance are time, that is, $\Omega \cdot \text{F} = \text{s}$.

Exercise:

Problem:

The RC time constant in heart defibrillation is crucial to limiting the time the current flows. If the capacitance in the defibrillation unit is fixed, how would you manipulate resistance in the circuit to adjust the RC constant τ ? Would an adjustment of the applied voltage also be needed to ensure that the current delivered has an appropriate value?

Exercise:**Problem:**

When making an ECG measurement, it is important to measure voltage variations over small time intervals. The time is limited by the RC constant of the circuit—it is not possible to measure time variations shorter than RC. How would you manipulate R and C in the circuit to allow the necessary measurements?

Exercise:**Problem:**

Draw two graphs of charge versus time on a capacitor. Draw one for charging an initially uncharged capacitor in series with a resistor, as in the circuit in [\[link\]](#), starting from $t = 0$. Draw the other for discharging a capacitor through a resistor, as in the circuit in [\[link\]](#), starting at $t = 0$, with an initial charge Q_0 . Show at least two intervals of τ .

Exercise:**Problem:**

When charging a capacitor, as discussed in conjunction with [\[link\]](#), how long does it take for the voltage on the capacitor to reach emf? Is this a problem?

Exercise:

Problem:

When discharging a capacitor, as discussed in conjunction with [\[link\]](#), how long does it take for the voltage on the capacitor to reach zero? Is this a problem?

Exercise:**Problem:**

Referring to [\[link\]](#), draw a graph of potential difference across the resistor versus time, showing at least two intervals of τ . Also draw a graph of current versus time for this situation.

Exercise:**Problem:**

A long, inexpensive extension cord is connected from inside the house to a refrigerator outside. The refrigerator doesn't run as it should. What might be the problem?

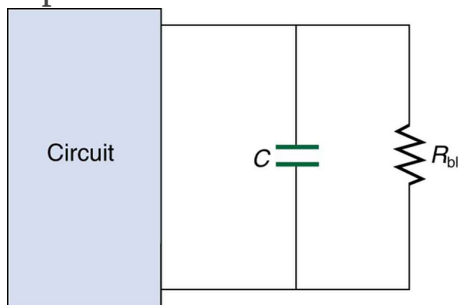
Exercise:**Problem:**

In [\[link\]](#), does the graph indicate the time constant is shorter for discharging than for charging? Would you expect ionized gas to have low resistance? How would you adjust R to get a longer time between flashes? Would adjusting R affect the discharge time?

Exercise:

Problem:

An electronic apparatus may have large capacitors at high voltage in the power supply section, presenting a shock hazard even when the apparatus is switched off. A “bleeder resistor” is therefore placed across such a capacitor, as shown schematically in [\[link\]](#), to bleed the charge from it after the apparatus is off. Why must the bleeder resistance be much greater than the effective resistance of the rest of the circuit? How does this affect the time constant for discharging the capacitor?



A bleeder resistor R_{bl} discharges the capacitor in this electronic device once it is switched off.

Problem Exercises**Exercise:****Problem:**

The timing device in an automobile’s intermittent wiper system is based on an RC time constant and utilizes a $0.500\text{-}\mu\text{F}$ capacitor and a variable resistor. Over what range must R be made to vary to achieve time constants from 2.00 to 15.0 s?

Solution:

range 4.00 to 30.0 M Ω

Exercise:**Problem:**

A heart pacemaker fires 72 times a minute, each time a 25.0-nF capacitor is charged (by a battery in series with a resistor) to 0.632 of its full voltage. What is the value of the resistance?

Exercise:**Problem:**

The duration of a photographic flash is related to an RC time constant, which is 0.100 μ s for a certain camera. (a) If the resistance of the flash lamp is 0.0400 Ω during discharge, what is the size of the capacitor supplying its energy? (b) What is the time constant for charging the capacitor, if the charging resistance is 800 k Ω ?

Solution:

(a) 2.50 μ F

(b) 2.00 s

Exercise:**Problem:**

A 2.00- and a 7.50- μ F capacitor can be connected in series or parallel, as can a 25.0- and a 100-k Ω resistor. Calculate the four RC time constants possible from connecting the resulting capacitance and resistance in series.

Exercise:

Problem:

After two time constants, what percentage of the final voltage, emf, is on an initially uncharged capacitor C , charged through a resistance R ?

Solution:

86.5%

Exercise:**Problem:**

A $500\text{-}\Omega$ resistor, an uncharged $1.50\text{-}\mu\text{F}$ capacitor, and a 6.16-V emf are connected in series. (a) What is the initial current? (b) What is the RC time constant? (c) What is the current after one time constant? (d) What is the voltage on the capacitor after one time constant?

Exercise:**Problem:**

A heart defibrillator being used on a patient has an RC time constant of 10.0 ms due to the resistance of the patient and the capacitance of the defibrillator. (a) If the defibrillator has an $8.00\text{-}\mu\text{F}$ capacitance, what is the resistance of the path through the patient? (You may neglect the capacitance of the patient and the resistance of the defibrillator.) (b) If the initial voltage is 12.0 kV , how long does it take to decline to $6.00 \times 10^2\text{ V}$?

Solution:

(a) $1.25\text{ k}\Omega$

(b) 30.0 ms

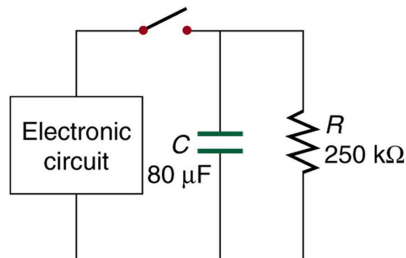
Exercise:

Problem:

An ECG monitor must have an RC time constant less than $1.00 \times 10^2 \mu\text{s}$ to be able to measure variations in voltage over small time intervals. (a) If the resistance of the circuit (due mostly to that of the patient's chest) is $1.00 \text{ k}\Omega$, what is the maximum capacitance of the circuit? (b) Would it be difficult in practice to limit the capacitance to less than the value found in (a)?

Exercise:**Problem:**

[\[link\]](#) shows how a bleeder resistor is used to discharge a capacitor after an electronic device is shut off, allowing a person to work on the electronics with less risk of shock. (a) What is the time constant? (b) How long will it take to reduce the voltage on the capacitor to 0.250% (5% of 5%) of its full value once discharge begins? (c) If the capacitor is charged to a voltage V_0 through a $100\text{-}\Omega$ resistance, calculate the time it takes to rise to $0.865V_0$ (This is about two time constants.)

**Solution:**

- (a) 20.0 s
- (b) 120 s
- (c) 16.0 ms

Exercise:

Problem:

Using the exact exponential treatment, find how much time is required to discharge a $250\text{-}\mu\text{F}$ capacitor through a $500\text{-}\Omega$ resistor down to 1.00% of its original voltage.

Exercise:**Problem:**

Using the exact exponential treatment, find how much time is required to charge an initially uncharged 100-pF capacitor through a $75.0\text{-M}\Omega$ resistor to 90.0% of its final voltage.

Solution:

$$1.73 \times 10^{-2} \text{ s}$$

Exercise:**Problem: Integrated Concepts**

If you wish to take a picture of a bullet traveling at 500 m/s , then a very brief flash of light produced by an RC discharge through a flash tube can limit blurring. Assuming 1.00 mm of motion during one RC constant is acceptable, and given that the flash is driven by a $600\text{-}\mu\text{F}$ capacitor, what is the resistance in the flash tube?

Solution:

$$3.33 \times 10^{-3} \Omega$$

Exercise:**Problem: Integrated Concepts**

A flashing lamp in a Christmas earring is based on an RC discharge of a capacitor through its resistance. The effective duration of the flash is 0.250 s , during which it produces an average 0.500 W from an average

3.00 V. (a) What energy does it dissipate? (b) How much charge moves through the lamp? (c) Find the capacitance. (d) What is the resistance of the lamp?

Exercise:

Problem: Integrated Concepts

A 160- μF capacitor charged to 450 V is discharged through a 31.2-k Ω resistor. (a) Find the time constant. (b) Calculate the temperature increase of the resistor, given that its mass is 2.50 g and its specific heat is $1.67 \frac{\text{kJ}}{\text{kg} \cdot ^\circ\text{C}}$, noting that most of the thermal energy is retained in the short time of the discharge. (c) Calculate the new resistance, assuming it is pure carbon. (d) Does this change in resistance seem significant?

Solution:

- (a) 4.99 s
- (b) 3.87 $^\circ\text{C}$
- (c) 31.1 k Ω
- (d) No

Exercise:

Problem: Unreasonable Results

(a) Calculate the capacitance needed to get an RC time constant of 1.00×10^3 s with a 0.100- Ω resistor. (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Construct Your Own Problem

Consider a camera's flash unit. Construct a problem in which you calculate the size of the capacitor that stores energy for the flash lamp. Among the things to be considered are the voltage applied to the capacitor, the energy needed in the flash and the associated charge needed on the capacitor, the resistance of the flash lamp during discharge, and the desired RC time constant.

Exercise:

Problem: Construct Your Own Problem

Consider a rechargeable lithium cell that is to be used to power a camcorder. Construct a problem in which you calculate the internal resistance of the cell during normal operation. Also, calculate the minimum voltage output of a battery charger to be used to recharge your lithium cell. Among the things to be considered are the emf and useful terminal voltage of a lithium cell and the current it should be able to supply to a camcorder.

Glossary

RC circuit

a circuit that contains both a resistor and a capacitor

capacitor

an electrical component used to store energy by separating electric charge on two opposing plates

capacitance

the maximum amount of electric potential energy that can be stored (or separated) for a given electric potential

Introduction to Magnetism

class="introduction"

The
magnificent
spectacle
of the
Aurora
Borealis, or
northern
lights,
glows in
the
northern
sky above
Bear Lake
near
Eielson Air
Force Base,
Alaska.
Shaped by
the Earth's
magnetic
field, this
light is
produced
by
radiation
spewed
from solar
storms.
(credit:
Senior
Airman
Joshua
Strang, via
Flickr)



One evening, an Alaskan sticks a note to his refrigerator with a small magnet. Through the kitchen window, the Aurora Borealis glows in the night sky. This grand spectacle is shaped by the same force that holds the note to the refrigerator.

People have been aware of magnets and magnetism for thousands of years. The earliest records date to well before the time of Christ, particularly in a region of Asia Minor called Magnesia (the name of this region is the source of words like *magnetic*). Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. A practical application for magnets was found later, when they were employed as navigational compasses. The use of magnets in compasses resulted not only in improved long-distance sailing, but also in the names of “north” and “south” being given to the two types of magnetic poles.

Today magnetism plays many important roles in our lives. Physicists’ understanding of magnetism has enabled the development of technologies that affect our everyday lives. The iPod in your purse or backpack, for example, wouldn’t have been possible without the applications of magnetism and electricity on a small scale.

The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology. The 2007 Nobel Prize in Physics went to Albert Fert from France and Peter Grunberg from Germany for this discovery of *giant magnetoresistance* and its applications to computer memory.

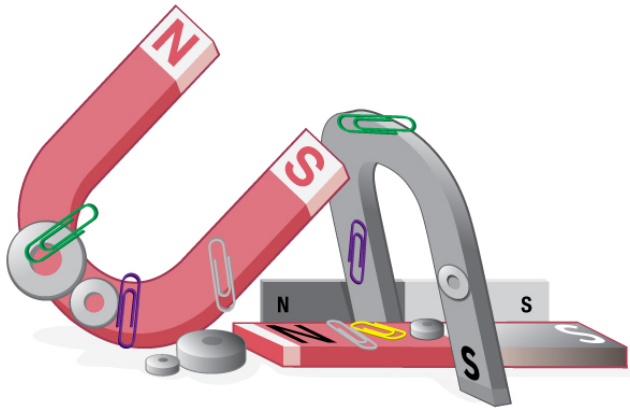
All electric motors, with uses as diverse as powering refrigerators, starting cars, and moving elevators, contain magnets. Generators, whether producing hydroelectric power or running bicycle lights, use magnetic fields. Recycling facilities employ magnets to separate iron from other refuse. Hundreds of millions of dollars are spent annually on magnetic containment of fusion as a future energy source. Magnetic resonance imaging (MRI) has become an important diagnostic tool in the field of medicine, and the use of magnetism to explore brain activity is a subject of contemporary research and development. The list of applications also includes computer hard drives, tape recording, detection of inhaled asbestos, and levitation of high-speed trains. Magnetism is used to explain atomic energy levels, cosmic rays, and charged particles trapped in the Van Allen belts. Once again, we will find all these disparate phenomena are linked by a small number of underlying physical principles.



Engineering of
technology like iPods
would not be possible
without a deep
understanding
magnetism. (credit: Jesse!
S?, Flickr)

Magnets

- Describe the difference between the north and south poles of a magnet.
- Describe how magnetic poles interact with each other.



Magnets come in various shapes, sizes, and strengths. All have both a north pole and a south pole. There is never an isolated pole (a monopole).

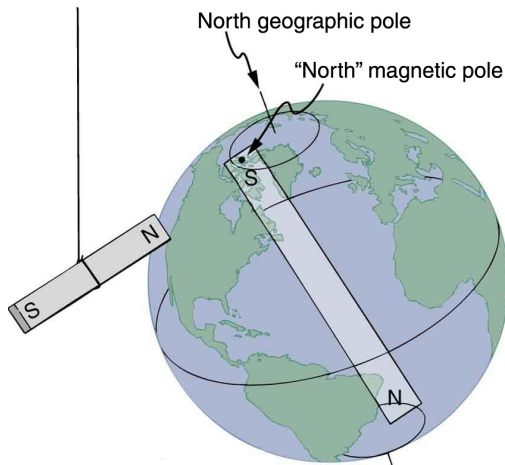
All magnets attract iron, such as that in a refrigerator door. However, magnets may attract or repel other magnets. Experimentation shows that all magnets have two poles. If freely suspended, one pole will point toward the north. The two poles are thus named the **north magnetic pole** and the **south magnetic pole** (or more properly, north-seeking and south-seeking poles, for the attractions in those directions).

Note:

Universal Characteristics of Magnets and Magnetic Poles

It is a universal characteristic of all magnets that *like poles repel and unlike poles attract*. (Note the similarity with electrostatics: unlike charges attract and like charges repel.)

Further experimentation shows that it is *impossible to separate north and south poles* in the manner that + and – charges can be separated.



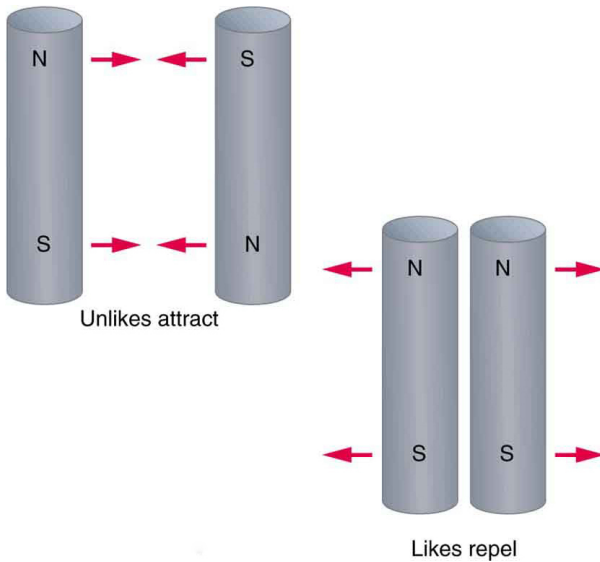
One end of a bar magnet is suspended from a thread that points toward north. The magnet's two poles are labeled N and S for north-seeking and south-seeking poles, respectively.

Note:

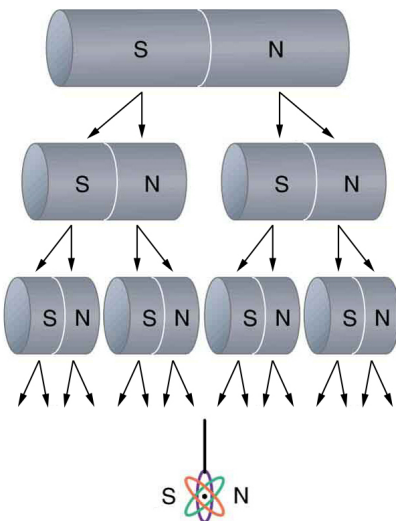
Misconception Alert: Earth's Geographic North Pole Hides an S

The Earth acts like a very large bar magnet with its south-seeking pole near the geographic North Pole. That is why the north pole of your compass is attracted toward the geographic north pole of the Earth—because the magnetic pole that is near the geographic North Pole is actually a south magnetic pole! Confusion arises because the geographic term “North Pole” has come to be used (incorrectly) for the magnetic pole that is near the

North Pole. Thus, “North magnetic pole” is actually a misnomer—it should be called the South magnetic pole.



Unlike poles attract, whereas
like poles repel.



North and south
poles always occur
in pairs. Attempts

to separate them
result in more pairs
of poles. If we
continue to split the
magnet, we will
eventually get
down to an iron
atom with a north
pole and a south
pole—these, too,
cannot be
separated.

The fact that magnetic poles always occur in pairs of north and south is true from the very large scale—for example, sunspots always occur in pairs that are north and south magnetic poles—all the way down to the very small scale. Magnetic atoms have both a north pole and a south pole, as do many types of subatomic particles, such as electrons, protons, and neutrons.

Note:

Making Connections: Take-Home Experiment—Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the door anyway? What can you say about the magnetic properties of the door next to the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

Section Summary

- Magnetism is a subject that includes the properties of magnets, the effect of the magnetic force on moving charges and currents, and the

- creation of magnetic fields by currents.
- There are two types of magnetic poles, called the north magnetic pole and south magnetic pole.
 - North magnetic poles are those that are attracted toward the Earth's geographic north pole.
 - Like poles repel and unlike poles attract.
 - Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.

Conceptual Questions

Exercise:

Problem:

Volcanic and other such activity at the mid-Atlantic ridge extrudes material to fill the gap between separating tectonic plates associated with continental drift. The magnetization of rocks is found to reverse in a coordinated manner with distance from the ridge. What does this imply about the Earth's magnetic field and how could the knowledge of the spreading rate be used to give its historical record?

Glossary

north magnetic pole

the end or the side of a magnet that is attracted toward Earth's geographic north pole

south magnetic pole

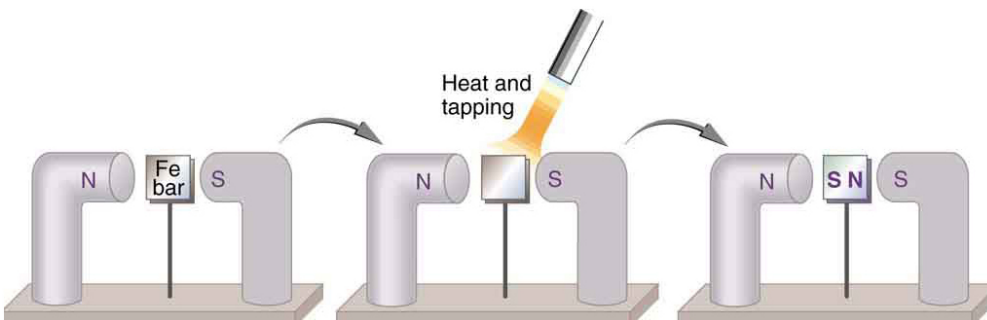
the end or the side of a magnet that is attracted toward Earth's geographic south pole

Ferromagnets and Electromagnets

- Define ferromagnet.
- Describe the role of magnetic domains in magnetization.
- Explain the significance of the Curie temperature.
- Describe the relationship between electricity and magnetism.

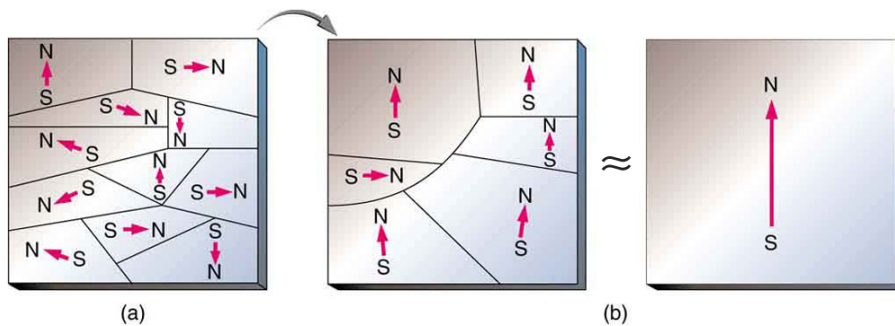
Ferromagnets

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word for iron, *ferrum*. A group of materials made from the alloys of the rare earth elements are also used as strong and permanent magnets; a popular one is neodymium. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets (the way iron is attracted to magnets), they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets.



An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that there are attractive forces between the magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in [\[link\]](#). (This results in the attraction of the previously unmagnetized material to the magnet.) What happens on a microscopic scale is illustrated in [\[link\]](#). The regions within the material called **domains** act like small bar magnets. Within domains, the poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves as shown in [\[link\]](#)(b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.



(a) An unmagnetized piece of iron (or other ferromagnetic material) has randomly oriented domains. (b) When magnetized by an external field, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and the size of

the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1043 K (770°C), which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

Electromagnets

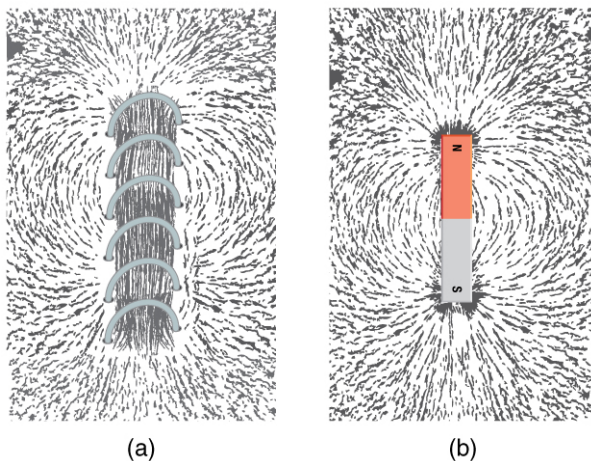
Early in the 19th century, it was discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of charges had any connection with magnets. **Electromagnetism** is the use of electric current to make magnets. These temporarily induced magnets are called **electromagnets**. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical imaging machines (See [\[link\]](#)).



Instrument for magnetic resonance imaging (MRI). The device uses a superconducting

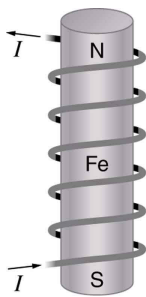
cylindrical coil for the main magnetic field. The patient goes into this “tunnel” on the gurney.
(credit: Bill McChesney, Flickr)

[\[link\]](#) shows that the response of iron filings to a current-carrying coil and to a permanent bar magnet. The patterns are similar. In fact, electromagnets and ferromagnets have the same basic characteristics—for example, they have north and south poles that cannot be separated and for which like poles repel and unlike poles attract.



Iron filings near (a) a current-carrying coil and (b) a magnet act like tiny compass needles, showing the shape of their fields. Their response to a current-carrying coil and a permanent magnet is seen to be very similar, especially near the ends of the coil and the magnet.

Combining a ferromagnet with an electromagnet can produce particularly strong magnetic effects. (See [\[link\]](#).) Whenever strong magnetic effects are needed, such as lifting scrap metal, or in particle accelerators, electromagnets are enhanced by ferromagnetic materials. Limits to how strong the magnets can be made are imposed by coil resistance (it will overheat and melt at sufficiently high current), and so superconducting magnets may be employed. These are still limited, because superconducting properties are destroyed by too great a magnetic field.

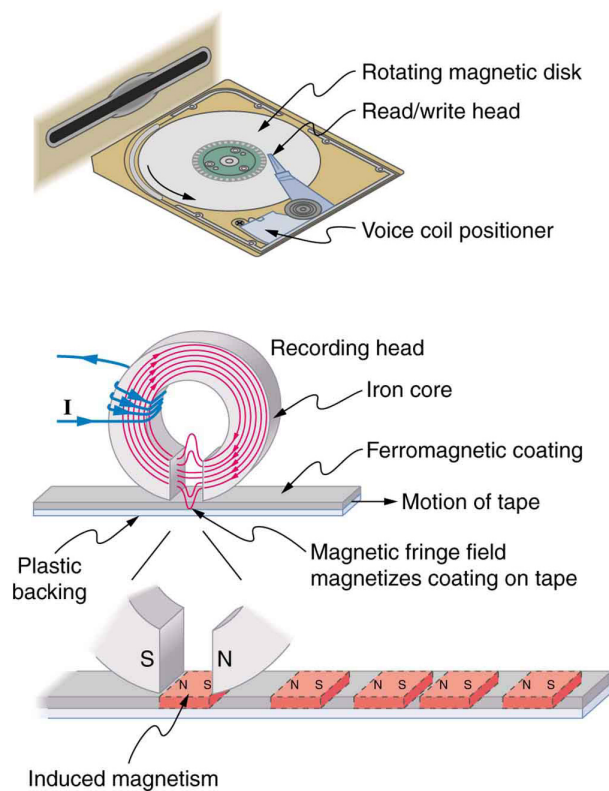


An
electromagnet
with a
ferromagnetic
core can
produce very
strong
magnetic
effects.

Alignment of
domains in the
core produces
a magnet, the
poles of which
are aligned
with the
electromagnet

.

[\[link\]](#) shows a few uses of combinations of electromagnets and ferromagnets. Ferromagnetic materials can act as memory devices, because the orientation of the magnetic fields of small domains can be reversed or erased. Magnetic information storage on videotapes and computer hard drives are among the most common applications. This property is vital in our digital world.



An electromagnet induces regions of permanent magnetism on a floppy disk coated with a ferromagnetic material. The information stored here is digital (a region is either magnetic or not); in other applications, it can be analog

(with a varying strength), such
as on audiotapes.

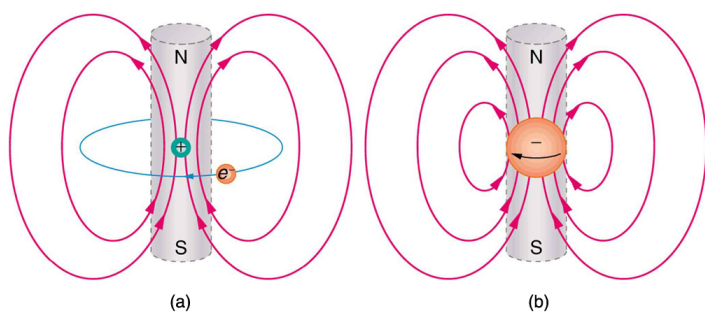
Current: The Source of All Magnetism

An electromagnet creates magnetism with an electric current. In later sections we explore this more quantitatively, finding the strength and direction of magnetic fields created by various currents. But what about ferromagnets? [\[link\]](#) shows models of how electric currents create magnetism at the submicroscopic level. (Note that we cannot directly observe the paths of individual electrons about atoms, and so a model or visual image, consistent with all direct observations, is made. We can directly observe the electron's orbital angular momentum, its spin momentum, and subsequent magnetic moments, all of which are explained with electric-current-creating subatomic magnetism.) Currents, including those associated with other submicroscopic particles like protons, allow us to explain ferromagnetism and all other magnetic effects. Ferromagnetism, for example, results from an internal cooperative alignment of electron spins, possible in some materials but not in others.

Crucial to the statement that electric current is the source of all magnetism is the fact that it is impossible to separate north and south magnetic poles. (This is far different from the case of positive and negative charges, which are easily separated.) A current loop always produces a magnetic dipole—that is, a magnetic field that acts like a north pole and south pole pair. Since isolated north and south magnetic poles, called **magnetic monopoles**, are not observed, currents are used to explain all magnetic effects. If magnetic monopoles did exist, then we would have to modify this underlying connection that all magnetism is due to electrical current. There is no known reason that magnetic monopoles should not exist—they are simply never observed—and so searches at the subnuclear level continue. If they *do not* exist, we would like to find out why not. If they *do* exist, we would like to see evidence of them.

Note:**Electric Currents and Magnetism**

Electric current is the source of all magnetism.



(a) In the planetary model of the atom, an electron orbits a nucleus, forming a closed-current loop and producing a magnetic field with a north pole and a south pole. (b) Electrons have spin and can be crudely pictured as rotating charge, forming a current that produces a magnetic field with a north pole and a south pole. Neither the planetary model nor the image of a spinning electron is completely consistent with modern physics. However, they do provide a useful way of understanding phenomena.

Note:**PhET Explorations: Magnets and Electromagnets**

Explore the interactions between a compass and bar magnet. Discover how you can use a battery and wire to make a magnet! Can you make it a stronger magnet? Can you make the magnetic field reverse?

Section Summary

- Magnetic poles always occur in pairs of north and south—it is not possible to isolate north and south poles.
- All magnetism is created by electric current.
- Ferromagnetic materials, such as iron, are those that exhibit strong magnetic effects.
- The atoms in ferromagnetic materials act like small magnets (due to currents within the atoms) and can be aligned, usually in millimeter-sized regions called domains.
- Domains can grow and align on a larger scale, producing permanent magnets. Such a material is magnetized, or induced to be magnetic.
- Above a material's Curie temperature, thermal agitation destroys the alignment of atoms, and ferromagnetism disappears.
- Electromagnets employ electric currents to make magnetic fields, often aided by induced fields in ferromagnetic materials.

Glossary

ferromagnetic

materials, such as iron, cobalt, nickel, and gadolinium, that exhibit strong magnetic effects

magnetized

to be turned into a magnet; to be induced to be magnetic

domains

regions within a material that behave like small bar magnets

Curie temperature

the temperature above which a ferromagnetic material cannot be magnetized

electromagnetism

the use of electrical currents to induce magnetism

electromagnet

an object that is temporarily magnetic when an electrical current is passed through it

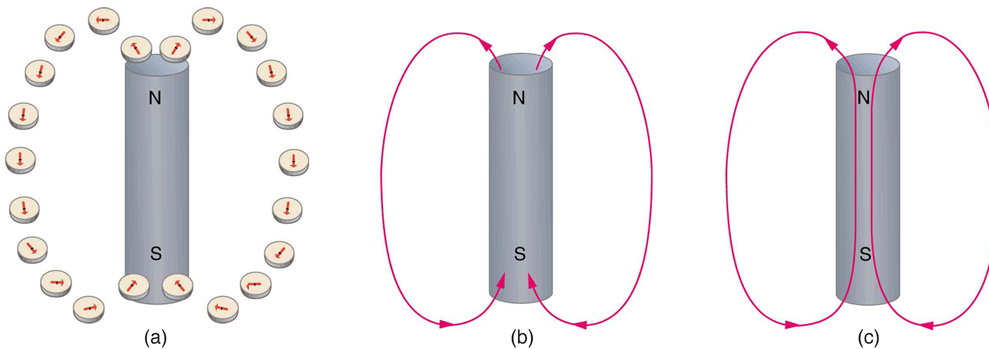
magnetic monopoles

an isolated magnetic pole; a south pole without a north pole, or vice versa (no magnetic monopole has ever been observed)

Magnetic Fields and Magnetic Field Lines

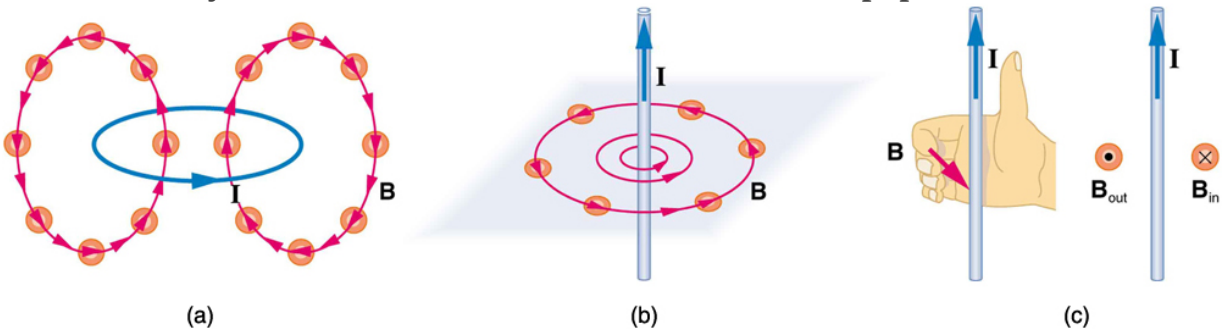
- Define magnetic field and describe the magnetic field lines of various magnetic fields.

Einstein is said to have been fascinated by a compass as a child, perhaps musing on how the needle felt a force without direct physical contact. His ability to think deeply and clearly about action at a distance, particularly for gravitational, electric, and magnetic forces, later enabled him to create his revolutionary theory of relativity. Since magnetic forces act at a distance, we define a **magnetic field** to represent magnetic forces. The pictorial representation of **magnetic field lines** is very useful in visualizing the strength and direction of the magnetic field. As shown in [\[link\]](#), the **direction of magnetic field lines** is defined to be the direction in which the north end of a compass needle points. The magnetic field is traditionally called the ***B*-field**.



Magnetic field lines are defined to have the direction that a small compass points when placed at a location. (a) If small compasses are used to map the magnetic field around a bar magnet, they will point in the directions shown: away from the north pole of the magnet, toward the south pole of the magnet. (Recall that the Earth's north magnetic pole is really a south pole in terms of definitions of poles on a bar magnet.) (b) Connecting the arrows gives continuous magnetic field lines. The strength of the field is proportional to the closeness (or density) of the lines. (c) If the interior of the magnet could be probed, the field lines would be found to form continuous closed loops.

Small compasses used to test a magnetic field will not disturb it. (This is analogous to the way we tested electric fields with a small test charge. In both cases, the fields represent only the object creating them and not the probe testing them.) [\[link\]](#) shows how the magnetic field appears for a current loop and a long straight wire, as could be explored with small compasses. A small compass placed in these fields will align itself parallel to the field line at its location, with its north pole pointing in the direction of B . Note the symbols used for field into and out of the paper.



Small compasses could be used to map the fields shown here. (a) The magnetic field of a circular current loop is similar to that of a bar magnet. (b) A long and straight wire creates a field with magnetic field lines forming circular loops. (c) When the wire is in the plane of the paper, the field is perpendicular to the paper. Note that the symbols used for the field pointing inward (like the tail of an arrow) and the field pointing outward (like the tip of an arrow).

Note:

Making Connections: Concept of a Field

A field is a way of mapping forces surrounding any object that can act on another object at a distance without apparent physical connection. The field represents the object generating it. Gravitational fields map

gravitational forces, electric fields map electrical forces, and magnetic fields map magnetic forces.

Extensive exploration of magnetic fields has revealed a number of hard-and-fast rules. We use magnetic field lines to represent the field (the lines are a pictorial tool, not a physical entity in and of themselves). The properties of magnetic field lines can be summarized by these rules:

1. The direction of the magnetic field is tangent to the field line at any point in space. A small compass will point in the direction of the field line.
2. The strength of the field is proportional to the closeness of the lines. It is exactly proportional to the number of lines per unit area perpendicular to the lines (called the areal density).
3. Magnetic field lines can never cross, meaning that the field is unique at any point in space.
4. Magnetic field lines are continuous, forming closed loops without beginning or end. They go from the north pole to the south pole.

The last property is related to the fact that the north and south poles cannot be separated. It is a distinct difference from electric field lines, which begin and end on the positive and negative charges. If magnetic monopoles existed, then magnetic field lines would begin and end on them.

Section Summary

- Magnetic fields can be pictorially represented by magnetic field lines, the properties of which are as follows:
 1. The field is tangent to the magnetic field line.
 2. Field strength is proportional to the line density.
 3. Field lines cannot cross.
 4. Field lines are continuous loops.

Conceptual Questions

Exercise:**Problem:**

Explain why the magnetic field would not be unique (that is, not have a single value) at a point in space where magnetic field lines might cross. (Consider the direction of the field at such a point.)

Exercise:**Problem:**

List the ways in which magnetic field lines and electric field lines are similar. For example, the field direction is tangent to the line at any point in space. Also list the ways in which they differ. For example, electric force is parallel to electric field lines, whereas magnetic force on moving charges is perpendicular to magnetic field lines.

Exercise:**Problem:**

Noting that the magnetic field lines of a bar magnet resemble the electric field lines of a pair of equal and opposite charges, do you expect the magnetic field to rapidly decrease in strength with distance from the magnet? Is this consistent with your experience with magnets?

Exercise:**Problem:**

Is the Earth's magnetic field parallel to the ground at all locations? If not, where is it parallel to the surface? Is its strength the same at all locations? If not, where is it greatest?

Glossary

magnetic field

the representation of magnetic forces

B-field

another term for magnetic field

magnetic field lines

the pictorial representation of the strength and the direction of a magnetic field

direction of magnetic field lines

the direction that the north end of a compass needle points

Magnetic Field Strength: Force on a Moving Charge in a Magnetic Field

- Describe the effects of magnetic fields on moving charges.
- Use the right hand rule 1 to determine the velocity of a charge, the direction of the magnetic field, and the direction of the magnetic force on a moving charge.
- Calculate the magnetic force on a moving charge.

What is the mechanism by which one magnet exerts a force on another?

The answer is related to the fact that all magnetism is caused by current, the flow of charge. *Magnetic fields exert forces on moving charges*, and so they exert forces on other magnets, all of which have moving charges.

Right Hand Rule 1

The magnetic force on a moving charge is one of the most fundamental known. Magnetic force is as important as the electrostatic or Coulomb force. Yet the magnetic force is more complex, in both the number of factors that affects it and in its direction, than the relatively simple Coulomb force. The magnitude of the **magnetic force** F on a charge q moving at a speed v in a magnetic field of strength B is given by

Equation:

$$F = qvB \sin \theta,$$

where θ is the angle between the directions of $\mathbf{v}$ and $\mathbf{B}$. This force is often called the **Lorentz force**. In fact, this is how we define the magnetic field strength B —in terms of the force on a charged particle moving in a magnetic field. The SI unit for magnetic field strength B is called the **tesla** (T) after the eccentric but brilliant inventor Nikola Tesla (1856–1943). To determine how the tesla relates to other SI units, we solve $F = qvB \sin \theta$ for B .

Equation:

$$B = \frac{F}{qv \sin \theta}$$

Because $\sin \theta$ is unitless, the tesla is

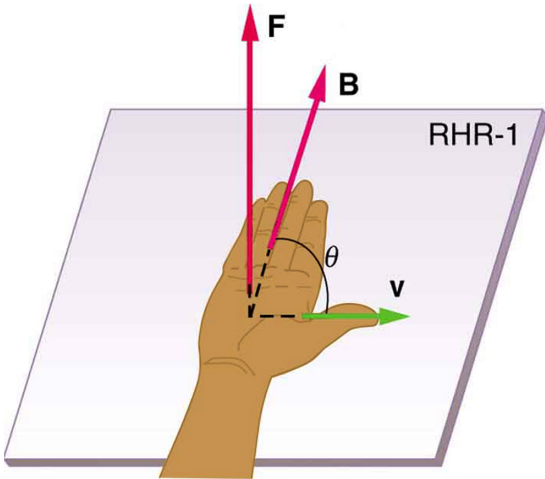
Equation:

$$1 \text{ T} = \frac{1 \text{ N}}{\text{C} \cdot \text{m/s}} = \frac{1 \text{ N}}{\text{A} \cdot \text{m}}$$

(note that $\text{C/s} = \text{A}$).

Another smaller unit, called the **gauss** (G), where $1 \text{ G} = 10^{-4} \text{ T}$, is sometimes used. The strongest permanent magnets have fields near 2 T; superconducting electromagnets may attain 10 T or more. The Earth's magnetic field on its surface is only about $5 \times 10^{-5} \text{ T}$, or 0.5 G.

The *direction* of the magnetic force **F** is perpendicular to the plane formed by **v** and **B**, as determined by the **right hand rule 1** (or RHR-1), which is illustrated in [\[link\]](#). RHR-1 states that, to determine the direction of the magnetic force on a positive moving charge, you point the thumb of the right hand in the direction of **v**, the fingers in the direction of **B**, and a perpendicular to the palm points in the direction of **F**. One way to remember this is that there is one velocity, and so the thumb represents it. There are many field lines, and so the fingers represent them. The force is in the direction you would push with your palm. The force on a negative charge is in exactly the opposite direction to that on a positive charge.



$$F = qvB \sin \theta$$

$\mathbf{F} \perp$ plane of $\mathbf{v}$ and $\mathbf{B}$

Magnetic fields exert forces on moving charges. This force is one of the most basic known. The direction of the magnetic force on a moving charge is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$ and follows right hand rule—1 (RHR-1) as shown. The magnitude of the force is proportional to q , v , B , and the sine of the angle between $\mathbf{v}$ and $\mathbf{B}$.

Note:

Making Connections: Charges and Magnets

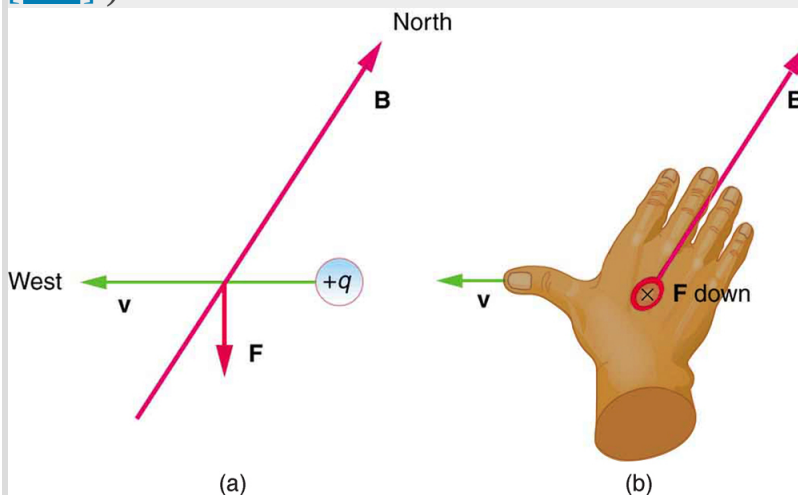
There is no magnetic force on static charges. However, there is a magnetic force on moving charges. When charges are stationary, their electric fields do not affect magnets. But, when charges move, they produce magnetic

fields that exert forces on other magnets. When there is relative motion, a connection between electric and magnetic fields emerges—each affects the other.

Example:

Calculating Magnetic Force: Earth's Magnetic Field on a Charged Glass Rod

With the exception of compasses, you seldom see or personally experience forces due to the Earth's small magnetic field. To illustrate this, suppose that in a physics lab you rub a glass rod with silk, placing a 20-nC positive charge on it. Calculate the force on the rod due to the Earth's magnetic field, if you throw it with a horizontal velocity of 10 m/s due west in a place where the Earth's field is due north parallel to the ground. (The direction of the force is determined with right hand rule 1 as shown in [\[link\]](#).)



A positively charged object moving due west in a region where the Earth's magnetic field is due north experiences a force that is straight down as shown. A negative charge moving in the same direction would feel a force straight up.

Strategy

We are given the charge, its velocity, and the magnetic field strength and direction. We can thus use the equation $F = qvB \sin \theta$ to find the force.

Solution

The magnetic force is

Equation:

$$F = qvb \sin \theta.$$

We see that $\sin \theta = 1$, since the angle between the velocity and the direction of the field is 90° . Entering the other given quantities yields

Equation:

$$\begin{aligned} F &= (20 \times 10^{-9} \text{ C})(10 \text{ m/s})(5 \times 10^{-5} \text{ T}) \\ &= 1 \times 10^{-11} (\text{C} \cdot \text{m/s}) \left(\frac{\text{N}}{\text{C} \cdot \text{m/s}} \right) = 1 \times 10^{-11} \text{ N}. \end{aligned}$$

Discussion

This force is completely negligible on any macroscopic object, consistent with experience. (It is calculated to only one digit, since the Earth's field varies with location and is given to only one digit.) The Earth's magnetic field, however, does produce very important effects, particularly on submicroscopic particles. Some of these are explored in [Force on a Moving Charge in a Magnetic Field: Examples and Applications](#).

Section Summary

- Magnetic fields exert a force on a moving charge q , the magnitude of which is

Equation:

$$F = qvB \sin \theta,$$

where θ is the angle between the directions of v and B .

- The SI unit for magnetic field strength B is the tesla (T), which is related to other units by

Equation:

$$1 \text{ T} = \frac{1 \text{ N}}{\text{C} \cdot \text{m/s}} = \frac{1 \text{ N}}{\text{A} \cdot \text{m}}.$$

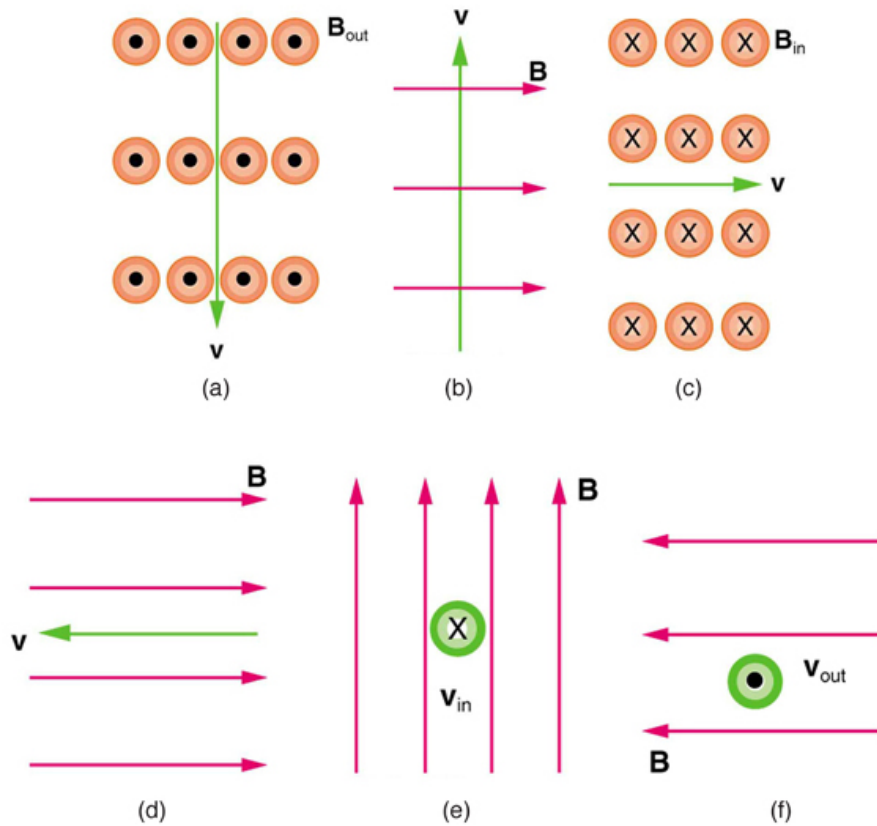
- The *direction* of the force on a moving charge is given by right hand rule 1 (RHR-1): Point the thumb of the right hand in the direction of v , the fingers in the direction of B , and a perpendicular to the palm points in the direction of F .
- The force is perpendicular to the plane formed by $\mathbf{v}$ and $\mathbf{B}$. Since the force is zero if $\mathbf{v}$ is parallel to $\mathbf{B}$, charged particles often follow magnetic field lines rather than cross them.

Conceptual Questions**Exercise:****Problem:**

If a charged particle moves in a straight line through some region of space, can you say that the magnetic field in that region is necessarily zero?

Problems & Exercises**Exercise:****Problem:**

What is the direction of the magnetic force on a positive charge that moves as shown in each of the six cases shown in [\[link\]](#)?



Solution:

- (a) Left (West)
- (b) Into the page
- (c) Up (North)
- (d) No force
- (e) Right (East)
- (f) Down (South)

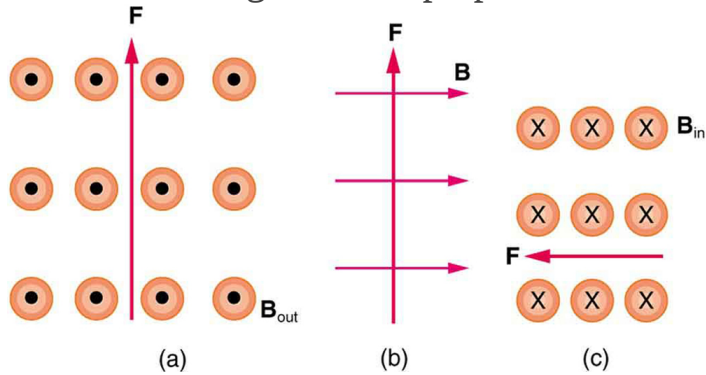
Exercise:

Problem: Repeat [\[link\]](#) for a negative charge.

Exercise:

Problem:

What is the direction of the velocity of a negative charge that experiences the magnetic force shown in each of the three cases in [\[link\]](#), assuming it moves perpendicular to $\mathbf{B}$?

**Solution:**

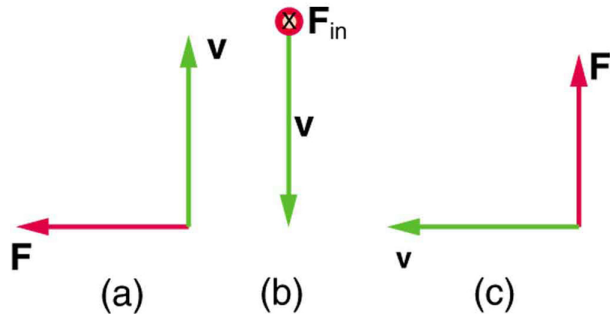
- (a) East (right)
- (b) Into page
- (c) South (down)

Exercise:

Problem: Repeat [\[link\]](#) for a positive charge.

Exercise:**Problem:**

What is the direction of the magnetic field that produces the magnetic force on a positive charge as shown in each of the three cases in the figure below, assuming $\mathbf{B}$ is perpendicular to $\mathbf{v}$?



Solution:

(a) Into page

(b) West (left)

(c) Out of page

Exercise:

Problem: Repeat [\[link\]](#) for a negative charge.

Exercise:

Problem:

What is the maximum magnitude of the force on an aluminum rod with a $0.100\text{-}\mu\text{C}$ charge that you pass between the poles of a 1.50-T permanent magnet at a speed of 5.00 m/s ? In what direction is the force?

Solution:

$7.50 \times 10^{-7}\text{ N}$ perpendicular to both the magnetic field lines and the velocity

Exercise:

Problem:

(a) Aircraft sometimes acquire small static charges. Suppose a supersonic jet has a $0.500\text{-}\mu\text{C}$ charge and flies due west at a speed of 660 m/s over the Earth's magnetic south pole (near Earth's geographic north pole), where the $8.00 \times 10^{-5}\text{-T}$ magnetic field points straight down. What are the direction and the magnitude of the magnetic force on the plane? (b) Discuss whether the value obtained in part (a) implies this is a significant or negligible effect.

Exercise:**Problem:**

(a) A cosmic ray proton moving toward the Earth at $5.00 \times 10^7\text{ m/s}$ experiences a magnetic force of $1.70 \times 10^{-16}\text{ N}$. What is the strength of the magnetic field if there is a 45° angle between it and the proton's velocity? (b) Is the value obtained in part (a) consistent with the known strength of the Earth's magnetic field on its surface? Discuss.

Solution:

(a) $3.01 \times 10^{-5}\text{ T}$

(b) This is slightly less than the magnetic field strength of $5 \times 10^{-5}\text{ T}$ at the surface of the Earth, so it is consistent.

Exercise:**Problem:**

An electron moving at $4.00 \times 10^3\text{ m/s}$ in a 1.25-T magnetic field experiences a magnetic force of $1.40 \times 10^{-16}\text{ N}$. What angle does the velocity of the electron make with the magnetic field? There are two answers.

Exercise:

Problem:

(a) A physicist performing a sensitive measurement wants to limit the magnetic force on a moving charge in her equipment to less than 1.00×10^{-12} N. What is the greatest the charge can be if it moves at a maximum speed of 30.0 m/s in the Earth's field? (b) Discuss whether it would be difficult to limit the charge to less than the value found in (a) by comparing it with typical static electricity and noting that static is often absent.

Solution:

(a) 6.67×10^{-10} C (taking the Earth's field to be 5.00×10^{-5} T)

(b) Less than typical static, therefore difficult

Glossary

right hand rule 1 (RHR-1)

the rule to determine the direction of the magnetic force on a positive moving charge: when the thumb of the right hand points in the direction of the charge's velocity $\mathbf{v}$ and the fingers point in the direction of the magnetic field $\mathbf{B}$, then the force on the charge is perpendicular and away from the palm; the force on a negative charge is perpendicular and into the palm

Lorentz force

the force on a charge moving in a magnetic field

tesla

T, the SI unit of the magnetic field strength; $1 \text{ T} = \frac{1 \text{ N}}{\text{A} \cdot \text{m}}$

magnetic force

the force on a charge produced by its motion through a magnetic field;
the Lorentz force

gauss

G, the unit of the magnetic field strength; $1 \text{ G} = 10^{-4} \text{ T}$

Force on a Moving Charge in a Magnetic Field: Examples and Applications

- Describe the effects of a magnetic field on a moving charge.
- Calculate the radius of curvature of the path of a charge that is moving in a magnetic field.

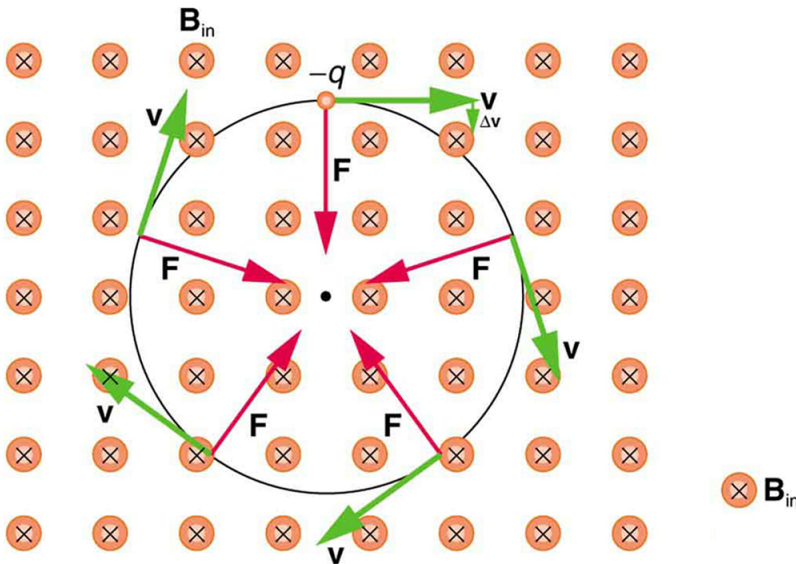
Magnetic force can cause a charged particle to move in a circular or spiral path. Cosmic rays are energetic charged particles in outer space, some of which approach the Earth. They can be forced into spiral paths by the Earth's magnetic field. Protons in giant accelerators are kept in a circular path by magnetic force. The bubble chamber photograph in [\[link\]](#) shows charged particles moving in such curved paths. The curved paths of charged particles in magnetic fields are the basis of a number of phenomena and can even be used analytically, such as in a mass spectrometer.



Trails of bubbles are produced by high-energy charged particles moving through the superheated liquid hydrogen in this artist's rendition of a bubble chamber. There is a strong magnetic field perpendicular to the page that causes the curved paths of the particles. The radius of the path can be

used to find the mass,
charge, and energy of the
particle.

So does the magnetic force cause circular motion? Magnetic force is always perpendicular to velocity, so that it does no work on the charged particle. The particle's kinetic energy and speed thus remain constant. The direction of motion is affected, but not the speed. This is typical of uniform circular motion. The simplest case occurs when a charged particle moves perpendicular to a uniform B -field, such as shown in [\[link\]](#). (If this takes place in a vacuum, the magnetic field is the dominant factor determining the motion.) Here, the magnetic force supplies the centripetal force $F_c = mv^2/r$. Noting that $\sin \theta = 1$, we see that $F = qvB$.



A negatively charged particle moves in the plane of the page in a region where the magnetic field is perpendicular into the page (represented by the small circles with x's—like the tails of arrows). The magnetic force is perpendicular to the velocity, and so velocity changes in direction but not magnitude. Uniform circular motion results.

Because the magnetic force F supplies the centripetal force F_c , we have
Equation:

$$qvB = \frac{mv^2}{r}.$$

Solving for r yields
Equation:

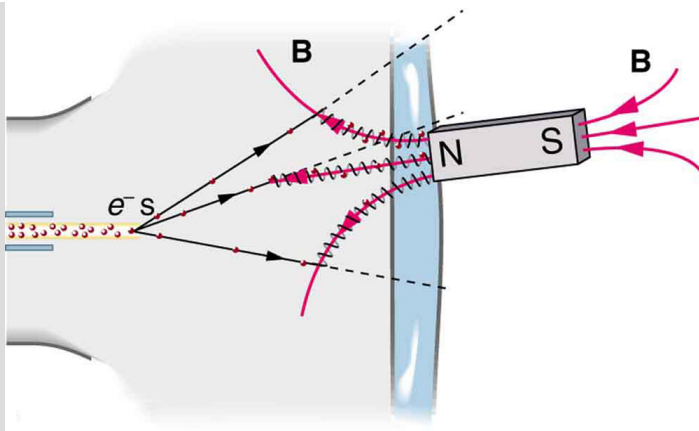
$$r = \frac{mv}{qB}.$$

Here, r is the radius of curvature of the path of a charged particle with mass m and charge q , moving at a speed v perpendicular to a magnetic field of strength B . If the velocity is not perpendicular to the magnetic field, then v is the component of the velocity perpendicular to the field. The component of the velocity parallel to the field is unaffected, since the magnetic force is zero for motion parallel to the field. This produces a spiral motion rather than a circular one.

Example:

Calculating the Curvature of the Path of an Electron Moving in a Magnetic Field: A Magnet on a TV Screen

A magnet brought near an old-fashioned TV screen such as in [\[link\]](#) (TV sets with cathode ray tubes instead of LCD screens) severely distorts its picture by altering the path of the electrons that make its phosphors glow. ***(Don't try this at home, as it will permanently magnetize and ruin the TV.)*** To illustrate this, calculate the radius of curvature of the path of an electron having a velocity of 6.00×10^7 m/s (corresponding to the accelerating voltage of about 10.0 kV used in some TVs) perpendicular to a magnetic field of strength $B = 0.500$ T (obtainable with permanent magnets).



Side view showing what happens when a magnet comes in contact with a computer monitor or TV screen. Electrons moving toward the screen spiral about magnetic field lines, maintaining the component of their velocity parallel to the field lines. This distorts the image on the screen.

Strategy

We can find the radius of curvature r directly from the equation $r = \frac{mv}{qB}$, since all other quantities in it are given or known.

Solution

Using known values for the mass and charge of an electron, along with the given values of v and B gives us

Equation:

$$\begin{aligned} r = \frac{mv}{qB} &= \frac{(9.11 \times 10^{-31} \text{ kg})(6.00 \times 10^7 \text{ m/s})}{(1.60 \times 10^{-19} \text{ C})(0.500 \text{ T})} \\ &= 6.83 \times 10^{-4} \text{ m} \end{aligned}$$

or

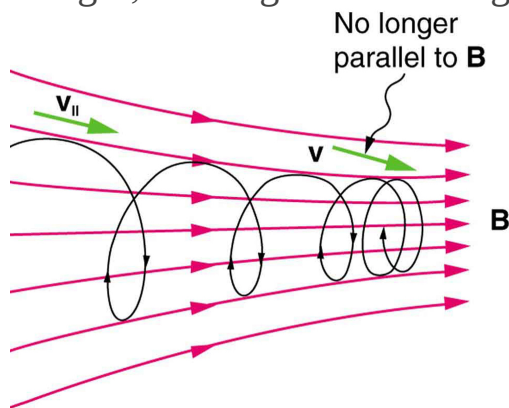
Equation:

$$r = 0.683 \text{ mm.}$$

Discussion

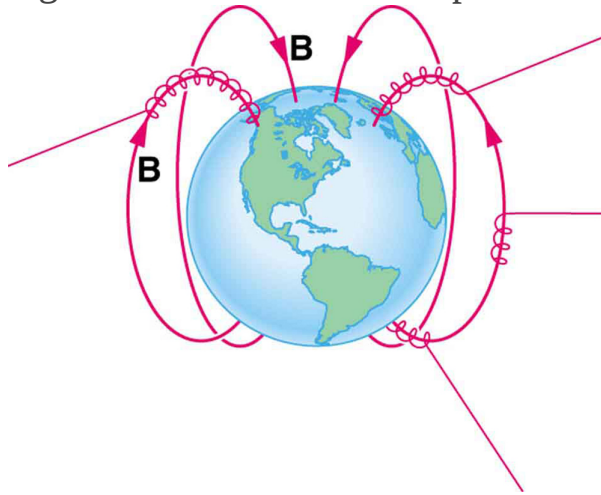
The small radius indicates a large effect. The electrons in the TV picture tube are made to move in very tight circles, greatly altering their paths and distorting the image.

[\[link\]](#) shows how electrons not moving perpendicular to magnetic field lines follow the field lines. The component of velocity parallel to the lines is unaffected, and so the charges spiral along the field lines. If field strength increases in the direction of motion, the field will exert a force to slow the charges, forming a kind of magnetic mirror, as shown below.



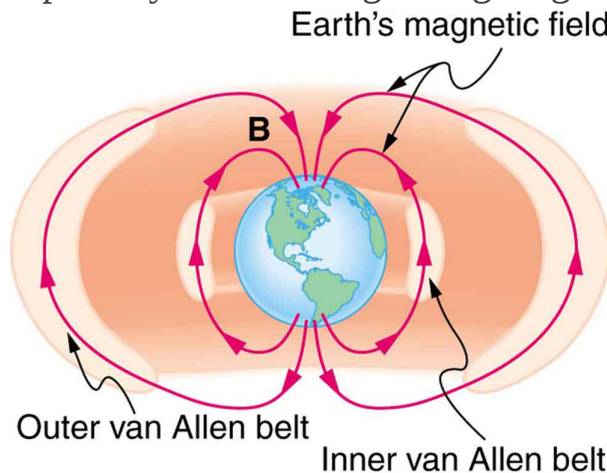
When a charged particle moves along a magnetic field line into a region where the field becomes stronger, the particle experiences a force that reduces the component of velocity parallel to the field. This force slows the motion along the field line and here reverses it, forming a “magnetic mirror.”

The properties of charged particles in magnetic fields are related to such different things as the Aurora Australis or Aurora Borealis and particle accelerators. *Charged particles approaching magnetic field lines may get trapped in spiral orbits about the lines rather than crossing them*, as seen above. Some cosmic rays, for example, follow the Earth's magnetic field lines, entering the atmosphere near the magnetic poles and causing the southern or northern lights through their ionization of molecules in the atmosphere. This glow of energized atoms and molecules is seen in [\[link\]](#). Those particles that approach middle latitudes must cross magnetic field lines, and many are prevented from penetrating the atmosphere. Cosmic rays are a component of background radiation; consequently, they give a higher radiation dose at the poles than at the equator.



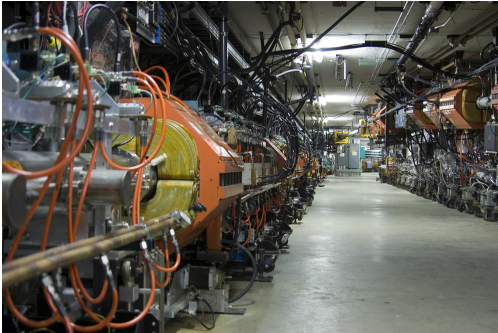
Energetic electrons and protons, components of cosmic rays, from the Sun and deep outer space often follow the Earth's magnetic field lines rather than cross them. (Recall that the Earth's north magnetic pole is really a south pole in terms of a bar magnet.)

Some incoming charged particles become trapped in the Earth's magnetic field, forming two belts above the atmosphere known as the Van Allen radiation belts after the discoverer James A. Van Allen, an American astrophysicist. (See [\[link\]](#).) Particles trapped in these belts form radiation fields (similar to nuclear radiation) so intense that manned space flights avoid them and satellites with sensitive electronics are kept out of them. In the few minutes it took lunar missions to cross the Van Allen radiation belts, astronauts received radiation doses more than twice the allowed annual exposure for radiation workers. Other planets have similar belts, especially those having strong magnetic fields like Jupiter.



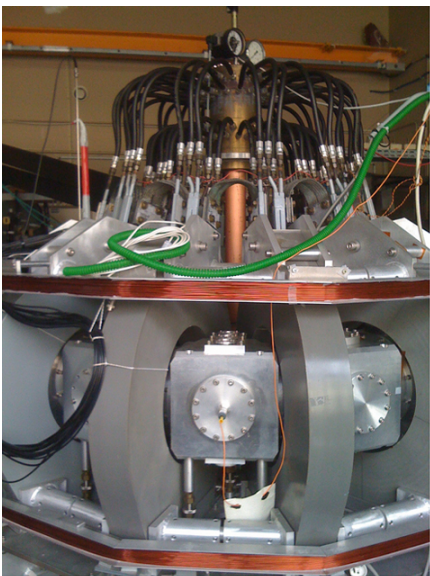
The Van Allen radiation belts are two regions in which energetic charged particles are trapped in the Earth's magnetic field. One belt lies about 300 km above the Earth's surface, the other about 16,000 km. Charged particles in these belts migrate along magnetic field lines and are partially reflected away from the poles by the stronger fields there. The charged particles that enter the atmosphere are replenished by the Sun and sources in deep outer space.

Back on Earth, we have devices that employ magnetic fields to contain charged particles. Among them are the giant particle accelerators that have been used to explore the substructure of matter. (See [\[link\]](#).) Magnetic fields not only control the direction of the charged particles, they also are used to focus particles into beams and overcome the repulsion of like charges in these beams.

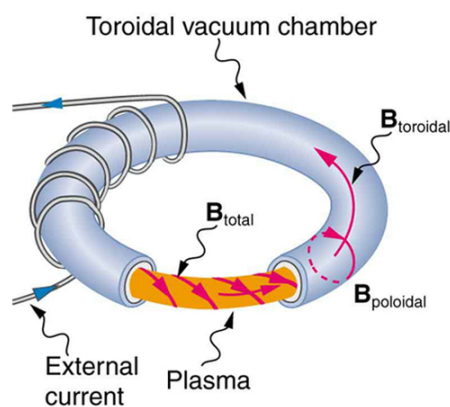


The Fermilab facility in Illinois has a large particle accelerator (the most powerful in the world until 2008) that employs magnetic fields (magnets seen here in orange) to contain and direct its beam. This and other accelerators have been in use for several decades and have allowed us to discover some of the laws underlying all matter. (credit: ammcgrim, Flickr)

Thermonuclear fusion (like that occurring in the Sun) is a hope for a future clean energy source. One of the most promising devices is the *tokamak*, which uses magnetic fields to contain (or trap) and direct the reactive charged particles. (See [\[link\]](#).) Less exotic, but more immediately practical, amplifiers in microwave ovens use a magnetic field to contain oscillating electrons. These oscillating electrons generate the microwaves sent into the oven.



(a)



(b)

Tokamaks such as the one shown in the figure are being studied with the goal of economical production of energy by nuclear fusion. Magnetic fields in the doughnut-shaped device contain and direct the reactive charged particles. (credit: David Mellis, Flickr)

Mass spectrometers have a variety of designs, and many use magnetic fields to measure mass. The curvature of a charged particle's path in the field is related to its mass and is measured to obtain mass information. (See [More Applications of Magnetism](#).) Historically, such techniques were employed in the first direct observations of electron charge and mass. Today, mass

spectrometers (sometimes coupled with gas chromatographs) are used to determine the make-up and sequencing of large biological molecules.

Section Summary

- Magnetic force can supply centripetal force and cause a charged particle to move in a circular path of radius

Equation:

$$r = \frac{mv}{qB},$$

where v is the component of the velocity perpendicular to B for a charged particle with mass m and charge q .

Conceptual Questions

Exercise:

Problem:

How can the motion of a charged particle be used to distinguish between a magnetic and an electric field?

Exercise:

Problem:

High-velocity charged particles can damage biological cells and are a component of radiation exposure in a variety of locations ranging from research facilities to natural background. Describe how you could use a magnetic field to shield yourself.

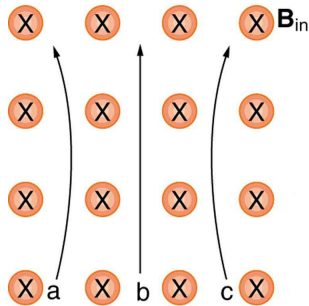
Exercise:

Problem:

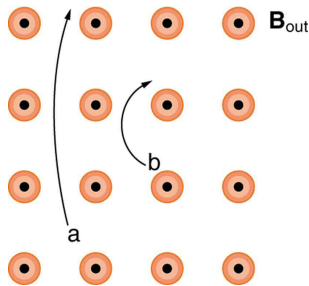
If a cosmic ray proton approaches the Earth from outer space along a line toward the center of the Earth that lies in the plane of the equator, in what direction will it be deflected by the Earth's magnetic field? What about an electron? A neutron?

Exercise:

Problem: What are the signs of the charges on the particles in [\[link\]](#)?

**Exercise:****Problem:**

Which of the particles in [\[link\]](#) has the greatest velocity, assuming they have identical charges and masses?

**Exercise:****Problem:**

Which of the particles in [\[link\]](#) has the greatest mass, assuming all have identical charges and velocities?

Exercise:

Problem:

While operating, a high-precision TV monitor is placed on its side during maintenance. The image on the monitor changes color and blurs slightly. Discuss the possible relation of these effects to the Earth's magnetic field.

Problems & Exercises

If you need additional support for these problems, see [More Applications of Magnetism](#).

Exercise:**Problem:**

A cosmic ray electron moves at 7.50×10^6 m/s perpendicular to the Earth's magnetic field at an altitude where field strength is 1.00×10^{-5} T. What is the radius of the circular path the electron follows?

Solution:

4.27 m

Exercise:**Problem:**

A proton moves at 7.50×10^7 m/s perpendicular to a magnetic field. The field causes the proton to travel in a circular path of radius 0.800 m. What is the field strength?

Exercise:

Problem:

(a) Viewers of *Star Trek* hear of an antimatter drive on the Starship *Enterprise*. One possibility for such a futuristic energy source is to store antimatter charged particles in a vacuum chamber, circulating in a magnetic field, and then extract them as needed. Antimatter annihilates with normal matter, producing pure energy. What strength magnetic field is needed to hold antiprotons, moving at 5.00×10^7 m/s in a circular path 2.00 m in radius? Antiprotons have the same mass as protons but the opposite (negative) charge. (b) Is this field strength obtainable with today's technology or is it a futuristic possibility?

Solution:

(a) 0.261 T

(b) This strength is definitely obtainable with today's technology. Magnetic field strengths of 0.500 T are obtainable with permanent magnets.

Exercise:**Problem:**

(a) An oxygen-16 ion with a mass of 2.66×10^{-26} kg travels at 5.00×10^6 m/s perpendicular to a 1.20-T magnetic field, which makes it move in a circular arc with a 0.231-m radius. What positive charge is on the ion? (b) What is the ratio of this charge to the charge of an electron? (c) Discuss why the ratio found in (b) should be an integer.

Exercise:**Problem:**

What radius circular path does an electron travel if it moves at the same speed and in the same magnetic field as the proton in [\[link\]](#)?

Solution:

$$4.36 \times 10^{-4} \text{ m}$$

Exercise:**Problem:**

A velocity selector in a mass spectrometer uses a 0.100-T magnetic field. (a) What electric field strength is needed to select a speed of $4.00 \times 10^6 \text{ m/s}$? (b) What is the voltage between the plates if they are separated by 1.00 cm?

Exercise:**Problem:**

An electron in a TV CRT moves with a speed of $6.00 \times 10^7 \text{ m/s}$, in a direction perpendicular to the Earth's field, which has a strength of $5.00 \times 10^{-5} \text{ T}$. (a) What strength electric field must be applied perpendicular to the Earth's field to make the electron moves in a straight line? (b) If this is done between plates separated by 1.00 cm, what is the voltage applied? (Note that TVs are usually surrounded by a ferromagnetic material to shield against external magnetic fields and avoid the need for such a correction.)

Solution:

(a) 3.00 kV/m

(b) 30.0 V

Exercise:

Problem:

(a) At what speed will a proton move in a circular path of the same radius as the electron in [\[link\]](#)? (b) What would the radius of the path be if the proton had the same speed as the electron? (c) What would the radius be if the proton had the same kinetic energy as the electron? (d) The same momentum?

Exercise:**Problem:**

A mass spectrometer is being used to separate common oxygen-16 from the much rarer oxygen-18, taken from a sample of old glacial ice. (The relative abundance of these oxygen isotopes is related to climatic temperature at the time the ice was deposited.) The ratio of the masses of these two ions is 16 to 18, the mass of oxygen-16 is 2.66×10^{-26} kg, and they are singly charged and travel at 5.00×10^6 m/s in a 1.20-T magnetic field. What is the separation between their paths when they hit a target after traversing a semicircle?

Solution:

0.173 m

Exercise:**Problem:**

(a) Triply charged uranium-235 and uranium-238 ions are being separated in a mass spectrometer. (The much rarer uranium-235 is used as reactor fuel.) The masses of the ions are 3.90×10^{-25} kg and 3.95×10^{-25} kg, respectively, and they travel at 3.00×10^5 m/s in a 0.250-T field. What is the separation between their paths when they hit a target after traversing a semicircle? (b) Discuss whether this distance between their paths seems to be big enough to be practical in the separation of uranium-235 from uranium-238.

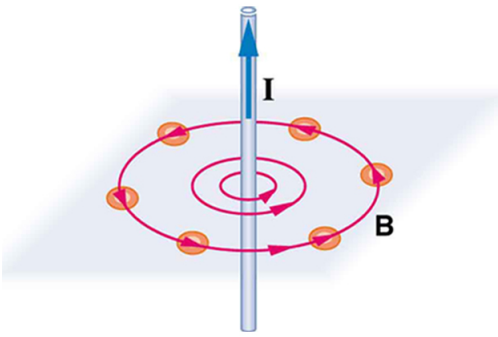
Magnetic Fields Produced by Currents: Ampere's Law

- Calculate current that produces a magnetic field.
- Use the right hand rule 2 to determine the direction of current or the direction of magnetic field loops.

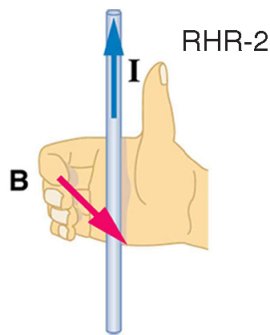
How much current is needed to produce a significant magnetic field, perhaps as strong as the Earth's field? Surveyors will tell you that overhead electric power lines create magnetic fields that interfere with their compass readings. Indeed, when Oersted discovered in 1820 that a current in a wire affected a compass needle, he was not dealing with extremely large currents. How does the shape of wires carrying current affect the shape of the magnetic field created? We noted earlier that a current loop created a magnetic field similar to that of a bar magnet, but what about a straight wire or a toroid (doughnut)? How is the direction of a current-created field related to the direction of the current? Answers to these questions are explored in this section, together with a brief discussion of the law governing the fields created by currents.

Magnetic Field Created by a Long Straight Current-Carrying Wire: Right Hand Rule 2

Magnetic fields have both direction and magnitude. As noted before, one way to explore the direction of a magnetic field is with compasses, as shown for a long straight current-carrying wire in [\[link\]](#). Hall probes can determine the magnitude of the field. The field around a long straight wire is found to be in circular loops. The **right hand rule 2** (RHR-2) emerges from this exploration and is valid for any current segment—*point the thumb in the direction of the current, and the fingers curl in the direction of the magnetic field loops* created by it.



(a)



(b)

(a) Compasses placed near a long straight current-carrying wire indicate that field lines form circular loops centered on the wire. (b) Right hand rule 2 states that, if the right hand thumb points in the direction of the current, the fingers curl in the direction of the field. This rule is consistent with the field mapped for the long straight wire and is valid for any current segment.

The **magnetic field strength (magnitude)** produced by a long straight current-carrying wire is found by experiment to be

Equation:

$$B = \frac{\mu_0 I}{2\pi r} \text{ (long straight wire),}$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the **permeability of free space**. (μ_0 is one of the basic constants in nature. We will see later that μ_0 is related to the speed of light.) Since the wire is very long, the magnitude of the field depends only on distance from the wire r , not on position along the wire.

Example:

Calculating Current that Produces a Magnetic Field

Find the current in a long straight wire that would produce a magnetic field twice the strength of the Earth's at a distance of 5.0 cm from the wire.

Strategy

The Earth's field is about $5.0 \times 10^{-5} \text{ T}$, and so here B due to the wire is taken to be $1.0 \times 10^{-4} \text{ T}$. The equation $B = \frac{\mu_0 I}{2\pi r}$ can be used to find I , since all other quantities are known.

Solution

Solving for I and entering known values gives

Equation:

$$\begin{aligned} I &= \frac{2\pi r B}{\mu_0} = \frac{2\pi(5.0 \times 10^{-2} \text{ m})(1.0 \times 10^{-4} \text{ T})}{4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}} \\ &= 25 \text{ A.} \end{aligned}$$

Discussion

So a moderately large current produces a significant magnetic field at a distance of 5.0 cm from a long straight wire. Note that the answer is stated to only two digits, since the Earth's field is specified to only two digits in this example.

Ampere's Law and Others

The magnetic field of a long straight wire has more implications than you might at first suspect. *Each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment.* The formal statement of the direction and magnitude of the field due to each segment is called the **Biot-Savart law**. Integral calculus is needed to sum the field for an arbitrary shape current. This results in a more complete law, called **Ampere's law**, which relates magnetic field and current in a general way. Ampere's law in turn is a part of **Maxwell's equations**, which give a complete theory of all electromagnetic phenomena. Considerations of how Maxwell's equations appear to different observers led to the modern theory of relativity, and the realization that electric and magnetic fields are different manifestations of the same thing. Most of this is beyond the scope of this text in both mathematical level, requiring calculus, and in the amount of space that can be devoted to it. But for the interested student, and particularly for those who continue in physics, engineering, or similar pursuits, delving into these matters further will reveal descriptions of nature that are elegant as well as profound. In this text, we shall keep the general features in mind, such as RHR-2 and the rules for magnetic field lines listed in [Magnetic Fields and Magnetic Field Lines](#), while concentrating on the fields created in certain important situations.

Note:

Making Connections: Relativity

Hearing all we do about Einstein, we sometimes get the impression that he invented relativity out of nothing. On the contrary, one of Einstein's motivations was to solve difficulties in knowing how different observers see magnetic and electric fields.

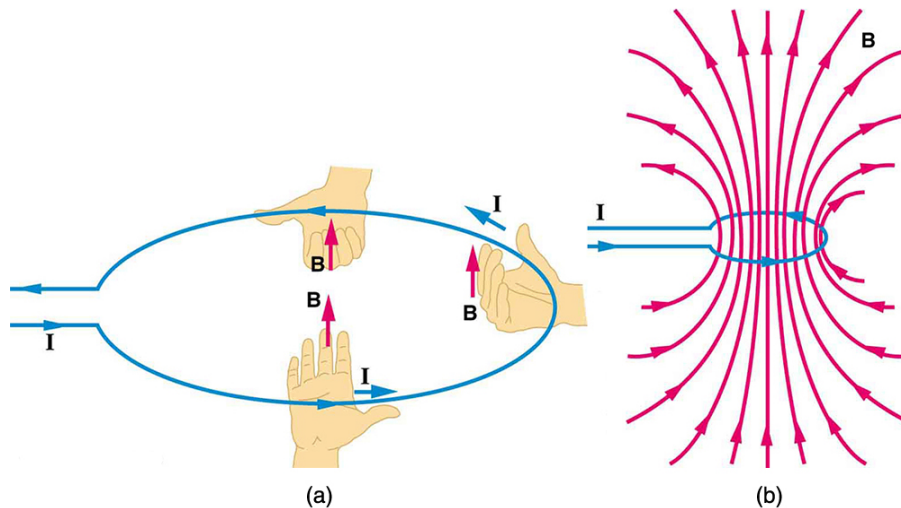
Magnetic Field Produced by a Current-Carrying Circular Loop

The magnetic field near a current-carrying loop of wire is shown in [\[link\]](#). Both the direction and the magnitude of the magnetic field produced by a current-carrying loop are complex. RHR-2 can be used to give the direction of the field near the loop, but mapping with compasses and the rules about field lines given in [Magnetic Fields and Magnetic Field Lines](#) are needed for more detail. There is a simple formula for the **magnetic field strength at the center of a circular loop**. It is

Equation:

$$B = \frac{\mu_0 I}{2R} \text{ (at center of loop),}$$

where R is the radius of the loop. This equation is very similar to that for a straight wire, but it is valid *only* at the center of a circular loop of wire. The similarity of the equations does indicate that similar field strength can be obtained at the center of a loop. One way to get a larger field is to have N loops; then, the field is $B = N\mu_0 I / (2R)$. Note that the larger the loop, the smaller the field at its center, because the current is farther away.

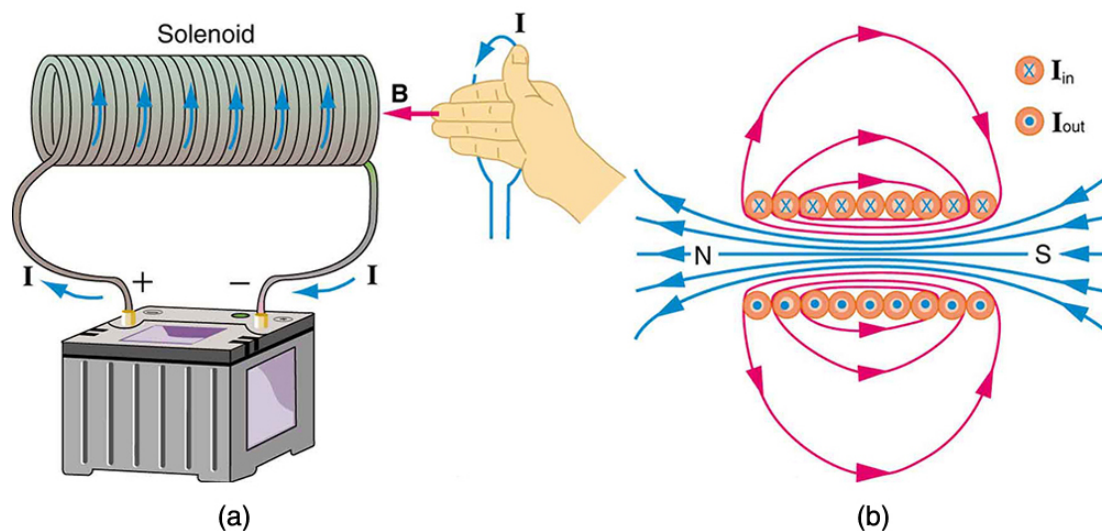


- (a) RHR-2 gives the direction of the magnetic field inside and outside a current-carrying loop. (b) More detailed mapping with compasses or with a

Hall probe completes the picture. The field is similar to that of a bar magnet.

Magnetic Field Produced by a Current-Carrying Solenoid

A **solenoid** is a long coil of wire (with many turns or loops, as opposed to a flat loop). Because of its shape, the field inside a solenoid can be very uniform, and also very strong. The field just outside the coils is nearly zero. [\[link\]](#) shows how the field looks and how its direction is given by RHR-2.



(a) Because of its shape, the field inside a solenoid of length l is remarkably uniform in magnitude and direction, as indicated by the straight and uniformly spaced field lines. The field outside the coils is nearly zero. (b) This cutaway shows the magnetic field generated by the current in the solenoid.

The magnetic field inside of a current-carrying solenoid is very uniform in direction and magnitude. Only near the ends does it begin to weaken and change direction. The field outside has similar complexities to flat loops

and bar magnets, but the **magnetic field strength inside a solenoid** is simply

Equation:

$$B = \mu_0 n I \quad (\text{inside a solenoid}),$$

where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length). Note that B is the field strength anywhere in the uniform region of the interior and not just at the center. Large uniform fields spread over a large volume are possible with solenoids, as [\[link\]](#) implies.

Example:

Calculating Field Strength inside a Solenoid

What is the field inside a 2.00-m-long solenoid that has 2000 loops and carries a 1600-A current?

Strategy

To find the field strength inside a solenoid, we use $B = \mu_0 n I$. First, we note the number of loops per unit length is

Equation:

$$n = \frac{N}{l} = \frac{2000}{2.00 \text{ m}} = 1000 \text{ m}^{-1} = 10 \text{ cm}^{-1}.$$

Solution

Substituting known values gives

Equation:

$$\begin{aligned} B &= \mu_0 n I = (4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}) (1000 \text{ m}^{-1}) (1600 \text{ A}) \\ &= 2.01 \text{ T}. \end{aligned}$$

Discussion

This is a large field strength that could be established over a large-diameter solenoid, such as in medical uses of magnetic resonance imaging (MRI).

The very large current is an indication that the fields of this strength are not

easily achieved, however. Such a large current through 1000 loops squeezed into a meter's length would produce significant heating. Higher currents can be achieved by using superconducting wires, although this is expensive. There is an upper limit to the current, since the superconducting state is disrupted by very large magnetic fields.

There are interesting variations of the flat coil and solenoid. For example, the toroidal coil used to confine the reactive particles in tokamaks is much like a solenoid bent into a circle. The field inside a toroid is very strong but circular. Charged particles travel in circles, following the field lines, and collide with one another, perhaps inducing fusion. But the charged particles do not cross field lines and escape the toroid. A whole range of coil shapes are used to produce all sorts of magnetic field shapes. Adding ferromagnetic materials produces greater field strengths and can have a significant effect on the shape of the field. Ferromagnetic materials tend to trap magnetic fields (the field lines bend into the ferromagnetic material, leaving weaker fields outside it) and are used as shields for devices that are adversely affected by magnetic fields, including the Earth's magnetic field.

Note:

PhET Explorations: Generator

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

[Generato](#)

[r](#)

Section Summary

- The strength of the magnetic field created by current in a long straight wire is given by

Equation:

$$B = \frac{\mu_0 I}{2\pi r} (\text{long straight wire}),$$

where I is the current, r is the shortest distance to the wire, and the constant $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space.

- The direction of the magnetic field created by a long straight wire is given by right hand rule 2 (RHR-2): *Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops* created by it.
- The magnetic field created by current following any path is the sum (or integral) of the fields due to segments along the path (magnitude and direction as for a straight wire), resulting in a general relationship between current and field known as Ampere's law.
- The magnetic field strength at the center of a circular loop is given by

Equation:

$$B = \frac{\mu_0 I}{2R} (\text{at center of loop}),$$

where R is the radius of the loop. This equation becomes $B = \mu_0 nI / (2R)$ for a flat coil of N loops. RHR-2 gives the direction of the field about the loop. A long coil is called a solenoid.

- The magnetic field strength inside a solenoid is

Equation:

$$B = \mu_0 nI \text{ (inside a solenoid),}$$

where n is the number of loops per unit length of the solenoid. The field inside is very uniform in magnitude and direction.

Conceptual Questions

Exercise:

Problem:

Make a drawing and use RHR-2 to find the direction of the magnetic field of a current loop in a motor (such as in [\[link\]](#)). Then show that the direction of the torque on the loop is the same as produced by like poles repelling and unlike poles attracting.

Glossary

right hand rule 2 (RHR-2)

a rule to determine the direction of the magnetic field induced by a current-carrying wire: Point the thumb of the right hand in the direction of current, and the fingers curl in the direction of the magnetic field loops

magnetic field strength (magnitude) produced by a long straight current-carrying wire

defined as $B = \frac{\mu_0 I}{2\pi r}$, where I is the current, r is the shortest distance to the wire, and μ_0 is the permeability of free space

permeability of free space

the measure of the ability of a material, in this case free space, to support a magnetic field; the constant $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$

magnetic field strength at the center of a circular loop

defined as $B = \frac{\mu_0 I}{2R}$ where R is the radius of the loop

solenoid

a thin wire wound into a coil that produces a magnetic field when an electric current is passed through it

magnetic field strength inside a solenoid

defined as $B = \mu_0 n I$ where n is the number of loops per unit length of the solenoid ($n = N/l$, with N being the number of loops and l the length)

Biot-Savart law

a physical law that describes the magnetic field generated by an electric current in terms of a specific equation

Ampere's law

the physical law that states that the magnetic field around an electric current is proportional to the current; each segment of current produces a magnetic field like that of a long straight wire, and the total field of any shape current is the vector sum of the fields due to each segment

Maxwell's equations

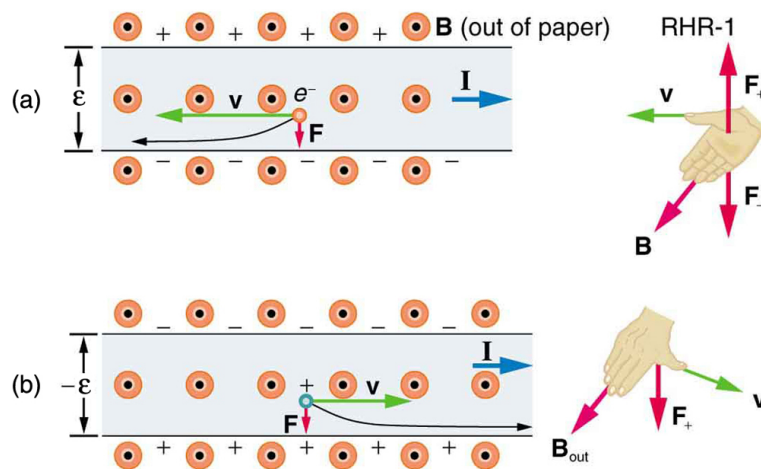
a set of four equations that describe electromagnetic phenomena

The Hall Effect

- Describe the Hall effect.
- Calculate the Hall emf across a current-carrying conductor.

We have seen effects of a magnetic field on free-moving charges. The magnetic field also affects charges moving in a conductor. One result is the Hall effect, which has important implications and applications.

[\[link\]](#) shows what happens to charges moving through a conductor in a magnetic field. The field is perpendicular to the electron drift velocity and to the width of the conductor. Note that conventional current is to the right in both parts of the figure. In part (a), electrons carry the current and move to the left. In part (b), positive charges carry the current and move to the right. Moving electrons feel a magnetic force toward one side of the conductor, leaving a net positive charge on the other side. This separation of charge *creates a voltage* ε , known as the **Hall emf**, *across* the conductor. The creation of a voltage *across* a current-carrying conductor by a magnetic field is known as the **Hall effect**, after Edwin Hall, the American physicist who discovered it in 1879.



The Hall effect. (a) Electrons move to the left in this flat conductor (conventional current to the right). The magnetic field is directly out of the page, represented by circled dots; it exerts a force on the moving charges, causing a voltage ε , the

Hall emf, across the conductor. (b)
Positive charges moving to the right
(conventional current also to the right) are
moved to the side, producing a Hall emf
of the opposite sign, $-\varepsilon$. Thus, if the
direction of the field and current are
known, the sign of the charge carriers can
be determined from the Hall effect.

One very important use of the Hall effect is to determine whether positive or negative charges carries the current. Note that in [\[link\]](#)(b), where positive charges carry the current, the Hall emf has the sign opposite to when negative charges carry the current. Historically, the Hall effect was used to show that electrons carry current in metals and it also shows that positive charges carry current in some semiconductors. The Hall effect is used today as a research tool to probe the movement of charges, their drift velocities and densities, and so on, in materials. In 1980, it was discovered that the Hall effect is quantized, an example of quantum behavior in a macroscopic object.

The Hall effect has other uses that range from the determination of blood flow rate to precision measurement of magnetic field strength. To examine these quantitatively, we need an expression for the Hall emf, ε , across a conductor. Consider the balance of forces on a moving charge in a situation where B , v , and l are mutually perpendicular, such as shown in [\[link\]](#). Although the magnetic force moves negative charges to one side, they cannot build up without limit. The electric field caused by their separation opposes the magnetic force, $F = qvB$, and the electric force, $F_e = qE$, eventually grows to equal it. That is,

Equation:

$$qE = qvB$$

or

Equation:

$$E = vB.$$

Note that the electric field E is uniform across the conductor because the magnetic field B is uniform, as is the conductor. For a uniform electric field, the relationship between electric field and voltage is $E = \varepsilon/l$, where l is the width of the conductor and ε is the Hall emf. Entering this into the last expression gives

Equation:

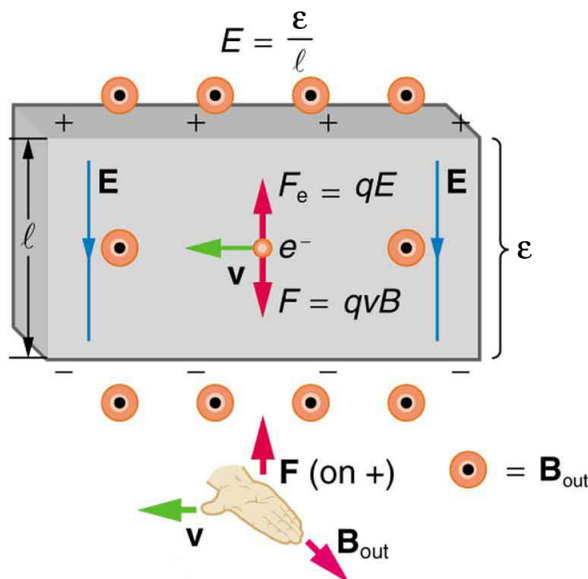
$$\frac{\varepsilon}{l} = vB.$$

Solving this for the Hall emf yields

Equation:

$$\varepsilon = Blv \text{ (} B, v, \text{ and } l, \text{ mutually perpendicular),}$$

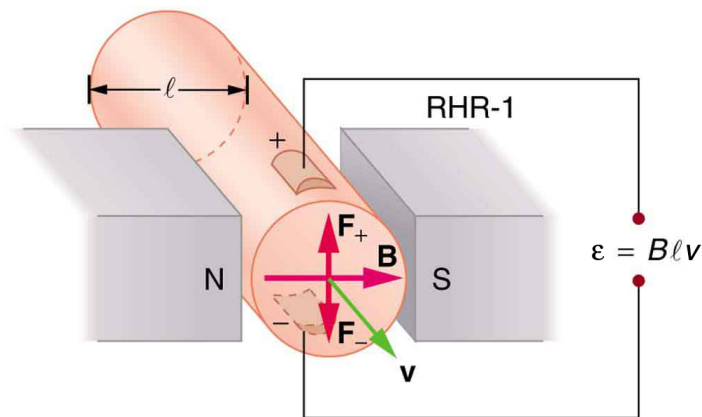
where ε is the Hall effect voltage across a conductor of width l through which charges move at a speed v .



The Hall emf ε produces an electric force that balances the magnetic force on the moving

charges. The magnetic force produces charge separation, which builds up until it is balanced by the electric force, an equilibrium that is quickly reached.

One of the most common uses of the Hall effect is in the measurement of magnetic field strength B . Such devices, called *Hall probes*, can be made very small, allowing fine position mapping. Hall probes can also be made very accurate, usually accomplished by careful calibration. Another application of the Hall effect is to measure fluid flow in any fluid that has free charges (most do). (See [\[link\]](#).) A magnetic field applied perpendicular to the flow direction produces a Hall emf ε as shown. Note that the sign of ε depends not on the sign of the charges, but only on the directions of B and v . The magnitude of the Hall emf is $\varepsilon = B\ell v$, where ℓ is the pipe diameter, so that the average velocity v can be determined from ε providing the other factors are known.



The Hall effect can be used to measure fluid flow in any fluid having free charges, such as blood. The Hall emf ε is measured across the tube perpendicular to the applied magnetic field and is proportional to the average velocity v .

Example:**Calculating the Hall emf: Hall Effect for Blood Flow**

A Hall effect flow probe is placed on an artery, applying a 0.100-T magnetic field across it, in a setup similar to that in [\[link\]](#). What is the Hall emf, given the vessel's inside diameter is 4.00 mm and the average blood velocity is 20.0 cm/s?

Strategy

Because B , v , and l are mutually perpendicular, the equation $\varepsilon = Blv$ can be used to find ε .

Solution

Entering the given values for B , v , and l gives

Equation:

$$\begin{aligned}\varepsilon &= Blv = (0.100 \text{ T})(4.00 \times 10^{-3} \text{ m})(0.200 \text{ m/s}) \\ &= 80.0 \text{ } \mu\text{V}\end{aligned}$$

Discussion

This is the average voltage output. Instantaneous voltage varies with pulsating blood flow. The voltage is small in this type of measurement. ε is particularly difficult to measure, because there are voltages associated with heart action (ECG voltages) that are on the order of millivolts. In practice, this difficulty is overcome by applying an AC magnetic field, so that the Hall emf is AC with the same frequency. An amplifier can be very selective in picking out only the appropriate frequency, eliminating signals and noise at other frequencies.

Section Summary

- The Hall effect is the creation of voltage ε , known as the Hall emf, across a current-carrying conductor by a magnetic field.
- The Hall emf is given by

Equation:

$$\varepsilon = Blv \text{ (} B, v, \text{ and } l, \text{ mutually perpendicular)}$$

for a conductor of width l through which charges move at a speed v .

Conceptual Questions

Exercise:

Problem:

Discuss how the Hall effect could be used to obtain information on free charge density in a conductor. (Hint: Consider how drift velocity and current are related.)

Problems & Exercises

Exercise:

Problem:

A large water main is 2.50 m in diameter and the average water velocity is 6.00 m/s. Find the Hall voltage produced if the pipe runs perpendicular to the Earth's 5.00×10^{-5} -T field.

Solution:

$$7.50 \times 10^{-4} \text{ V}$$

Exercise:

Problem:

What Hall voltage is produced by a 0.200-T field applied across a 2.60-cm-diameter aorta when blood velocity is 60.0 cm/s?

Exercise:

Problem:

(a) What is the speed of a supersonic aircraft with a 17.0-m wingspan, if it experiences a 1.60-V Hall voltage between its wing tips when in level flight over the north magnetic pole, where the Earth's field strength is $8.00 \times 10^{-5} \text{ T}$? (b) Explain why very little current flows as a result of this Hall voltage.

Solution:

(a) $1.18 \times 10^3 \text{ m/s}$

(b) Once established, the Hall emf pushes charges one direction and the magnetic force acts in the opposite direction resulting in no net force on the charges. Therefore, no current flows in the direction of the Hall emf. This is the same as in a current-carrying conductor—current does not flow in the direction of the Hall emf.

Exercise:**Problem:**

A nonmechanical water meter could utilize the Hall effect by applying a magnetic field across a metal pipe and measuring the Hall voltage produced. What is the average fluid velocity in a 3.00-cm-diameter pipe, if a 0.500-T field across it creates a 60.0-mV Hall voltage?

Exercise:**Problem:**

Calculate the Hall voltage induced on a patient's heart while being scanned by an MRI unit. Approximate the conducting path on the heart wall by a wire 7.50 cm long that moves at 10.0 cm/s perpendicular to a 1.50-T magnetic field.

Solution:

11.3 mV

Exercise:**Problem:**

A Hall probe calibrated to read $1.00\ \mu\text{V}$ when placed in a 2.00-T field is placed in a 0.150-T field. What is its output voltage?

Exercise:**Problem:**

Using information in [\[link\]](#), what would the Hall voltage be if a 2.00-T field is applied across a 10-gauge copper wire (2.588 mm in diameter) carrying a 20.0-A current?

Solution:

$1.16\ \mu\text{V}$

Exercise:**Problem:**

Show that the Hall voltage across wires made of the same material, carrying identical currents, and subjected to the same magnetic field is inversely proportional to their diameters. (Hint: Consider how drift velocity depends on wire diameter.)

Exercise:**Problem:**

A patient with a pacemaker is mistakenly being scanned for an MRI image. A 10.0-cm -long section of pacemaker wire moves at a speed of 10.0 cm/s perpendicular to the MRI unit's magnetic field and a 20.0-mV Hall voltage is induced. What is the magnetic field strength?

Solution:

2.00 T

Glossary

Hall effect

the creation of voltage across a current-carrying conductor by a magnetic field

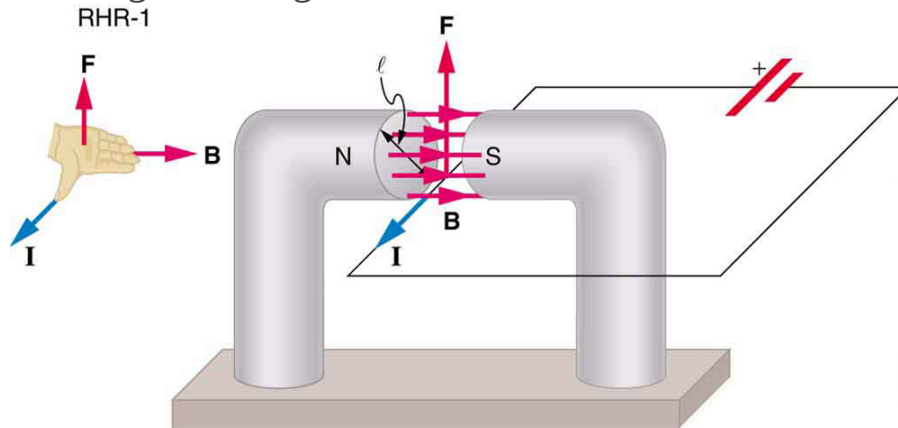
Hall emf

the electromotive force created by a current-carrying conductor by a magnetic field, $\varepsilon = Blv$

Magnetic Force on a Current-Carrying Conductor

- Describe the effects of a magnetic force on a current-carrying conductor.
- Calculate the magnetic force on a current-carrying conductor.

Because charges ordinarily cannot escape a conductor, the magnetic force on charges moving in a conductor is transmitted to the conductor itself.



The magnetic field exerts a force on a current-carrying wire in a direction given by the right hand rule 1 (the same direction as that on the individual moving charges). This force can easily be large enough to move the wire, since typical currents consist of very large numbers of moving charges.

We can derive an expression for the magnetic force on a current by taking a sum of the magnetic forces on individual charges. (The forces add because they are in the same direction.) The force on an individual charge moving at the drift velocity v_d is given by $F = qv_d B \sin \theta$. Taking B to be uniform over a length of wire l and zero elsewhere, the total magnetic force on the wire is then $F = (qv_d B \sin \theta)(N)$, where N is the number of charge carriers in the section of wire of length l . Now, $N = nV$, where n is the number of charge carriers per unit volume and V is the volume of wire in the field. Noting that $V = Al$, where A is the cross-sectional area of the

wire, then the force on the wire is $F = (qv_d B \sin \theta)(nAl)$. Gathering terms,

Equation:

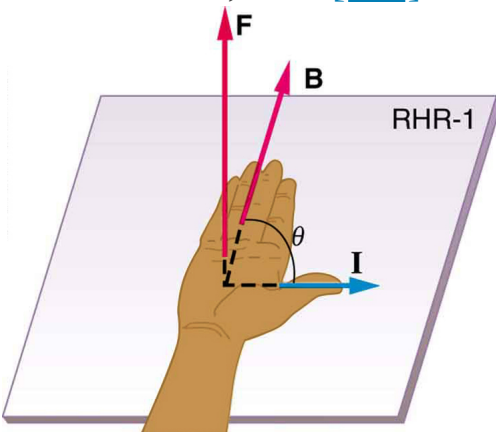
$$F = (nqAv_d)lB \sin \theta.$$

Because $nqAv_d = I$ (see [Current](#)),

Equation:

$$F = IlB \sin \theta$$

is the equation for *magnetic force on a length l of wire carrying a current I in a uniform magnetic field B* , as shown in [\[link\]](#). If we divide both sides of this expression by l , we find that the magnetic force per unit length of wire in a uniform field is $\frac{F}{l} = IB \sin \theta$. The direction of this force is given by RHR-1, with the thumb in the direction of the current I . Then, with the fingers in the direction of B , a perpendicular to the palm points in the direction of F , as in [\[link\]](#).



$$F = IlB \sin \theta$$

$\mathbf{F} \perp$ plane of $\mathbf{I}$ and $\mathbf{B}$

The force on a current-carrying wire in a magnetic field is $F = IlB \sin \theta$. Its

direction is given by
RHR-1.

Example:

Calculating Magnetic Force on a Current-Carrying Wire: A Strong Magnetic Field

Calculate the force on the wire shown in [\[link\]](#), given $B = 1.50 \text{ T}$, $l = 5.00 \text{ cm}$, and $I = 20.0 \text{ A}$.

Strategy

The force can be found with the given information by using $F = IlB \sin \theta$ and noting that the angle θ between I and B is 90° , so that $\sin \theta = 1$.

Solution

Entering the given values into $F = IlB \sin \theta$ yields

Equation:

$$F = IlB \sin \theta = (20.0 \text{ A})(0.0500 \text{ m})(1.50 \text{ T})(1).$$

The units for tesla are $1 \text{ T} = \frac{\text{N}}{\text{A}\cdot\text{m}}$; thus,

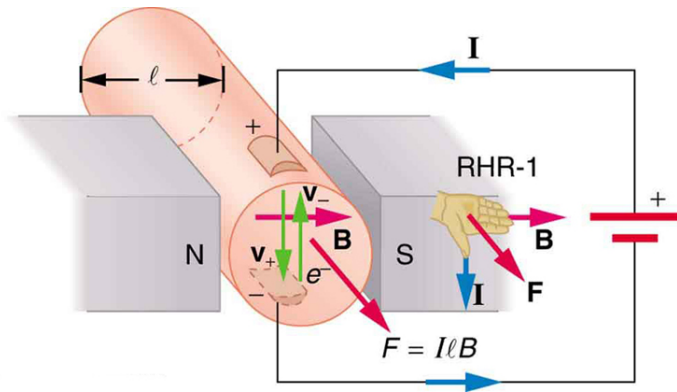
Equation:

$$F = 1.50 \text{ N}.$$

Discussion

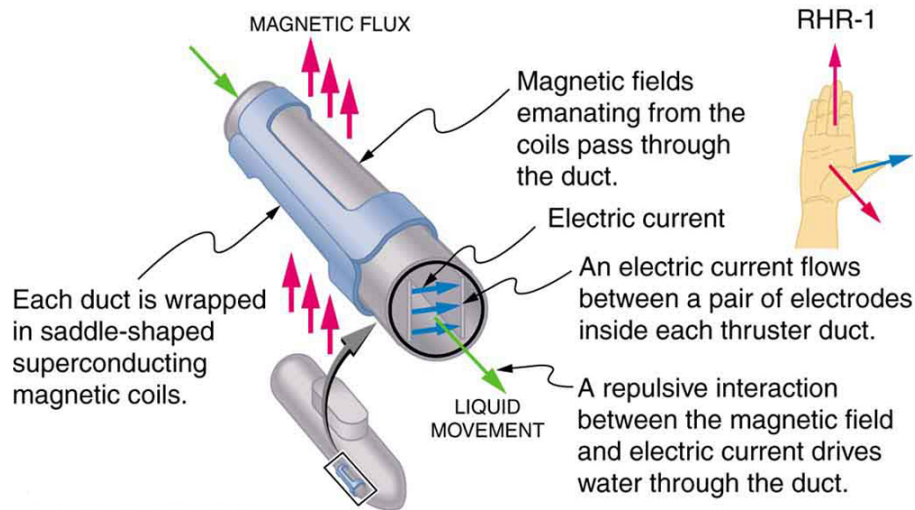
This large magnetic field creates a significant force on a small length of wire.

Magnetic force on current-carrying conductors is used to convert electric energy to work. (Motors are a prime example—they employ loops of wire and are considered in the next section.) Magnetohydrodynamics (MHD) is the technical name given to a clever application where magnetic force pumps fluids without moving mechanical parts. (See [\[link\]](#).)



Magnetohydrodynamics. The magnetic force on the current passed through this fluid can be used as a nonmechanical pump.

A strong magnetic field is applied across a tube and a current is passed through the fluid at right angles to the field, resulting in a force on the fluid parallel to the tube axis as shown. The absence of moving parts makes this attractive for moving a hot, chemically active substance, such as the liquid sodium employed in some nuclear reactors. Experimental artificial hearts are testing with this technique for pumping blood, perhaps circumventing the adverse effects of mechanical pumps. (Cell membranes, however, are affected by the large fields needed in MHD, delaying its practical application in humans.) MHD propulsion for nuclear submarines has been proposed, because it could be considerably quieter than conventional propeller drives. The deterrent value of nuclear submarines is based on their ability to hide and survive a first or second nuclear strike. As we slowly disassemble our nuclear weapons arsenals, the submarine branch will be the last to be decommissioned because of this ability (See [\[link\]](#).) Existing MHD drives are heavy and inefficient—much development work is needed.



An MHD propulsion system in a nuclear submarine could produce significantly less turbulence than propellers and allow it to run more silently. The development of a silent drive submarine was dramatized in the book and the film *The Hunt for Red October*.

Section Summary

- The magnetic force on current-carrying conductors is given by **Equation:**

$$F = IlB \sin \theta,$$

where I is the current, l is the length of a straight conductor in a uniform magnetic field B , and θ is the angle between I and B . The force follows RHR-1 with the thumb in the direction of I .

Conceptual Questions

Exercise:

Problem:

Draw a sketch of the situation in [\[link\]](#) showing the direction of electrons carrying the current, and use RHR-1 to verify the direction of the force on the wire.

Exercise:**Problem:**

Verify that the direction of the force in an MHD drive, such as that in [\[link\]](#), does not depend on the sign of the charges carrying the current across the fluid.

Exercise:**Problem:**

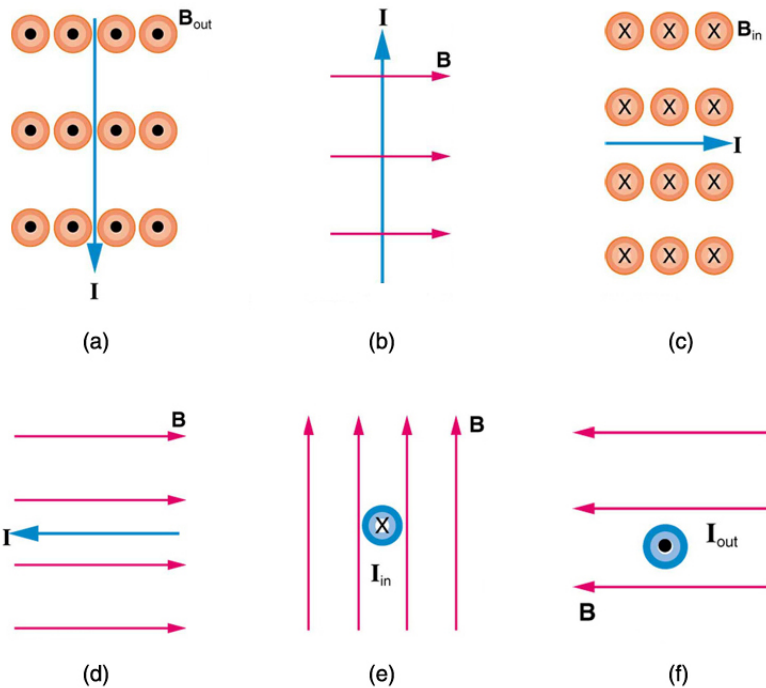
Why would a magnetohydrodynamic drive work better in ocean water than in fresh water? Also, why would superconducting magnets be desirable?

Exercise:**Problem:**

Which is more likely to interfere with compass readings, AC current in your refrigerator or DC current when you start your car? Explain.

Problems & Exercises**Exercise:****Problem:**

What is the direction of the magnetic force on the current in each of the six cases in [\[link\]](#)?



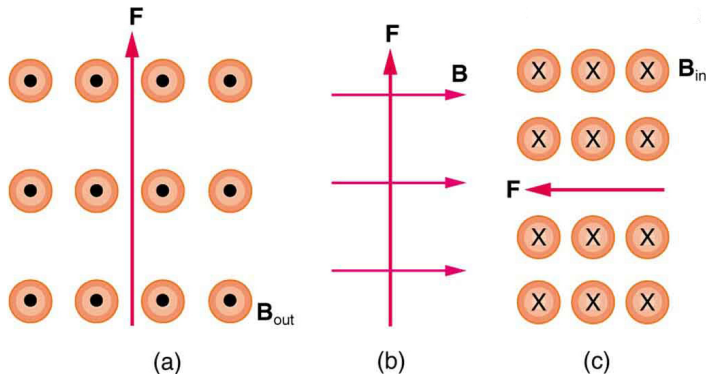
Solution:

- (a) west (left)
- (b) into page
- (c) north (up)
- (d) no force
- (e) east (right)
- (f) south (down)

Exercise:

Problem:

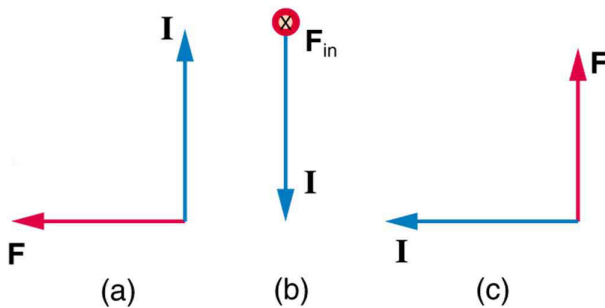
What is the direction of a current that experiences the magnetic force shown in each of the three cases in [\[link\]](#), assuming the current runs perpendicular to B ?



Exercise:

Problem:

What is the direction of the magnetic field that produces the magnetic force shown on the currents in each of the three cases in [\[link\]](#), assuming $\mathbf{I}$ is perpendicular to $\mathbf{B}$?



Solution:

(a) into page

(b) west (left)

(c) out of page

Exercise:

Problem:

(a) What is the force per meter on a lightning bolt at the equator that carries 20,000 A perpendicular to the Earth's 3.00×10^{-5} -T field? (b) What is the direction of the force if the current is straight up and the Earth's field direction is due north, parallel to the ground?

Exercise:**Problem:**

(a) A DC power line for a light-rail system carries 1000 A at an angle of 30.0° to the Earth's 5.00×10^{-5} -T field. What is the force on a 100-m section of this line? (b) Discuss practical concerns this presents, if any.

Solution:

(a) 2.50 N

(b) This is about half a pound of force per 100 m of wire, which is much less than the weight of the wire itself. Therefore, it does not cause any special concerns.

Exercise:**Problem:**

What force is exerted on the water in an MHD drive utilizing a 25.0-cm-diameter tube, if 100-A current is passed across the tube that is perpendicular to a 2.00-T magnetic field? (The relatively small size of this force indicates the need for very large currents and magnetic fields to make practical MHD drives.)

Exercise:**Problem:**

A wire carrying a 30.0-A current passes between the poles of a strong magnet that is perpendicular to its field and experiences a 2.16-N force on the 4.00 cm of wire in the field. What is the average field strength?

Solution:

1.80 T

Exercise:

Problem:

(a) A 0.750-m-long section of cable carrying current to a car starter motor makes an angle of 60° with the Earth's 5.50×10^{-5} T field. What is the current when the wire experiences a force of 7.00×10^{-3} N? (b) If you run the wire between the poles of a strong horseshoe magnet, subjecting 5.00 cm of it to a 1.75-T field, what force is exerted on this segment of wire?

Exercise:**Problem:**

(a) What is the angle between a wire carrying an 8.00-A current and the 1.20-T field it is in if 50.0 cm of the wire experiences a magnetic force of 2.40 N? (b) What is the force on the wire if it is rotated to make an angle of 90° with the field?

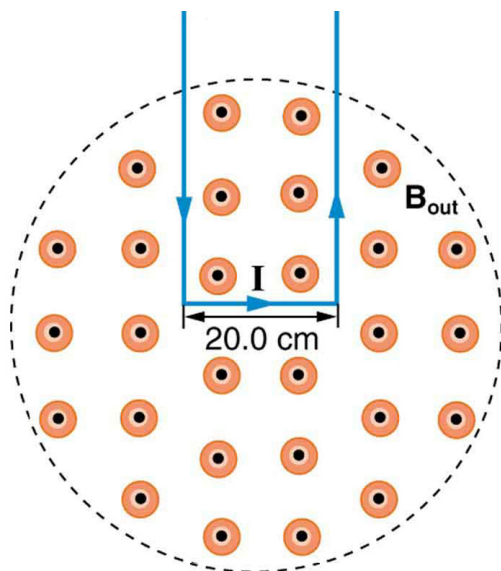
Solution:

(a) 30°

(b) 4.80 N

Exercise:**Problem:**

The force on the rectangular loop of wire in the magnetic field in [\[link\]](#) can be used to measure field strength. The field is uniform, and the plane of the loop is perpendicular to the field. (a) What is the direction of the magnetic force on the loop? Justify the claim that the forces on the sides of the loop are equal and opposite, independent of how much of the loop is in the field and do not affect the net force on the loop. (b) If a current of 5.00 A is used, what is the force per tesla on the 20.0-cm-wide loop?



A rectangular loop of wire carrying a current is perpendicular to a magnetic field. The field is uniform in the region shown and is zero outside that region.

Magnetic Force between Two Parallel Conductors

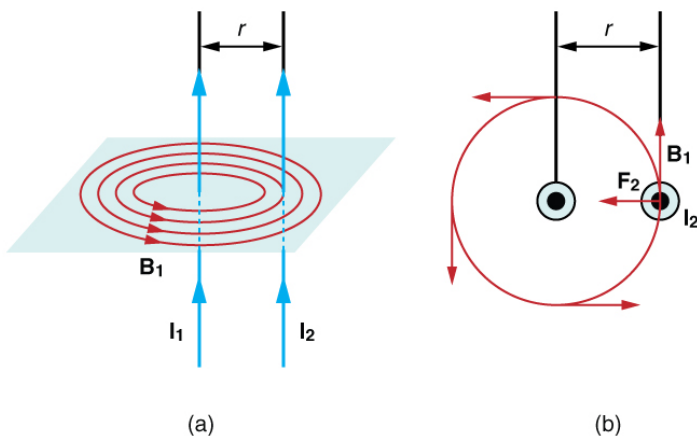
- Describe the effects of the magnetic force between two conductors.
- Calculate the force between two parallel conductors.

You might expect that there are significant forces between current-carrying wires, since ordinary currents produce significant magnetic fields and these fields exert significant forces on ordinary currents. But you might not expect that the force between wires is used to *define* the ampere. It might also surprise you to learn that this force has something to do with why large circuit breakers burn up when they attempt to interrupt large currents.

The force between two long straight and parallel conductors separated by a distance r can be found by applying what we have developed in preceding sections. [\[link\]](#) shows the wires, their currents, the fields they create, and the subsequent forces they exert on one another. Let us consider the field produced by wire 1 and the force it exerts on wire 2 (call the force F_2). The field due to I_1 at a distance r is given to be

Equation:

$$B_1 = \frac{\mu_0 I_1}{2\pi r}.$$



(a) The magnetic field produced by a long straight conductor is perpendicular to a parallel conductor, as indicated by RHR-2. (b) A view

from above of the two wires shown in (a), with one magnetic field line shown for each wire. RHR-1 shows that the force between the parallel conductors is attractive when the currents are in the same direction. A similar analysis shows that the force is repulsive between currents in opposite directions.

This field is uniform along wire 2 and perpendicular to it, and so the force F_2 it exerts on wire 2 is given by $F = IlB \sin \theta$ with $\sin \theta = 1$:

Equation:

$$F_2 = I_2 l B_1.$$

By Newton's third law, the forces on the wires are equal in magnitude, and so we just write F for the magnitude of F_2 . (Note that $F_1 = -F_2$.) Since the wires are very long, it is convenient to think in terms of F/l , the force per unit length. Substituting the expression for B_1 into the last equation and rearranging terms gives

Equation:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

F/l is the force per unit length between two parallel currents I_1 and I_2 separated by a distance r . The force is attractive if the currents are in the same direction and repulsive if they are in opposite directions.

This force is responsible for the *pinch effect* in electric arcs and plasmas. The force exists whether the currents are in wires or not. In an electric arc, where currents are moving parallel to one another, there is an attraction that squeezes currents into a smaller tube. In large circuit breakers, like those

used in neighborhood power distribution systems, the pinch effect can concentrate an arc between plates of a switch trying to break a large current, burn holes, and even ignite the equipment. Another example of the pinch effect is found in the solar plasma, where jets of ionized material, such as solar flares, are shaped by magnetic forces.

The *operational definition of the ampere* is based on the force between current-carrying wires. Note that for parallel wires separated by 1 meter with each carrying 1 ampere, the force per meter is

Equation:

$$\frac{F}{l} = \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(1 \text{ A})^2}{(2\pi)(1 \text{ m})} = 2 \times 10^{-7} \text{ N/m}.$$

Since μ_0 is exactly $4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ by definition, and because $1 \text{ T} = 1 \text{ N}/(\text{A} \cdot \text{m})$, the force per meter is exactly $2 \times 10^{-7} \text{ N/m}$. This is the basis of the operational definition of the ampere.

Note:

The Ampere

The official definition of the ampere is:

One ampere of current through each of two parallel conductors of infinite length, separated by one meter in empty space free of other magnetic fields, causes a force of exactly $2 \times 10^{-7} \text{ N/m}$ on each conductor.

Infinite-length straight wires are impractical and so, in practice, a current balance is constructed with coils of wire separated by a few centimeters. Force is measured to determine current. This also provides us with a method for measuring the coulomb. We measure the charge that flows for a current of one ampere in one second. That is, $1 \text{ C} = 1 \text{ A} \cdot \text{s}$. For both the ampere and the coulomb, the method of measuring force between conductors is the most accurate in practice.

Section Summary

- The force between two parallel currents I_1 and I_2 , separated by a distance r , has a magnitude per unit length given by

Equation:

$$\frac{F}{l} = \frac{\mu_0 I_1 I_2}{2\pi r}.$$

- The force is attractive if the currents are in the same direction, repulsive if they are in opposite directions.

Conceptual Questions

Exercise:

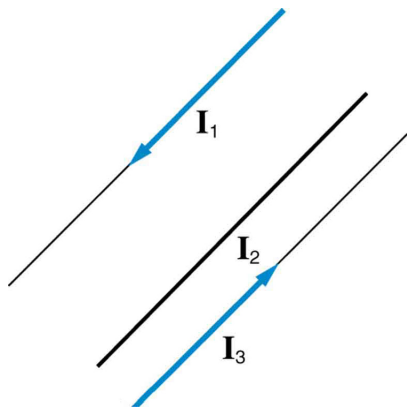
Problem:

Is the force attractive or repulsive between the hot and neutral lines hung from power poles? Why?

Exercise:

Problem:

If you have three parallel wires in the same plane, as in [\[link\]](#), with currents in the outer two running in opposite directions, is it possible for the middle wire to be repelled by both? Attracted by both? Explain.



Three parallel

coplanar wires with
currents in the
outer two in
opposite directions.

Exercise:

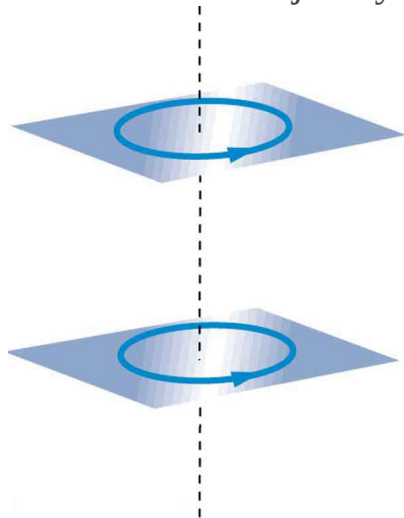
Problem:

Suppose two long straight wires run perpendicular to one another without touching. Does one exert a net force on the other? If so, what is its direction? Does one exert a net torque on the other? If so, what is its direction? Justify your responses by using the right hand rules.

Exercise:

Problem:

Use the right hand rules to show that the force between the two loops in [\[link\]](#) is attractive if the currents are in the same direction and repulsive if they are in opposite directions. Is this consistent with like poles of the loops repelling and unlike poles of the loops attracting? Draw sketches to justify your answers.



Two loops of wire
carrying currents

can exert forces
and torques on one
another.

Exercise:

Problem:

If one of the loops in [\[link\]](#) is tilted slightly relative to the other and their currents are in the same direction, what are the directions of the torques they exert on each other? Does this imply that the poles of the bar magnet-like fields they create will line up with each other if the loops are allowed to rotate?

Exercise:

Problem:

Electric field lines can be shielded by the Faraday cage effect. Can we have magnetic shielding? Can we have gravitational shielding?

Problems & Exercises

Exercise:

Problem:

- (a) The hot and neutral wires supplying DC power to a light-rail commuter train carry 800 A and are separated by 75.0 cm. What is the magnitude and direction of the force between 50.0 m of these wires?
- (b) Discuss the practical consequences of this force, if any.

Solution:

- (a) 8.53 N, repulsive
- (b) This force is repulsive and therefore there is never a risk that the two wires will touch and short circuit.

Exercise:**Problem:**

The force per meter between the two wires of a jumper cable being used to start a stalled car is 0.225 N/m. (a) What is the current in the wires, given they are separated by 2.00 cm? (b) Is the force attractive or repulsive?

Exercise:**Problem:**

A 2.50-m segment of wire supplying current to the motor of a submerged submarine carries 1000 A and feels a 4.00-N repulsive force from a parallel wire 5.00 cm away. What is the direction and magnitude of the current in the other wire?

Solution:

400 A in the opposite direction

Exercise:**Problem:**

The wire carrying 400 A to the motor of a commuter train feels an attractive force of 4.00×10^{-3} N/m due to a parallel wire carrying 5.00 A to a headlight. (a) How far apart are the wires? (b) Are the currents in the same direction?

Exercise:**Problem:**

An AC appliance cord has its hot and neutral wires separated by 3.00 mm and carries a 5.00-A current. (a) What is the average force per meter between the wires in the cord? (b) What is the maximum force per meter between the wires? (c) Are the forces attractive or repulsive? (d) Do appliance cords need any special design features to compensate for these forces?

Solution:

(a) $1.67 \times 10^{-3} \text{ N/m}$

(b) $3.33 \times 10^{-3} \text{ N/m}$

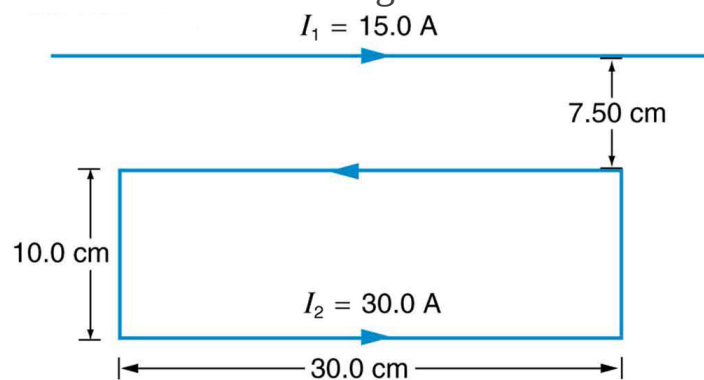
(c) Repulsive

(d) No, these are very small forces

Exercise:

Problem:

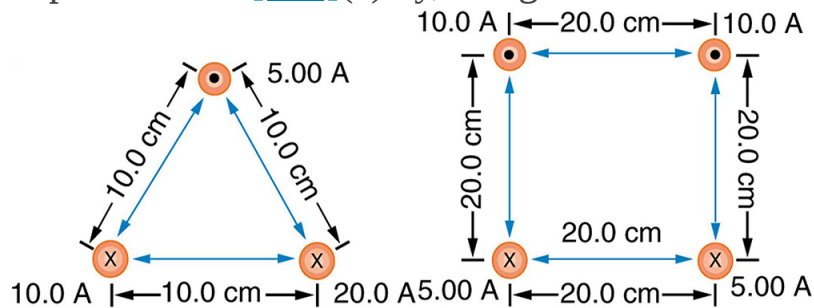
[\[link\]](#) shows a long straight wire near a rectangular current loop. What is the direction and magnitude of the total force on the loop?



Exercise:

Problem:

Find the direction and magnitude of the force that each wire experiences in [\[link\]](#)(a) by, using vector addition.



Solution:

(a) Top wire: $2.65 \times 10^{-4} \text{ N/m}$, 10.9° to left of up

(b) Lower left wire: $3.61 \times 10^{-4} \text{ N/m}$, 13.9° down from right

(c) Lower right wire: $3.46 \times 10^{-4} \text{ N/m}$, 30.0° down from left

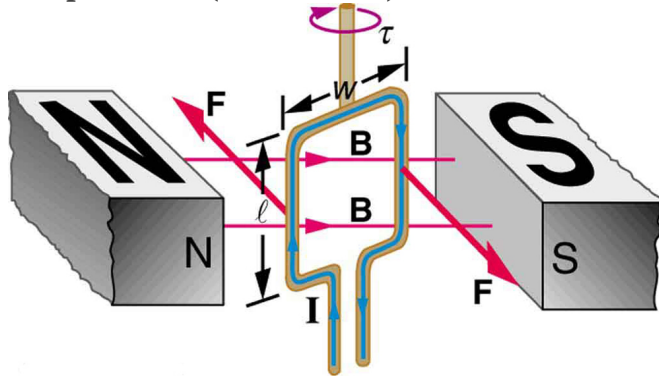
Exercise:**Problem:**

Find the direction and magnitude of the force that each wire experiences in [\[link\]](#)(b), using vector addition.

Torque on a Current Loop: Motors and Meters

- Describe how motors and meters work in terms of torque on a current loop.
- Calculate the torque on a current-carrying loop in a magnetic field.

Motors are the most common application of magnetic force on current-carrying wires. Motors have loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. (See [\[link\]](#).)



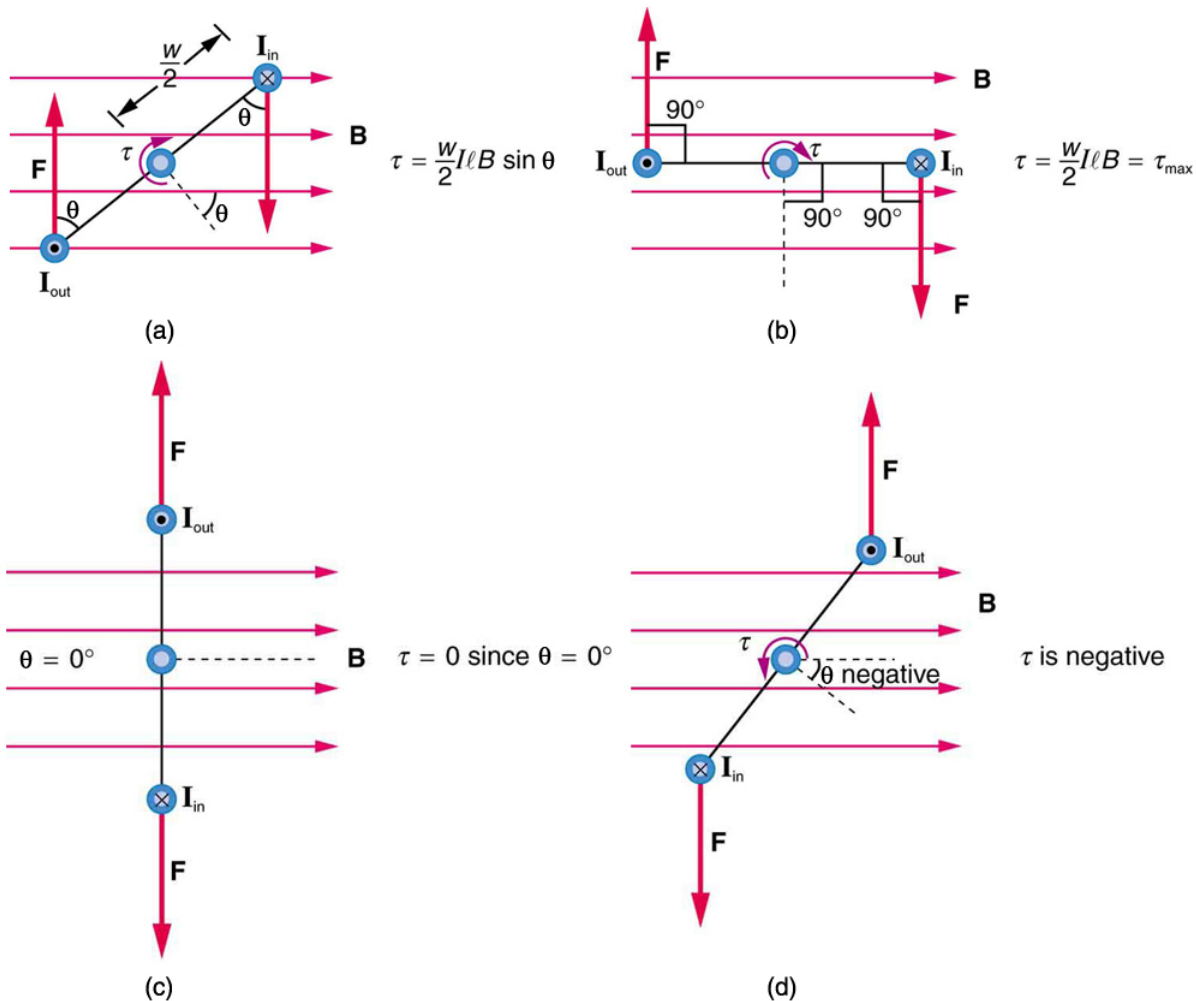
Torque on a current loop. A current-carrying loop of wire attached to a vertically rotating shaft feels magnetic forces that produce a clockwise torque as viewed from above.

Let us examine the force on each segment of the loop in [\[link\]](#) to find the torques produced about the axis of the vertical shaft. (This will lead to a useful equation for the torque on the loop.) We take the magnetic field to be uniform over the rectangular loop, which has width w and height l . First, we note that the forces on the top and bottom segments are vertical and, therefore, parallel to the shaft, producing no torque. Those vertical forces are equal in magnitude and opposite in direction, so that they also produce no net force on the loop. [\[link\]](#) shows views of the loop from above. Torque

is defined as $\tau = rF \sin \theta$, where F is the force, r is the distance from the pivot that the force is applied, and θ is the angle between r and F . As seen in [link](a), right hand rule 1 gives the forces on the sides to be equal in magnitude and opposite in direction, so that the net force is again zero. However, each force produces a clockwise torque. Since $r = w/2$, the torque on each vertical segment is $(w/2)F \sin \theta$, and the two add to give a total torque.

Equation:

$$\tau = \frac{w}{2} F \sin \theta + \frac{w}{2} F \sin \theta = wF \sin \theta$$



Top views of a current-carrying loop in a magnetic field. (a) The equation for torque is derived using this view. Note that the perpendicular to the loop makes an angle θ with the field that is the

same as the angle between $w/2$ and $\mathbf{F}$. (b) The maximum torque occurs when θ is a right angle and $\sin \theta = 1$. (c) Zero (minimum) torque occurs when θ is zero and $\sin \theta = 0$. (d) The torque reverses once the loop rotates past $\theta = 0$.

Now, each vertical segment has a length l that is perpendicular to B , so that the force on each is $F = IlB$. Entering F into the expression for torque yields

Equation:

$$\tau = wIlB \sin \theta.$$

If we have a multiple loop of N turns, we get N times the torque of one loop. Finally, note that the area of the loop is $A = wl$; the expression for the torque becomes

Equation:

$$\tau = NIAB \sin \theta.$$

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape. The loop carries a current I , has N turns, each of area A , and the perpendicular to the loop makes an angle θ with the field B . The net force on the loop is zero.

Example:

Calculating Torque on a Current-Carrying Loop in a Strong Magnetic Field

Find the maximum torque on a 100-turn square loop of a wire of 10.0 cm on a side that carries 15.0 A of current in a 2.00-T field.

Strategy

Torque on the loop can be found using $\tau = NIAB \sin \theta$. Maximum torque occurs when $\theta = 90^\circ$ and $\sin \theta = 1$.

Solution

For $\sin \theta = 1$, the maximum torque is

Equation:

$$\tau_{\max} = NIAB.$$

Entering known values yields

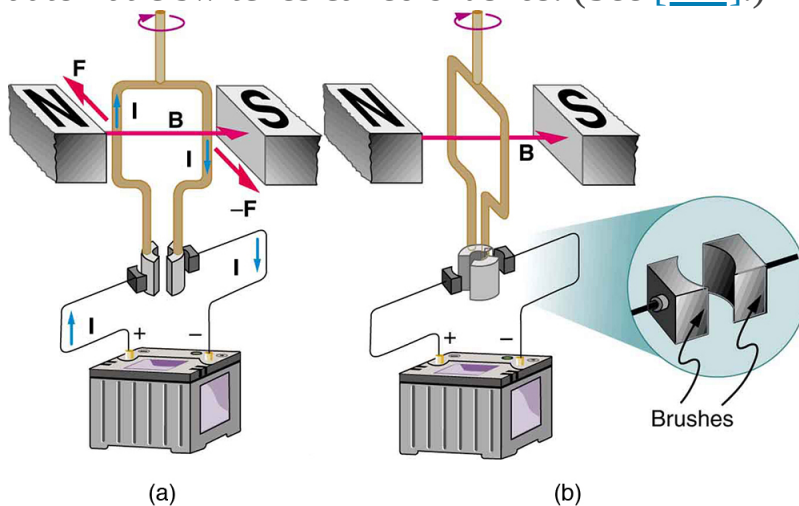
Equation:

$$\begin{aligned}\tau_{\max} &= (100)(15.0 \text{ A})(0.100 \text{ m}^2)(2.00 \text{ T}) \\ &= 30.0 \text{ N} \cdot \text{m}.\end{aligned}$$

Discussion

This torque is large enough to be useful in a motor.

The torque found in the preceding example is the maximum. As the coil rotates, the torque decreases to zero at $\theta = 0$. The torque then *reverses* its direction once the coil rotates past $\theta = 0$. (See [\[link\]](#)(d).) This means that, unless we do something, the coil will oscillate back and forth about equilibrium at $\theta = 0$. To get the coil to continue rotating in the same direction, we can reverse the current as it passes through $\theta = 0$ with automatic switches called *brushes*. (See [\[link\]](#).)

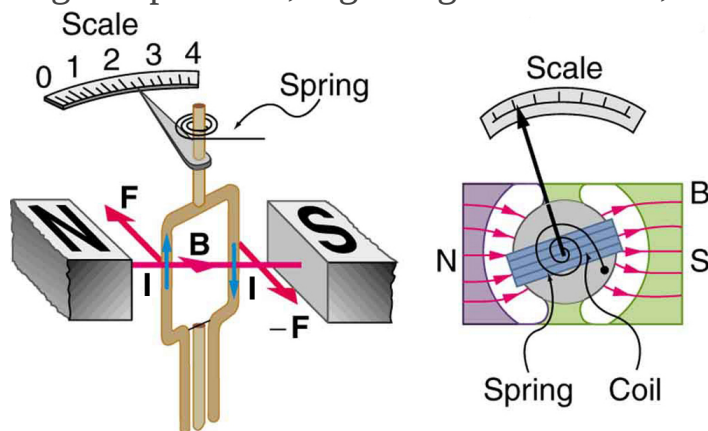


(a) As the angular momentum of the coil carries it through $\theta = 0$, the brushes reverse

the current to keep the torque clockwise. (b)

The coil will rotate continuously in the clockwise direction, with the current reversing each half revolution to maintain the clockwise torque.

Meters, such as those in analog fuel gauges on a car, are another common application of magnetic torque on a current-carrying loop. [\[link\]](#) shows that a meter is very similar in construction to a motor. The meter in the figure has its magnets shaped to limit the effect of θ by making B perpendicular to the loop over a large angular range. Thus the torque is proportional to I and not θ . A linear spring exerts a counter-torque that balances the current-produced torque. This makes the needle deflection proportional to I . If an exact proportionality cannot be achieved, the gauge reading can be calibrated. To produce a galvanometer for use in analog voltmeters and ammeters that have a low resistance and respond to small currents, we use a large loop area A , high magnetic field B , and low-resistance coils.



Meters are very similar to motors but only rotate through a part of a revolution. The magnetic poles of this meter are shaped to keep the component of B perpendicular to the loop constant, so that the torque does not depend on θ and the deflection

against the return spring is proportional only to the current I .

Section Summary

- The torque τ on a current-carrying loop of any shape in a uniform magnetic field. is

Equation:

$$\tau = NIAB \sin \theta,$$

where N is the number of turns, I is the current, A is the area of the loop, B is the magnetic field strength, and θ is the angle between the perpendicular to the loop and the magnetic field.

Conceptual Questions

Exercise:

Problem:

Draw a diagram and use RHR-1 to show that the forces on the top and bottom segments of the motor's current loop in [\[link\]](#) are vertical and produce no torque about the axis of rotation.

Problems & Exercises

Exercise:

Problem:

(a) By how many percent is the torque of a motor decreased if its permanent magnets lose 5.0% of their strength? (b) How many percent would the current need to be increased to return the torque to original values?

Solution:

(a) τ decreases by 5.00% if B decreases by 5.00%

(b) 5.26% increase

Exercise:

Problem:

(a) What is the maximum torque on a 150-turn square loop of wire 18.0 cm on a side that carries a 50.0-A current in a 1.60-T field? (b) What is the torque when θ is 10.9° ?

Exercise:

Problem:

Find the current through a loop needed to create a maximum torque of $9.00 \text{ N} \cdot \text{m}$. The loop has 50 square turns that are 15.0 cm on a side and is in a uniform 0.800-T magnetic field.

Solution:

10.0 A

Exercise:

Problem:

Calculate the magnetic field strength needed on a 200-turn square loop 20.0 cm on a side to create a maximum torque of $300 \text{ N} \cdot \text{m}$ if the loop is carrying 25.0 A.

Exercise:

Problem:

Since the equation for torque on a current-carrying loop is $\tau = NIAB \sin \theta$, the units of $\text{N} \cdot \text{m}$ must equal units of $\text{A} \cdot \text{m}^2 \text{ T}$. Verify this.

Solution:

$$A \cdot m^2 \cdot T = A \cdot m^2 \left(\frac{N}{A \cdot m} \right) = N \cdot m.$$

Exercise:

Problem:

(a) At what angle θ is the torque on a current loop 90.0% of maximum? (b) 50.0% of maximum? (c) 10.0% of maximum?

Exercise:

Problem:

A proton has a magnetic field due to its spin on its axis. The field is similar to that created by a circular current loop 0.650×10^{-15} m in radius with a current of 1.05×10^4 A (no kidding). Find the maximum torque on a proton in a 2.50-T field. (This is a significant torque on a small particle.)

Solution:

$$3.48 \times 10^{-26} \text{ N} \cdot \text{m}$$

Exercise:

Problem:

(a) A 200-turn circular loop of radius 50.0 cm is vertical, with its axis on an east-west line. A current of 100 A circulates clockwise in the loop when viewed from the east. The Earth's field here is due north, parallel to the ground, with a strength of 3.00×10^{-5} T. What are the direction and magnitude of the torque on the loop? (b) Does this device have any practical applications as a motor?

Exercise:

Problem:

Repeat [\[link\]](#), but with the loop lying flat on the ground with its current circulating counterclockwise (when viewed from above) in a location where the Earth's field is north, but at an angle 45.0° below the horizontal and with a strength of 6.00×10^{-5} T.

Solution:

(a) $0.666 \text{ N} \cdot \text{m}$ west

(b) This is not a very significant torque, so practical use would be limited. Also, the current would need to be alternated to make the loop rotate (otherwise it would oscillate).

Glossary

motor

loop of wire in a magnetic field; when current is passed through the loops, the magnetic field exerts torque on the loops, which rotates a shaft; electrical energy is converted to mechanical work in the process

meter

common application of magnetic torque on a current-carrying loop that is very similar in construction to a motor; by design, the torque is proportional to I and not θ , so the needle deflection is proportional to the current

More Applications of Magnetism

- Describe some applications of magnetism.

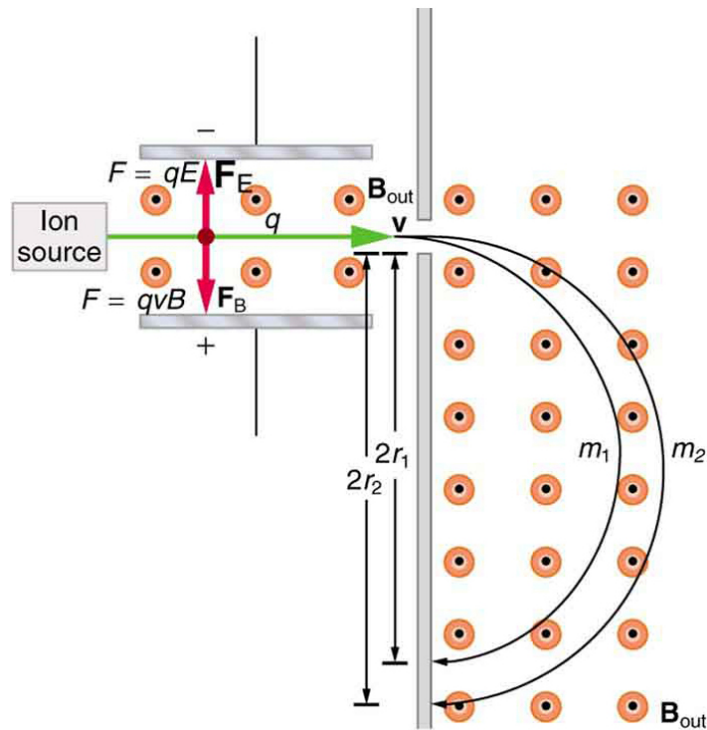
Mass Spectrometry

The curved paths followed by charged particles in magnetic fields can be put to use. A charged particle moving perpendicular to a magnetic field travels in a circular path having a radius r .

Equation:

$$r = \frac{mv}{qB}$$

It was noted that this relationship could be used to measure the mass of charged particles such as ions. A mass spectrometer is a device that measures such masses. Most mass spectrometers use magnetic fields for this purpose, although some of them have extremely sophisticated designs. Since there are five variables in the relationship, there are many possibilities. However, if v , q , and B can be fixed, then the radius of the path r is simply proportional to the mass m of the charged particle. Let us examine one such mass spectrometer that has a relatively simple design. (See [\[link\]](#).) The process begins with an ion source, a device like an electron gun. The ion source gives ions their charge, accelerates them to some velocity v , and directs a beam of them into the next stage of the spectrometer. This next region is a *velocity selector* that only allows particles with a particular value of v to get through.



This mass spectrometer uses a velocity selector to fix v so that the radius of the path is proportional to mass.

The velocity selector has both an electric field and a magnetic field, perpendicular to one another, producing forces in opposite directions on the ions. Only those ions for which the forces balance travel in a straight line into the next region. If the forces balance, then the electric force $F = qE$ equals the magnetic force $F = qvB$, so that $qE = qvB$. Noting that q

Equation:

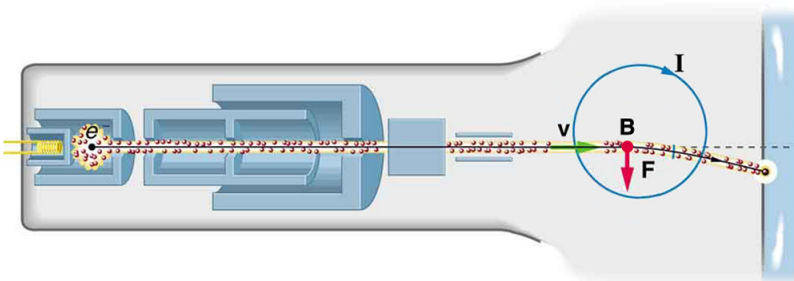
$$v = \frac{E}{B}$$

is the velocity particles must have to make it through the velocity selector, and further, that v can be selected by varying E and B . In the final region, there is only a uniform magnetic field, and so the charged particles move in circular arcs with radii proportional to particle mass. The paths also depend on charge q , but since q is in multiples of electron charges, it is easy to determine and to discriminate between ions in different charge states.

Mass spectrometry today is used extensively in chemistry and biology laboratories to identify chemical and biological substances according to their mass-to-charge ratios. In medicine, mass spectrometers are used to measure the concentration of isotopes used as tracers. Usually, biological molecules such as proteins are very large, so they are broken down into smaller fragments before analyzing. Recently, large virus particles have been analyzed as a whole on mass spectrometers. Sometimes a gas chromatograph or high-performance liquid chromatograph provides an initial separation of the large molecules, which are then input into the mass spectrometer.

Cathode Ray Tubes—CRTs—and the Like

What do non-flat-screen TVs, old computer monitors, x-ray machines, and the 2-mile-long Stanford Linear Accelerator have in common? All of them accelerate electrons, making them different versions of the electron gun. Many of these devices use magnetic fields to steer the accelerated electrons. [\[link\]](#) shows the construction of the type of cathode ray tube (CRT) found in some TVs, oscilloscopes, and old computer monitors. Two pairs of coils are used to steer the electrons, one vertically and the other horizontally, to their desired destination.



The cathode ray tube (CRT) is so named because rays of electrons originate at the cathode in the electron gun. Magnetic coils are used to steer the beam in many CRTs. In this case, the beam is moved down. Another pair of horizontal coils would steer the beam horizontally.

Magnetic Resonance Imaging

Magnetic resonance imaging (MRI) is one of the most useful and rapidly growing medical imaging tools. It non-invasively produces two-dimensional and three-dimensional images of the body that provide important medical information with none of the hazards of x-rays. MRI is based on an effect called **nuclear magnetic resonance (NMR)** in which an externally applied magnetic field interacts with the nuclei of certain atoms, particularly those of hydrogen (protons). These nuclei possess their own small magnetic fields, similar to those of electrons and the current loops discussed earlier in this chapter.

When placed in an external magnetic field, such nuclei experience a torque that pushes or aligns the nuclei into one of two new energy states—depending on the orientation of its spin (analogous to the N pole and S pole in a bar magnet). Transitions from the lower to higher energy state can be achieved by using an external radio frequency signal to “flip” the orientation of the small magnets. (This is actually a quantum mechanical process. The direction of the nuclear magnetic field is quantized as is energy in the radio waves. We will return to these topics in later chapters.) The specific frequency of the radio waves that are absorbed and reemitted depends sensitively on the type of nucleus, the chemical environment, and the external magnetic field strength. Therefore, this is a *resonance* phenomenon in which *nuclei* in a *magnetic* field act like resonators (analogous to those discussed in the treatment of sound in [Oscillatory](#)

[Motion and Waves](#)) that absorb and reemit only certain frequencies. Hence, the phenomenon is named *nuclear magnetic resonance (NMR)*.

NMR has been used for more than 50 years as an analytical tool. It was formulated in 1946 by F. Bloch and E. Purcell, with the 1952 Nobel Prize in Physics going to them for their work. Over the past two decades, NMR has been developed to produce detailed images in a process now called magnetic resonance imaging (MRI), a name coined to avoid the use of the word “nuclear” and the concomitant implication that nuclear radiation is involved. (It is not.) The 2003 Nobel Prize in Medicine went to P. Lauterbur and P. Mansfield for their work with MRI applications.

The largest part of the MRI unit is a superconducting magnet that creates a magnetic field, typically between 1 and 2 T in strength, over a relatively large volume. MRI images can be both highly detailed and informative about structures and organ functions. It is helpful that normal and non-normal tissues respond differently for slight changes in the magnetic field. In most medical images, the protons that are hydrogen nuclei are imaged. (About 2/3 of the atoms in the body are hydrogen.) Their location and density give a variety of medically useful information, such as organ function, the condition of tissue (as in the brain), and the shape of structures, such as vertebral disks and knee-joint surfaces. MRI can also be used to follow the movement of certain ions across membranes, yielding information on active transport, osmosis, dialysis, and other phenomena. With excellent spatial resolution, MRI can provide information about tumors, strokes, shoulder injuries, infections, etc.

An image requires position information as well as the density of a nuclear type (usually protons). By varying the magnetic field slightly over the volume to be imaged, the resonant frequency of the protons is made to vary with position. Broadcast radio frequencies are swept over an appropriate range and nuclei absorb and reemit them only if the nuclei are in a magnetic field with the correct strength. The imaging receiver gathers information through the body almost point by point, building up a tissue map. The reception of reemitted radio waves as a function of frequency thus gives position information. These “slices” or cross sections through the body are only several mm thick. The intensity of the reemitted radio waves is

proportional to the concentration of the nuclear type being flipped, as well as information on the chemical environment in that area of the body.

Various techniques are available for enhancing contrast in images and for obtaining more information. Scans called T1, T2, or proton density scans rely on different relaxation mechanisms of nuclei. Relaxation refers to the time it takes for the protons to return to equilibrium after the external field is turned off. This time depends upon tissue type and status (such as inflammation).

While MRI images are superior to x rays for certain types of tissue and have none of the hazards of x rays, they do not completely supplant x-ray images. MRI is less effective than x rays for detecting breaks in bone, for example, and in imaging breast tissue, so the two diagnostic tools complement each other. MRI images are also expensive compared to simple x-ray images and tend to be used most often where they supply information not readily obtained from x rays. Another disadvantage of MRI is that the patient is totally enclosed with detectors close to the body for about 30 minutes or more, leading to claustrophobia. It is also difficult for the obese patient to be in the magnet tunnel. New “open-MRI” machines are now available in which the magnet does not completely surround the patient.

Over the last decade, the development of much faster scans, called “functional MRI” (fMRI), has allowed us to map the functioning of various regions in the brain responsible for thought and motor control. This technique measures the change in blood flow for activities (thought, experiences, action) in the brain. The nerve cells increase their consumption of oxygen when active. Blood hemoglobin releases oxygen to active nerve cells and has somewhat different magnetic properties when oxygenated than when deoxygenated. With MRI, we can measure this and detect a blood oxygen-dependent signal. Most of the brain scans today use fMRI.

Other Medical Uses of Magnetic Fields

Currents in nerve cells and the heart create magnetic fields like any other currents. These can be measured but with some difficulty since their strengths are about 10^{-6} to 10^{-8} less than the Earth’s magnetic field. Recording of the heart’s magnetic field as it beats is called a

magnetocardiogram (MCG), while measurements of the brain's magnetic field is called a **magnetoencephalogram (MEG)**. Both give information that differs from that obtained by measuring the electric fields of these organs (ECGs and EEGs), but they are not yet of sufficient importance to make these difficult measurements common.

In both of these techniques, the sensors do not touch the body. MCG can be used in fetal studies, and is probably more sensitive than echocardiography. MCG also looks at the heart's electrical activity whose voltage output is too small to be recorded by surface electrodes as in EKG. It has the potential of being a rapid scan for early diagnosis of cardiac ischemia (obstruction of blood flow to the heart) or problems with the fetus.

MEG can be used to identify abnormal electrical discharges in the brain that produce weak magnetic signals. Therefore, it looks at brain activity, not just brain structure. It has been used for studies of Alzheimer's disease and epilepsy. Advances in instrumentation to measure very small magnetic fields have allowed these two techniques to be used more in recent years. What is used is a sensor called a SQUID, for superconducting quantum interference device. This operates at liquid helium temperatures and can measure magnetic fields thousands of times smaller than the Earth's.

Finally, there is a burgeoning market for magnetic cures in which magnets are applied in a variety of ways to the body, from magnetic bracelets to magnetic mattresses. The best that can be said for such practices is that they are apparently harmless, unless the magnets get close to the patient's computer or magnetic storage disks. Claims are made for a broad spectrum of benefits from cleansing the blood to giving the patient more energy, but clinical studies have not verified these claims, nor is there an identifiable mechanism by which such benefits might occur.

Note:

PhET Explorations: Magnet and Compass

Ever wonder how a compass worked to point you to the Arctic? Explore the interactions between a compass and bar magnet, and then add the Earth and find the surprising answer! Vary the magnet's strength, and see how

things change both inside and outside. Use the field meter to measure how the magnetic field changes.

<https://archive.cnx.org/specials/5ca3e2cc-ae74-11e5-b6d3-f3c228f04b5c/magnet-and-compass/#sim-bar-magnet>

Section Summary

- Crossed (perpendicular) electric and magnetic fields act as a velocity filter, giving equal and opposite forces on any charge with velocity perpendicular to the fields and of magnitude

Equation:

$$v = \frac{E}{B}.$$

Conceptual Questions

Exercise:

Problem:

Measurements of the weak and fluctuating magnetic fields associated with brain activity are called magnetoencephalograms (MEGs). Do the brain's magnetic fields imply coordinated or uncoordinated nerve impulses? Explain.

Exercise:

Problem:

Discuss the possibility that a Hall voltage would be generated on the moving heart of a patient during MRI imaging. Also discuss the same effect on the wires of a pacemaker. (The fact that patients with pacemakers are not given MRIs is significant.)

Exercise:

Problem:

A patient in an MRI unit turns his head quickly to one side and experiences momentary dizziness and a strange taste in his mouth. Discuss the possible causes.

Exercise:**Problem:**

You are told that in a certain region there is either a uniform electric or magnetic field. What measurement or observation could you make to determine the type? (Ignore the Earth's magnetic field.)

Exercise:**Problem:**

An example of magnetohydrodynamics (MHD) comes from the flow of a river (salty water). This fluid interacts with the Earth's magnetic field to produce a potential difference between the two river banks. How would you go about calculating the potential difference?

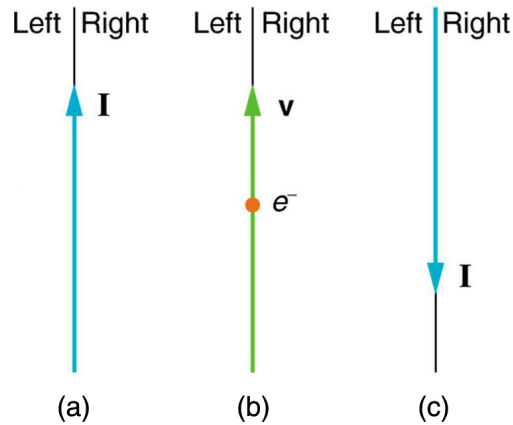
Exercise:**Problem:**

Draw gravitational field lines between 2 masses, electric field lines between a positive and a negative charge, electric field lines between 2 positive charges and magnetic field lines around a magnet. Qualitatively describe the differences between the fields and the entities responsible for the field lines.

Problems & Exercises**Exercise:**

Problem:

Indicate whether the magnetic field created in each of the three situations shown in [\[link\]](#) is into or out of the page on the left and right of the current.

**Solution:**

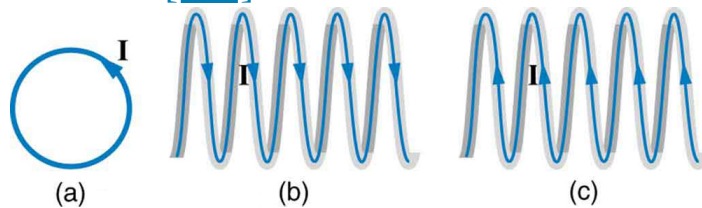
(a) right-into page, left-out of page

(b) right-out of page, left-into page

(c) right-out of page, left-into page

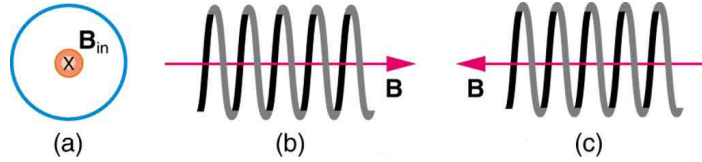
Exercise:**Problem:**

What are the directions of the fields in the center of the loop and coils shown in [\[link\]](#)?

**Exercise:**

Problem:

What are the directions of the currents in the loop and coils shown in [\[link\]](#)?

**Solution:**

- (a) clockwise
- (b) clockwise as seen from the left
- (c) clockwise as seen from the right

Exercise:**Problem:**

To see why an MRI utilizes iron to increase the magnetic field created by a coil, calculate the current needed in a 400-loop-per-meter circular coil 0.660 m in radius to create a 1.20-T field (typical of an MRI instrument) at its center with no iron present. The magnetic field of a proton is approximately like that of a circular current loop 0.650×10^{-15} m in radius carrying 1.05×10^4 A. What is the field at the center of such a loop?

Solution:

$$1.01 \times 10^{13} \text{ T}$$

Exercise:**Problem:**

Inside a motor, 30.0 A passes through a 250-turn circular loop that is 10.0 cm in radius. What is the magnetic field strength created at its center?

Exercise:**Problem:**

Nonnuclear submarines use batteries for power when submerged. (a) Find the magnetic field 50.0 cm from a straight wire carrying 1200 A from the batteries to the drive mechanism of a submarine. (b) What is the field if the wires to and from the drive mechanism are side by side? (c) Discuss the effects this could have for a compass on the submarine that is not shielded.

Solution:

(a) $4.80 \times 10^{-4} \text{ T}$

(b) Zero

(c) If the wires are not paired, the field is about 10 times stronger than Earth's magnetic field and so could severely disrupt the use of a compass.

Exercise:**Problem:**

How strong is the magnetic field inside a solenoid with 10,000 turns per meter that carries 20.0 A?

Exercise:**Problem:**

What current is needed in the solenoid described in [\[link\]](#) to produce a magnetic field 10^4 times the Earth's magnetic field of $5.00 \times 10^{-5} \text{ T}$?

Solution:

39.8 A

Exercise:

Problem:

How far from the starter cable of a car, carrying 150 A, must you be to experience a field less than the Earth's ($5.00 \times 10^{-5} \text{ T}$)? Assume a long straight wire carries the current. (In practice, the body of your car shields the dashboard compass.)

Exercise:**Problem:**

Measurements affect the system being measured, such as the current loop in [\[link\]](#). (a) Estimate the field the loop creates by calculating the field at the center of a circular loop 20.0 cm in diameter carrying 5.00 A. (b) What is the smallest field strength this loop can be used to measure, if its field must alter the measured field by less than 0.0100%?

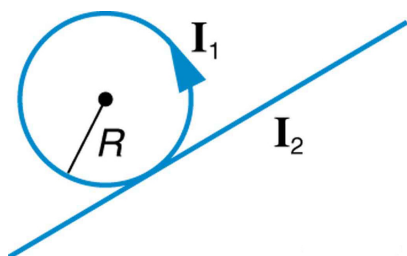
Solution:

(a) $3.14 \times 10^{-5} \text{ T}$

(b) 0.314 T

Exercise:**Problem:**

[\[link\]](#) shows a long straight wire just touching a loop carrying a current I_1 . Both lie in the same plane. (a) What direction must the current I_2 in the straight wire have to create a field at the center of the loop in the direction opposite to that created by the loop? (b) What is the ratio of I_1/I_2 that gives zero field strength at the center of the loop? (c) What is the direction of the field directly above the loop under this circumstance?



Exercise:

Problem:

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [\[link\]\(a\)](#), using the rules of vector addition to sum the contributions from each wire.

Solution:

$$7.55 \times 10^{-5} \text{ T}, 23.4^\circ$$

Exercise:

Problem:

Find the magnitude and direction of the magnetic field at the point equidistant from the wires in [\[link\]\(b\)](#), using the rules of vector addition to sum the contributions from each wire.

Exercise:

Problem:

What current is needed in the top wire in [\[link\]\(a\)](#) to produce a field of zero at the point equidistant from the wires, if the currents in the bottom two wires are both 10.0 A into the page?

Solution:

$$10.0 \text{ A}$$

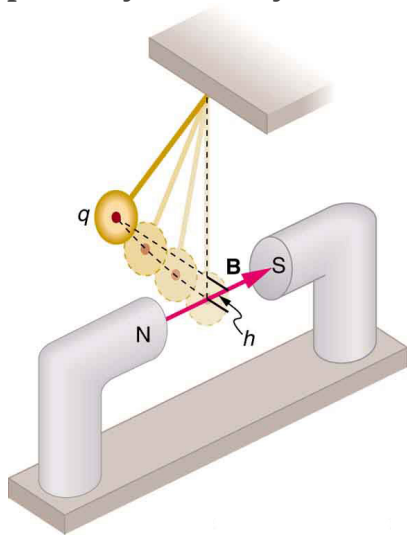
Exercise:

Problem:

Calculate the size of the magnetic field 20 m below a high voltage power line. The line carries 450 MW at a voltage of 300,000 V.

Exercise:**Problem: Integrated Concepts**

(a) A pendulum is set up so that its bob (a thin copper disk) swings between the poles of a permanent magnet as shown in [\[link\]](#). What is the magnitude and direction of the magnetic force on the bob at the lowest point in its path, if it has a positive $0.250\ \mu\text{C}$ charge and is released from a height of 30.0 cm above its lowest point? The magnetic field strength is 1.50 T. (b) What is the acceleration of the bob at the bottom of its swing if its mass is 30.0 grams and it is hung from a flexible string? Be certain to include a free-body diagram as part of your analysis.

**Solution:**

(a) $9.09 \times 10^{-7}\ \text{N}$ upward

(b) $3.03 \times 10^{-5}\ \text{m/s}^2$

Exercise:

Problem: Integrated Concepts

(a) What voltage will accelerate electrons to a speed of $6.00 \times 10^{-7} \text{ m/s}$? (b) Find the radius of curvature of the path of a *proton* accelerated through this potential in a 0.500-T field and compare this with the radius of curvature of an electron accelerated through the same potential.

Exercise:**Problem: Integrated Concepts**

Find the radius of curvature of the path of a 25.0-MeV proton moving perpendicularly to the 1.20-T field of a cyclotron.

Solution:

60.2 cm

Exercise:**Problem: Integrated Concepts**

To construct a nonmechanical water meter, a 0.500-T magnetic field is placed across the supply water pipe to a home and the Hall voltage is recorded. (a) Find the flow rate in liters per second through a 3.00-cm-diameter pipe if the Hall voltage is 60.0 mV. (b) What would the Hall voltage be for the same flow rate through a 10.0-cm-diameter pipe with the same field applied?

Exercise:**Problem: Integrated Concepts**

(a) Using the values given for an MHD drive in [\[link\]](#), and assuming the force is uniformly applied to the fluid, calculate the pressure created in N/m^2 . (b) Is this a significant fraction of an atmosphere?

Solution:

(a) $1.02 \times 10^3 \text{ N/m}^2$

(b) Not a significant fraction of an atmosphere

Exercise:**Problem: Integrated Concepts**

(a) Calculate the maximum torque on a 50-turn, 1.50 cm radius circular current loop carrying 50 μA in a 0.500-T field. (b) If this coil is to be used in a galvanometer that reads 50 μA full scale, what force constant spring must be used, if it is attached 1.00 cm from the axis of rotation and is stretched by the 60° arc moved?

Exercise:**Problem: Integrated Concepts**

A current balance used to define the ampere is designed so that the current through it is constant, as is the distance between wires. Even so, if the wires change length with temperature, the force between them will change. What percent change in force per degree will occur if the wires are copper?

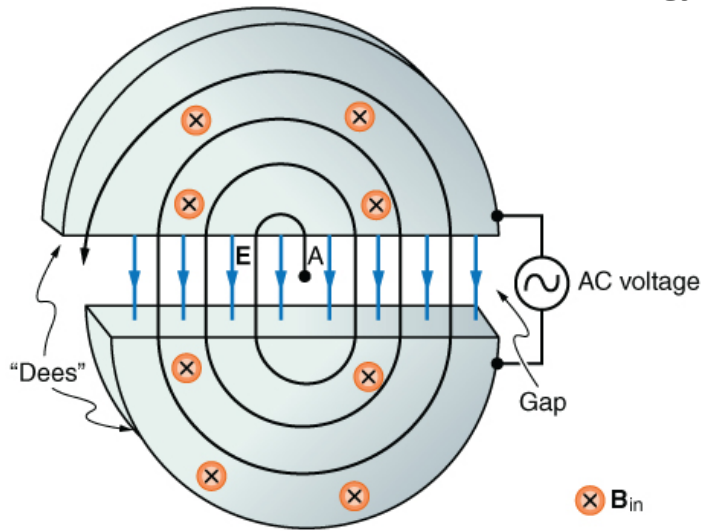
Solution:

$$17.0 \times 10^{-4} \% / ^\circ\text{C}$$

Exercise:**Problem: Integrated Concepts**

(a) Show that the period of the circular orbit of a charged particle moving perpendicularly to a uniform magnetic field is $T = 2\pi m / (qB)$. (b) What is the frequency f ? (c) What is the angular

velocity ω ? Note that these results are independent of the velocity and radius of the orbit and, hence, of the energy of the particle. ([link](#).)



Cyclotrons accelerate charged particles orbiting in a magnetic field by placing an AC voltage on the metal Dees, between which the particles move, so that energy is added twice each orbit. The frequency is constant, since it is independent of the particle energy—the radius of the orbit simply increases with energy until the particles approach the edge and are extracted for various experiments and applications.

Exercise:

Problem: Integrated Concepts

A cyclotron accelerates charged particles as shown in [link](#). Using the results of the previous problem, calculate the frequency of the accelerating voltage needed for a proton in a 1.20-T field.

Solution:

18.3 MHz

Exercise:**Problem: Integrated Concepts**

(a) A 0.140-kg baseball, pitched at 40.0 m/s horizontally and perpendicular to the Earth's horizontal 5.00×10^{-5} T field, has a 100-nC charge on it. What distance is it deflected from its path by the magnetic force, after traveling 30.0 m horizontally? (b) Would you suggest this as a secret technique for a pitcher to throw curve balls?

Exercise:**Problem: Integrated Concepts**

(a) What is the direction of the force on a wire carrying a current due east in a location where the Earth's field is due north? Both are parallel to the ground. (b) Calculate the force per meter if the wire carries 20.0 A and the field strength is 3.00×10^{-5} T. (c) What diameter copper wire would have its weight supported by this force? (d) Calculate the resistance per meter and the voltage per meter needed.

Solution:

(a) Straight up

(b) 6.00×10^{-4} N/m

(c) 94.1 μ m

(d) 2.47 Ω /m, 49.4 V/m

Exercise:**Problem: Integrated Concepts**

One long straight wire is to be held directly above another by repulsion between their currents. The lower wire carries 100 A and the wire 7.50 cm above it is 10-gauge (2.588 mm diameter) copper wire. (a) What current must flow in the upper wire, neglecting the Earth's field? (b) What is the smallest current if the Earth's 3.00×10^{-5} T field is parallel to the ground and is not neglected? (c) Is the supported wire in a stable or unstable equilibrium if displaced vertically? If displaced horizontally?

Exercise:

Problem: Unreasonable Results

(a) Find the charge on a baseball, thrown at 35.0 m/s perpendicular to the Earth's 5.00×10^{-5} T field, that experiences a 1.00-N magnetic force. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) 571 C

(b) Impossible to have such a large separated charge on such a small object.

(c) The 1.00-N force is much too great to be realistic in the Earth's field.

Exercise:

Problem: Unreasonable Results

A charged particle having mass 6.64×10^{-27} kg (that of a helium atom) moving at 8.70×10^5 m/s perpendicular to a 1.50-T magnetic field travels in a circular path of radius 16.0 mm. (a) What is the charge of the particle? (b) What is unreasonable about this result? (c) Which assumptions are responsible?

Exercise:

Problem: Unreasonable Results

An inventor wants to generate 120-V power by moving a 1.00-m-long wire perpendicular to the Earth's 5.00×10^{-5} T field. (a) Find the speed with which the wire must move. (b) What is unreasonable about this result? (c) Which assumption is responsible?

Solution:

(a) 2.40×10^6 m/s

(b) The speed is too high to be practical $\leq 1\%$ speed of light

(c) The assumption that you could reasonably generate such a voltage with a single wire in the Earth's field is unreasonable

Exercise:**Problem: Unreasonable Results**

Frustrated by the small Hall voltage obtained in blood flow measurements, a medical physicist decides to increase the applied magnetic field strength to get a 0.500-V output for blood moving at 30.0 cm/s in a 1.50-cm-diameter vessel. (a) What magnetic field strength is needed? (b) What is unreasonable about this result? (c) Which premise is responsible?

Exercise:**Problem: Unreasonable Results**

A surveyor 100 m from a long straight 200-kV DC power line suspects that its magnetic field may equal that of the Earth and affect compass readings. (a) Calculate the current in the wire needed to create a 5.00×10^{-5} T field at this distance. (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) 25.0 kA

(b) This current is unreasonably high. It implies a total power delivery in the line of 50.0×10^9 W, which is much too high for standard transmission lines.

(c) 100 meters is a long distance to obtain the required field strength. Also coaxial cables are used for transmission lines so that there is virtually no field for DC power lines, because of cancellation from opposing currents. The surveyor's concerns are not a problem for his magnetic field measurements.

Exercise:**Problem: Construct Your Own Problem**

Consider a mass separator that applies a magnetic field perpendicular to the velocity of ions and separates the ions based on the radius of curvature of their paths in the field. Construct a problem in which you calculate the magnetic field strength needed to separate two ions that differ in mass, but not charge, and have the same initial velocity. Among the things to consider are the types of ions, the velocities they can be given before entering the magnetic field, and a reasonable value for the radius of curvature of the paths they follow. In addition, calculate the separation distance between the ions at the point where they are detected.

Exercise:**Problem: Construct Your Own Problem**

Consider using the torque on a current-carrying coil in a magnetic field to detect relatively small magnetic fields (less than the field of the Earth, for example). Construct a problem in which you calculate the maximum torque on a current-carrying loop in a magnetic field. Among the things to be considered are the size of the coil, the number

of loops it has, the current you pass through the coil, and the size of the field you wish to detect. Discuss whether the torque produced is large enough to be effectively measured. Your instructor may also wish for you to consider the effects, if any, of the field produced by the coil on the surroundings that could affect detection of the small field.

Glossary

magnetic resonance imaging (MRI)

a medical imaging technique that uses magnetic fields create detailed images of internal tissues and organs

nuclear magnetic resonance (NMR)

a phenomenon in which an externally applied magnetic field interacts with the nuclei of certain atoms

magnetocardiogram (MCG)

a recording of the heart's magnetic field as it beats

magnetoencephalogram (MEG)

a measurement of the brain's magnetic field

Introduction to Electromagnetic Induction, AC Circuits and Electrical Technologies

class="introduction"

These wind turbines in the Thames Estuary in the UK are an example of induction at work.

Wind pushes the blades of the turbine, spinning a shaft attached to magnets.

The magnets spin around a conductive coil, inducing an electric current in the coil, and eventually feeding the electrical grid. (credit: modification of work by Petr Kratochvil)



Nature's displays of symmetry are beautiful and alluring. A butterfly's wings exhibit an appealing symmetry in a complex system. (See [\[link\]](#).) The laws of physics display symmetries at the most basic level—these symmetries are a source of wonder and imply deeper meaning. Since we place a high value on symmetry, we look for it when we explore nature. The remarkable thing is that we find it.



Physics, like this butterfly, has inherent symmetries. (credit: Thomas Bresson)

The hint of symmetry between electricity and magnetism found in the preceding chapter will be elaborated upon in this chapter. Specifically, we know that a current creates a magnetic field. If nature is symmetric here, then perhaps a magnetic field can create a current. The Hall effect is a voltage caused by a magnetic force. That voltage could drive a current. Historically, it was very shortly after Oersted discovered currents cause magnetic fields that other scientists asked the following question: Can magnetic fields cause currents? The answer was soon found by experiment to be yes. In 1831, some 12 years after Oersted's discovery, the English scientist Michael Faraday (1791–1862) and the American scientist Joseph Henry (1797–1878) independently demonstrated that magnetic fields can produce currents. The basic process of generating emfs (electromotive force) and, hence, currents with magnetic fields is known as **induction**; this process is also called magnetic induction to distinguish it from charging by induction, which utilizes the Coulomb force.

Today, currents induced by magnetic fields are essential to our technological society. The ubiquitous generator—found in automobiles, on bicycles, in nuclear power plants, and so on—uses magnetism to generate current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances. Not so familiar perhaps, but important nevertheless, is that the behavior of AC circuits depends strongly on the effect of magnetic fields on currents.

Glossary

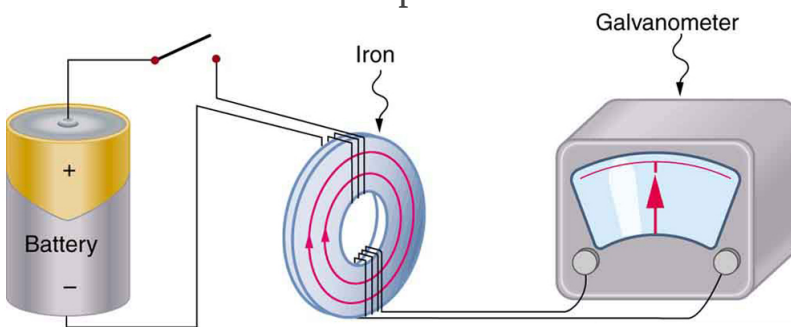
induction

(magnetic induction) the creation of emfs and hence currents by magnetic fields

Induced Emf and Magnetic Flux

- Calculate the flux of a uniform magnetic field through a loop of arbitrary orientation.
- Describe methods to produce an electromotive force (emf) with a magnetic field or magnet and a loop of wire.

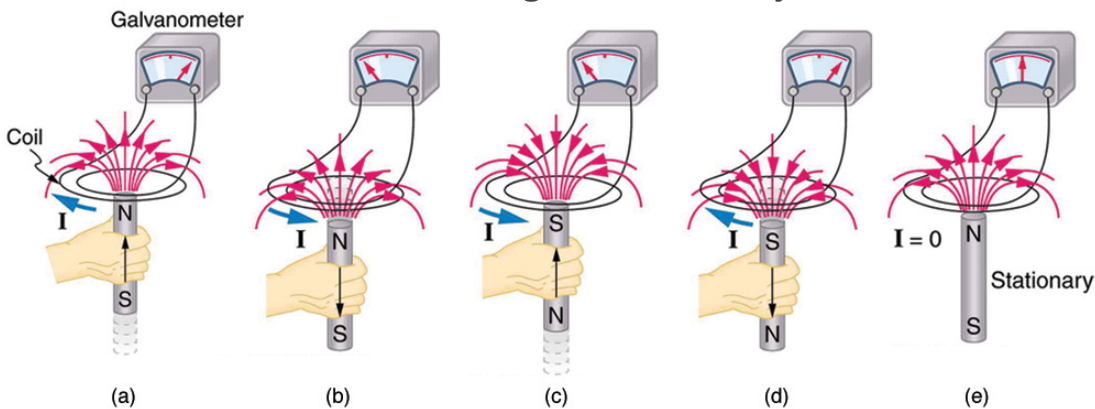
The apparatus used by Faraday to demonstrate that magnetic fields can create currents is illustrated in [\[link\]](#). When the switch is closed, a magnetic field is produced in the coil on the top part of the iron ring and transmitted to the coil on the bottom part of the ring. The galvanometer is used to detect any current induced in the coil on the bottom. It was found that each time the switch is closed, the galvanometer detects a current in one direction in the coil on the bottom. (You can also observe this in a physics lab.) Each time the switch is opened, the galvanometer detects a current in the opposite direction. Interestingly, if the switch remains closed or open for any length of time, there is no current through the galvanometer. *Closing and opening the switch* induces the current. It is the *change* in magnetic field that creates the current. More basic than the current that flows is the emf that causes it. The current is a result of an *emf induced by a changing magnetic field*, whether or not there is a path for current to flow.



Faraday's apparatus for demonstrating that a magnetic field can produce a current. A change in the field produced by the top coil induces an emf and, hence, a current in the bottom coil. When the switch is opened and closed, the galvanometer registers currents in opposite directions. No current flows

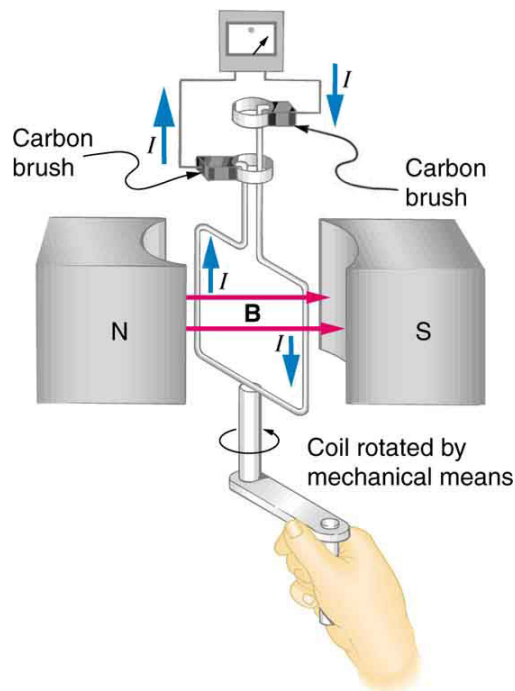
through the galvanometer when the switch remains closed or open.

An experiment easily performed and often done in physics labs is illustrated in [\[link\]](#). An emf is induced in the coil when a bar magnet is pushed in and out of it. Emfs of opposite signs are produced by motion in opposite directions, and the emfs are also reversed by reversing poles. The same results are produced if the coil is moved rather than the magnet—it is the relative motion that is important. The faster the motion, the greater the emf, and there is no emf when the magnet is stationary relative to the coil.



Movement of a magnet relative to a coil produces emfs as shown. The same emfs are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the emf, and the emf is zero when there is no motion.

The method of inducing an emf used in most electric generators is shown in [\[link\]](#). A coil is rotated in a magnetic field, producing an alternating current emf, which depends on rotation rate and other factors that will be explored in later sections. Note that the generator is remarkably similar in construction to a motor (another symmetry).



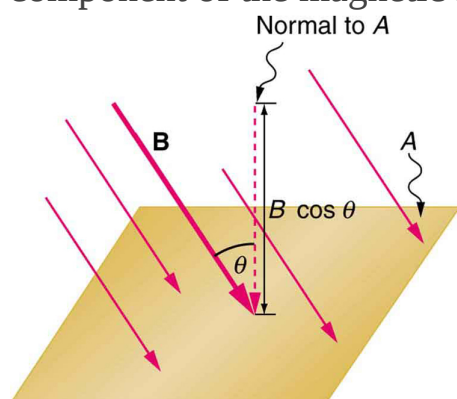
Rotation of a coil in a magnetic field produces an emf. This is the basic construction of a generator, where work done to turn the coil is converted to electric energy. Note the generator is very similar in construction to a motor.

So we see that changing the magnitude or direction of a magnetic field produces an emf. Experiments revealed that there is a crucial quantity called the **magnetic flux**, Φ , given by

Equation:

$$\Phi = BA \cos \theta,$$

where B is the magnetic field strength over an area A , at an angle θ with the perpendicular to the area as shown in [\[link\]](#). **Any change in magnetic flux Φ induces an emf.** This process is defined to be **electromagnetic induction**. Units of magnetic flux Φ are $\text{T} \cdot \text{m}^2$. As seen in [\[link\]](#), $B \cos \theta = B_{\perp}$, which is the component of B perpendicular to the area A . Thus magnetic flux is $\Phi = B_{\perp} A$, the product of the area and the component of the magnetic field perpendicular to it.



$$\Phi = BA \cos \theta = B_{\perp} A$$

Magnetic flux Φ is related to the magnetic field and the area over which it exists. The flux $\Phi = BA \cos \theta$ is related to induction; any change in Φ induces an emf.

All induction, including the examples given so far, arises from some change in magnetic flux Φ . For example, Faraday changed B and hence Φ when opening and closing the switch in his apparatus (shown in [\[link\]](#)). This is also true for the bar magnet and coil shown in [\[link\]](#). When rotating the coil of a generator, the angle θ and, hence, Φ is changed. Just how great an emf and what direction it takes depend on the change in Φ and how rapidly the change is made, as examined in the next section.

Section Summary

- The crucial quantity in induction is magnetic flux Φ , defined to be $\Phi = BA \cos \theta$, where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area.
- Units of magnetic flux Φ are $\text{T} \cdot \text{m}^2$.
- Any change in magnetic flux Φ induces an emf—the process is defined to be electromagnetic induction.

Conceptual Questions

Exercise:

Problem:

How do the multiple-loop coils and iron ring in the version of Faraday's apparatus shown in [\[link\]](#) enhance the observation of induced emf?

Exercise:

Problem:

When a magnet is thrust into a coil as in [\[link\]](#)(a), what is the direction of the force exerted by the coil on the magnet? Draw a diagram showing the direction of the current induced in the coil and the magnetic field it produces, to justify your response. How does the magnitude of the force depend on the resistance of the galvanometer?

Exercise:

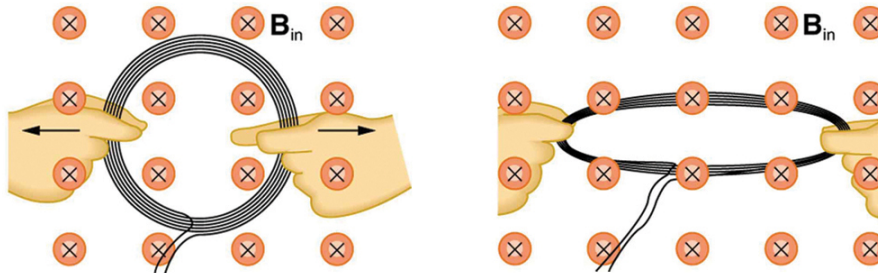
Problem:

Explain how magnetic flux can be zero when the magnetic field is not zero.

Exercise:

Problem:

Is an emf induced in the coil in [\[link\]](#) when it is stretched? If so, state why and give the direction of the induced current.



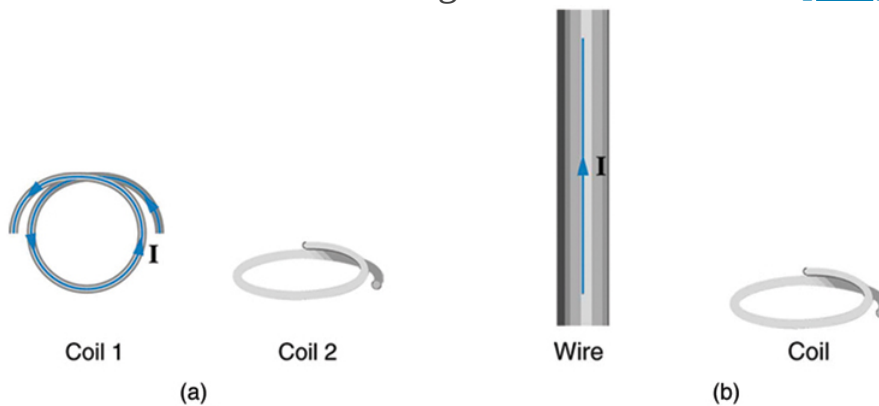
A circular coil of wire is stretched in a magnetic field.

Problems & Exercises

Exercise:

Problem:

What is the value of the magnetic flux at coil 2 in [\[link\]](#) due to coil 1?



(a) The planes of the two coils are perpendicular.

(b) The wire is perpendicular to the plane of the coil.

Solution:

Zero

Exercise:**Problem:**

What is the value of the magnetic flux through the coil in [\[link\]](#)(b) due to the wire?

Glossary

magnetic flux

the amount of magnetic field going through a particular area, calculated with $\Phi = BA \cos \theta$ where B is the magnetic field strength over an area A at an angle θ with the perpendicular to the area

electromagnetic induction

the process of inducing an emf (voltage) with a change in magnetic flux

Faraday's Law of Induction: Lenz's Law

- Calculate emf, current, and magnetic fields using Faraday's Law.
- Explain the physical results of Lenz's Law

Faraday's and Lenz's Law

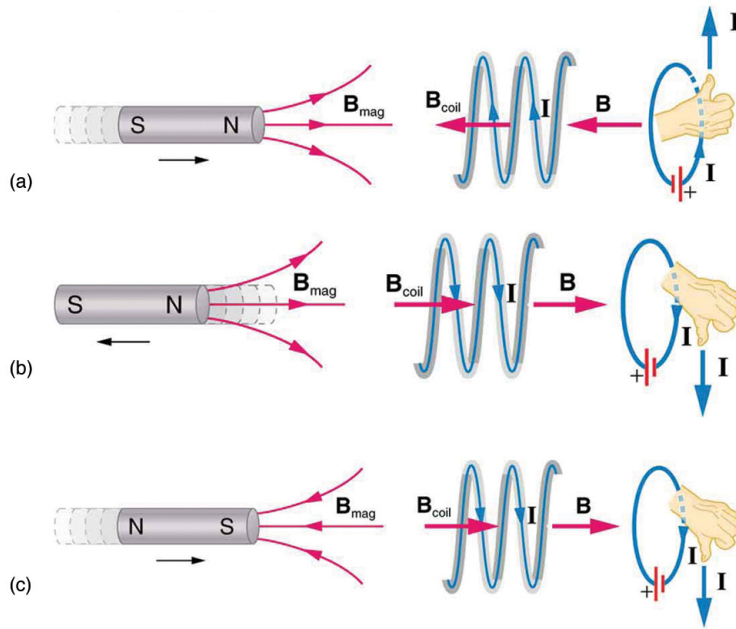
Faraday's experiments showed that the emf induced by a change in magnetic flux depends on only a few factors. First, emf is directly proportional to the change in flux $\Delta\Phi$. Second, emf is greatest when the change in time Δt is smallest—that is, emf is inversely proportional to Δt . Finally, if a coil has N turns, an emf will be produced that is N times greater than for a single coil, so that emf is directly proportional to N . The equation for the emf induced by a change in magnetic flux is

Equation:

$$\text{emf} = -N \frac{\Delta\Phi}{\Delta t}.$$

This relationship is known as **Faraday's law of induction**. The units for emf are volts, as is usual.

The minus sign in Faraday's law of induction is very important. The minus means that *the emf creates a current I and magnetic field B that oppose the change in flux $\Delta\Phi$ —this is known as Lenz's law*. The direction (given by the minus sign) of the emf is so important that it is called **Lenz's law** after the Russian Heinrich Lenz (1804–1865), who, like Faraday and Henry, independently investigated aspects of induction. Faraday was aware of the direction, but Lenz stated it so clearly that he is credited for its discovery. (See [\[link\]](#).)



(a) When this bar magnet is thrust into the coil, the strength of the magnetic field increases in the coil. The current induced in the coil creates another field, in the opposite direction of the bar magnet's to oppose the increase. This is one aspect of *Lenz's law—induction opposes any change in flux*. (b) and (c) are two other situations. Verify for yourself that the direction of the induced B_{coil} shown indeed opposes the change in flux and that the current direction shown is consistent with RHR-2.

Note:

Problem-Solving Strategy for Lenz's Law

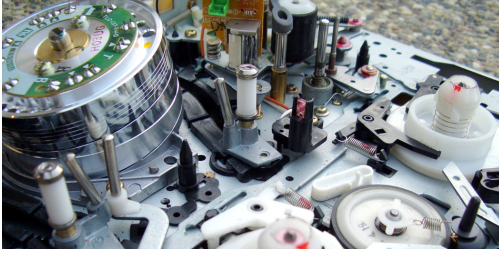
To use Lenz's law to determine the directions of the induced magnetic fields, currents, and emfs:

1. Make a sketch of the situation for use in visualizing and recording directions.
2. Determine the direction of the magnetic field B .
3. Determine whether the flux is increasing or decreasing.
4. Now determine the direction of the induced magnetic field B . It opposes the *change* in flux by adding or subtracting from the original field.
5. Use RHR-2 to determine the direction of the induced current I that is responsible for the induced magnetic field B .
6. The direction (or polarity) of the induced emf will now drive a current in this direction and can be represented as current emerging from the positive terminal of the emf and returning to its negative terminal.

For practice, apply these steps to the situations shown in [\[link\]](#) and to others that are part of the following text material.

Applications of Electromagnetic Induction

There are many applications of Faraday's Law of induction, as we will explore in this chapter and others. At this juncture, let us mention several that have to do with data storage and magnetic fields. A very important application has to do with audio and video *recording tapes*. A plastic tape, coated with iron oxide, moves past a recording head. This recording head is basically a round iron ring about which is wrapped a coil of wire—an electromagnet ([\[link\]](#)). A signal in the form of a varying input current from a microphone or camera goes to the recording head. These signals (which are a function of the signal amplitude and frequency) produce varying magnetic fields at the recording head. As the tape moves past the recording head, the magnetic field orientations of the iron oxide molecules on the tape are changed thus recording the signal. In the playback mode, the magnetized tape is run past another head, similar in structure to the recording head. The different magnetic field orientations of the iron oxide molecules on the tape induces an emf in the coil of wire in the playback head. This signal then is sent to a loudspeaker or video player.



Recording and playback
heads used with audio
and video magnetic tapes.
(credit: Steve Jurvetson)

Similar principles apply to computer hard drives, except at a much faster rate. Here recordings are on a coated, spinning disk. Read heads historically were made to work on the principle of induction. However, the input information is carried in digital rather than analog form – a series of 0's or 1's are written upon the spinning hard drive. Today, most hard drive readout devices do not work on the principle of induction, but use a technique known as *giant magnetoresistance*. (The discovery that weak changes in a magnetic field in a thin film of iron and chromium could bring about much larger changes in electrical resistance was one of the first large successes of nanotechnology.) Another application of induction is found on the magnetic stripe on the back of your personal credit card as used at the grocery store or the ATM machine. This works on the same principle as the audio or video tape mentioned in the last paragraph in which a head reads personal information from your card.

Another application of electromagnetic induction is when electrical signals need to be transmitted across a barrier. Consider the *cochlear implant* shown below. Sound is picked up by a microphone on the outside of the skull and is used to set up a varying magnetic field. A current is induced in a receiver secured in the bone beneath the skin and transmitted to electrodes in the inner ear. Electromagnetic induction can be used in other instances where electric signals need to be conveyed across various media.



Electromagnetic induction used in transmitting electric currents across mediums. The device on the baby's head induces an electrical current in a receiver secured in the bone beneath the skin. (credit: Bjorn Knetsch)

Another contemporary area of research in which electromagnetic induction is being successfully implemented (and with substantial potential) is transcranial magnetic stimulation. A host of disorders, including depression and hallucinations can be traced to irregular localized electrical activity in the brain. In *transcranial magnetic stimulation*, a rapidly varying and very localized magnetic field is placed close to certain sites identified in the brain. Weak electric currents are induced in the identified sites and can result in recovery of electrical functioning in the brain tissue.

Sleep apnea (“the cessation of breath”) affects both adults and infants (especially premature babies and it may be a cause of sudden infant deaths [SID]). In such individuals, breath can stop repeatedly during their sleep. A cessation of more than 20 seconds can be very dangerous. Stroke, heart failure, and tiredness are just some of the possible consequences for a

person having sleep apnea. The concern in infants is the stopping of breath for these longer times. One type of monitor to alert parents when a child is not breathing uses electromagnetic induction. A wire wrapped around the infant's chest has an alternating current running through it. The expansion and contraction of the infant's chest as the infant breathes changes the area through the coil. A pickup coil located nearby has an alternating current induced in it due to the changing magnetic field of the initial wire. If the child stops breathing, there will be a change in the induced current, and so a parent can be alerted.

Note:**Making Connections: Conservation of Energy**

Lenz's law is a manifestation of the conservation of energy. The induced emf produces a current that opposes the change in flux, because a change in flux means a change in energy. Energy can enter or leave, but not instantaneously. Lenz's law is a consequence. As the change begins, the law says induction opposes and, thus, slows the change. In fact, if the induced emf were in the same direction as the change in flux, there would be a positive feedback that would give us free energy from no apparent source—conservation of energy would be violated.

Example:**Calculating Emf: How Great Is the Induced Emf?**

Calculate the magnitude of the induced emf when the magnet in [\[link\]](#)(a) is thrust into the coil, given the following information: the single loop coil has a radius of 6.00 cm and the average value of $B \cos \theta$ (this is given, since the bar magnet's field is complex) increases from 0.0500 T to 0.250 T in 0.100 s.

Strategy

To find the *magnitude* of emf, we use Faraday's law of induction as stated by $\text{emf} = -N \frac{\Delta \Phi}{\Delta t}$, but without the minus sign that indicates direction:

Equation:

$$\text{emf} = N \frac{\Delta\Phi}{\Delta t}.$$

Solution

We are given that $N = 1$ and $\Delta t = 0.100$ s, but we must determine the change in flux $\Delta\Phi$ before we can find emf. Since the area of the loop is fixed, we see that

Equation:

$$\Delta\Phi = \Delta(BA \cos \theta) = A\Delta(B \cos \theta).$$

Now $\Delta(B \cos \theta) = 0.200$ T, since it was given that $B \cos \theta$ changes from 0.0500 to 0.250 T. The area of the loop is

$A = \pi r^2 = (3.14...)(0.060 \text{ m})^2 = 1.13 \times 10^{-2} \text{ m}^2$. Thus,

Equation:

$$\Delta\Phi = (1.13 \times 10^{-2} \text{ m}^2)(0.200 \text{ T}).$$

Entering the determined values into the expression for emf gives

Equation:

$$\text{Emf} = N \frac{\Delta\Phi}{\Delta t} = \frac{(1.13 \times 10^{-2} \text{ m}^2)(0.200 \text{ T})}{0.100 \text{ s}} = 22.6 \text{ mV}.$$

Discussion

While this is an easily measured voltage, it is certainly not large enough for most practical applications. More loops in the coil, a stronger magnet, and faster movement make induction the practical source of voltages that it is.

Note:

PhET Explorations: Faraday's Electromagnetic Lab

Play with a bar magnet and coils to learn about Faraday's law. Move a bar magnet near one or two coils to make a light bulb glow. View the magnetic field lines. A meter shows the direction and magnitude of the current. View the magnetic field lines or use a meter to show the direction and magnitude of the current. You can also play with electromagnets, generators and

transformers!

<https://archive.cnx.org/specials/70b14c10-ae73-11e5-8eb2-b7fbe0c5c7a4/faraday/#sim-bar-magnet>

Section Summary

- Faraday's law of induction states that the emf induced by a change in magnetic flux is

Equation:

$$\text{emf} = -N \frac{\Delta\Phi}{\Delta t}$$

when flux changes by $\Delta\Phi$ in a time Δt .

- If emf is induced in a coil, N is its number of turns.
- The minus sign means that the emf creates a current I and magnetic field B that *oppose the change in flux* $\Delta\Phi$ —this opposition is known as Lenz's law.

Conceptual Questions

Exercise:

Problem:

A person who works with large magnets sometimes places her head inside a strong field. She reports feeling dizzy as she quickly turns her head. How might this be associated with induction?

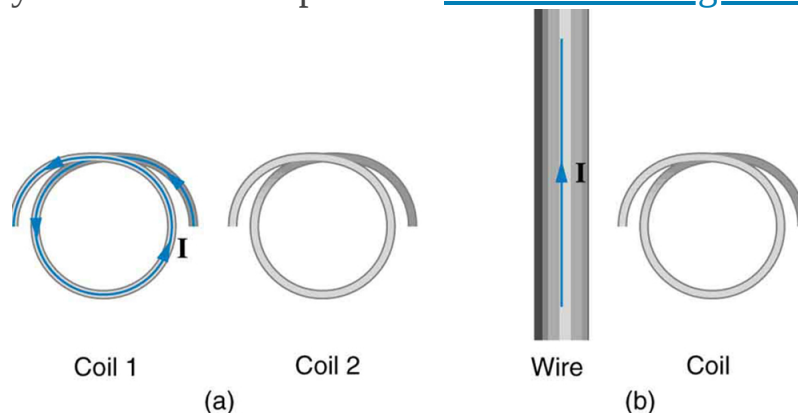
Exercise:

Problem:

A particle accelerator sends high-velocity charged particles down an evacuated pipe. Explain how a coil of wire wrapped around the pipe could detect the passage of individual particles. Sketch a graph of the voltage output of the coil as a single particle passes through it.

Problems & Exercises**Exercise:****Problem:**

Referring to [\[link\]](#)(a), what is the direction of the current induced in coil 2: (a) If the current in coil 1 increases? (b) If the current in coil 1 decreases? (c) If the current in coil 1 is constant? Explicitly show how you follow the steps in the [Problem-Solving Strategy for Lenz's Law](#).



(a) The coils lie in the same plane. (b) The wire is in the plane of the coil

Solution:

(a) CCW

(b) CW

(c) No current induced

Exercise:

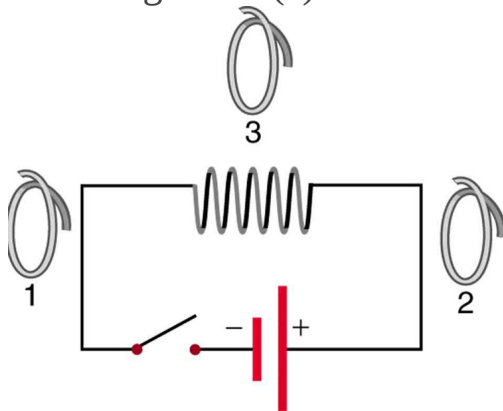
Problem:

Referring to [\[link\]](#)(b), what is the direction of the current induced in the coil: (a) If the current in the wire increases? (b) If the current in the wire decreases? (c) If the current in the wire suddenly changes direction? Explicitly show how you follow the steps in the [Problem-Solving Strategy for Lenz's Law](#).

Exercise:

Problem:

Referring to [\[link\]](#), what are the directions of the currents in coils 1, 2, and 3 (assume that the coils are lying in the plane of the circuit): (a) When the switch is first closed? (b) When the switch has been closed for a long time? (c) Just after the switch is opened?



Solution:

(a) 1 CCW, 2 CCW, 3 CW

(b) 1, 2, and 3 no current induced

(c) 1 CW, 2 CW, 3 CCW

Exercise:

Problem: Repeat the previous problem with the battery reversed.

Exercise:

Problem:

Verify that the units of $\Delta\Phi/\Delta t$ are volts. That is, show that $1 \text{ T} \cdot \text{m}^2/\text{s} = 1 \text{ V}$.

Exercise:

Problem:

Suppose a 50-turn coil lies in the plane of the page in a uniform magnetic field that is directed into the page. The coil originally has an area of 0.250 m^2 . It is stretched to have no area in 0.100 s . What is the direction and magnitude of the induced emf if the uniform magnetic field has a strength of 1.50 T ?

Exercise:

Problem:

(a) An MRI technician moves his hand from a region of very low magnetic field strength into an MRI scanner's 2.00 T field with his fingers pointing in the direction of the field. Find the average emf induced in his wedding ring, given its diameter is 2.20 cm and assuming it takes 0.250 s to move it into the field. (b) Discuss whether this current would significantly change the temperature of the ring.

Solution:

(a) 3.04 mV

(b) As a lower limit on the ring, estimate $R = 1.00 \text{ m}\Omega$. The heat transferred will be 2.31 mJ . This is not a significant amount of heat.

Exercise:

Problem: Integrated Concepts

Referring to the situation in the previous problem: (a) What current is induced in the ring if its resistance is $0.0100\ \Omega$? (b) What average power is dissipated? (c) What magnetic field is induced at the center of the ring? (d) What is the direction of the induced magnetic field relative to the MRI's field?

Exercise:

Problem:

An emf is induced by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's $5.00 \times 10^{-5}\ \text{T}$ magnetic field. What average emf is induced, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

Solution:

0.157 V

Exercise:

Problem:

A 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.) Find the magnetic field strength needed to induce an average emf of 10,000 V.

Exercise:

Problem: Integrated Concepts

Approximately how does the emf induced in the loop in [\[link\]](#)(b) depend on the distance of the center of the loop from the wire?

Solution:

proportional to $\frac{1}{r}$

Exercise:

Problem: Integrated Concepts

- (a) A lightning bolt produces a rapidly varying magnetic field. If the bolt strikes the earth vertically and acts like a current in a long straight wire, it will induce a voltage in a loop aligned like that in [\[link\]](#)(b). What voltage is induced in a 1.00 m diameter loop 50.0 m from a 2.00×10^6 A lightning strike, if the current falls to zero in 25.0 μ s?
- (b) Discuss circumstances under which such a voltage would produce noticeable consequences.

Glossary

Faraday's law of induction

the means of calculating the emf in a coil due to changing magnetic flux, given by $\text{emf} = -N \frac{\Delta\Phi}{\Delta t}$

Lenz's law

the minus sign in Faraday's law, signifying that the emf induced in a coil opposes the change in magnetic flux

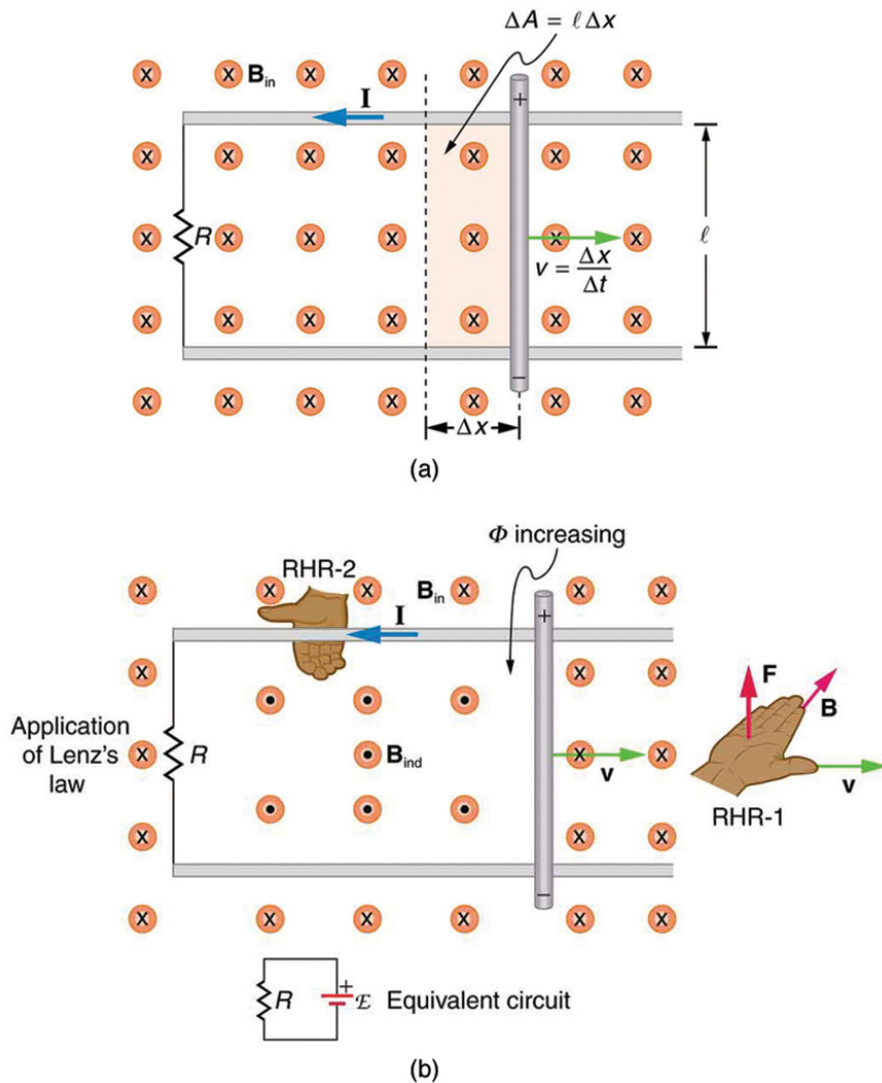
Motional Emf

- Calculate emf, force, magnetic field, and work due to the motion of an object in a magnetic field.

As we have seen, any change in magnetic flux induces an emf opposing that change—a process known as induction. Motion is one of the major causes of induction. For example, a magnet moved toward a coil induces an emf, and a coil moved toward a magnet produces a similar emf. In this section, we concentrate on motion in a magnetic field that is stationary relative to the Earth, producing what is loosely called *motional emf*.

One situation where motional emf occurs is known as the Hall effect and has already been examined. Charges moving in a magnetic field experience the magnetic force $F = qvB \sin \theta$, which moves opposite charges in opposite directions and produces an $\text{emf} = B\ell v$. We saw that the Hall effect has applications, including measurements of B and v . We will now see that the Hall effect is one aspect of the broader phenomenon of induction, and we will find that motional emf can be used as a power source.

Consider the situation shown in [\[link\]](#). A rod is moved at a speed v along a pair of conducting rails separated by a distance ℓ in a uniform magnetic field B . The rails are stationary relative to B and are connected to a stationary resistor R . The resistor could be anything from a light bulb to a voltmeter. Consider the area enclosed by the moving rod, rails, and resistor. B is perpendicular to this area, and the area is increasing as the rod moves. Thus the magnetic flux enclosed by the rails, rod, and resistor is increasing. When flux changes, an emf is induced according to Faraday's law of induction.



(a) A motional emf $= B\ell v$ is induced between the rails when this rod moves to the right in the uniform magnetic field. The magnetic field B is into the page, perpendicular to the moving rod and rails and, hence, to the area enclosed by them. (b) Lenz's law gives the directions of the induced field and current, and the polarity of the induced emf. Since the flux is increasing, the induced field is in the opposite direction, or out of the page. RHR-2 gives the current direction shown, and the polarity of the rod will drive such a current. RHR-1 also indicates the same polarity for the rod. (Note that

the script E symbol used in the equivalent circuit at the bottom of part (b) represents emf.)

To find the magnitude of emf induced along the moving rod, we use Faraday's law of induction without the sign:

Equation:

$$\text{emf} = N \frac{\Delta \Phi}{\Delta t}.$$

Here and below, “emf” implies the magnitude of the emf. In this equation, $N = 1$ and the flux $\Phi = BA \cos \theta$. We have $\theta = 0^\circ$ and $\cos \theta = 1$, since B is perpendicular to A . Now $\Delta \Phi = \Delta(BA) = B \Delta A$, since B is uniform. Note that the area swept out by the rod is $\Delta A = \ell \Delta x$. Entering these quantities into the expression for emf yields

Equation:

$$\text{emf} = \frac{B \Delta A}{\Delta t} = B \frac{\ell \Delta x}{\Delta t}.$$

Finally, note that $\Delta x / \Delta t = v$, the velocity of the rod. Entering this into the last expression shows that

Equation:

$$\text{emf} = B\ell v \quad (B, \ell, \text{ and } v \text{ perpendicular})$$

is the motional emf. This is the same expression given for the Hall effect previously.

Note:

Making Connections: Unification of Forces

There are many connections between the electric force and the magnetic force. The fact that a moving electric field produces a magnetic field and, conversely, a moving magnetic field produces an electric field is part of why electric and magnetic forces are now considered to be different manifestations of the same force. This classic unification of electric and magnetic forces into what is called the electromagnetic force is the inspiration for contemporary efforts to unify other basic forces.

To find the direction of the induced field, the direction of the current, and the polarity of the induced emf, we apply Lenz's law as explained in [Faraday's Law of Induction: Lenz's Law](#). (See [\[link\]](#)(b).) Flux is increasing, since the area enclosed is increasing. Thus the induced field must oppose the existing one and be out of the page. And so the RHR-2 requires that I be counterclockwise, which in turn means the top of the rod is positive as shown.

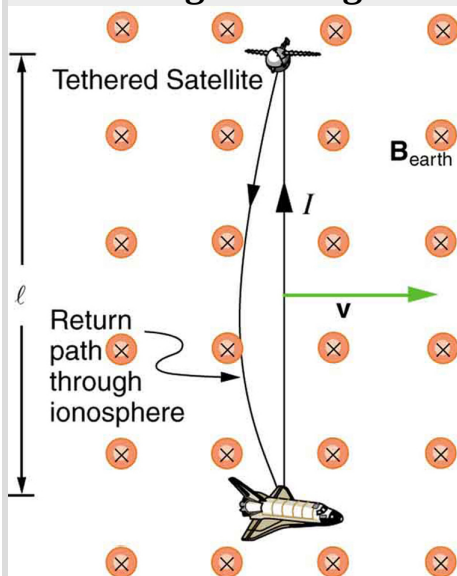
Motional emf also occurs if the magnetic field moves and the rod (or other object) is stationary relative to the Earth (or some observer). We have seen an example of this in the situation where a moving magnet induces an emf in a stationary coil. It is the relative motion that is important. What is emerging in these observations is a connection between magnetic and electric fields. A moving magnetic field produces an electric field through its induced emf. We already have seen that a moving electric field produces a magnetic field—moving charge implies moving electric field and moving charge produces a magnetic field.

Motional emfs in the Earth's weak magnetic field are not ordinarily very large, or we would notice voltage along metal rods, such as a screwdriver, during ordinary motions. For example, a simple calculation of the motional emf of a 1 m rod moving at 3.0 m/s perpendicular to the Earth's field gives $\text{emf} = B\ell v = (5.0 \times 10^{-5} \text{ T})(1.0 \text{ m})(3.0 \text{ m/s}) = 150 \text{ } \mu\text{V}$. This small value is consistent with experience. There is a spectacular exception, however. In 1992 and 1996, attempts were made with the space shuttle to create large motional emfs. The Tethered Satellite was to be let out on a 20 km length of wire as shown in [\[link\]](#), to create a 5 kV emf by moving at

orbital speed through the Earth's field. This emf could be used to convert some of the shuttle's kinetic and potential energy into electrical energy if a complete circuit could be made. To complete the circuit, the stationary ionosphere was to supply a return path for the current to flow. (The ionosphere is the rarefied and partially ionized atmosphere at orbital altitudes. It conducts because of the ionization. The ionosphere serves the same function as the stationary rails and connecting resistor in [\[link\]](#), without which there would not be a complete circuit.) Drag on the current in the cable due to the magnetic force $F = I\ell B \sin \theta$ does the work that reduces the shuttle's kinetic and potential energy and allows it to be converted to electrical energy. The tests were both unsuccessful. In the first, the cable hung up and could only be extended a couple of hundred meters; in the second, the cable broke when almost fully extended. [\[link\]](#) indicates feasibility in principle.

Example:

Calculating the Large Motional Emf of an Object in Orbit



Motional emf as electrical power conversion for the space shuttle is the motivation for the

Tethered Satellite experiment. A 5 kV emf was predicted to be induced in the 20 km long tether while moving at orbital speed in the Earth's magnetic field. The circuit is completed by a return path through the stationary ionosphere.

Calculate the motional emf induced along a 20.0 km long conductor moving at an orbital speed of 7.80 km/s perpendicular to the Earth's 5.00×10^{-5} T magnetic field.

Strategy

This is a straightforward application of the expression for motional emf— $\text{emf} = B\ell v$.

Solution

Entering the given values into $\text{emf} = B\ell v$ gives

Equation:

$$\begin{aligned}\text{emf} &= B\ell v \\ &= (5.00 \times 10^{-5} \text{ T})(2.0 \times 10^4 \text{ m})(7.80 \times 10^3 \text{ m/s}) \\ &= 7.80 \times 10^3 \text{ V}.\end{aligned}$$

Discussion

The value obtained is greater than the 5 kV measured voltage for the shuttle experiment, since the actual orbital motion of the tether is not perpendicular to the Earth's field. The 7.80 kV value is the maximum emf obtained when $\theta = 90^\circ$ and $\sin \theta = 1$.

Section Summary

- An emf induced by motion relative to a magnetic field B is called a *motional emf* and is given by

Equation:

$$\text{emf} = B\ell v \quad (B, \ell, \text{ and } v \text{ perpendicular}),$$

where ℓ is the length of the object moving at speed v relative to the field.

Conceptual Questions

Exercise:

Problem:

Why must part of the circuit be moving relative to other parts, to have usable motional emf? Consider, for example, that the rails in [\[link\]](#) are stationary relative to the magnetic field, while the rod moves.

Exercise:

Problem:

A powerful induction cannon can be made by placing a metal cylinder inside a solenoid coil. The cylinder is forcefully expelled when solenoid current is turned on rapidly. Use Faraday's and Lenz's laws to explain how this works. Why might the cylinder get live/hot when the cannon is fired?

Exercise:

Problem:

An induction stove heats a pot with a coil carrying an alternating current located beneath the pot (and without a hot surface). Can the stove surface be a conductor? Why won't a coil carrying a direct current work?

Exercise:

Problem:

Explain how you could thaw out a frozen water pipe by wrapping a coil carrying an alternating current around it. Does it matter whether or not the pipe is a conductor? Explain.

Problems & Exercises**Exercise:****Problem:**

Use Faraday's law, Lenz's law, and RHR-1 to show that the magnetic force on the current in the moving rod in [\[link\]](#) is in the opposite direction of its velocity.

Exercise:**Problem:**

If a current flows in the Satellite Tether shown in [\[link\]](#), use Faraday's law, Lenz's law, and RHR-1 to show that there is a magnetic force on the tether in the direction opposite to its velocity.

Exercise:**Problem:**

(a) A jet airplane with a 75.0 m wingspan is flying at 280 m/s. What emf is induced between wing tips if the vertical component of the Earth's field is $3.00 \times 10^{-5} \text{ T}$? (b) Is an emf of this magnitude likely to have any consequences? Explain.

Solution:

(a) 0.630 V

(b) No, this is a very small emf.

Exercise:

Problem:

(a) A nonferrous screwdriver is being used in a 2.00 T magnetic field. What maximum emf can be induced along its 12.0 cm length when it moves at 6.00 m/s? (b) Is it likely that this emf will have any consequences or even be noticed?

Exercise:**Problem:**

At what speed must the sliding rod in [\[link\]](#) move to produce an emf of 1.00 V in a 1.50 T field, given the rod's length is 30.0 cm?

Solution:

2.22 m/s

Exercise:**Problem:**

The 12.0 cm long rod in [\[link\]](#) moves at 4.00 m/s. What is the strength of the magnetic field if a 95.0 V emf is induced?

Exercise:**Problem:**

Prove that when B , ℓ , and v are not mutually perpendicular, motional emf is given by $\text{emf} = B\ell v \sin \theta$. If v is perpendicular to B , then θ is the angle between ℓ and B . If ℓ is perpendicular to B , then θ is the angle between v and B .

Exercise:

Problem:

In the August 1992 space shuttle flight, only 250 m of the conducting tether considered in [\[link\]](#) could be let out. A 40.0 V motional emf was generated in the Earth's 5.00×10^{-5} T field, while moving at 7.80×10^3 m/s. What was the angle between the shuttle's velocity and the Earth's field, assuming the conductor was perpendicular to the field?

Exercise:**Problem: Integrated Concepts**

Derive an expression for the current in a system like that in [\[link\]](#), under the following conditions. The resistance between the rails is R , the rails and the moving rod are identical in cross section A and have the same resistivity ρ . The distance between the rails is l , and the rod moves at constant speed v perpendicular to the uniform field B . At time zero, the moving rod is next to the resistance R .

Exercise:**Problem: Integrated Concepts**

The Tethered Satellite in [\[link\]](#) has a mass of 525 kg and is at the end of a 20.0 km long, 2.50 mm diameter cable with the tensile strength of steel. (a) How much does the cable stretch if a 100 N force is exerted to pull the satellite in? (Assume the satellite and shuttle are at the same altitude above the Earth.) (b) What is the effective force constant of the cable? (c) How much energy is stored in it when stretched by the 100 N force?

Exercise:**Problem: Integrated Concepts**

The Tethered Satellite discussed in this module is producing 5.00 kV, and a current of 10.0 A flows. (a) What magnetic drag force does this

produce if the system is moving at 7.80 km/s? (b) How much kinetic energy is removed from the system in 1.00 h, neglecting any change in altitude or velocity during that time? (c) What is the change in velocity if the mass of the system is 100,000 kg? (d) Discuss the long term consequences (say, a week-long mission) on the space shuttle's orbit, noting what effect a decrease in velocity has and assessing the magnitude of the effect.

Solution:

(a) 10.0 N

(b) 2.81×10^8 J

(c) 0.36 m/s

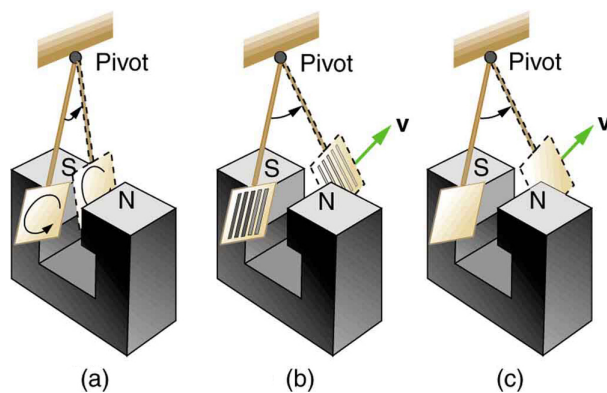
(d) For a week-long mission (168 hours), the change in velocity will be 60 m/s, or approximately 1%. In general, a decrease in velocity would cause the orbit to start spiraling inward because the velocity would no longer be sufficient to keep the circular orbit. The long-term consequences are that the shuttle would require a little more fuel to maintain the desired speed, otherwise the orbit would spiral slightly inward.

Eddy Currents and Magnetic Damping

- Explain the magnitude and direction of an induced eddy current, and the effect this will have on the object it is induced in.
- Describe several applications of magnetic damping.

Eddy Currents and Magnetic Damping

As discussed in [Motional Emf](#), motional emf is induced when a conductor moves in a magnetic field or when a magnetic field moves relative to a conductor. If motional emf can cause a current loop in the conductor, we refer to that current as an **eddy current**. Eddy currents can produce significant drag, called **magnetic damping**, on the motion involved. Consider the apparatus shown in [\[link\]](#), which swings a pendulum bob between the poles of a strong magnet. (This is another favorite physics lab activity.) If the bob is metal, there is significant drag on the bob as it enters and leaves the field, quickly damping the motion. If, however, the bob is a slotted metal plate, as shown in [\[link\]\(b\)](#), there is a much smaller effect due to the magnet. There is no discernible effect on a bob made of an insulator. Why is there drag in both directions, and are there any uses for magnetic drag?

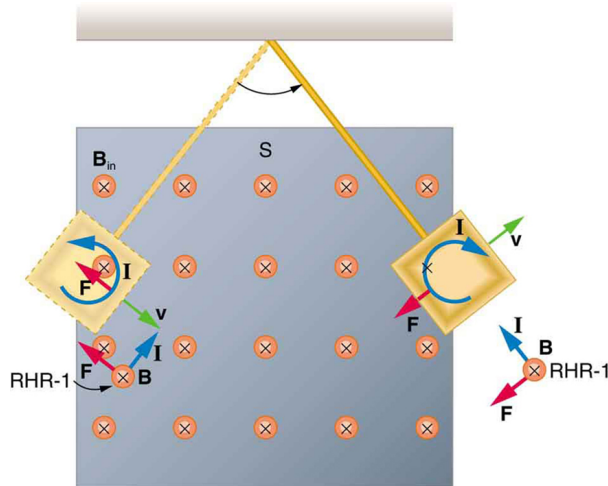


A common physics demonstration device for exploring eddy currents and magnetic damping. (a) The

motion of a metal pendulum bob swinging between the poles of a magnet is quickly damped by the action of eddy currents.

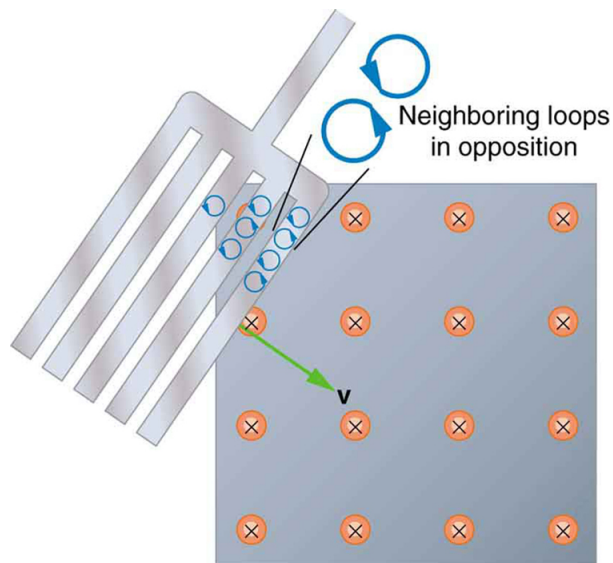
(b) There is little effect on the motion of a slotted metal bob, implying that eddy currents are made less effective. (c) There is also no magnetic damping on a nonconducting bob, since the eddy currents are extremely small.

[\[link\]](#) shows what happens to the metal plate as it enters and leaves the magnetic field. In both cases, it experiences a force opposing its motion. As it enters from the left, flux increases, and so an eddy current is set up (Faraday's law) in the counterclockwise direction (Lenz's law), as shown. Only the right-hand side of the current loop is in the field, so that there is an unopposed force on it to the left (RHR-1). When the metal plate is completely inside the field, there is no eddy current if the field is uniform, since the flux remains constant in this region. But when the plate leaves the field on the right, flux decreases, causing an eddy current in the clockwise direction that, again, experiences a force to the left, further slowing the motion. A similar analysis of what happens when the plate swings from the right toward the left shows that its motion is also damped when entering and leaving the field.



A more detailed look at the conducting plate passing between the poles of a magnet. As it enters and leaves the field, the change in flux produces an eddy current. Magnetic force on the current loop opposes the motion. There is no current and no magnetic drag when the plate is completely inside the uniform field.

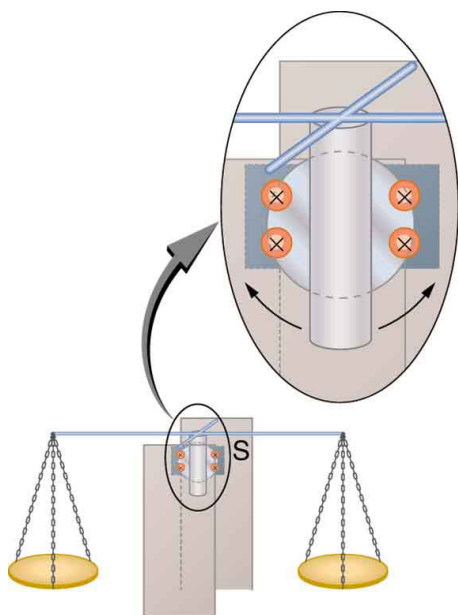
When a slotted metal plate enters the field, as shown in [\[link\]](#), an emf is induced by the change in flux, but it is less effective because the slots limit the size of the current loops. Moreover, adjacent loops have currents in opposite directions, and their effects cancel. When an insulating material is used, the eddy current is extremely small, and so magnetic damping on insulators is negligible. If eddy currents are to be avoided in conductors, then they can be slotted or constructed of thin layers of conducting material separated by insulating sheets.



Eddy currents induced in a slotted metal plate entering a magnetic field form small loops, and the forces on them tend to cancel, thereby making magnetic drag almost zero.

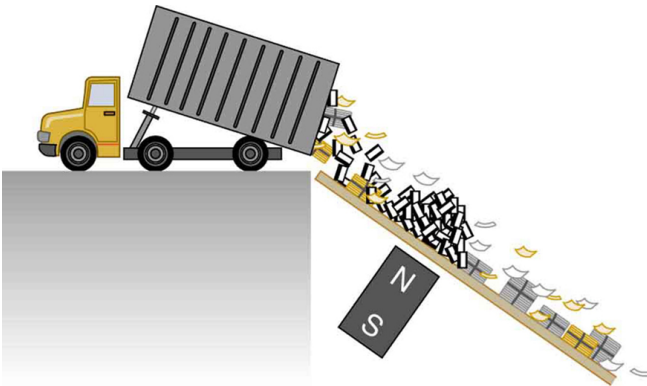
Applications of Magnetic Damping

One use of magnetic damping is found in sensitive laboratory balances. To have maximum sensitivity and accuracy, the balance must be as friction-free as possible. But if it is friction-free, then it will oscillate for a very long time. Magnetic damping is a simple and ideal solution. With magnetic damping, drag is proportional to speed and becomes zero at zero velocity. Thus the oscillations are quickly damped, after which the damping force disappears, allowing the balance to be very sensitive. (See [\[link\]](#).) In most balances, magnetic damping is accomplished with a conducting disc that rotates in a fixed field.



Magnetic damping of this sensitive balance slows its oscillations. Since Faraday's law of induction gives the greatest effect for the most rapid change, damping is greatest for large oscillations and goes to zero as the motion stops.

Since eddy currents and magnetic damping occur only in conductors, recycling centers can use magnets to separate metals from other materials. Trash is dumped in batches down a ramp, beneath which lies a powerful magnet. Conductors in the trash are slowed by magnetic damping while nonmetals in the trash move on, separating from the metals. (See [\[link\]](#).) This works for all metals, not just ferromagnetic ones. A magnet can separate out the ferromagnetic materials alone by acting on stationary trash.



Metals can be separated from other trash by magnetic drag. Eddy currents and magnetic drag are created in the metals sent down this ramp by the powerful magnet beneath it. Nonmetals move on.

Other major applications of eddy currents are in metal detectors and braking systems in trains and roller coasters. Portable metal detectors ([\[link\]](#)) consist of a primary coil carrying an alternating current and a secondary coil in which a current is induced. An eddy current will be induced in a piece of metal close to the detector which will cause a change in the induced current within the secondary coil, leading to some sort of signal like a shrill noise. Braking using eddy currents is safer because factors such as rain do not affect the braking and the braking is smoother. However, eddy currents cannot bring the motion to a complete stop, since the force produced decreases with speed. Thus, speed can be reduced from say 20 m/s to 5 m/s, but another form of braking is needed to completely stop the vehicle. Generally, powerful rare earth magnets such as neodymium magnets are used in roller coasters. [\[link\]](#) shows rows of magnets in such an application. The vehicle has metal fins (normally containing copper) which pass through the magnetic field slowing the vehicle down in much the same way as with the pendulum bob shown in [\[link\]](#).



A soldier in Iraq uses a metal detector to search for explosives and weapons. (credit: U.S. Army)



The rows of rare earth magnets (protruding horizontally) are used for magnetic braking in roller coasters. (credit: Stefan Scheer, Wikimedia Commons)

Induction cooktops have electromagnets under their surface. The magnetic field is varied rapidly producing eddy currents in the base of the pot, causing the pot and its contents to increase in temperature. Induction cooktops have high efficiencies and good response times but the base of the pot needs to be ferromagnetic, iron or steel for induction to work.

Section Summary

- Current loops induced in moving conductors are called eddy currents.
- They can create significant drag, called magnetic damping.

Conceptual Questions

Exercise:

Problem:

Explain why magnetic damping might not be effective on an object made of several thin conducting layers separated by insulation.

Exercise:

Problem:

Explain how electromagnetic induction can be used to detect metals? This technique is particularly important in detecting buried landmines for disposal, geophysical prospecting and at airports.

Problems & Exercises

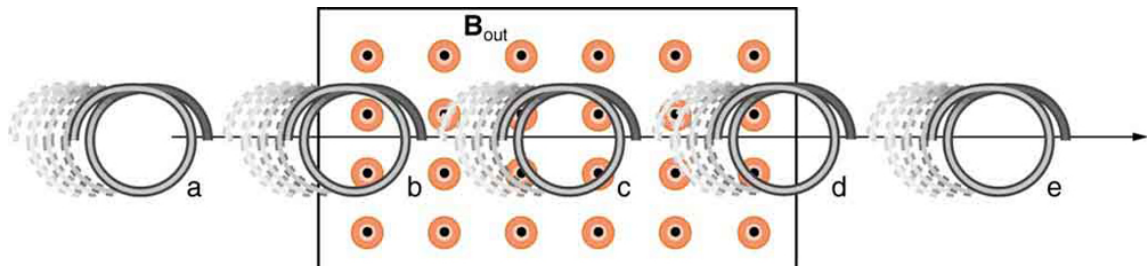
Exercise:

Problem:

Make a drawing similar to [\[link\]](#), but with the pendulum moving in the opposite direction. Then use Faraday's law, Lenz's law, and RHR-1 to show that magnetic force opposes motion.

Exercise:

Problem:



A coil is moved into and out of a region of uniform magnetic field.

A coil is moved through a magnetic field as shown in [\[link\]](#). The field is uniform inside the rectangle and zero outside. What is the direction of the induced current and what is the direction of the magnetic force on the coil at each position shown?

Glossary

eddy current

a current loop in a conductor caused by motional emf

magnetic damping

the drag produced by eddy currents

Electric Generators

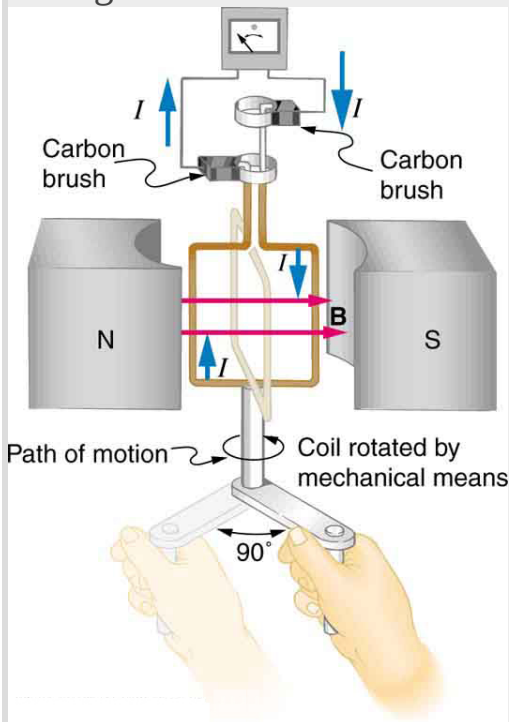
- Calculate the emf induced in a generator.
- Calculate the peak emf which can be induced in a particular generator system.

Electric generators induce an emf by rotating a coil in a magnetic field, as briefly discussed in [Induced Emf and Magnetic Flux](#). We will now explore generators in more detail. Consider the following example.

Example:

Calculating the Emf Induced in a Generator Coil

The generator coil shown in [\[link\]](#) is rotated through one-fourth of a revolution (from $\theta = 0^\circ$ to $\theta = 90^\circ$) in 15.0 ms. The 200-turn circular coil has a 5.00 cm radius and is in a uniform 1.25 T magnetic field. What is the average emf induced?



When this generator coil is rotated through one-fourth of a revolution, the

magnetic flux Φ changes
from its maximum to
zero, inducing an emf.

Strategy

We use Faraday's law of induction to find the average emf induced over a time Δt :

Equation:

$$\text{emf} = -N \frac{\Delta \Phi}{\Delta t}.$$

We know that $N = 200$ and $\Delta t = 15.0$ ms, and so we must determine the change in flux $\Delta \Phi$ to find emf.

Solution

Since the area of the loop and the magnetic field strength are constant, we see that

Equation:

$$\Delta \Phi = \Delta(BA \cos \theta) = AB \Delta(\cos \theta).$$

Now, $\Delta(\cos \theta) = -1.0$, since it was given that θ goes from 0° to 90° .

Thus $\Delta \Phi = -AB$, and

Equation:

$$\text{emf} = N \frac{AB}{\Delta t}.$$

The area of the loop is

$A = \pi r^2 = (3.14...)(0.0500 \text{ m})^2 = 7.85 \times 10^{-3} \text{ m}^2$. Entering this value gives

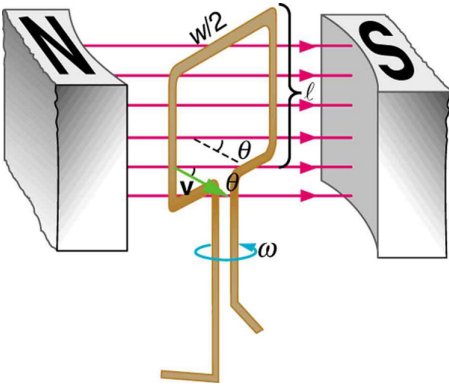
Equation:

$$\text{emf} = 200 \frac{(7.85 \times 10^{-3} \text{ m}^2)(1.25 \text{ T})}{15.0 \times 10^{-3} \text{ s}} = 131 \text{ V}.$$

Discussion

This is a practical average value, similar to the 120 V used in household power.

The emf calculated in [\[link\]](#) is the average over one-fourth of a revolution. What is the emf at any given instant? It varies with the angle between the magnetic field and a perpendicular to the coil. We can get an expression for emf as a function of time by considering the motional emf on a rotating rectangular coil of width w and height ℓ in a uniform magnetic field, as illustrated in [\[link\]](#).



A generator with a single rectangular coil rotated at constant angular velocity in a uniform magnetic field produces an emf that varies sinusoidally in time. Note the generator is similar to a motor, except the shaft is rotated to produce a current rather than the other way around.

Charges in the wires of the loop experience the magnetic force, because they are moving in a magnetic field. Charges in the vertical wires experience forces parallel to the wire, causing currents. But those in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. We can thus find the induced emf by considering only the side wires. Motional emf is given to be $\text{emf} = B\ell v$, where the velocity v is perpendicular to the magnetic field B . Here the velocity is at an angle θ with B , so that its component perpendicular to B is $v \sin \theta$ (see [\[link\]](#)). Thus in this case the emf induced on each side is $\text{emf} = B\ell v \sin \theta$, and they are in the same direction. The total emf around the loop is then

Equation:

$$\text{emf} = 2B\ell v \sin \theta.$$

This expression is valid, but it does not give emf as a function of time. To find the time dependence of emf, we assume the coil rotates at a constant angular velocity ω . The angle θ is related to angular velocity by $\theta = \omega t$, so that

Equation:

$$\text{emf} = 2B\ell v \sin \omega t.$$

Now, linear velocity v is related to angular velocity ω by $v = r\omega$. Here $r = w/2$, so that $v = (w/2)\omega$, and

Equation:

$$\text{emf} = 2B\ell \frac{w}{2} \omega \sin \omega t = (\ell w) B \omega \sin \omega t.$$

Noting that the area of the loop is $A = \ell w$, and allowing for N loops, we find that

Equation:

$$\text{emf} = NAB\omega \sin \omega t$$

is the **emf induced in a generator coil** of N turns and area A rotating at a constant angular velocity ω in a uniform magnetic field B . This can also be expressed as

Equation:

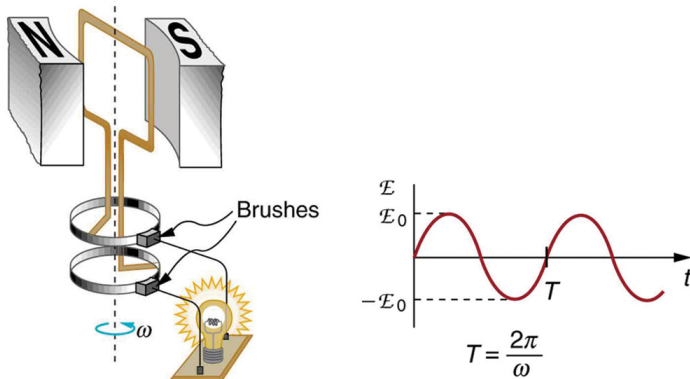
$$\text{emf} = \text{emf}_0 \sin \omega t,$$

where

Equation:

$$\text{emf}_0 = NAB\omega$$

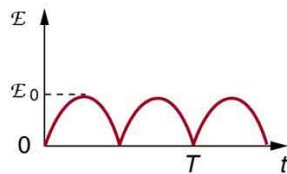
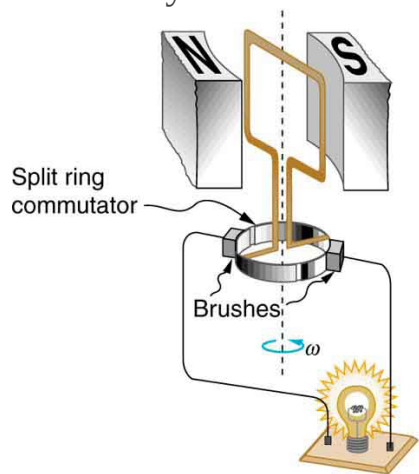
is the maximum **(peak) emf**. Note that the frequency of the oscillation is $f = \omega/2\pi$, and the period is $T = 1/f = 2\pi/\omega$. [\[link\]](#) shows a graph of emf as a function of time, and it now seems reasonable that AC voltage is sinusoidal.



The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time. emf_0 is the peak emf. The period is $T = 1/f = 2\pi/\omega$, where f is the frequency. Note that the script $\mathcal{E}$ stands for emf.

The fact that the peak emf, $\text{emf}_0 = NAB\omega$, makes good sense. The greater the number of coils, the larger their area, and the stronger the field, the greater the output voltage. It is interesting that the faster the generator is spun (greater ω), the greater the emf. This is noticeable on bicycle generators—at least the cheaper varieties. One of the authors as a juvenile found it amusing to ride his bicycle fast enough to burn out his lights, until he had to ride home lightless one dark night.

[\[link\]](#) shows a scheme by which a generator can be made to produce pulsed DC. More elaborate arrangements of multiple coils and split rings can produce smoother DC, although electronic rather than mechanical means are usually used to make ripple-free DC.



Split rings, called commutators, produce a pulsed DC emf output in this configuration.

Example:**Calculating the Maximum Emf of a Generator**

Calculate the maximum emf, emf_0 , of the generator that was the subject of [\[link\]](#).

Strategy

Once ω , the angular velocity, is determined, $\text{emf}_0 = NAB\omega$ can be used to find emf_0 . All other quantities are known.

Solution

Angular velocity is defined to be the change in angle per unit time:

Equation:

$$\omega = \frac{\Delta\theta}{\Delta t}.$$

One-fourth of a revolution is $\pi/2$ radians, and the time is 0.0150 s; thus,

Equation:

$$\begin{aligned}\omega &= \frac{\pi/2 \text{ rad}}{0.0150 \text{ s}} \\ &= 104.7 \text{ rad/s}.\end{aligned}$$

104.7 rad/s is exactly 1000 rpm. We substitute this value for ω and the information from the previous example into $\text{emf}_0 = NAB\omega$, yielding

Equation:

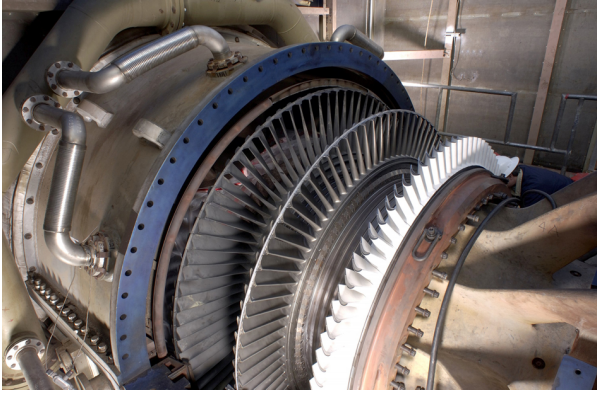
$$\begin{aligned}\text{emf}_0 &= NAB\omega \\ &= 200(7.85 \times 10^{-3} \text{ m}^2)(1.25 \text{ T})(104.7 \text{ rad/s}). \\ &= 206 \text{ V}\end{aligned}$$

Discussion

The maximum emf is greater than the average emf of 131 V found in the previous example, as it should be.

In real life, electric generators look a lot different than the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water (hydropower), steam produced by the

burning of fossil fuels, or the kinetic energy of wind. [\[link\]](#) shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator.



Steam turbine/generator. The steam produced by burning coal impacts the turbine blades, turning the shaft which is connected to the generator.
(credit: Nabonaco, Wikimedia Commons)

Generators illustrated in this section look very much like the motors illustrated previously. This is not coincidental. In fact, a motor becomes a generator when its shaft rotates. Certain early automobiles used their starter motor as a generator. In [Back Emf](#), we shall further explore the action of a motor as a generator.

Section Summary

- An electric generator rotates a coil in a magnetic field, inducing an emf given as a function of time by

Equation:

$$\text{emf} = NAB\omega \sin \omega t,$$

where A is the area of an N -turn coil rotated at a constant angular velocity ω in a uniform magnetic field B .

- The peak emf emf_0 of a generator is

Equation:

$$\text{emf}_0 = NAB\omega.$$

Conceptual Questions

Exercise:

Problem:

Using RHR-1, show that the emfs in the sides of the generator loop in [\[link\]](#) are in the same sense and thus add.

Exercise:

Problem:

The source of a generator's electrical energy output is the work done to turn its coils. How is the work needed to turn the generator related to Lenz's law?

Problems & Exercises

Exercise:

Problem:

Calculate the peak voltage of a generator that rotates its 200-turn, 0.100 m diameter coil at 3600 rpm in a 0.800 T field.

Solution:

474 V

Exercise:

Problem:

At what angular velocity in rpm will the peak voltage of a generator be 480 V, if its 500-turn, 8.00 cm diameter coil rotates in a 0.250 T field?

Exercise:**Problem:**

What is the peak emf generated by rotating a 1000-turn, 20.0 cm diameter coil in the Earth's 5.00×10^{-5} T magnetic field, given the plane of the coil is originally perpendicular to the Earth's field and is rotated to be parallel to the field in 10.0 ms?

Solution:

0.247 V

Exercise:**Problem:**

What is the peak emf generated by a 0.250 m radius, 500-turn coil is rotated one-fourth of a revolution in 4.17 ms, originally having its plane perpendicular to a uniform magnetic field. (This is 60 rev/s.)

Exercise:**Problem:**

(a) A bicycle generator rotates at 1875 rad/s, producing an 18.0 V peak emf. It has a 1.00 by 3.00 cm rectangular coil in a 0.640 T field. How many turns are in the coil? (b) Is this number of turns of wire practical for a 1.00 by 3.00 cm coil?

Solution:

(a) 50

(b) yes

Exercise:**Problem: Integrated Concepts**

This problem refers to the bicycle generator considered in the previous problem. It is driven by a 1.60 cm diameter wheel that rolls on the outside rim of the bicycle tire. (a) What is the velocity of the bicycle if the generator's angular velocity is 1875 rad/s? (b) What is the maximum emf of the generator when the bicycle moves at 10.0 m/s, noting that it was 18.0 V under the original conditions? (c) If the sophisticated generator can vary its own magnetic field, what field strength will it need at 5.00 m/s to produce a 9.00 V maximum emf?

Exercise:**Problem:**

(a) A car generator turns at 400 rpm when the engine is idling. Its 300-turn, 5.00 by 8.00 cm rectangular coil rotates in an adjustable magnetic field so that it can produce sufficient voltage even at low rpms. What is the field strength needed to produce a 24.0 V peak emf? (b) Discuss how this required field strength compares to those available in permanent and electromagnets.

Solution:

(a) 0.477 T

(b) This field strength is small enough that it can be obtained using either a permanent magnet or an electromagnet.

Exercise:**Problem:**

Show that if a coil rotates at an angular velocity ω , the period of its AC output is $2\pi/\omega$.

Exercise:

Problem:

A 75-turn, 10.0 cm diameter coil rotates at an angular velocity of 8.00 rad/s in a 1.25 T field, starting with the plane of the coil parallel to the field. (a) What is the peak emf? (b) At what time is the peak emf first reached? (c) At what time is the emf first at its most negative? (d) What is the period of the AC voltage output?

Solution:

- (a) 5.89 V
- (b) At $t=0$
- (c) 0.393 s
- (d) 0.785 s

Exercise:**Problem:**

(a) If the emf of a coil rotating in a magnetic field is zero at $t = 0$, and increases to its first peak at $t = 0.100$ ms, what is the angular velocity of the coil? (b) At what time will its next maximum occur? (c) What is the period of the output? (d) When is the output first one-fourth of its maximum? (e) When is it next one-fourth of its maximum?

Exercise:**Problem: Unreasonable Results**

A 500-turn coil with a 0.250 m^2 area is spun in the Earth's $5.00 \times 10^{-5} \text{ T}$ field, producing a 12.0 kV maximum emf. (a) At what angular velocity must the coil be spun? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) $1.92 \times 10^6 \text{ rad/s}$

(b) This angular velocity is unreasonably high, higher than can be obtained for any mechanical system.

(c) The assumption that a voltage as great as 12.0 kV could be obtained is unreasonable.

Glossary

electric generator

a device for converting mechanical work into electric energy; it induces an emf by rotating a coil in a magnetic field

emf induced in a generator coil

$\text{emf} = NAB\omega \sin \omega t$, where A is the area of an N -turn coil rotated at a constant angular velocity ω in a uniform magnetic field B , over a period of time t

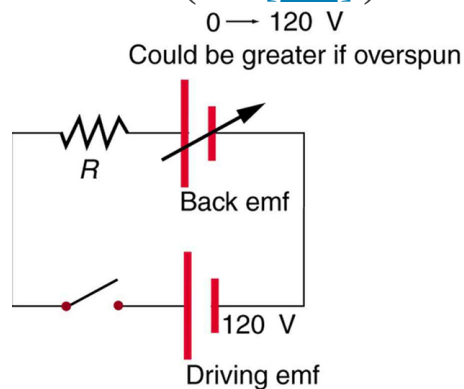
peak emf

$$\text{emf}_0 = NAB\omega$$

Back Emf

- Explain what back emf is and how it is induced.

It has been noted that motors and generators are very similar. Generators convert mechanical energy into electrical energy, whereas motors convert electrical energy into mechanical energy. Furthermore, motors and generators have the same construction. When the coil of a motor is turned, magnetic flux changes, and an emf (consistent with Faraday's law of induction) is induced. The motor thus acts as a generator whenever its coil rotates. This will happen whether the shaft is turned by an external input, like a belt drive, or by the action of the motor itself. That is, when a motor is doing work and its shaft is turning, an emf is generated. Lenz's law tells us the emf opposes any change, so that the input emf that powers the motor will be opposed by the motor's self-generated emf, called the **back emf** of the motor. (See [\[link\]](#).)



The coil of a DC motor is represented as a resistor in this schematic. The back emf is represented as a variable emf that opposes the one driving the motor. Back emf is zero when the motor is not turning, and it increases

proportionally to the
motor's angular
velocity.

Back emf is the generator output of a motor, and so it is proportional to the motor's angular velocity ω . It is zero when the motor is first turned on, meaning that the coil receives the full driving voltage and the motor draws maximum current when it is on but not turning. As the motor turns faster and faster, the back emf grows, always opposing the driving emf, and reduces the voltage across the coil and the amount of current it draws. This effect is noticeable in a number of situations. When a vacuum cleaner, refrigerator, or washing machine is first turned on, lights in the same circuit dim briefly due to the IR drop produced in feeder lines by the large current drawn by the motor. When a motor first comes on, it draws more current than when it runs at its normal operating speed. When a mechanical load is placed on the motor, like an electric wheelchair going up a hill, the motor slows, the back emf drops, more current flows, and more work can be done. If the motor runs at too low a speed, the larger current can overheat it (via resistive power in the coil, $P = I^2 R$), perhaps even burning it out. On the other hand, if there is no mechanical load on the motor, it will increase its angular velocity ω until the back emf is nearly equal to the driving emf. Then the motor uses only enough energy to overcome friction.

Consider, for example, the motor coils represented in [\[link\]](#). The coils have a $0.400\ \Omega$ equivalent resistance and are driven by a $48.0\ \text{V}$ emf. Shortly after being turned on, they draw a current $I = V/R = (48.0\ \text{V}) / (0.400\ \Omega) = 120\ \text{A}$ and, thus, dissipate $P = I^2 R = 5.76\ \text{kW}$ of energy as heat transfer. Under normal operating conditions for this motor, suppose the back emf is $40.0\ \text{V}$. Then at operating speed, the total voltage across the coils is $8.0\ \text{V}$ ($48.0\ \text{V}$ minus the $40.0\ \text{V}$ back emf), and the current drawn is $I = V/R = (8.0\ \text{V}) / (0.400\ \Omega) = 20\ \text{A}$. Under normal load, then, the power dissipated is $P = IV = (20\ \text{A})(8.0\ \text{V}) = 160\ \text{W}$. The latter will not cause a problem for this motor, whereas the former $5.76\ \text{kW}$ would burn out the coils if sustained.

Section Summary

- Any rotating coil will have an induced emf—in motors, this is called back emf, since it opposes the emf input to the motor.

Conceptual Questions

Exercise:

Problem:

Suppose you find that the belt drive connecting a powerful motor to an air conditioning unit is broken and the motor is running freely. Should you be worried that the motor is consuming a great deal of energy for no useful purpose? Explain why or why not.

Problems & Exercises

Exercise:

Problem:

Suppose a motor connected to a 120 V source draws 10.0 A when it first starts. (a) What is its resistance? (b) What current does it draw at its normal operating speed when it develops a 100 V back emf?

Solution:

(a) $12.00\ \Omega$

(b) 1.67 A

Exercise:

Problem:

A motor operating on 240 V electricity has a 180 V back emf at operating speed and draws a 12.0 A current. (a) What is its resistance? (b) What current does it draw when it is first started?

Exercise:**Problem:**

What is the back emf of a 120 V motor that draws 8.00 A at its normal speed and 20.0 A when first starting?

Solution:

72.0 V

Exercise:**Problem:**

The motor in a toy car operates on 6.00 V, developing a 4.50 V back emf at normal speed. If it draws 3.00 A at normal speed, what current does it draw when starting?

Exercise:**Problem: Integrated Concepts**

The motor in a toy car is powered by four batteries in series, which produce a total emf of 6.00 V. The motor draws 3.00 A and develops a 4.50 V back emf at normal speed. Each battery has a $0.100\ \Omega$ internal resistance. What is the resistance of the motor?

Solution:

$0.100\ \Omega$

Glossary**back emf**

the emf generated by a running motor, because it consists of a coil turning in a magnetic field; it opposes the voltage powering the motor

Transformers

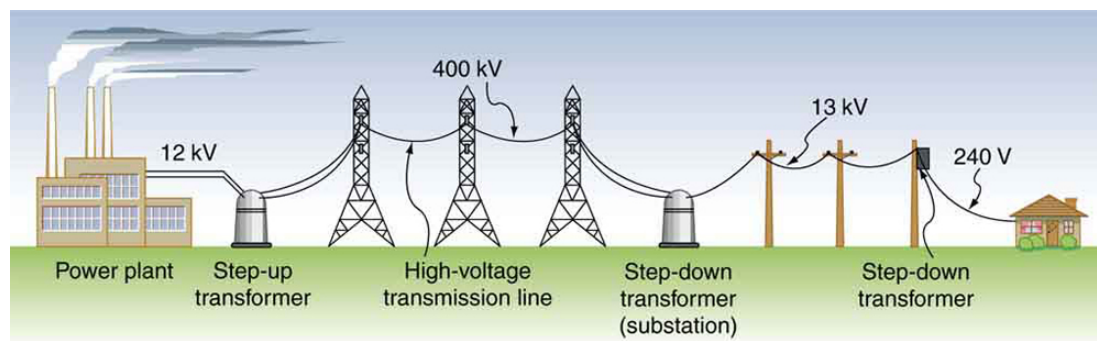
- Explain how a transformer works.
- Calculate voltage, current, and/or number of turns given the other quantities.

Transformers do what their name implies—they transform voltages from one value to another (The term voltage is used rather than emf, because transformers have internal resistance). For example, many cell phones, laptops, video games, and power tools and small appliances have a transformer built into their plug-in unit (like that in [\[link\]](#)) that changes 120 V or 240 V AC into whatever voltage the device uses. Transformers are also used at several points in the power distribution systems, such as illustrated in [\[link\]](#). Power is sent long distances at high voltages, because less current is required for a given amount of power, and this means less line loss, as was discussed previously. But high voltages pose greater hazards, so that transformers are employed to produce lower voltage at the user's location.



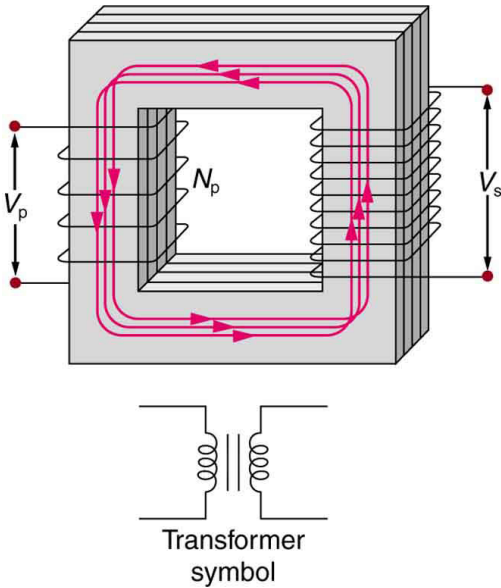
The plug-in transformer has become increasingly familiar with the proliferation of electronic devices that operate on voltages other than common 120 V

AC. Most are in the
3 to 12 V range.
(credit: Shop
Xtreme)



Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages over 200 kV—sometimes as great as 700 kV—to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

The type of transformer considered in this text—see [\[link\]](#)—is based on Faraday’s law of induction and is very similar in construction to the apparatus Faraday used to demonstrate magnetic fields could cause currents. The two coils are called the *primary* and *secondary coils*. In normal use, the input voltage is placed on the primary, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, its magnetization increases the field strength. Since the input voltage is AC, a time-varying magnetic flux is sent to the secondary, inducing its AC output voltage.



A typical construction of a simple transformer has two coils wound on a ferromagnetic core that is laminated to minimize eddy currents. The magnetic field created by the primary is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary induces a current in the secondary.

For the simple transformer shown in [\[link\]](#), the output voltage V_s depends almost entirely on the input voltage V_p and the ratio of the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_s to be

Equation:

$$V_s = -N_s \frac{\Delta\Phi}{\Delta t},$$

where N_s is the number of loops in the secondary coil and $\Delta\Phi/\Delta t$ is the rate of change of magnetic flux. Note that the output voltage equals the induced emf ($V_s = \text{emf}_s$), provided coil resistance is small (a reasonable assumption for transformers). The cross-sectional area of the coils is the same on either side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on either side. The input primary voltage V_p is also related to changing flux by

Equation:

$$V_p = -N_p \frac{\Delta\Phi}{\Delta t}.$$

The reason for this is a little more subtle. Lenz's law tells us that the primary coil opposes the change in flux caused by the input voltage V_p , hence the minus sign (This is an example of *self-inductance*, a topic to be explored in some detail in later sections). Assuming negligible coil resistance, Kirchhoff's loop rule tells us that the induced emf exactly equals the input voltage. Taking the ratio of these last two equations yields a useful relationship:

Equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p}.$$

This is known as the **transformer equation**, and it simply states that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils.

The output voltage of a transformer can be less than, greater than, or equal to the input voltage, depending on the ratio of the number of loops in their coils. Some transformers even provide a variable output by allowing connection to be made at different points on the secondary coil. A **step-up**

transformer is one that increases voltage, whereas a **step-down transformer** decreases voltage. Assuming, as we have, that resistance is negligible, the electrical power output of a transformer equals its input. This is nearly true in practice—transformer efficiency often exceeds 99%.

Equating the power input and output,

Equation:

$$P_p = I_p V_p = I_s V_s = P_s.$$

Rearranging terms gives

Equation:

$$\frac{V_s}{V_p} = \frac{I_p}{I_s}.$$

Combining this with $\frac{V_s}{V_p} = \frac{N_s}{N_p}$, we find that

Equation:

$$\frac{I_s}{I_p} = \frac{N_p}{N_s}$$

is the relationship between the output and input currents of a transformer. So if voltage increases, current decreases. Conversely, if voltage decreases, current increases.

Example:

Calculating Characteristics of a Step-Up Transformer

A portable x-ray unit has a step-up transformer, the 120 V input of which is transformed to the 100 kV output needed by the x-ray tube. The primary has 50 loops and draws a current of 10.00 A when in use. (a) What is the number of loops in the secondary? (b) Find the current output of the secondary.

Strategy and Solution for (a)

We solve $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s , the number of loops in the secondary, and enter the known values. This gives

Equation:

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (50) \frac{100,000 \text{ V}}{120 \text{ V}} = 4.17 \times 10^4. \end{aligned}$$

Discussion for (a)

A large number of loops in the secondary (compared with the primary) is required to produce such a large voltage. This would be true for neon sign transformers and those supplying high voltage inside TVs and CRTs.

Strategy and Solution for (b)

We can similarly find the output current of the secondary by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_s and entering known values. This gives

Equation:

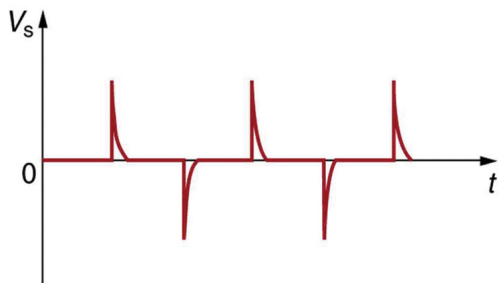
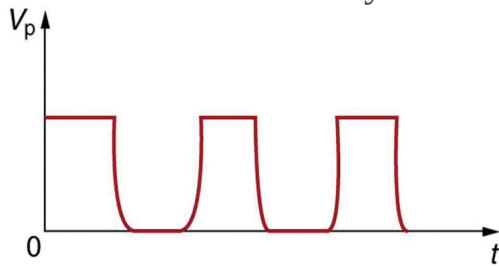
$$\begin{aligned} I_s &= I_p \frac{N_p}{N_s} \\ &= (10.00 \text{ A}) \frac{50}{4.17 \times 10^4} = 12.0 \text{ mA}. \end{aligned}$$

Discussion for (b)

As expected, the current output is significantly less than the input. In certain spectacular demonstrations, very large voltages are used to produce long arcs, but they are relatively safe because the transformer output does not supply a large current. Note that the power input here is $P_p = I_p V_p = (10.00 \text{ A})(120 \text{ V}) = 1.20 \text{ kW}$. This equals the power output $P_p = I_s V_s = (12.0 \text{ mA})(100 \text{ kV}) = 1.20 \text{ kW}$, as we assumed in the derivation of the equations used.

The fact that transformers are based on Faraday's law of induction makes it clear why we cannot use transformers to change DC voltages. If there is no change in primary voltage, there is no voltage induced in the secondary. One possibility is to connect DC to the primary coil through a switch. As the switch is opened and closed, the secondary produces a voltage like that

in [\[link\]](#). This is not really a practical alternative, and AC is in common use wherever it is necessary to increase or decrease voltages.



Transformers do not work for pure DC voltage input, but if it is switched on and off as on the top graph, the output will look something like that on the bottom graph. This is not the sinusoidal AC most AC appliances need.

Example:

Calculating Characteristics of a Step-Down Transformer

A battery charger meant for a series connection of ten nickel-cadmium batteries (total emf of 12.5 V DC) needs to have a 15.0 V output to charge the batteries. It uses a step-down transformer with a 200-loop primary and a 120 V input. (a) How many loops should there be in the secondary coil? (b) If the charging current is 16.0 A, what is the input current?

Strategy and Solution for (a)

You would expect the secondary to have a small number of loops. Solving $\frac{V_s}{V_p} = \frac{N_s}{N_p}$ for N_s and entering known values gives

Equation:

$$\begin{aligned} N_s &= N_p \frac{V_s}{V_p} \\ &= (200) \frac{15.0 \text{ V}}{120 \text{ V}} = 25. \end{aligned}$$

Strategy and Solution for (b)

The current input can be obtained by solving $\frac{I_s}{I_p} = \frac{N_p}{N_s}$ for I_p and entering known values. This gives

Equation:

$$\begin{aligned} I_p &= I_s \frac{N_s}{N_p} \\ &= (16.0 \text{ A}) \frac{25}{200} = 2.00 \text{ A}. \end{aligned}$$

Discussion

The number of loops in the secondary is small, as expected for a step-down transformer. We also see that a small input current produces a larger output current in a step-down transformer. When transformers are used to operate large magnets, they sometimes have a small number of very heavy loops in the secondary. This allows the secondary to have low internal resistance and produce large currents. Note again that this solution is based on the assumption of 100% efficiency—or power out equals power in ($P_p = P_s$)—reasonable for good transformers. In this case the primary and secondary power is 240 W. (Verify this for yourself as a consistency check.) Note that the Ni-Cd batteries need to be charged from a DC power source (as would a 12 V battery). So the AC output of the secondary coil needs to be converted into DC. This is done using something called a rectifier, which uses devices called diodes that allow only a one-way flow of current.

Transformers have many applications in electrical safety systems, which are discussed in [Electrical Safety: Systems and Devices](#).

Note:**PhET Explorations: Generator**

Generate electricity with a bar magnet! Discover the physics behind the phenomena by exploring magnets and how you can use them to make a bulb light.

<https://archive.cnx.org/specials/1e9b7292-ae74-11e5-a9dc-c7c8521ba8e6/generator/#sim-generator>

Section Summary

- Transformers use induction to transform voltages from one value to another.
- For a transformer, the voltages across the primary and secondary coils are related by

Equation:

$$\frac{V_s}{V_p} = \frac{N_s}{N_p},$$

where V_p and V_s are the voltages across primary and secondary coils having N_p and N_s turns.

- The currents I_p and I_s in the primary and secondary coils are related by $\frac{I_s}{I_p} = \frac{N_p}{N_s}$.
- A step-up transformer increases voltage and decreases current, whereas a step-down transformer decreases voltage and increases current.

Conceptual Questions

Exercise:**Problem:**

Explain what causes physical vibrations in transformers at twice the frequency of the AC power involved.

Problems & Exercises

Exercise:

Problem:

A plug-in transformer, like that in [\[link\]](#), supplies 9.00 V to a video game system. (a) How many turns are in its secondary coil, if its input voltage is 120 V and the primary coil has 400 turns? (b) What is its input current when its output is 1.30 A?

Solution:

(a) 30.0

(b) 9.75×10^{-2} A

Exercise:

Problem:

An American traveler in New Zealand carries a transformer to convert New Zealand's standard 240 V to 120 V so that she can use some small appliances on her trip. (a) What is the ratio of turns in the primary and secondary coils of her transformer? (b) What is the ratio of input to output current? (c) How could a New Zealander traveling in the United States use this same transformer to power her 240 V appliances from 120 V?

Exercise:

Problem:

A cassette recorder uses a plug-in transformer to convert 120 V to 12.0 V, with a maximum current output of 200 mA. (a) What is the current input? (b) What is the power input? (c) Is this amount of power reasonable for a small appliance?

Solution:

(a) 20.0 mA

(b) 2.40 W

(c) Yes, this amount of power is quite reasonable for a small appliance.

Exercise:

Problem:

(a) What is the voltage output of a transformer used for rechargeable flashlight batteries, if its primary has 500 turns, its secondary 4 turns, and the input voltage is 120 V? (b) What input current is required to produce a 4.00 A output? (c) What is the power input?

Exercise:

Problem:

(a) The plug-in transformer for a laptop computer puts out 7.50 V and can supply a maximum current of 2.00 A. What is the maximum input current if the input voltage is 240 V? Assume 100% efficiency. (b) If the actual efficiency is less than 100%, would the input current need to be greater or smaller? Explain.

Solution:

(a) 0.063 A

(b) Greater input current needed.

Exercise:

Problem:

A multipurpose transformer has a secondary coil with several points at which a voltage can be extracted, giving outputs of 5.60, 12.0, and 480 V. (a) The input voltage is 240 V to a primary coil of 280 turns. What are the numbers of turns in the parts of the secondary used to produce the output voltages? (b) If the maximum input current is 5.00 A, what are the maximum output currents (each used alone)?

Exercise:**Problem:**

A large power plant generates electricity at 12.0 kV. Its old transformer once converted the voltage to 335 kV. The secondary of this transformer is being replaced so that its output can be 750 kV for more efficient cross-country transmission on upgraded transmission lines.

(a) What is the ratio of turns in the new secondary compared with the old secondary? (b) What is the ratio of new current output to old output (at 335 kV) for the same power? (c) If the upgraded transmission lines have the same resistance, what is the ratio of new line power loss to old?

Solution:

(a) 2.2

(b) 0.45

(c) 0.20, or 20.0%

Exercise:**Problem:**

If the power output in the previous problem is 1000 MW and line resistance is $2.00\ \Omega$, what were the old and new line losses?

Exercise:**Problem: Unreasonable Results**

The 335 kV AC electricity from a power transmission line is fed into the primary coil of a transformer. The ratio of the number of turns in the secondary to the number in the primary is $N_s/N_p = 1000$. (a) What voltage is induced in the secondary? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Solution:

(a) 335 MV

(b) way too high, well beyond the breakdown voltage of air over reasonable distances

(c) input voltage is too high

Exercise:

Problem: Construct Your Own Problem

Consider a double transformer to be used to create very large voltages. The device consists of two stages. The first is a transformer that produces a much larger output voltage than its input. The output of the first transformer is used as input to a second transformer that further increases the voltage. Construct a problem in which you calculate the output voltage of the final stage based on the input voltage of the first stage and the number of turns or loops in both parts of both transformers (four coils in all). Also calculate the maximum output current of the final stage based on the input current. Discuss the possibility of power losses in the devices and the effect on the output current and power.

Glossary

transformer

a device that transforms voltages from one value to another using induction

transformer equation

the equation showing that the ratio of the secondary to primary voltages in a transformer equals the ratio of the number of loops in their coils; $\frac{V_s}{V_p} = \frac{N_s}{N_p}$

step-up transformer

a transformer that increases voltage

step-down transformer
a transformer that decreases voltage

Electrical Safety: Systems and Devices

- Explain how various modern safety features in electric circuits work, with an emphasis on how induction is employed.

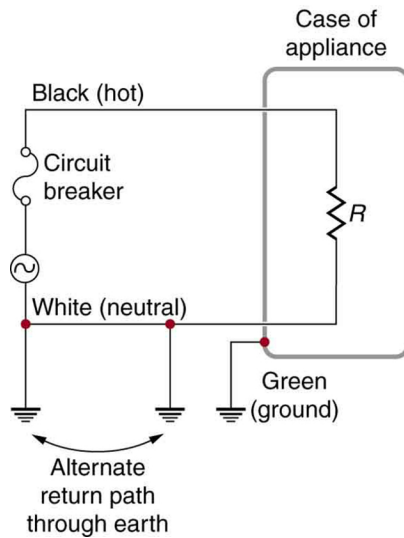
Electricity has two hazards. A **thermal hazard** occurs when there is electrical overheating. A **shock hazard** occurs when electric current passes through a person. Both hazards have already been discussed. Here we will concentrate on systems and devices that prevent electrical hazards.

[\[link\]](#) shows the schematic for a simple AC circuit with no safety features. This is not how power is distributed in practice. Modern household and industrial wiring requires the **three-wire system**, shown schematically in [\[link\]](#), which has several safety features. First is the familiar *circuit breaker* (or *fuse*) to prevent thermal overload. Second, there is a protective *case* around the appliance, such as a toaster or refrigerator. The case's safety feature is that it prevents a person from touching exposed wires and coming into electrical contact with the circuit, helping prevent shocks.



Schematic of a simple AC circuit with a voltage source and a single appliance represented by the resistance R .

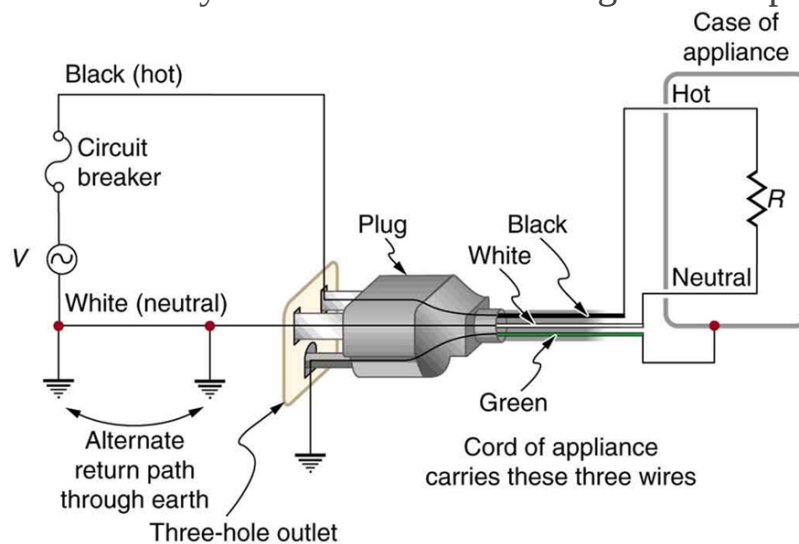
There are no safety features in this circuit.



The three-wire system connects the neutral wire to the earth at the voltage source and user location, forcing it to be at zero volts and supplying an alternative return path for the current through the earth. Also grounded to zero volts is the case of the appliance. A circuit breaker or fuse protects against thermal overload and is in series on the active (live/hot) wire. Note that wire insulation colors vary with region and it is essential to

check locally to determine which color codes are in use (and even if they were followed in the particular installation).

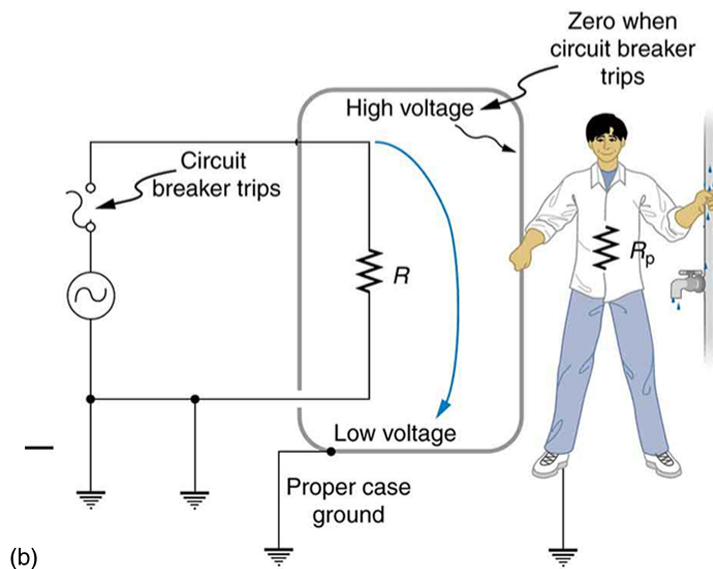
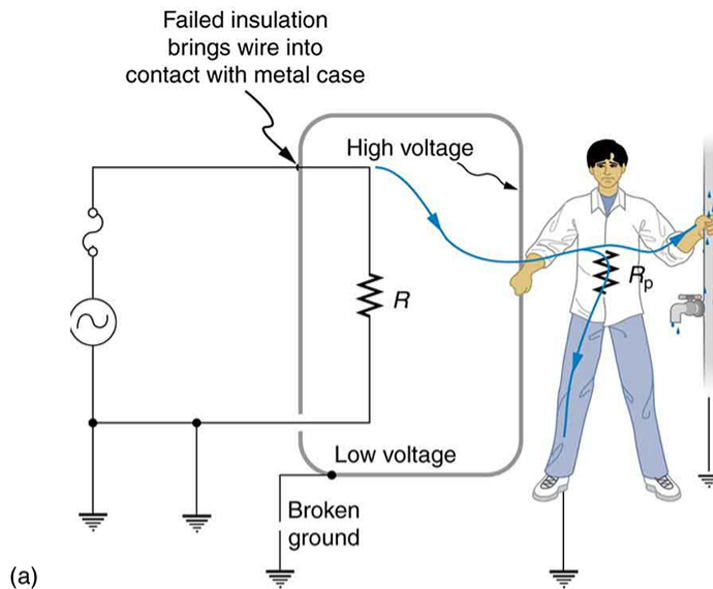
There are *three connections to earth or ground* (hereafter referred to as “earth/ground”) shown in [\[link\]](#). Recall that an earth/ground connection is a low-resistance path directly to the earth. The two earth/ground connections on the *neutral wire* force it to be at zero volts relative to the earth, giving the wire its name. This wire is therefore safe to touch even if its insulation, usually white, is missing. The neutral wire is the return path for the current to follow to complete the circuit. Furthermore, the two earth/ground connections supply an alternative path through the earth, a good conductor, to complete the circuit. The earth/ground connection closest to the power source could be at the generating plant, while the other is at the user’s location. The third earth/ground is to the case of the appliance, through the green *earth/ground wire*, forcing the case, too, to be at zero volts. The *live* or *hot wire* (hereafter referred to as “live/hot”) supplies voltage and current to operate the appliance. [\[link\]](#) shows a more pictorial version of how the three-wire system is connected through a three-prong plug to an appliance.



The standard three-prong plug can only be inserted in one way, to assure proper function of the three-wire system.

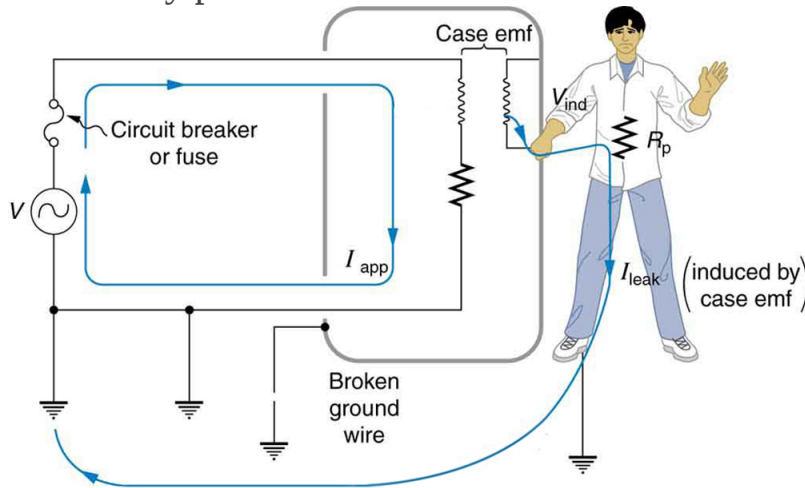
A note on insulation color-coding: Insulating plastic is color-coded to identify live/hot, neutral and ground wires but these codes vary around the world. Live/hot wires may be brown, red, black, blue or grey. Neutral wire may be blue, black or white. Since the same color may be used for live/hot or neutral in different parts of the world, it is essential to determine the color code in your region. The only exception is the earth/ground wire which is often green but may be yellow or just bare wire. Striped coatings are sometimes used for the benefit of those who are colorblind.

The three-wire system replaced the older two-wire system, which lacks an earth/ground wire. Under ordinary circumstances, insulation on the live/hot and neutral wires prevents the case from being directly in the circuit, so that the earth/ground wire may seem like double protection. Grounding the case solves more than one problem, however. The simplest problem is worn insulation on the live/hot wire that allows it to contact the case, as shown in [\[link\]](#). Lacking an earth/ground connection (some people cut the third prong off the plug because they only have outdated two hole receptacles), a severe shock is possible. This is particularly dangerous in the kitchen, where a good connection to earth/ground is available through water on the floor or a water faucet. With the earth/ground connection intact, the circuit breaker will trip, forcing repair of the appliance. Why are some appliances still sold with two-prong plugs? These have nonconducting cases, such as power tools with impact resistant plastic cases, and are called *doubly insulated*. Modern two-prong plugs can be inserted into the asymmetric standard outlet in only one way, to ensure proper connection of live/hot and neutral wires.



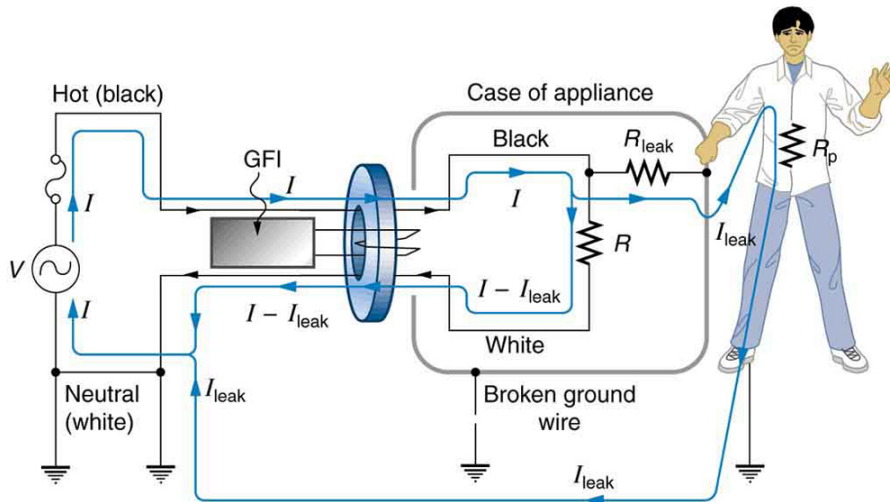
Worn insulation allows the live/hot wire to come into direct contact with the metal case of this appliance. (a) The earth/ground connection being broken, the person is severely shocked. The appliance may operate normally in this situation. (b) With a proper earth/ground, the circuit breaker trips, forcing repair of the appliance.

Electromagnetic induction causes a more subtle problem that is solved by grounding the case. The AC current in appliances can induce an emf on the case. If grounded, the case voltage is kept near zero, but if the case is not grounded, a shock can occur as pictured in [\[link\]](#). Current driven by the induced case emf is called a *leakage current*, although current does not necessarily pass from the resistor to the case.



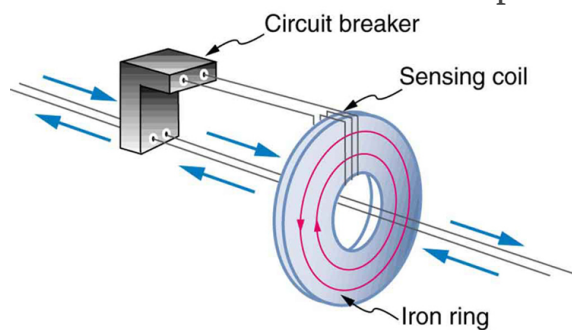
AC currents can induce an emf on the case of an appliance. The voltage can be large enough to cause a shock. If the case is grounded, the induced emf is kept near zero.

A *ground fault interrupter* (GFI) is a safety device found in updated kitchen and bathroom wiring that works based on electromagnetic induction. GFIs compare the currents in the live/hot and neutral wires. When live/hot and neutral currents are not equal, it is almost always because current in the neutral is less than in the live/hot wire. Then some of the current, again called a leakage current, is returning to the voltage source by a path other than through the neutral wire. It is assumed that this path presents a hazard, such as shown in [\[link\]](#). GFIs are usually set to interrupt the circuit if the leakage current is greater than 5 mA, the accepted maximum harmless shock. Even if the leakage current goes safely to earth/ground through an intact earth/ground wire, the GFI will trip, forcing repair of the leakage.



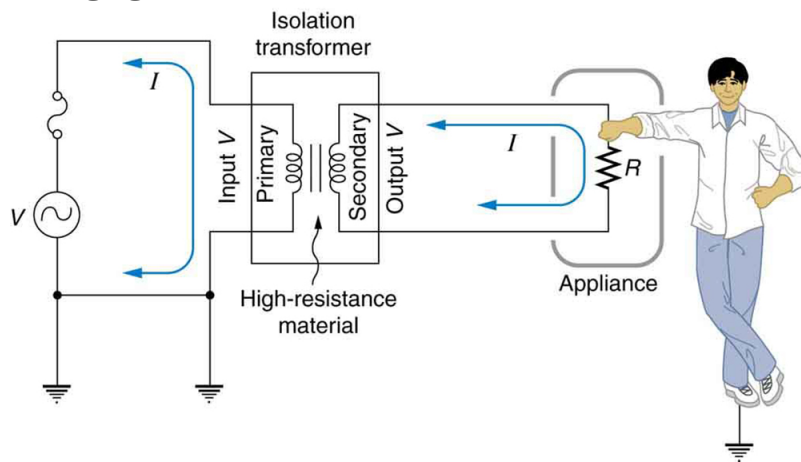
A ground fault interrupter (GFI) compares the currents in the live/hot and neutral wires and will trip if their difference exceeds a safe value. The leakage current here follows a hazardous path that could have been prevented by an intact earth/ground wire.

[\[link\]](#) shows how a GFI works. If the currents in the live/hot and neutral wires are equal, then they induce equal and opposite emfs in the coil. If not, then the circuit breaker will trip.



A GFI compares currents by using both to induce an emf in the same coil. If the currents are equal, they will induce equal but opposite emfs.

Another induction-based safety device is the *isolation transformer*, shown in [\[link\]](#). Most isolation transformers have equal input and output voltages. Their function is to put a large resistance between the original voltage source and the device being operated. This prevents a complete circuit between them, even in the circumstance shown. There is a complete circuit through the appliance. But there is not a complete circuit for current to flow through the person in the figure, who is touching only one of the transformer's output wires, and neither output wire is grounded. The appliance is isolated from the original voltage source by the high resistance of the material between the transformer coils, hence the name isolation transformer. For current to flow through the person, it must pass through the high-resistance material between the coils, through the wire, the person, and back through the earth—a path with such a large resistance that the current is negligible.



An isolation transformer puts a large resistance between the original voltage source and the device, preventing a complete circuit between them.

The basics of electrical safety presented here help prevent many electrical hazards. Electrical safety can be pursued to greater depths. There are, for

example, problems related to different earth/ground connections for appliances in close proximity. Many other examples are found in hospitals. Microshock-sensitive patients, for instance, require special protection. For these people, currents as low as 0.1 mA may cause ventricular fibrillation. The interested reader can use the material presented here as a basis for further study.

Section Summary

- Electrical safety systems and devices are employed to prevent thermal and shock hazards.
- Circuit breakers and fuses interrupt excessive currents to prevent thermal hazards.
- The three-wire system guards against thermal and shock hazards, utilizing live/hot, neutral, and earth/ground wires, and grounding the neutral wire and case of the appliance.
- A ground fault interrupter (GFI) prevents shock by detecting the loss of current to unintentional paths.
- An isolation transformer insulates the device being powered from the original source, also to prevent shock.
- Many of these devices use induction to perform their basic function.

Conceptual Questions

Exercise:

Problem:

Does plastic insulation on live/hot wires prevent shock hazards, thermal hazards, or both?

Exercise:

Problem:

Why are ordinary circuit breakers and fuses ineffective in preventing shocks?

Exercise:

Problem:

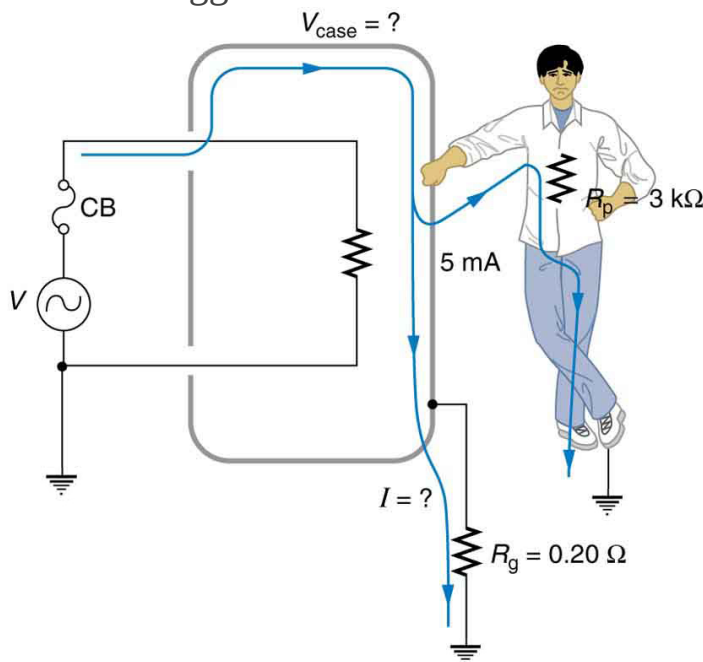
A GFI may trip just because the live/hot and neutral wires connected to it are significantly different in length. Explain why.

Problems & Exercises

Exercise:

Problem: Integrated Concepts

A short circuit to the grounded metal case of an appliance occurs as shown in [\[link\]](#). The person touching the case is wet and only has a $3.00 \text{ k}\Omega$ resistance to earth/ground. (a) What is the voltage on the case if 5.00 mA flows through the person? (b) What is the current in the short circuit if the resistance of the earth/ground wire is 0.200Ω ? (c) Will this trigger the 20.0 A circuit breaker supplying the appliance?



A person can be shocked even when the case of an appliance is grounded.

The large short circuit current

produces a voltage on the case of the appliance, since the resistance of the earth/ground wire is not zero.

Solution:

(a) 15.0 V

(b) 75.0 A

(c) yes

Glossary

thermal hazard

the term for electrical hazards due to overheating

shock hazard

the term for electrical hazards due to current passing through a human

three-wire system

the wiring system used at present for safety reasons, with live, neutral, and ground wires

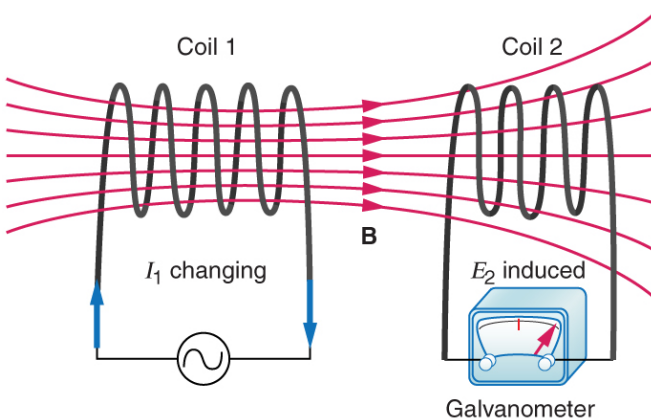
Inductance

- Calculate the inductance of an inductor.
- Calculate the energy stored in an inductor.
- Calculate the emf generated in an inductor.

Inductors

Induction is the process in which an emf is induced by changing magnetic flux. Many examples have been discussed so far, some more effective than others. Transformers, for example, are designed to be particularly effective at inducing a desired voltage and current with very little loss of energy to other forms. Is there a useful physical quantity related to how “effective” a given device is? The answer is yes, and that physical quantity is called **inductance**.

Mutual inductance is the effect of Faraday’s law of induction for one device upon another, such as the primary coil in transmitting energy to the secondary in a transformer. See [\[link\]](#), where simple coils induce emfs in one another.



These coils can induce emfs in one another like an inefficient transformer. Their mutual inductance M indicates the

effectiveness of the coupling between them. Here a change in current in coil 1 is seen to induce an emf in coil 2. (Note that " E_2 induced" represents the induced emf in coil 2.)

In the many cases where the geometry of the devices is fixed, flux is changed by varying current. We therefore concentrate on the rate of change of current, $\Delta I / \Delta t$, as the cause of induction. A change in the current I_1 in one device, coil 1 in the figure, induces an emf_2 in the other. We express this in equation form as

Equation:

$$\text{emf}_2 = -M \frac{\Delta I_1}{\Delta t},$$

where M is defined to be the mutual inductance between the two devices. The minus sign is an expression of Lenz's law. The larger the mutual inductance M , the more effective the coupling. For example, the coils in [\[link\]](#) have a small M compared with the transformer coils in [\[link\]](#). Units for M are $(\text{V} \cdot \text{s}) / \text{A} = \Omega \cdot \text{s}$, which is named a **henry** (H), after Joseph Henry. That is, $1 \text{ H} = 1 \Omega \cdot \text{s}$.

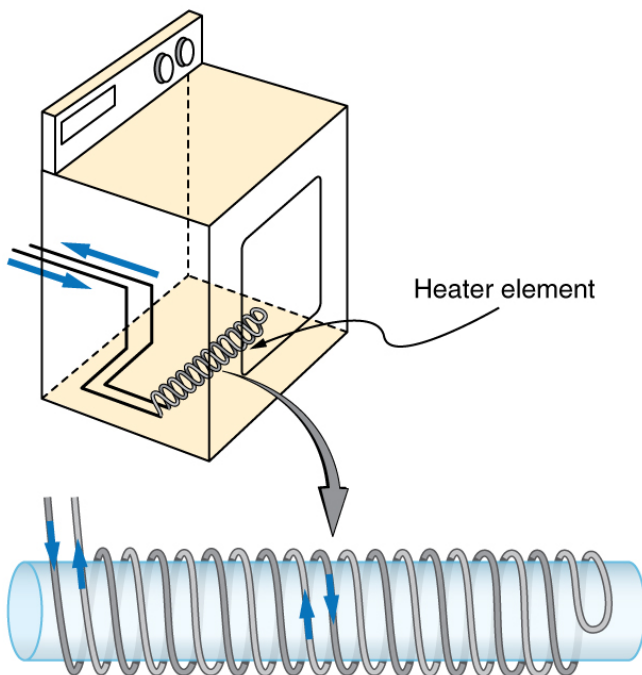
Nature is symmetric here. If we change the current I_2 in coil 2, we induce an emf_1 in coil 1, which is given by

Equation:

$$\text{emf}_1 = -M \frac{\Delta I_2}{\Delta t},$$

where M is the same as for the reverse process. Transformers run backward with the same effectiveness, or mutual inductance M .

A large mutual inductance M may or may not be desirable. We want a transformer to have a large mutual inductance. But an appliance, such as an electric clothes dryer, can induce a dangerous emf on its case if the mutual inductance between its coils and the case is large. One way to reduce mutual inductance M is to counterwind coils to cancel the magnetic field produced. (See [\[link\]](#).)



The heating coils of an electric clothes dryer can be counterwound so that their magnetic fields cancel one another, greatly reducing the mutual inductance with the case of the dryer.

Self-inductance, the effect of Faraday's law of induction of a device on itself, also exists. When, for example, current through a coil is increased, the magnetic field and flux also increase, inducing a counter emf, as

required by Lenz's law. Conversely, if the current is decreased, an emf is induced that opposes the decrease. Most devices have a fixed geometry, and so the change in flux is due entirely to the change in current ΔI through the device. The induced emf is related to the physical geometry of the device and the rate of change of current. It is given by

Equation:

$$\text{emf} = -L \frac{\Delta I}{\Delta t},$$

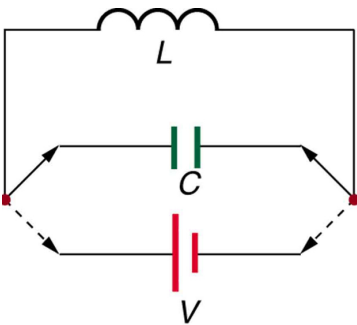
where L is the self-inductance of the device. A device that exhibits significant self-inductance is called an **inductor**, and given the symbol in [\[link\]](#).



The minus sign is an expression of Lenz's law, indicating that emf opposes the change in current. Units of self-inductance are henries (H) just as for mutual inductance. The larger the self-inductance L of a device, the greater its opposition to any change in current through it. For example, a large coil with many turns and an iron core has a large L and will not allow current to change quickly. To avoid this effect, a small L must be achieved, such as by counterwinding coils as in [\[link\]](#).

A 1 H inductor is a large inductor. To illustrate this, consider a device with $L = 1.0$ H that has a 10 A current flowing through it. What happens if we try to shut off the current rapidly, perhaps in only 1.0 ms? An emf, given by $\text{emf} = -L(\Delta I/\Delta t)$, will oppose the change. Thus an emf will be induced given by $\text{emf} = -L(\Delta I/\Delta t) = (1.0 \text{ H})[(10 \text{ A})/(1.0 \text{ ms})] = 10,000 \text{ V}$. The positive sign means this large voltage is in the same direction as the current, opposing its decrease. Such large emfs can cause arcs, damaging switching equipment, and so it may be necessary to change current more slowly.

There are uses for such a large induced voltage. Camera flashes use a battery, two inductors that function as a transformer, and a switching system or oscillator to induce large voltages. (Remember that we need a changing magnetic field, brought about by a changing current, to induce a voltage in another coil.) The oscillator system will do this many times as the battery voltage is boosted to over one thousand volts. (You may hear the high pitched whine from the transformer as the capacitor is being charged.) A capacitor stores the high voltage for later use in powering the flash. (See [\[link\]](#).)



Through rapid switching of an inductor, 1.5 V batteries can be used to induce emfs of several thousand volts. This voltage can be used to store charge in a capacitor for later use, such as in a camera flash attachment.

It is possible to calculate L for an inductor given its geometry (size and shape) and knowing the magnetic field that it produces. This is difficult in most cases, because of the complexity of the field created. So in this text the inductance L is usually a given quantity. One exception is the solenoid, because it has a very uniform field inside, a nearly zero field outside, and a simple shape. It is instructive to derive an equation for its inductance. We start by noting that the induced emf is given by Faraday's law of induction as $\text{emf} = -N(\Delta\Phi/\Delta t)$ and, by the definition of self-inductance, as $\text{emf} = -L(\Delta I/\Delta t)$. Equating these yields

Equation:

$$\text{emf} = -N \frac{\Delta\Phi}{\Delta t} = -L \frac{\Delta I}{\Delta t}.$$

Solving for L gives

Equation:

$$L = N \frac{\Delta\Phi}{\Delta I}.$$

This equation for the self-inductance L of a device is always valid. It means that self-inductance L depends on how effective the current is in creating flux; the more effective, the greater $\Delta\Phi/\Delta I$ is.

Let us use this last equation to find an expression for the inductance of a solenoid. Since the area A of a solenoid is fixed, the change in flux is $\Delta\Phi = \Delta(BA) = A\Delta B$. To find ΔB , we note that the magnetic field of a solenoid is given by $B = \mu_0 nI = \mu_0 \frac{NI}{\ell}$. (Here $n = N/\ell$, where N is the number of coils and ℓ is the solenoid's length.) Only the current changes, so that $\Delta\Phi = A\Delta B = \mu_0 NA \frac{\Delta I}{\ell}$. Substituting $\Delta\Phi$ into $L = N \frac{\Delta\Phi}{\Delta I}$ gives

Equation:

$$L = N \frac{\Delta\Phi}{\Delta I} = N \frac{\mu_0 NA \frac{\Delta I}{\ell}}{\Delta I}.$$

This simplifies to

Equation:

$$L = \frac{\mu_0 N^2 A}{\ell} \text{ (solenoid).}$$

This is the self-inductance of a solenoid of cross-sectional area A and length ℓ . Note that the inductance depends only on the physical characteristics of the solenoid, consistent with its definition.

Example:

Calculating the Self-inductance of a Moderate Size Solenoid

Calculate the self-inductance of a 10.0 cm long, 4.00 cm diameter solenoid that has 200 coils.

Strategy

This is a straightforward application of $L = \frac{\mu_0 N^2 A}{\ell}$, since all quantities in the equation except L are known.

Solution

Use the following expression for the self-inductance of a solenoid:

Equation:

$$L = \frac{\mu_0 N^2 A}{\ell}.$$

The cross-sectional area in this example is

$A = \pi r^2 = (3.14...)(0.0200 \text{ m})^2 = 1.26 \times 10^{-3} \text{ m}^2$, N is given to be 200, and the length ℓ is 0.100 m. We know the permeability of free space is $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$. Substituting these into the expression for L gives

Equation:

$$\begin{aligned} L &= \frac{(4\pi \times 10^{-7} \text{ T} \cdot \text{m/A})(200)^2(1.26 \times 10^{-3} \text{ m}^2)}{0.100 \text{ m}} \\ &= 0.632 \text{ mH.} \end{aligned}$$

Discussion

This solenoid is moderate in size. Its inductance of nearly a millihenry is also considered moderate.

One common application of inductance is used in traffic lights that can tell when vehicles are waiting at the intersection. An electrical circuit with an inductor is placed in the road under the place a waiting car will stop over. The body of the car increases the inductance and the circuit changes sending a signal to the traffic lights to change colors. Similarly, metal detectors used for airport security employ the same technique. A coil or inductor in the metal detector frame acts as both a transmitter and a receiver. The pulsed signal in the transmitter coil induces a signal in the receiver. The self-inductance of the circuit is affected by any metal object in the path. Such detectors can be adjusted for sensitivity and also can indicate the approximate location of metal found on a person. (But they will not be able to detect any plastic explosive such as that found on the “underwear bomber.”) See [\[link\]](#).



The familiar security gate at an airport can not only detect metals but also indicate their approximate height above the floor.

(credit: Alexbuidrs,
Wikimedia Commons)

Energy Stored in an Inductor

We know from Lenz's law that inductances oppose changes in current. There is an alternative way to look at this opposition that is based on energy. Energy is stored in a magnetic field. It takes time to build up energy, and it also takes time to deplete energy; hence, there is an opposition to rapid change. In an inductor, the magnetic field is directly proportional to current and to the inductance of the device. It can be shown that the **energy stored in an inductor** E_{ind} is given by

Equation:

$$E_{\text{ind}} = \frac{1}{2}LI^2.$$

This expression is similar to that for the energy stored in a capacitor.

Example:

Calculating the Energy Stored in the Field of a Solenoid

How much energy is stored in the 0.632 mH inductor of the preceding example when a 30.0 A current flows through it?

Strategy

The energy is given by the equation $E_{\text{ind}} = \frac{1}{2}LI^2$, and all quantities except E_{ind} are known.

Solution

Substituting the value for L found in the previous example and the given current into $E_{\text{ind}} = \frac{1}{2}LI^2$ gives

Equation:

$$\begin{aligned} E_{\text{ind}} &= \frac{1}{2}LI^2 \\ &= 0.5(0.632 \times 10^{-3} \text{ H})(30.0 \text{ A})^2 = 0.284 \text{ J.} \end{aligned}$$

Discussion

This amount of energy is certainly enough to cause a spark if the current is suddenly switched off. It cannot be built up instantaneously unless the power input is infinite.

Section Summary

- Inductance is the property of a device that tells how effectively it induces an emf in another device.
- Mutual inductance is the effect of two devices in inducing emfs in each other.
- A change in current $\Delta I_1 / \Delta t$ in one induces an emf emf_2 in the second:

Equation:

$$\text{emf}_2 = -M \frac{\Delta I_1}{\Delta t},$$

where M is defined to be the mutual inductance between the two devices, and the minus sign is due to Lenz's law.

- Symmetrically, a change in current $\Delta I_2 / \Delta t$ through the second device induces an emf emf_1 in the first:

Equation:

$$\text{emf}_1 = -M \frac{\Delta I_2}{\Delta t},$$

where M is the same mutual inductance as in the reverse process.

- Current changes in a device induce an emf in the device itself.
- Self-inductance is the effect of the device inducing emf in itself.
- The device is called an inductor, and the emf induced in it by a change in current through it is

Equation:

$$\text{emf} = -L \frac{\Delta I}{\Delta t},$$

where L is the self-inductance of the inductor, and $\Delta I/\Delta t$ is the rate of change of current through it. The minus sign indicates that emf opposes the change in current, as required by Lenz's law.

- The unit of self- and mutual inductance is the henry (H), where $1 \text{ H} = 1 \Omega \cdot \text{s}$.
- The self-inductance L of an inductor is proportional to how much flux changes with current. For an N -turn inductor,

Equation:

$$L = N \frac{\Delta \Phi}{\Delta I}.$$

- The self-inductance of a solenoid is

Equation:

$$L = \frac{\mu_0 N^2 A}{\ell} \text{ (solenoid)},$$

where N is its number of turns in the solenoid, A is its cross-sectional area, ℓ is its length, and $\mu_0 = 4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$ is the permeability of free space.

- The energy stored in an inductor E_{ind} is

Equation:

$$E_{\text{ind}} = \frac{1}{2} LI^2.$$

Conceptual Questions

Exercise:

Problem:

How would you place two identical flat coils in contact so that they had the greatest mutual inductance? The least?

Exercise:

Problem:

How would you shape a given length of wire to give it the greatest self-inductance? The least?

Exercise:**Problem:**

Verify, as was concluded without proof in [\[link\]](#), that units of $\text{T} \cdot \text{m}^2/\text{A} = \Omega \cdot \text{s} = \text{H}$.

Problems & Exercises**Exercise:****Problem:**

Two coils are placed close together in a physics lab to demonstrate Faraday's law of induction. A current of 5.00 A in one is switched off in 1.00 ms, inducing a 9.00 V emf in the other. What is their mutual inductance?

Solution:

1.80 mH

Exercise:**Problem:**

If two coils placed next to one another have a mutual inductance of 5.00 mH, what voltage is induced in one when the 2.00 A current in the other is switched off in 30.0 ms?

Exercise:**Problem:**

The 4.00 A current through a 7.50 mH inductor is switched off in 8.33 ms. What is the emf induced opposing this?

Solution:

3.60 V

Exercise:**Problem:**

A device is turned on and 3.00 A flows through it 0.100 ms later. What is the self-inductance of the device if an induced 150 V emf opposes this?

Exercise:**Problem:**

Starting with $\text{emf}_2 = -M \frac{\Delta I_1}{\Delta t}$, show that the units of inductance are $(\text{V} \cdot \text{s})/\text{A} = \Omega \cdot \text{s}$.

Exercise:**Problem:**

Camera flashes charge a capacitor to high voltage by switching the current through an inductor on and off rapidly. In what time must the 0.100 A current through a 2.00 mH inductor be switched on or off to induce a 500 V emf?

Exercise:**Problem:**

A large research solenoid has a self-inductance of 25.0 H. (a) What induced emf opposes shutting it off when 100 A of current through it is switched off in 80.0 ms? (b) How much energy is stored in the inductor at full current? (c) At what rate in watts must energy be dissipated to switch the current off in 80.0 ms? (d) In view of the answer to the last part, is it surprising that shutting it down this quickly is difficult?

Solution:

(a) 31.3 kV

(b) 125 kJ

(c) 1.56 MW

(d) No, it is not surprising since this power is very high.

Exercise:

Problem:

(a) Calculate the self-inductance of a 50.0 cm long, 10.0 cm diameter solenoid having 1000 loops. (b) How much energy is stored in this inductor when 20.0 A of current flows through it? (c) How fast can it be turned off if the induced emf cannot exceed 3.00 V?

Exercise:

Problem:

A precision laboratory resistor is made of a coil of wire 1.50 cm in diameter and 4.00 cm long, and it has 500 turns. (a) What is its self-inductance? (b) What average emf is induced if the 12.0 A current through it is turned on in 5.00 ms (one-fourth of a cycle for 50 Hz AC)? (c) What is its inductance if it is shortened to half its length and counter-wound (two layers of 250 turns in opposite directions)?

Solution:

(a) 1.39 mH

(b) 3.33 V

(c) Zero

Exercise:

Problem:

The heating coils in a hair dryer are 0.800 cm in diameter, have a combined length of 1.00 m, and a total of 400 turns. (a) What is their total self-inductance assuming they act like a single solenoid? (b) How much energy is stored in them when 6.00 A flows? (c) What average emf opposes shutting them off if this is done in 5.00 ms (one-fourth of a cycle for 50 Hz AC)?

Exercise:**Problem:**

When the 20.0 A current through an inductor is turned off in 1.50 ms, an 800 V emf is induced, opposing the change. What is the value of the self-inductance?

Solution:

60.0 mH

Exercise:**Problem:**

How fast can the 150 A current through a 0.250 H inductor be shut off if the induced emf cannot exceed 75.0 V?

Exercise:**Problem: Integrated Concepts**

A very large, superconducting solenoid such as one used in MRI scans, stores 1.00 MJ of energy in its magnetic field when 100 A flows. (a) Find its self-inductance. (b) If the coils “go normal,” they gain resistance and start to dissipate thermal energy. What temperature increase is produced if all the stored energy goes into heating the 1000 kg magnet, given its average specific heat is 200 J/kg·°C?

Solution:

(a) 200 H

(b) 5.00°C

Exercise:

Problem: Unreasonable Results

A 25.0 H inductor has 100 A of current turned off in 1.00 ms. (a) What voltage is induced to oppose this? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Glossary

inductance

a property of a device describing how efficient it is at inducing emf in another device

mutual inductance

how effective a pair of devices are at inducing emfs in each other

henry

the unit of inductance; $1 \text{ H} = 1 \Omega \cdot \text{s}$

self-inductance

how effective a device is at inducing emf in itself

inductor

a device that exhibits significant self-inductance

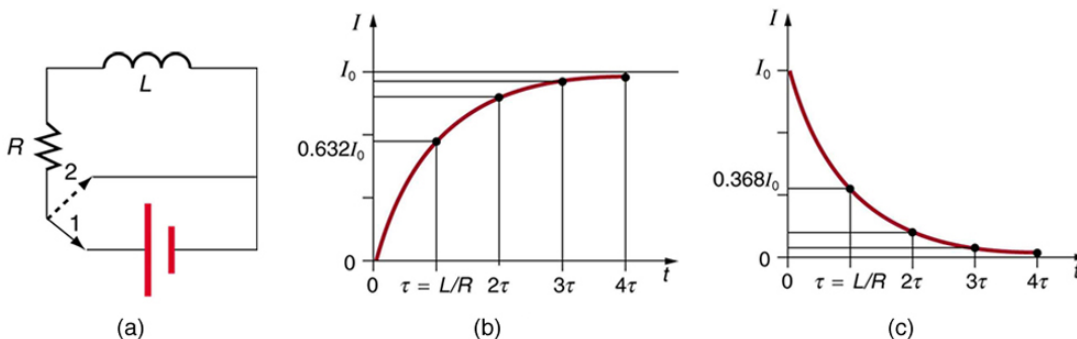
energy stored in an inductor

self-explanatory; calculated by $E_{\text{ind}} = \frac{1}{2}LI^2$

RL Circuits

- Calculate the current in an RL circuit after a specified number of characteristic time steps.
- Calculate the characteristic time of an RL circuit.
- Sketch the current in an RL circuit over time.

We know that the current through an inductor L cannot be turned on or off instantaneously. The change in current changes flux, inducing an emf opposing the change (Lenz's law). How long does the opposition last? Current *will* flow and *can* be turned off, but how long does it take? [\[link\]](#) shows a switching circuit that can be used to examine current through an inductor as a function of time.



(a) An RL circuit with a switch to turn current on and off.

When in position 1, the battery, resistor, and inductor are in series and a current is established. In position 2, the battery is removed and the current eventually stops because of energy loss in the resistor. (b) A graph of current growth versus time when the switch is moved to position 1. (c) A graph of current decay when the switch is moved to position 2.

When the switch is first moved to position 1 (at $t = 0$), the current is zero and it eventually rises to $I_0 = V/R$, where R is the total resistance of the circuit. The opposition of the inductor L is greatest at the beginning, because the amount of change is greatest. The opposition it poses is in the form of an induced emf, which decreases to zero as the current approaches its final value. The opposing emf is proportional to the amount of change

left. This is the hallmark of an exponential behavior, and it can be shown with calculus that

Equation:

$$I = I_0(1 - e^{-t/\tau}) \quad (\text{turning on}),$$

is the current in an RL circuit when switched on (Note the similarity to the exponential behavior of the voltage on a charging capacitor). The initial current is zero and approaches $I_0 = V/R$ with a **characteristic time constant** τ for an RL circuit, given by

Equation:

$$\tau = \frac{L}{R},$$

where τ has units of seconds, since $1 \text{ H} = 1 \Omega \cdot \text{s}$. In the first period of time τ , the current rises from zero to $0.632I_0$, since $I = I_0(1 - e^{-1}) = I_0(1 - 0.368) = 0.632I_0$. The current will go 0.632 of the remainder in the next time τ . A well-known property of the exponential is that the final value is never exactly reached, but 0.632 of the remainder to that value is achieved in every characteristic time τ . In just a few multiples of the time τ , the final value is very nearly achieved, as the graph in [\[link\]](#)(b) illustrates.

The characteristic time τ depends on only two factors, the inductance L and the resistance R . The greater the inductance L , the greater τ is, which makes sense since a large inductance is very effective in opposing change. The smaller the resistance R , the greater τ is. Again this makes sense, since a small resistance means a large final current and a greater change to get there. In both cases—large L and small R —more energy is stored in the inductor and more time is required to get it in and out.

When the switch in [\[link\]](#)(a) is moved to position 2 and cuts the battery out of the circuit, the current drops because of energy dissipation by the resistor. But this is also not instantaneous, since the inductor opposes the decrease in current by inducing an emf in the same direction as the battery that drove

the current. Furthermore, there is a certain amount of energy, $(1/2)LI_0^2$, stored in the inductor, and it is dissipated at a finite rate. As the current approaches zero, the rate of decrease slows, since the energy dissipation rate is I^2R . Once again the behavior is exponential, and I is found to be

Equation:

$$I = I_0 e^{-t/\tau} \quad (\text{turning off}).$$

(See [\[link\]](#)(c).) In the first period of time $\tau = L/R$ after the switch is closed, the current falls to 0.368 of its initial value, since $I = I_0 e^{-1} = 0.368 I_0$. In each successive time τ , the current falls to 0.368 of the preceding value, and in a few multiples of τ , the current becomes very close to zero, as seen in the graph in [\[link\]](#)(c).

Example:

Calculating Characteristic Time and Current in an RL Circuit

(a) What is the characteristic time constant for a 7.50 mH inductor in series with a $3.00 \, \Omega$ resistor? (b) Find the current 5.00 ms after the switch is moved to position 2 to disconnect the battery, if it is initially 10.0 A.

Strategy for (a)

The time constant for an RL circuit is defined by $\tau = L/R$.

Solution for (a)

Entering known values into the expression for τ given in $\tau = L/R$ yields

Equation:

$$\tau = \frac{L}{R} = \frac{7.50 \text{ mH}}{3.00 \, \Omega} = 2.50 \text{ ms}.$$

Discussion for (a)

This is a small but definitely finite time. The coil will be very close to its full current in about ten time constants, or about 25 ms.

Strategy for (b)

We can find the current by using $I = I_0 e^{-t/\tau}$, or by considering the decline in steps. Since the time is twice the characteristic time, we consider the process in steps.

Solution for (b)

In the first 2.50 ms, the current declines to 0.368 of its initial value, which is

Equation:

$$\begin{aligned} I &= 0.368I_0 = (0.368)(10.0 \text{ A}) \\ &= 3.68 \text{ A at } t = 2.50 \text{ ms.} \end{aligned}$$

After another 2.50 ms, or a total of 5.00 ms, the current declines to 0.368 of the value just found. That is,

Equation:

$$\begin{aligned} I' &= 0.368I = (0.368)(3.68 \text{ A}) \\ &= 1.35 \text{ A at } t = 5.00 \text{ ms.} \end{aligned}$$

Discussion for (b)

After another 5.00 ms has passed, the current will be 0.183 A (see [\[link\]](#)); so, although it does die out, the current certainly does not go to zero instantaneously.

In summary, when the voltage applied to an inductor is changed, the current also changes, *but the change in current lags the change in voltage in an RL circuit*. In [Reactance, Inductive and Capacitive](#), we explore how an *RL* circuit behaves when a sinusoidal AC voltage is applied.

Section Summary

- When a series connection of a resistor and an inductor—an *RL* circuit—is connected to a voltage source, the time variation of the current is

Equation:

$$I = I_0(1 - e^{-t/\tau}) \quad (\text{turning on}).$$

where $I_0 = V/R$ is the final current.

- The characteristic time constant τ is $\tau = \frac{L}{R}$, where L is the inductance and R is the resistance.
- In the first time constant τ , the current rises from zero to $0.632I_0$, and 0.632 of the remainder in every subsequent time interval τ .
- When the inductor is shorted through a resistor, current decreases as

Equation:

$$I = I_0 e^{-t/\tau} \quad (\text{turning off}).$$

Here I_0 is the initial current.

- Current falls to $0.368I_0$ in the first time interval τ , and 0.368 of the remainder toward zero in each subsequent time τ .

Problem Exercises

Exercise:

Problem:

If you want a characteristic RL time constant of 1.00 s, and you have a $500\ \Omega$ resistor, what value of self-inductance is needed?

Solution:

500 H

Exercise:

Problem:

Your RL circuit has a characteristic time constant of 20.0 ns, and a resistance of $5.00\ \text{M}\Omega$. (a) What is the inductance of the circuit? (b) What resistance would give you a 1.00 ns time constant, perhaps needed for quick response in an oscilloscope?

Exercise:

Problem:

A large superconducting magnet, used for magnetic resonance imaging, has a 50.0 H inductance. If you want current through it to be adjustable with a 1.00 s characteristic time constant, what is the minimum resistance of system?

Solution:

50.0 Ω

Exercise:**Problem:**

Verify that after a time of 10.0 ms, the current for the situation considered in [\[link\]](#) will be 0.183 A as stated.

Exercise:**Problem:**

Suppose you have a supply of inductors ranging from 1.00 nH to 10.0 H, and resistors ranging from 0.100 Ω to 1.00 M Ω . What is the range of characteristic RL time constants you can produce by connecting a single resistor to a single inductor?

Solution:

1.00×10^{-18} s to 0.100 s

Exercise:**Problem:**

(a) What is the characteristic time constant of a 25.0 mH inductor that has a resistance of 4.00 Ω ? (b) If it is connected to a 12.0 V battery, what is the current after 12.5 ms?

Exercise:

Problem:

What percentage of the final current I_0 flows through an inductor L in series with a resistor R , three time constants after the circuit is completed?

Solution:

95.0%

Exercise:**Problem:**

The 5.00 A current through a 1.50 H inductor is dissipated by a $2.00\ \Omega$ resistor in a circuit like that in [\[link\]](#) with the switch in position 2. (a) What is the initial energy in the inductor? (b) How long will it take the current to decline to 5.00% of its initial value? (c) Calculate the average power dissipated, and compare it with the initial power dissipated by the resistor.

Exercise:**Problem:**

(a) Use the exact exponential treatment to find how much time is required to bring the current through an 80.0 mH inductor in series with a $15.0\ \Omega$ resistor to 99.0% of its final value, starting from zero. (b) Compare your answer to the approximate treatment using integral numbers of τ . (c) Discuss how significant the difference is.

Solution:

(a) 24.6 ms

(b) 26.7 ms

(c) 9% difference, which is greater than the inherent uncertainty in the given parameters.

Exercise:**Problem:**

(a) Using the exact exponential treatment, find the time required for the current through a 2.00 H inductor in series with a $0.500\ \Omega$ resistor to be reduced to 0.100% of its original value. (b) Compare your answer to the approximate treatment using integral numbers of τ . (c) Discuss how significant the difference is.

Glossary

characteristic time constant

denoted by τ , of a particular series RL circuit is calculated by $\tau = \frac{L}{R}$, where L is the inductance and R is the resistance

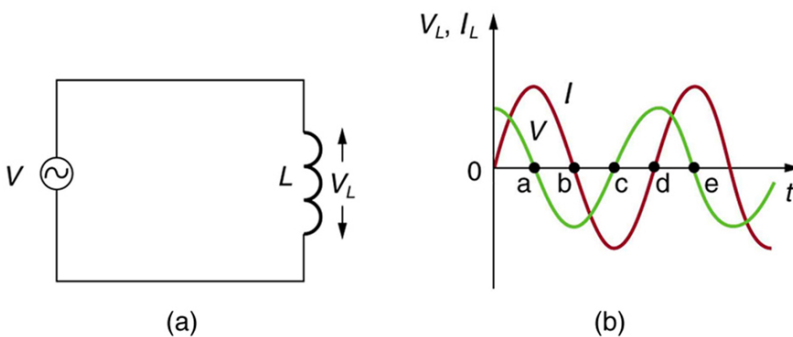
Reactance, Inductive and Capacitive

- Sketch voltage and current versus time in simple inductive, capacitive, and resistive circuits.
- Calculate inductive and capacitive reactance.
- Calculate current and/or voltage in simple inductive, capacitive, and resistive circuits.

Many circuits also contain capacitors and inductors, in addition to resistors and an AC voltage source. We have seen how capacitors and inductors respond to DC voltage when it is switched on and off. We will now explore how inductors and capacitors react to sinusoidal AC voltage.

Inductors and Inductive Reactance

Suppose an inductor is connected directly to an AC voltage source, as shown in [\[link\]](#). It is reasonable to assume negligible resistance, since in practice we can make the resistance of an inductor so small that it has a negligible effect on the circuit. Also shown is a graph of voltage and current as functions of time.



(a) An AC voltage source in series with an inductor having negligible resistance. (b) Graph of current and voltage across the inductor as functions of time.

The graph in [\[link\]](#)(b) starts with voltage at a maximum. Note that the current starts at zero and rises to its peak *after* the voltage that drives it, just as was the case when DC voltage was switched on in the preceding section. When the voltage becomes negative at point a, the current begins to decrease; it becomes zero at point b, where voltage is its most negative. The current then becomes negative, again following the voltage. The voltage becomes positive at point c and begins to make the current less negative. At point d, the current goes through zero just as the voltage reaches its positive peak to start another cycle. This behavior is summarized as follows:

Note:

AC Voltage in an Inductor

When a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90° phase angle.

Current lags behind voltage, since inductors oppose change in current. Changing current induces a back emf $V = -L(\Delta I/\Delta t)$. This is considered to be an effective resistance of the inductor to AC. The rms current I through an inductor L is given by a version of Ohm's law:

Equation:

$$I = \frac{V}{X_L},$$

where V is the rms voltage across the inductor and X_L is defined to be

Equation:

$$X_L = 2\pi fL,$$

with f the frequency of the AC voltage source in hertz (An analysis of the circuit using Kirchhoff's loop rule and calculus actually produces this expression). X_L is called the **inductive reactance**, because the inductor

reacts to impede the current. X_L has units of ohms ($1 \text{ H} = 1 \Omega \cdot \text{s}$, so that frequency times inductance has units of $(\text{cycles/s})(\Omega \cdot \text{s}) = \Omega$), consistent with its role as an effective resistance. It makes sense that X_L is proportional to L , since the greater the induction the greater its resistance to change. It is also reasonable that X_L is proportional to frequency f , since greater frequency means greater change in current. That is, $\Delta I/\Delta t$ is large for large frequencies (large f , small Δt). The greater the change, the greater the opposition of an inductor.

Example:**Calculating Inductive Reactance and then Current**

(a) Calculate the inductive reactance of a 3.00 mH inductor when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current at each frequency if the applied rms voltage is 120 V?

Strategy

The inductive reactance is found directly from the expression $X_L = 2\pi fL$. Once X_L has been found at each frequency, Ohm's law as stated in the Equation $I = V/X_L$ can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and inductance into Equation $X_L = 2\pi fL$ gives

Equation:

$$X_L = 2\pi fL = 6.28(60.0/\text{s})(3.00 \text{ mH}) = 1.13 \Omega \text{ at } 60 \text{ Hz.}$$

Similarly, at 10 kHz,

Equation:

$$X_L = 2\pi fL = 6.28(1.00 \times 10^4/\text{s})(3.00 \text{ mH}) = 188 \Omega \text{ at } 10 \text{ kHz.}$$

Solution for (b)

The rms current is now found using the version of Ohm's law in Equation $I = V/X_L$, given the applied rms voltage is 120 V. For the first frequency, this yields

Equation:

$$I = \frac{V}{X_L} = \frac{120 \text{ V}}{1.13 \Omega} = 106 \text{ A at } 60 \text{ Hz.}$$

Similarly, at 10 kHz,

Equation:

$$I = \frac{V}{X_L} = \frac{120 \text{ V}}{188 \Omega} = 0.637 \text{ A at } 10 \text{ kHz.}$$

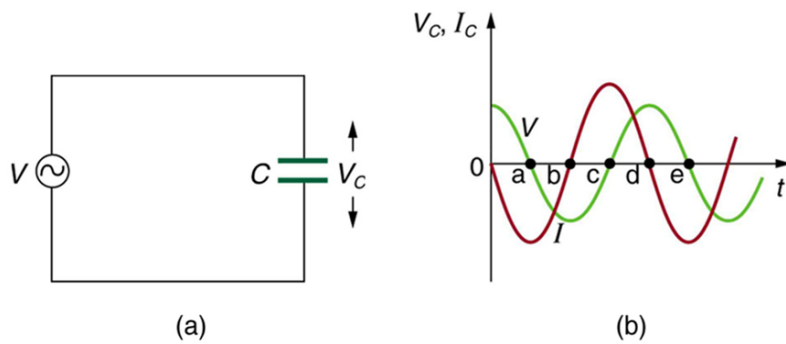
Discussion

The inductor reacts very differently at the two different frequencies. At the higher frequency, its reactance is large and the current is small, consistent with how an inductor impedes rapid change. Thus high frequencies are impeded the most. Inductors can be used to filter out high frequencies; for example, a large inductor can be put in series with a sound reproduction system or in series with your home computer to reduce high-frequency sound output from your speakers or high-frequency power spikes into your computer.

Note that although the resistance in the circuit considered is negligible, the AC current is not extremely large because inductive reactance impedes its flow. With AC, there is no time for the current to become extremely large.

Capacitors and Capacitive Reactance

Consider the capacitor connected directly to an AC voltage source as shown in [\[link\]](#). The resistance of a circuit like this can be made so small that it has a negligible effect compared with the capacitor, and so we can assume negligible resistance. Voltage across the capacitor and current are graphed as functions of time in the figure.



(a) An AC voltage source in series with a capacitor C having negligible resistance. (b) Graph of current and voltage across the capacitor as functions of time.

The graph in [\[link\]](#) starts with voltage across the capacitor at a maximum. The current is zero at this point, because the capacitor is fully charged and halts the flow. Then voltage drops and the current becomes negative as the capacitor discharges. At point a, the capacitor has fully discharged ($Q = 0$ on it) and the voltage across it is zero. The current remains negative between points a and b, causing the voltage on the capacitor to reverse. This is complete at point b, where the current is zero and the voltage has its most negative value. The current becomes positive after point b, neutralizing the charge on the capacitor and bringing the voltage to zero at point c, which allows the current to reach its maximum. Between points c and d, the current drops to zero as the voltage rises to its peak, and the process starts to repeat. Throughout the cycle, the voltage follows what the current is doing by one-fourth of a cycle:

Note:

AC Voltage in a Capacitor

When a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or by a 90° phase angle.

The capacitor is affecting the current, having the ability to stop it altogether when fully charged. Since an AC voltage is applied, there is an rms current, but it is limited by the capacitor. This is considered to be an effective resistance of the capacitor to AC, and so the rms current I in the circuit containing only a capacitor C is given by another version of Ohm's law to be

Equation:

$$I = \frac{V}{X_C},$$

where V is the rms voltage and X_C is defined (As with X_L , this expression for X_C results from an analysis of the circuit using Kirchhoff's rules and calculus) to be

Equation:

$$X_C = \frac{1}{2\pi fC},$$

where X_C is called the **capacitive reactance**, because the capacitor reacts to impede the current. X_C has units of ohms (verification left as an exercise for the reader). X_C is inversely proportional to the capacitance C ; the larger the capacitor, the greater the charge it can store and the greater the current that can flow. It is also inversely proportional to the frequency f ; the greater the frequency, the less time there is to fully charge the capacitor, and so it impedes current less.

Example:

Calculating Capacitive Reactance and then Current

(a) Calculate the capacitive reactance of a 5.00 mF capacitor when 60.0 Hz and 10.0 kHz AC voltages are applied. (b) What is the rms current if the applied rms voltage is 120 V?

Strategy

The capacitive reactance is found directly from the expression in $X_C = \frac{1}{2\pi fC}$. Once X_C has been found at each frequency, Ohm's law stated as $I = V/X_C$ can be used to find the current at each frequency.

Solution for (a)

Entering the frequency and capacitance into $X_C = \frac{1}{2\pi fC}$ gives

Equation:

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} \\ &= \frac{1}{6.28(60.0/\text{s})(5.00 \mu\text{F})} = 531 \Omega \text{ at } 60 \text{ Hz.} \end{aligned}$$

Similarly, at 10 kHz,

Equation:

$$\begin{aligned} X_C &= \frac{1}{2\pi fC} = \frac{1}{6.28(1.00 \times 10^4/\text{s})(5.00 \mu\text{F})} \\ &= 3.18 \Omega \text{ at } 10 \text{ kHz} \end{aligned}$$

Solution for (b)

The rms current is now found using the version of Ohm's law in $I = V/X_C$, given the applied rms voltage is 120 V. For the first frequency, this yields

Equation:

$$I = \frac{V}{X_C} = \frac{120 \text{ V}}{531 \Omega} = 0.226 \text{ A at } 60 \text{ Hz.}$$

Similarly, at 10 kHz,

Equation:

$$I = \frac{V}{X_C} = \frac{120 \text{ V}}{3.18 \Omega} = 37.7 \text{ A at } 10 \text{ kHz.}$$

Discussion

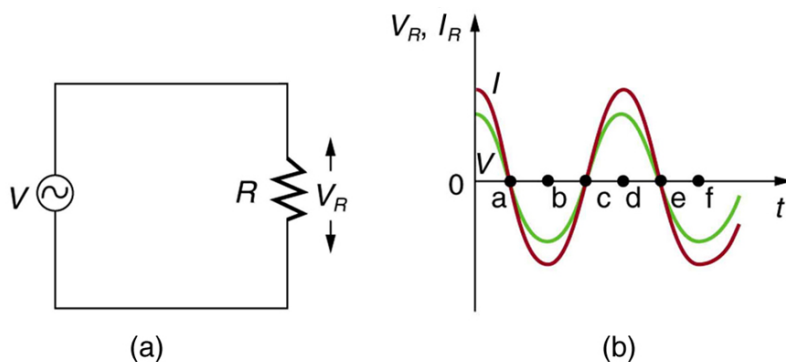
The capacitor reacts very differently at the two different frequencies, and in exactly the opposite way an inductor reacts. At the higher frequency, its reactance is small and the current is large. Capacitors favor change,

whereas inductors oppose change. Capacitors impede low frequencies the most, since low frequency allows them time to become charged and stop the current. Capacitors can be used to filter out low frequencies. For example, a capacitor in series with a sound reproduction system rids it of the 60 Hz hum.

Although a capacitor is basically an open circuit, there is an rms current in a circuit with an AC voltage applied to a capacitor. This is because the voltage is continually reversing, charging and discharging the capacitor. If the frequency goes to zero (DC), X_C tends to infinity, and the current is zero once the capacitor is charged. At very high frequencies, the capacitor's reactance tends to zero—it has a negligible reactance and does not impede the current (it acts like a simple wire). *Capacitors have the opposite effect on AC circuits that inductors have.*

Resistors in an AC Circuit

Just as a reminder, consider [\[link\]](#), which shows an AC voltage applied to a resistor and a graph of voltage and current versus time. The voltage and current are exactly *in phase* in a resistor. There is no frequency dependence to the behavior of plain resistance in a circuit:



(a) An AC voltage source in series with a resistor. (b) Graph of current and voltage

across the resistor as functions of time, showing them to be exactly in phase.

Note:

AC Voltage in a Resistor

When a sinusoidal voltage is applied to a resistor, the voltage is exactly in phase with the current—they have a 0° phase angle.

Section Summary

- For inductors in AC circuits, we find that when a sinusoidal voltage is applied to an inductor, the voltage leads the current by one-fourth of a cycle, or by a 90° phase angle.
- The opposition of an inductor to a change in current is expressed as a type of AC resistance.
- Ohm's law for an inductor is

Equation:

$$I = \frac{V}{X_L},$$

where V is the rms voltage across the inductor.

- X_L is defined to be the inductive reactance, given by

Equation:

$$X_L = 2\pi fL,$$

with f the frequency of the AC voltage source in hertz.

- Inductive reactance X_L has units of ohms and is greatest at high frequencies.
- For capacitors, we find that when a sinusoidal voltage is applied to a capacitor, the voltage follows the current by one-fourth of a cycle, or

by a 90° phase angle.

- Since a capacitor can stop current when fully charged, it limits current and offers another form of AC resistance; Ohm's law for a capacitor is

Equation:

$$I = \frac{V}{X_C},$$

where V is the rms voltage across the capacitor.

- X_C is defined to be the capacitive reactance, given by

Equation:

$$X_C = \frac{1}{2\pi f C}.$$

- X_C has units of ohms and is greatest at low frequencies.

Conceptual Questions

Exercise:

Problem:

Presbycusis is a hearing loss due to age that progressively affects higher frequencies. A hearing aid amplifier is designed to amplify all frequencies equally. To adjust its output for presbycusis, would you put a capacitor in series or parallel with the hearing aid's speaker? Explain.

Exercise:

Problem:

Would you use a large inductance or a large capacitance in series with a system to filter out low frequencies, such as the 100 Hz hum in a sound system? Explain.

Exercise:

Problem:

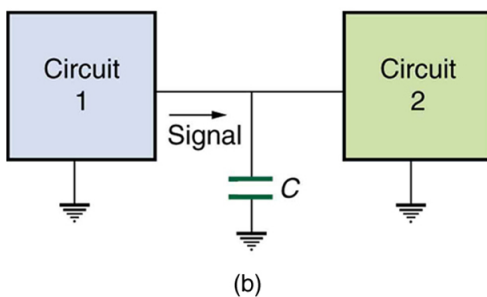
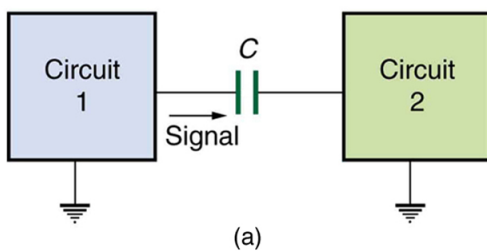
High-frequency noise in AC power can damage computers. Does the plug-in unit designed to prevent this damage use a large inductance or a large capacitance (in series with the computer) to filter out such high frequencies? Explain.

Exercise:**Problem:**

Does inductance depend on current, frequency, or both? What about inductive reactance?

Exercise:**Problem:**

Explain why the capacitor in [\[link\]\(a\)](#) acts as a low-frequency filter between the two circuits, whereas that in [\[link\]\(b\)](#) acts as a high-frequency filter.



Capacitors and inductors.

Capacitor with high frequency and low frequency.

Exercise:

Problem:

If the capacitors in [\[link\]](#) are replaced by inductors, which acts as a low-frequency filter and which as a high-frequency filter?

Problems & Exercises

Exercise:

Problem:

At what frequency will a 30.0 mH inductor have a reactance of 100 Ω ?

Solution:

531 Hz

Exercise:

Problem:

What value of inductance should be used if a 20.0 k Ω reactance is needed at a frequency of 500 Hz?

Exercise:

Problem:

What capacitance should be used to produce a 2.00 M Ω reactance at 60.0 Hz?

Solution:

1.33 nF

Exercise:

Problem:

At what frequency will an 80.0 mF capacitor have a reactance of $0.250\ \Omega$?

Exercise:

Problem:

(a) Find the current through a 0.500 H inductor connected to a 60.0 Hz, 480 V AC source. (b) What would the current be at 100 kHz?

Solution:

(a) 2.55 A

(b) 1.53 mA

Exercise:

Problem:

(a) What current flows when a 60.0 Hz, 480 V AC source is connected to a $0.250\ \mu\text{F}$ capacitor? (b) What would the current be at 25.0 kHz?

Exercise:

Problem:

A 20.0 kHz, 16.0 V source connected to an inductor produces a 2.00 A current. What is the inductance?

Solution:

63.7 μH

Exercise:

Problem:

A 20.0 Hz, 16.0 V source produces a 2.00 mA current when connected to a capacitor. What is the capacitance?

Exercise:**Problem:**

(a) An inductor designed to filter high-frequency noise from power supplied to a personal computer is placed in series with the computer. What minimum inductance should it have to produce a $2.00\text{ k}\Omega$ reactance for 15.0 kHz noise? (b) What is its reactance at 60.0 Hz?

Solution:

(a) 21.2 mH

(b) $8.00\ \Omega$

Exercise:**Problem:**

The capacitor in [\[link\]](#)(a) is designed to filter low-frequency signals, impeding their transmission between circuits. (a) What capacitance is needed to produce a $100\text{ k}\Omega$ reactance at a frequency of 120 Hz? (b) What would its reactance be at 1.00 MHz? (c) Discuss the implications of your answers to (a) and (b).

Exercise:**Problem:**

The capacitor in [\[link\]](#)(b) will filter high-frequency signals by shorting them to earth/ground. (a) What capacitance is needed to produce a reactance of $10.0\text{ m}\Omega$ for a 5.00 kHz signal? (b) What would its reactance be at 3.00 Hz? (c) Discuss the implications of your answers to (a) and (b).

Solution:

(a) 3.18 mF

(b) 16.7 Ω

Exercise:

Problem: Unreasonable Results

In a recording of voltages due to brain activity (an EEG), a 10.0 mV signal with a 0.500 Hz frequency is applied to a capacitor, producing a current of 100 mA. Resistance is negligible. (a) What is the capacitance? (b) What is unreasonable about this result? (c) Which assumption or premise is responsible?

Exercise:

Problem: Construct Your Own Problem

Consider the use of an inductor in series with a computer operating on 60 Hz electricity. Construct a problem in which you calculate the relative reduction in voltage of incoming high frequency noise compared to 60 Hz voltage. Among the things to consider are the acceptable series reactance of the inductor for 60 Hz power and the likely frequencies of noise coming through the power lines.

Glossary

inductive reactance

the opposition of an inductor to a change in current; calculated by

$$X_L = 2\pi fL$$

capacitive reactance

the opposition of a capacitor to a change in current; calculated by

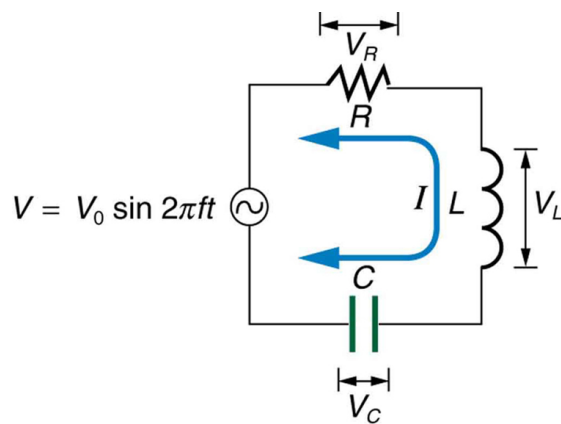
$$X_C = \frac{1}{2\pi fC}$$

RLC Series AC Circuits

- Calculate the impedance, phase angle, resonant frequency, power, power factor, voltage, and/or current in a RLC series circuit.
- Draw the circuit diagram for an RLC series circuit.
- Explain the significance of the resonant frequency.

Impedance

When alone in an AC circuit, inductors, capacitors, and resistors all impede current. How do they behave when all three occur together? Interestingly, their individual resistances in ohms do not simply add. Because inductors and capacitors behave in opposite ways, they partially to totally cancel each other's effect. [\[link\]](#) shows an *RLC* series circuit with an AC voltage source, the behavior of which is the subject of this section. The crux of the analysis of an *RLC* circuit is the frequency dependence of X_L and X_C , and the effect they have on the phase of voltage versus current (established in the preceding section). These give rise to the frequency dependence of the circuit, with important “resonance” features that are the basis of many applications, such as radio tuners.



An *RLC* series circuit with an AC voltage source.

The combined effect of resistance R , inductive reactance X_L , and capacitive reactance X_C is defined to be **impedance**, an AC analogue to resistance in a DC circuit. Current, voltage, and impedance in an RLC circuit are related by an AC version of Ohm's law:

Equation:

$$I_0 = \frac{V_0}{Z} \text{ or } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z}.$$

Here I_0 is the peak current, V_0 the peak source voltage, and Z is the impedance of the circuit. The units of impedance are ohms, and its effect on the circuit is as you might expect: the greater the impedance, the smaller the current. To get an expression for Z in terms of R , X_L , and X_C , we will now examine how the voltages across the various components are related to the source voltage. Those voltages are labeled V_R , V_L , and V_C in [\[link\]](#).

Conservation of charge requires current to be the same in each part of the circuit at all times, so that we can say the currents in R , L , and C are equal and in phase. But we know from the preceding section that the voltage across the inductor V_L leads the current by one-fourth of a cycle, the voltage across the capacitor V_C follows the current by one-fourth of a cycle, and the voltage across the resistor V_R is exactly in phase with the current. [\[link\]](#) shows these relationships in one graph, as well as showing the total voltage around the circuit $V = V_R + V_L + V_C$, where all four voltages are the instantaneous values. According to Kirchhoff's loop rule, the total voltage around the circuit V is also the voltage of the source.

You can see from [\[link\]](#) that while V_R is in phase with the current, V_L leads by 90° , and V_C follows by 90° . Thus V_L and V_C are 180° out of phase (crest to trough) and tend to cancel, although not completely unless they have the same magnitude. Since the peak voltages are not aligned (not in phase), the peak voltage V_0 of the source does *not* equal the sum of the peak voltages across R , L , and C . The actual relationship is

Equation:

$$V_0 = \sqrt{V_{0R}^2 + (V_{0L} - V_{0C})^2},$$

where V_{0R} , V_{0L} , and V_{0C} are the peak voltages across R , L , and C , respectively. Now, using Ohm's law and definitions from [Reactance, Inductive and Capacitive](#), we substitute $V_0 = I_0 Z$ into the above, as well as $V_{0R} = I_0 R$, $V_{0L} = I_0 X_L$, and $V_{0C} = I_0 X_C$, yielding

Equation:

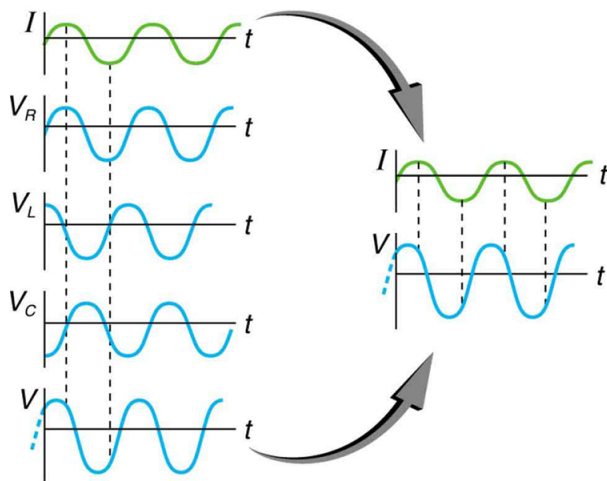
$$I_0 Z = \sqrt{I_0^2 R^2 + (I_0 X_L - I_0 X_C)^2} = I_0 \sqrt{R^2 + (X_L - X_C)^2}.$$

I_0 cancels to yield an expression for Z :

Equation:

$$Z = \sqrt{R^2 + (X_L - X_C)^2},$$

which is the impedance of an RLC series AC circuit. For circuits without a resistor, take $R = 0$; for those without an inductor, take $X_L = 0$; and for those without a capacitor, take $X_C = 0$.



This graph shows the relationships of the voltages in an RLC circuit to the current.

The voltages across the circuit elements add to equal the voltage of the source, which is seen to be out of phase with the current.

Example:

Calculating Impedance and Current

An RLC series circuit has a $40.0\ \Omega$ resistor, a $3.00\ \text{mH}$ inductor, and a $5.00\ \mu\text{F}$ capacitor. (a) Find the circuit's impedance at $60.0\ \text{Hz}$ and $10.0\ \text{kHz}$, noting that these frequencies and the values for L and C are the same as in [\[link\]](#) and [\[link\]](#). (b) If the voltage source has $V_{\text{rms}} = 120\ \text{V}$, what is I_{rms} at each frequency?

Strategy

For each frequency, we use $Z = \sqrt{R^2 + (X_L - X_C)^2}$ to find the impedance and then Ohm's law to find current. We can take advantage of the results of the previous two examples rather than calculate the reactances again.

Solution for (a)

At $60.0\ \text{Hz}$, the values of the reactances were found in [\[link\]](#) to be $X_L = 1.13\ \Omega$ and in [\[link\]](#) to be $X_C = 531\ \Omega$. Entering these and the given $40.0\ \Omega$ for resistance into $Z = \sqrt{R^2 + (X_L - X_C)^2}$ yields

Equation:

$$\begin{aligned} Z &= \sqrt{R^2 + (X_L - X_C)^2} \\ &= \sqrt{(40.0\ \Omega)^2 + (1.13\ \Omega - 531\ \Omega)^2} \\ &= 531\ \Omega \text{ at } 60.0\ \text{Hz}. \end{aligned}$$

Similarly, at $10.0\ \text{kHz}$, $X_L = 188\ \Omega$ and $X_C = 3.18\ \Omega$, so that

Equation:

$$\begin{aligned} Z &= \sqrt{(40.0 \, \Omega)^2 + (188 \, \Omega - 3.18 \, \Omega)^2} \\ &= 190 \, \Omega \text{ at } 10.0 \text{ kHz.} \end{aligned}$$

Discussion for (a)

In both cases, the result is nearly the same as the largest value, and the impedance is definitely not the sum of the individual values. It is clear that X_L dominates at high frequency and X_C dominates at low frequency.

Solution for (b)

The current I_{rms} can be found using the AC version of Ohm's law in Equation $I_{\text{rms}} = V_{\text{rms}}/Z$:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120 \text{ V}}{531 \, \Omega} = 0.226 \text{ A at } 60.0 \text{ Hz}$$

Finally, at 10.0 kHz, we find

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120 \text{ V}}{190 \, \Omega} = 0.633 \text{ A at } 10.0 \text{ kHz}$$

Discussion for (a)

The current at 60.0 Hz is the same (to three digits) as found for the capacitor alone in [\[link\]](#). The capacitor dominates at low frequency. The current at 10.0 kHz is only slightly different from that found for the inductor alone in [\[link\]](#). The inductor dominates at high frequency.

Resonance in *RLC* Series AC Circuits

How does an *RLC* circuit behave as a function of the frequency of the driving voltage source? Combining Ohm's law, $I_{\text{rms}} = V_{\text{rms}}/Z$, and the expression for impedance Z from $Z = \sqrt{R^2 + (X_L - X_C)^2}$ gives

Equation:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{\sqrt{R^2 + (X_L - X_C)^2}}.$$

The reactances vary with frequency, with X_L large at high frequencies and X_C large at low frequencies, as we have seen in three previous examples. At some intermediate frequency f_0 , the reactances will be equal and cancel,

giving $Z = R$ —this is a minimum value for impedance, and a maximum value for I_{rms} results. We can get an expression for f_0 by taking

Equation:

$$X_L = X_C.$$

Substituting the definitions of X_L and X_C ,

Equation:

$$2\pi f_0 L = \frac{1}{2\pi f_0 C}.$$

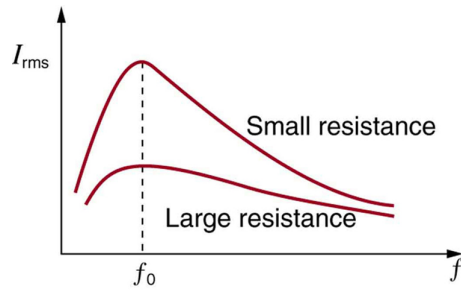
Solving this expression for f_0 yields

Equation:

$$f_0 = \frac{1}{2\pi\sqrt{LC}},$$

where f_0 is the **resonant frequency** of an RLC series circuit. This is also the *natural frequency* at which the circuit would oscillate if not driven by the voltage source. At f_0 , the effects of the inductor and capacitor cancel, so that $Z = R$, and I_{rms} is a maximum.

Resonance in AC circuits is analogous to mechanical resonance, where resonance is defined to be a forced oscillation—in this case, forced by the voltage source—at the natural frequency of the system. The receiver in a radio is an RLC circuit that oscillates best at its f_0 . A variable capacitor is often used to adjust f_0 to receive a desired frequency and to reject others. [\[link\]](#) is a graph of current as a function of frequency, illustrating a resonant peak in I_{rms} at f_0 . The two curves are for two different circuits, which differ only in the amount of resistance in them. The peak is lower and broader for the higher-resistance circuit. Thus the higher-resistance circuit does not resonate as strongly and would not be as selective in a radio receiver, for example.



A graph of current versus frequency for two RLC series circuits differing only in the amount of resistance. Both have a resonance at f_0 , but that for the higher resistance is lower and broader. The driving AC voltage source has a fixed amplitude V_0 .

Example:

Calculating Resonant Frequency and Current

For the same RLC series circuit having a $40.0\ \Omega$ resistor, a 3.00 mH inductor, and a $5.00\ \mu\text{F}$ capacitor: (a) Find the resonant frequency. (b) Calculate I_{rms} at resonance if V_{rms} is 120 V .

Strategy

The resonant frequency is found by using the expression in $f_0 = \frac{1}{2\pi\sqrt{LC}}$.

The current at that frequency is the same as if the resistor alone were in the circuit.

Solution for (a)

Entering the given values for L and C into the expression given for f_0 in $f_0 = \frac{1}{2\pi\sqrt{LC}}$ yields

Equation:

$$\begin{aligned} f_0 &= \frac{1}{2\pi\sqrt{LC}} \\ &= \frac{1}{2\pi\sqrt{(3.00 \times 10^{-3} \text{ H})(5.00 \times 10^{-6} \text{ F})}} = 1.30 \text{ kHz}. \end{aligned}$$

Discussion for (a)

We see that the resonant frequency is between 60.0 Hz and 10.0 kHz, the two frequencies chosen in earlier examples. This was to be expected, since the capacitor dominated at the low frequency and the inductor dominated at the high frequency. Their effects are the same at this intermediate frequency.

Solution for (b)

The current is given by Ohm's law. At resonance, the two reactances are equal and cancel, so that the impedance equals the resistance alone. Thus,

Equation:

$$I_{\text{rms}} = \frac{V_{\text{rms}}}{Z} = \frac{120 \text{ V}}{40.0 \Omega} = 3.00 \text{ A}.$$

Discussion for (b)

At resonance, the current is greater than at the higher and lower frequencies considered for the same circuit in the preceding example.

Power in *RLC* Series AC Circuits

If current varies with frequency in an *RLC* circuit, then the power delivered to it also varies with frequency. But the average power is not simply current times voltage, as it is in purely resistive circuits. As was seen in [\[link\]](#), voltage and current are out of phase in an *RLC* circuit. There is a **phase angle** ϕ between the source voltage V and the current I , which can be found from

Equation:

$$\cos \phi = \frac{R}{Z}.$$

For example, at the resonant frequency or in a purely resistive circuit $Z = R$, so that $\cos \phi = 1$. This implies that $\phi = 0^\circ$ and that voltage and current are in phase, as expected for resistors. At other frequencies, average power is less than at resonance. This is both because voltage and current are out of phase and because I_{rms} is lower. The fact that source voltage and current are out of phase affects the power delivered to the circuit. It can be shown that the *average power* is

Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos \phi,$$

Thus $\cos \phi$ is called the **power factor**, which can range from 0 to 1. Power factors near 1 are desirable when designing an efficient motor, for example. At the resonant frequency, $\cos \phi = 1$.

Example:

Calculating the Power Factor and Power

For the same RLC series circuit having a $40.0 \, \Omega$ resistor, a $3.00 \, \text{mH}$ inductor, a $5.00 \, \mu\text{F}$ capacitor, and a voltage source with a V_{rms} of $120 \, \text{V}$: (a) Calculate the power factor and phase angle for $f = 60.0 \, \text{Hz}$. (b) What is the average power at $50.0 \, \text{Hz}$? (c) Find the average power at the circuit's resonant frequency.

Strategy and Solution for (a)

The power factor at $60.0 \, \text{Hz}$ is found from

Equation:

$$\cos \phi = \frac{R}{Z}.$$

We know $Z = 531 \, \Omega$ from [\[link\]](#), so that

Equation:

$$\cos \phi = \frac{40.0 \, \Omega}{531 \, \Omega} = 0.0753 \text{ at } 60.0 \text{ Hz.}$$

This small value indicates the voltage and current are significantly out of phase. In fact, the phase angle is

Equation:

$$\phi = \cos^{-1} 0.0753 = 85.7^\circ \text{ at } 60.0 \text{ Hz.}$$

Discussion for (a)

The phase angle is close to 90° , consistent with the fact that the capacitor dominates the circuit at this low frequency (a pure RC circuit has its voltage and current 90° out of phase).

Strategy and Solution for (b)

The average power at 60.0 Hz is

Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos \phi.$$

I_{rms} was found to be 0.226 A in [\[link\]](#). Entering the known values gives

Equation:

$$P_{\text{ave}} = (0.226 \text{ A})(120 \text{ V})(0.0753) = 2.04 \text{ W at } 60.0 \text{ Hz.}$$

Strategy and Solution for (c)

At the resonant frequency, we know $\cos \phi = 1$, and I_{rms} was found to be 6.00 A in [\[link\]](#). Thus,

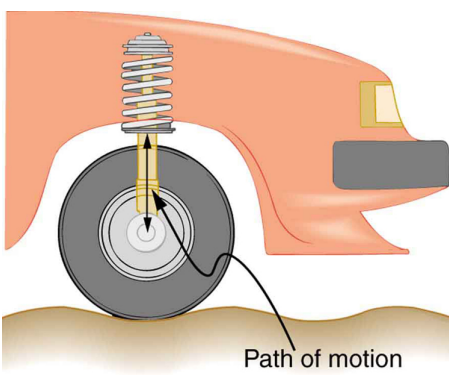
$$P_{\text{ave}} = (3.00 \text{ A})(120 \text{ V})(1) = 360 \text{ W at resonance (1.30 kHz)}$$

Discussion

Both the current and the power factor are greater at resonance, producing significantly greater power than at higher and lower frequencies.

Power delivered to an RLC series AC circuit is dissipated by the resistance alone. The inductor and capacitor have energy input and output but do not dissipate it out of the circuit. Rather they transfer energy back and forth to one another, with the resistor dissipating exactly what the voltage source

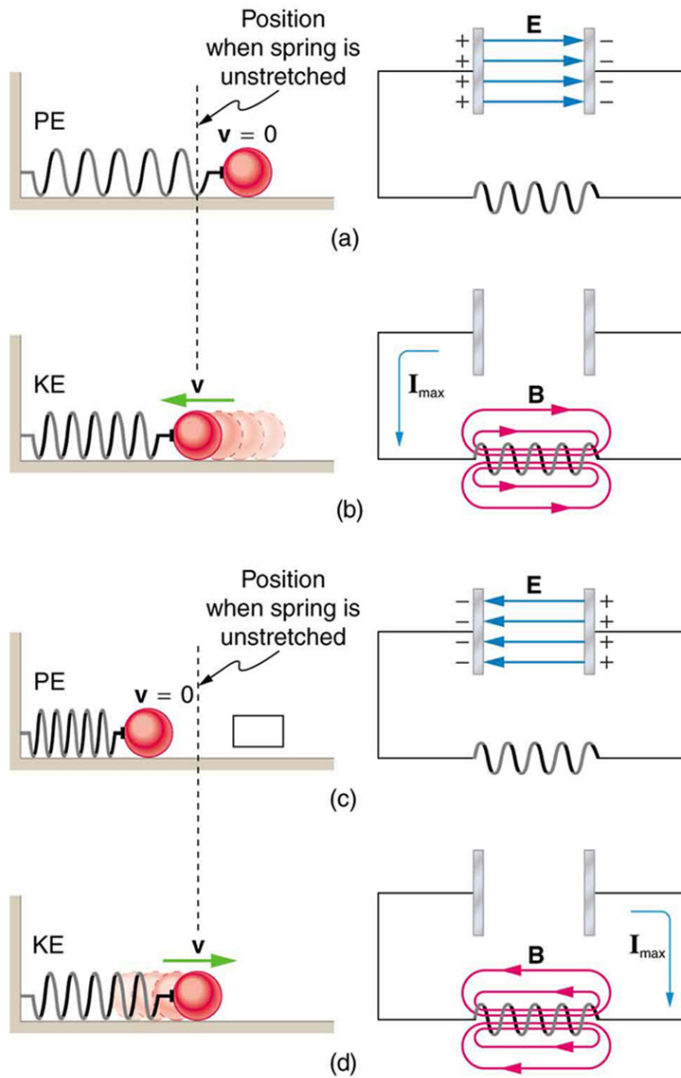
puts into the circuit. This assumes no significant electromagnetic radiation from the inductor and capacitor, such as radio waves. Such radiation can happen and may even be desired, as we will see in the next chapter on electromagnetic radiation, but it can also be suppressed as is the case in this chapter. The circuit is analogous to the wheel of a car driven over a corrugated road as shown in [\[link\]](#). The regularly spaced bumps in the road are analogous to the voltage source, driving the wheel up and down. The shock absorber is analogous to the resistance damping and limiting the amplitude of the oscillation. Energy within the system goes back and forth between kinetic (analogous to maximum current, and energy stored in an inductor) and potential energy stored in the car spring (analogous to no current, and energy stored in the electric field of a capacitor). The amplitude of the wheels' motion is a maximum if the bumps in the road are hit at the resonant frequency.



The forced but damped motion of the wheel on the car spring is analogous to an RLC series AC circuit. The shock absorber damps the motion and dissipates energy, analogous to the resistance in an RLC circuit. The mass

and spring determine
the resonant
frequency.

A pure LC circuit with negligible resistance oscillates at f_0 , the same resonant frequency as an RLC circuit. It can serve as a frequency standard or clock circuit—for example, in a digital wristwatch. With a very small resistance, only a very small energy input is necessary to maintain the oscillations. The circuit is analogous to a car with no shock absorbers. Once it starts oscillating, it continues at its natural frequency for some time. [\[link\]](#) shows the analogy between an LC circuit and a mass on a spring.



An LC circuit is analogous to a mass oscillating on a spring with no friction and no driving force. Energy moves back and forth between the inductor and capacitor, just as it moves from kinetic to potential in the mass-spring system.

Note:

PhET Explorations: Circuit Construction Kit (AC+DC), Virtual Lab
Build circuits with capacitors, inductors, resistors and AC or DC voltage sources, and inspect them using lab instruments such as voltmeters and ammeters.

https://phet.colorado.edu/sims/html/circuit-construction-kit-dc/latest/circuit-construction-kit-dc_en.html

Section Summary

- The AC analogy to resistance is impedance Z , the combined effect of resistors, inductors, and capacitors, defined by the AC version of Ohm's law:

Equation:

$$I_0 = \frac{V_0}{Z} \text{ or } I_{\text{rms}} = \frac{V_{\text{rms}}}{Z},$$

where I_0 is the peak current and V_0 is the peak source voltage.

- Impedance has units of ohms and is given by
 $Z = \sqrt{R^2 + (X_L - X_C)^2}$.
- The resonant frequency f_0 , at which $X_L = X_C$, is

Equation:

$$f_0 = \frac{1}{2\pi\sqrt{LC}}.$$

- In an AC circuit, there is a phase angle ϕ between source voltage V and the current I , which can be found from

Equation:

$$\cos \phi = \frac{R}{Z},$$

- $\phi = 0^\circ$ for a purely resistive circuit or an RLC circuit at resonance.

- The average power delivered to an *RLC* circuit is affected by the phase angle and is given by

Equation:

$$P_{\text{ave}} = I_{\text{rms}} V_{\text{rms}} \cos \phi,$$

$\cos \phi$ is called the power factor, which ranges from 0 to 1.

Conceptual Questions

Exercise:

Problem:

Does the resonant frequency of an AC circuit depend on the peak voltage of the AC source? Explain why or why not.

Exercise:

Problem:

Suppose you have a motor with a power factor significantly less than 1. Explain why it would be better to improve the power factor as a method of improving the motor's output, rather than to increase the voltage input.

Problems & Exercises

Exercise:

Problem:

An *RL* circuit consists of a $40.0\ \Omega$ resistor and a $3.00\ \text{mH}$ inductor. (a) Find its impedance Z at $60.0\ \text{Hz}$ and $10.0\ \text{kHz}$. (b) Compare these values of Z with those found in [\[link\]](#) in which there was also a capacitor.

Solution:

(a) $40.02\ \Omega$ at 60.0 Hz, $193\ \Omega$ at 10.0 kHz

(b) At 60 Hz, with a capacitor, $Z=531\ \Omega$, over 13 times as high as without the capacitor. The capacitor makes a large difference at low frequencies. At 10 kHz, with a capacitor $Z=190\ \Omega$, about the same as without the capacitor. The capacitor has a smaller effect at high frequencies.

Exercise:

Problem:

An RC circuit consists of a $40.0\ \Omega$ resistor and a $5.00\ \mu\text{F}$ capacitor. (a) Find its impedance at 60.0 Hz and 10.0 kHz. (b) Compare these values of Z with those found in [\[link\]](#), in which there was also an inductor.

Exercise:

Problem:

An LC circuit consists of a $3.00\ \text{mH}$ inductor and a $5.00\ \mu\text{F}$ capacitor. (a) Find its impedance at 60.0 Hz and 10.0 kHz. (b) Compare these values of Z with those found in [\[link\]](#) in which there was also a resistor.

Solution:

(a) $529\ \Omega$ at 60.0 Hz, $185\ \Omega$ at 10.0 kHz

(b) These values are close to those obtained in [\[link\]](#) because at low frequency the capacitor dominates and at high frequency the inductor dominates. So in both cases the resistor makes little contribution to the total impedance.

Exercise:

Problem:

What is the resonant frequency of a $0.500\ \text{mH}$ inductor connected to a $40.0\ \mu\text{F}$ capacitor?

Exercise:

Problem:

To receive AM radio, you want an *RLC* circuit that can be made to resonate at any frequency between 500 and 1650 kHz. This is accomplished with a fixed $1.00\ \mu\text{H}$ inductor connected to a variable capacitor. What range of capacitance is needed?

Solution:

9.30 nF to 101 nF

Exercise:**Problem:**

Suppose you have a supply of inductors ranging from $1.00\ \text{nH}$ to $10.0\ \text{H}$, and capacitors ranging from $1.00\ \text{pF}$ to $0.100\ \text{F}$. What is the range of resonant frequencies that can be achieved from combinations of a single inductor and a single capacitor?

Exercise:**Problem:**

What capacitance do you need to produce a resonant frequency of $1.00\ \text{GHz}$, when using an $8.00\ \text{nH}$ inductor?

Solution:

$3.17\ \text{pF}$

Exercise:**Problem:**

What inductance do you need to produce a resonant frequency of $60.0\ \text{Hz}$, when using a $2.00\ \mu\text{F}$ capacitor?

Exercise:

Problem:

The lowest frequency in the FM radio band is 88.0 MHz. (a) What inductance is needed to produce this resonant frequency if it is connected to a 2.50 pF capacitor? (b) The capacitor is variable, to allow the resonant frequency to be adjusted to as high as 108 MHz. What must the capacitance be at this frequency?

Solution:

(a) 1.31 μH

(b) 1.66 pF

Exercise:**Problem:**

An RLC series circuit has a $2.50\ \Omega$ resistor, a $100\ \mu\text{H}$ inductor, and an $80.0\ \mu\text{F}$ capacitor. (a) Find the circuit's impedance at 120 Hz. (b) Find the circuit's impedance at 5.00 kHz. (c) If the voltage source has $V_{\text{rms}} = 5.60\ \text{V}$, what is I_{rms} at each frequency? (d) What is the resonant frequency of the circuit? (e) What is I_{rms} at resonance?

Exercise:**Problem:**

An RLC series circuit has a $1.00\ \text{k}\Omega$ resistor, a $150\ \mu\text{H}$ inductor, and a $25.0\ \text{nF}$ capacitor. (a) Find the circuit's impedance at 500 Hz. (b) Find the circuit's impedance at 7.50 kHz. (c) If the voltage source has $V_{\text{rms}} = 408\ \text{V}$, what is I_{rms} at each frequency? (d) What is the resonant frequency of the circuit? (e) What is I_{rms} at resonance?

Solution:

(a) 12.8 $\text{k}\Omega$

(b) 1.31 $\text{k}\Omega$

(c) 31.9 mA at 500 Hz, 312 mA at 7.50 kHz

(d) 82.2 kHz

(e) 0.408 A

Exercise:

Problem:

An RLC series circuit has a $2.50\ \Omega$ resistor, a $100\ \mu\text{H}$ inductor, and an $80.0\ \mu\text{F}$ capacitor. (a) Find the power factor at $f = 120\ \text{Hz}$. (b) What is the phase angle at 120 Hz? (c) What is the average power at 120 Hz? (d) Find the average power at the circuit's resonant frequency.

Exercise:

Problem:

An RLC series circuit has a $1.00\ \text{k}\Omega$ resistor, a $150\ \mu\text{H}$ inductor, and a $25.0\ \text{nF}$ capacitor. (a) Find the power factor at $f = 7.50\ \text{Hz}$. (b) What is the phase angle at this frequency? (c) What is the average power at this frequency? (d) Find the average power at the circuit's resonant frequency.

Solution:

(a) 0.159

(b) 80.9°

(c) 26.4 W

(d) 166 W

Exercise:

Problem:

An RLC series circuit has a $200\ \Omega$ resistor and a 25.0 mH inductor. At 8000 Hz , the phase angle is 45.0° . (a) What is the impedance? (b) Find the circuit's capacitance. (c) If $V_{\text{rms}} = 408\text{ V}$ is applied, what is the average power supplied?

Exercise:

Problem: Referring to [\[link\]](#), find the average power at 10.0 kHz .

Solution:

16.0 W

Glossary

impedance

the AC analogue to resistance in a DC circuit; it is the combined effect of resistance, inductive reactance, and capacitive reactance in the form

$$Z = \sqrt{R^2 + (X_L - X_C)^2}$$

resonant frequency

the frequency at which the impedance in a circuit is at a minimum, and also the frequency at which the circuit would oscillate if not driven by a voltage source; calculated by $f_0 = \frac{1}{2\pi\sqrt{LC}}$

phase angle

denoted by ϕ , the amount by which the voltage and current are out of phase with each other in a circuit

power factor

the amount by which the power delivered in the circuit is less than the theoretical maximum of the circuit due to voltage and current being out of phase; calculated by $\cos \phi$

Introduction to Electromagnetic Waves

class="introduction"

Human eyes
detect these
orange “sea
goldie” fish
swimming
over a coral
reef in the
blue waters
of the Gulf
of Eilat (Red
Sea) using
visible light.

(credit:
Daviddarom
, Wikimedia
Commons)



The beauty of a coral reef, the warm radiance of sunshine, the sting of sunburn, the X-ray revealing a broken bone, even microwave popcorn—all are brought to us by **electromagnetic waves**. The list of the various types of electromagnetic waves, ranging from radio transmission waves to nuclear gamma-ray (γ -ray) emissions, is interesting in itself.

Even more intriguing is that all of these widely varied phenomena are different manifestations of the same thing—electromagnetic waves. (See [\[link\]](#).) What are electromagnetic waves? How are they created, and how do they travel? How can we understand and organize their widely varying properties? What is their relationship to electric and magnetic effects? These and other questions will be explored.

Note:**Misconception Alert: Sound Waves vs. Radio Waves**

Many people confuse sound waves with **radio waves**, one type of electromagnetic (EM) wave. However, sound and radio waves are

completely different phenomena. Sound creates pressure variations (waves) in matter, such as air or water, or your eardrum. Conversely, radio waves are *electromagnetic waves*, like visible light, infrared, ultraviolet, X-rays, and gamma rays. EM waves don't need a medium in which to propagate; they can travel through a vacuum, such as outer space. A radio works because sound waves played by the D.J. at the radio station are converted into electromagnetic waves, then encoded and transmitted in the radio-frequency range. The radio in your car receives the radio waves, decodes the information, and uses a speaker to change it back into a sound wave, bringing sweet music to your ears.

Discovering a New Phenomenon

It is worth noting at the outset that the general phenomenon of electromagnetic waves was predicted by theory before it was realized that light is a form of electromagnetic wave. The prediction was made by James Clerk Maxwell in the mid-19th century when he formulated a single theory combining all the electric and magnetic effects known by scientists at that time. "Electromagnetic waves" was the name he gave to the phenomena his theory predicted.

Such a theoretical prediction followed by experimental verification is an indication of the power of science in general, and physics in particular. The underlying connections and unity of physics allow certain great minds to solve puzzles without having all the pieces. The prediction of electromagnetic waves is one of the most spectacular examples of this power. Certain others, such as the prediction of antimatter, will be discussed in later modules.



The
electromagnetic
waves sent
and received by
this 50-foot
radar dish
antenna at
Kennedy Space
Center in
Florida are not
visible, but
help track
expendable
launch vehicles
with high-
definition
imagery. The
first use of this
C-band radar
dish was for
the launch of
the Atlas V
rocket sending
the New
Horizons probe

toward Pluto.
(credit: NASA)

Maxwell's Equations: Electromagnetic Waves Predicted and Observed

- Restate Maxwell's equations.

The Scotsman James Clerk Maxwell (1831–1879) is regarded as the greatest theoretical physicist of the 19th century. (See [\[link\]](#).) Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by **Maxwell's equations**, he also developed the kinetic theory of gases and made significant contributions to the understanding of color vision and the nature of Saturn's rings.



James Clerk Maxwell, a 19th-century physicist, developed a theory that explained the relationship between electricity and magnetism and correctly predicted that visible light is caused by electromagnetic

waves. (credit:
G. J. Stodart)

Maxwell brought together all the work that had been done by brilliant physicists such as Oersted, Coulomb, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. Maxwell's equations are paraphrased here in words because their mathematical statement is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

Note:

Maxwell's Equations

1. **Electric field lines** originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge, and the strength of the force is related to the electric constant ϵ_0 , also known as the permittivity of free space. From Maxwell's first equation we obtain a special form of Coulomb's law known as Gauss's law for electricity.
2. **Magnetic field lines** are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the magnetic constant μ_0 , also known as the permeability of free space. This second of Maxwell's equations is known as Gauss's law for magnetism.
3. A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change. This third of Maxwell's equations is Faraday's law of induction, and includes Lenz's law.
4. Magnetic fields are generated by moving charges or by changing electric fields. This fourth of Maxwell's equations encompasses Ampere's law and adds another source of magnetism—changing electric fields.

Maxwell's equations encompass the major laws of electricity and magnetism. What is not so apparent is the symmetry that Maxwell introduced in his mathematical framework. Especially important is his addition of the hypothesis that changing electric fields create magnetic fields. This is exactly analogous (and symmetric) to Faraday's law of induction and had been suspected for some time, but fits beautifully into Maxwell's equations.

Symmetry is apparent in nature in a wide range of situations. In contemporary research, symmetry plays a major part in the search for subatomic particles using massive multinational particle accelerators such as the new Large Hadron Collider at CERN.

Note:

Making Connections: Unification of Forces

Maxwell's complete and symmetric theory showed that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electrical, strong, and weak nuclear forces.

Since changing electric fields create relatively weak magnetic fields, they could not be easily detected at the time of Maxwell's hypothesis. Maxwell realized, however, that oscillating charges, like those in AC circuits, produce changing electric fields. He predicted that these changing fields would propagate from the source like waves generated on a lake by a jumping fish.

The waves predicted by Maxwell would consist of oscillating electric and magnetic fields—defined to be an electromagnetic wave (EM wave). Electromagnetic waves would be capable of exerting forces on charges great distances from their source, and they might thus be detectable. Maxwell calculated that electromagnetic waves would propagate at a speed given by the equation

Equation:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}.$$

When the values for μ_0 and ϵ_0 are entered into the equation for c , we find that

Equation:

$$c = \frac{1}{\sqrt{(8.85 \times 10^{-12} \frac{\text{C}^2}{\text{N} \cdot \text{m}^2})(4\pi \times 10^{-7} \frac{\text{T} \cdot \text{m}}{\text{A}})}} = 3.00 \times 10^8 \text{ m/s},$$

which is the speed of light. In fact, Maxwell concluded that light is an electromagnetic wave having such wavelengths that it can be detected by the eye.

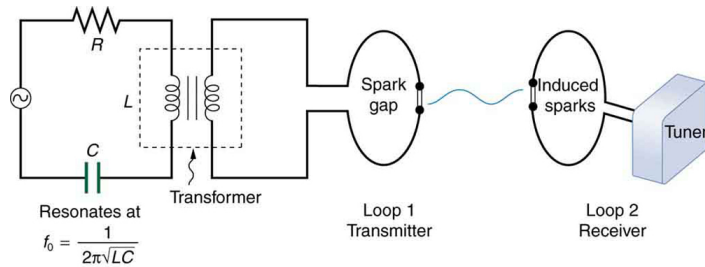
Other wavelengths should exist—it remained to be seen if they did. If so, Maxwell's theory and remarkable predictions would be verified, the greatest triumph of physics since Newton. Experimental verification came within a few years, but not before Maxwell's death.

Hertz's Observations

The German physicist Heinrich Hertz (1857–1894) was the first to generate and detect certain types of electromagnetic waves in the laboratory. Starting in 1887, he performed a series of experiments that not only confirmed the existence of electromagnetic waves, but also verified that they travel at the speed of light.

Hertz used an AC RLC (resistor-inductor-capacitor) circuit that resonates at a known frequency $f_0 = \frac{1}{2\pi\sqrt{LC}}$ and connected it to a loop of wire as shown in [\[link\]](#). High voltages induced across the gap in the loop produced sparks that were visible evidence of the current in the circuit and that helped generate electromagnetic waves.

Across the laboratory, Hertz had another loop attached to another RLC circuit, which could be tuned (as the dial on a radio) to the same resonant frequency as the first and could, thus, be made to receive electromagnetic waves. This loop also had a gap across which sparks were generated, giving solid evidence that electromagnetic waves had been received.



The apparatus used by Hertz in 1887 to generate and detect electromagnetic waves. An RLC circuit connected to the first loop caused sparks across a gap in the wire loop and generated electromagnetic waves. Sparks across a gap in the second loop located across the laboratory gave evidence that the waves had been received.

Hertz also studied the reflection, refraction, and interference patterns of the electromagnetic waves he generated, verifying their wave character. He was able to determine wavelength from the interference patterns, and knowing their frequency, he could calculate the propagation speed using the equation $v = f\lambda$ (velocity—or speed—equals frequency times wavelength). Hertz was thus able to prove that electromagnetic waves travel at the speed of light. The SI unit for frequency, the hertz (1 Hz = 1 cycle/sec), is named in his honor.

Section Summary

- Electromagnetic waves consist of oscillating electric and magnetic fields and propagate at the speed of light c . They were predicted by Maxwell, who also showed that

Equation:

$$c = \frac{1}{\sqrt{\mu_0 \epsilon_0}},$$

where μ_0 is the permeability of free space and ϵ_0 is the permittivity of free space.

- Maxwell's prediction of electromagnetic waves resulted from his formulation of a complete and symmetric theory of electricity and magnetism, known as Maxwell's equations.
- These four equations are paraphrased in this text, rather than presented numerically, and encompass the major laws of electricity and magnetism. First is Gauss's law for electricity, second is Gauss's law for magnetism, third is Faraday's law of induction, including Lenz's law, and fourth is Ampere's law in a symmetric formulation that adds another source of magnetism—changing electric fields.

Problems & Exercises

Exercise:

Problem:

Verify that the correct value for the speed of light c is obtained when numerical values for the permeability and permittivity of free space (μ_0 and ϵ_0) are entered into the equation $c = \frac{1}{\sqrt{\mu_0 \epsilon_0}}$.

Exercise:

Problem:

Show that, when SI units for μ_0 and ϵ_0 are entered, the units given by the right-hand side of the equation in the problem above are m/s.

Glossary

electromagnetic waves

radiation in the form of waves of electric and magnetic energy

Maxwell's equations

a set of four equations that comprise a complete, overarching theory of electromagnetism

RLC circuit

an electric circuit that includes a resistor, capacitor and inductor

hertz

an SI unit denoting the frequency of an electromagnetic wave, in cycles per second

speed of light

in a vacuum, such as space, the speed of light is a constant 3×10^8 m/s

electromotive force (emf)

energy produced per unit charge, drawn from a source that produces an electrical current

electric field lines

a pattern of imaginary lines that extend between an electric source and charged objects in the surrounding area, with arrows pointed away from positively charged objects and toward negatively charged objects. The more lines in the pattern, the stronger the electric field in that region

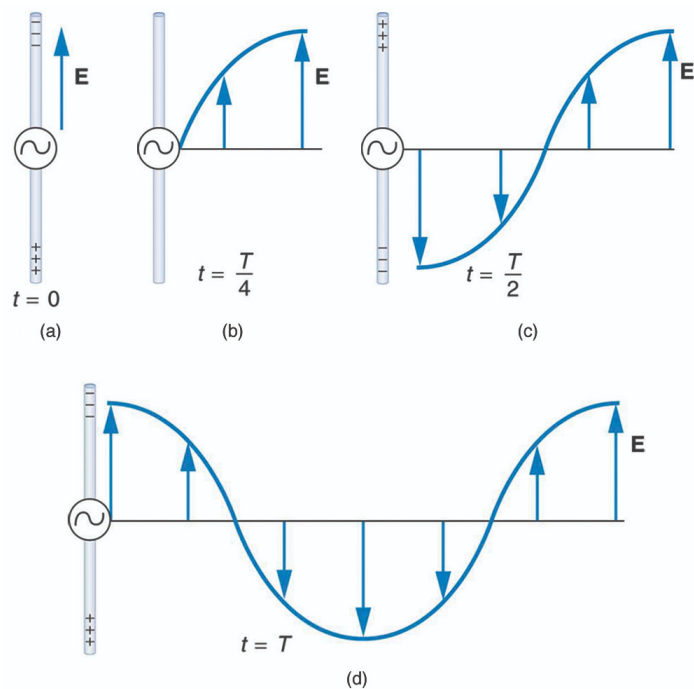
magnetic field lines

a pattern of continuous, imaginary lines that emerge from and enter into opposite magnetic poles. The density of the lines indicates the magnitude of the magnetic field

Production of Electromagnetic Waves

- Describe the electric and magnetic waves as they move out from a source, such as an AC generator.
- Explain the mathematical relationship between the magnetic field strength and the electrical field strength.
- Calculate the maximum strength of the magnetic field in an electromagnetic wave, given the maximum electric field strength.

We can get a good understanding of **electromagnetic waves** (EM) by considering how they are produced. Whenever a current varies, associated electric and magnetic fields vary, moving out from the source like waves. Perhaps the easiest situation to visualize is a varying current in a long straight wire, produced by an AC generator at its center, as illustrated in [\[link\]](#).



This long straight gray wire with an AC generator at its center becomes a broadcast antenna for electromagnetic waves. Shown here are the charge distributions at four different times. The electric field (**E**) propagates away

from the antenna at the speed of light,
forming part of an electromagnetic
wave.

The **electric field** (**E**) shown surrounding the wire is produced by the charge distribution on the wire. Both the **E** and the charge distribution vary as the current changes. The changing field propagates outward at the speed of light.

There is an associated **magnetic field** (**B**) which propagates outward as well (see [\[link\]](#)). The electric and magnetic fields are closely related and propagate as an electromagnetic wave. This is what happens in broadcast antennae such as those in radio and TV stations.

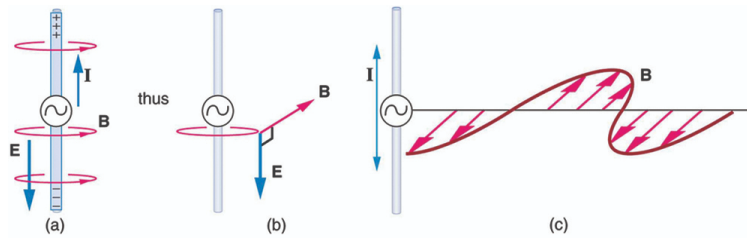
Closer examination of the one complete cycle shown in [\[link\]](#) reveals the periodic nature of the generator-driven charges oscillating up and down in the antenna and the electric field produced. At time $t = 0$, there is the maximum separation of charge, with negative charges at the top and positive charges at the bottom, producing the maximum magnitude of the electric field (or E -field) in the upward direction. One-fourth of a cycle later, there is no charge separation and the field next to the antenna is zero, while the maximum E -field has moved away at speed c .

As the process continues, the charge separation reverses and the field reaches its maximum downward value, returns to zero, and rises to its maximum upward value at the end of one complete cycle. The outgoing wave has an **amplitude** proportional to the maximum separation of charge. Its **wavelength**(λ) is proportional to the period of the oscillation and, hence, is smaller for short periods or high frequencies. (As usual, wavelength and **frequency**(f) are inversely proportional.)

Electric and Magnetic Waves: Moving Together

Following Ampere's law, current in the antenna produces a magnetic field, as shown in [\[link\]](#). The relationship between **E** and **B** is shown at one

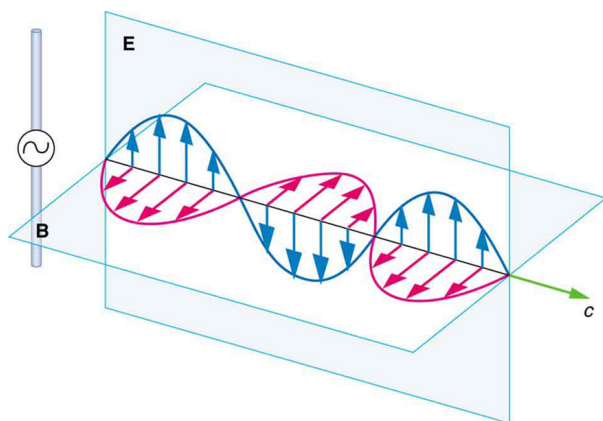
instant in [\[link\]](#) (a). As the current varies, the magnetic field varies in magnitude and direction.



(a) The current in the antenna produces the circular magnetic field lines. The current (I) produces the separation of charge along the wire, which in turn creates the electric field as shown. (b) The electric and magnetic fields ($\mathbf{E}$ and $\mathbf{B}$) near the wire are perpendicular; they are shown here for one point in space. (c) The magnetic field varies with current and propagates away from the antenna at the speed of light.

The magnetic field lines also propagate away from the antenna at the speed of light, forming the other part of the electromagnetic wave, as seen in [\[link\]](#) (b). The magnetic part of the wave has the same period and wavelength as the electric part, since they are both produced by the same movement and separation of charges in the antenna.

The electric and magnetic waves are shown together at one instant in time in [\[link\]](#). The electric and magnetic fields produced by a long straight wire antenna are exactly in phase. Note that they are perpendicular to one another and to the direction of propagation, making this a **transverse wave**.



A part of the electromagnetic wave sent out from the antenna at one instant in time. The electric and magnetic fields (**E** and **B**) are in phase, and they are perpendicular to one another and the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

Electromagnetic waves generally propagate out from a source in all directions, sometimes forming a complex radiation pattern. A linear antenna like this one will not radiate parallel to its length, for example. The wave is shown in one direction from the antenna in [\[link\]](#) to illustrate its basic characteristics.

Instead of the AC generator, the antenna can also be driven by an AC circuit. In fact, charges radiate whenever they are accelerated. But while a current in a circuit needs a complete path, an antenna has a varying charge distribution forming a **standing wave**, driven by the AC. The dimensions of the antenna are critical for determining the frequency of the radiated electromagnetic waves. This is a **resonant** phenomenon and when we tune

radios or TV, we vary electrical properties to achieve appropriate resonant conditions in the antenna.

Receiving Electromagnetic Waves

Electromagnetic waves carry energy away from their source, similar to a sound wave carrying energy away from a standing wave on a guitar string. An antenna for receiving EM signals works in reverse. And like antennas that produce EM waves, receiver antennas are specially designed to resonate at particular frequencies.

An incoming electromagnetic wave accelerates electrons in the antenna, setting up a standing wave. If the radio or TV is switched on, electrical components pick up and amplify the signal formed by the accelerating electrons. The signal is then converted to audio and/or video format. Sometimes big receiver dishes are used to focus the signal onto an antenna.

In fact, charges radiate whenever they are accelerated. When designing circuits, we often assume that energy does not quickly escape AC circuits, and mostly this is true. A broadcast antenna is specially designed to enhance the rate of electromagnetic radiation, and shielding is necessary to keep the radiation close to zero. Some familiar phenomena are based on the production of electromagnetic waves by varying currents. Your microwave oven, for example, sends electromagnetic waves, called microwaves, from a concealed antenna that has an oscillating current imposed on it.

Relating E -Field and B -Field Strengths

There is a relationship between the E - and B -field strengths in an electromagnetic wave. This can be understood by again considering the antenna just described. The stronger the E -field created by a separation of charge, the greater the current and, hence, the greater the B -field created.

Since current is directly proportional to voltage (Ohm's law) and voltage is directly proportional to E -field strength, the two should be directly proportional. It can be shown that the magnitudes of the fields do have a constant ratio, equal to the speed of light. That is,

Equation:

$$\frac{E}{B} = c$$

is the ratio of E -field strength to B -field strength in any electromagnetic wave. This is true at all times and at all locations in space. A simple and elegant result.

Example:**Calculating B -Field Strength in an Electromagnetic Wave**

What is the maximum strength of the B -field in an electromagnetic wave that has a maximum E -field strength of 1000 V/m?

Strategy

To find the B -field strength, we rearrange the above equation to solve for B , yielding

Equation:

$$B = \frac{E}{c}.$$

Solution

We are given E , and c is the speed of light. Entering these into the expression for B yields

Equation:

$$B = \frac{1000 \text{ V/m}}{3.00 \times 10^8 \text{ m/s}} = 3.33 \times 10^{-6} \text{ T},$$

Where T stands for Tesla, a measure of magnetic field strength.

Discussion

The B -field strength is less than a tenth of the Earth's admittedly weak magnetic field. This means that a relatively strong electric field of 1000 V/m is accompanied by a relatively weak magnetic field. Note that as this

wave spreads out, say with distance from an antenna, its field strengths become progressively weaker.

The result of this example is consistent with the statement made in the module [Maxwell's Equations: Electromagnetic Waves Predicted and Observed](#) that changing electric fields create relatively weak magnetic fields. They can be detected in electromagnetic waves, however, by taking advantage of the phenomenon of resonance, as Hertz did. A system with the same natural frequency as the electromagnetic wave can be made to oscillate. All radio and TV receivers use this principle to pick up and then amplify weak electromagnetic waves, while rejecting all others not at their resonant frequency.

Note:

Take-Home Experiment: Antennas

For your TV or radio at home, identify the antenna, and sketch its shape. If you don't have cable, you might have an outdoor or indoor TV antenna. Estimate its size. If the TV signal is between 60 and 216 MHz for basic channels, then what is the wavelength of those EM waves?

Try tuning the radio and note the small range of frequencies at which a reasonable signal for that station is received. (This is easier with digital readout.) If you have a car with a radio and extendable antenna, note the quality of reception as the length of the antenna is changed.

Note:

PhET Explorations: Radio Waves and Electromagnetic Fields

Broadcast radio waves from KPhET. Wiggle the transmitter electron manually or have it oscillate automatically. Display the field as a curve or vectors. The strip chart shows the electron positions at the transmitter and at the receiver.

<https://archive.cnx.org/specials/c8dd764c-ae74-11e5-af4c-3375261fa183/radio-waves/#sim-radio-waves>

Section Summary

- Electromagnetic waves are created by oscillating charges (which radiate whenever accelerated) and have the same frequency as the oscillation.
- Since the electric and magnetic fields in most electromagnetic waves are perpendicular to the direction in which the wave moves, it is ordinarily a transverse wave.
- The strengths of the electric and magnetic parts of the wave are related by
Equation:

$$\frac{E}{B} = c,$$

which implies that the magnetic field B is very weak relative to the electric field E .

Conceptual Questions

Exercise:

Problem:

The direction of the electric field shown in each part of [\[link\]](#) is that produced by the charge distribution in the wire. Justify the direction shown in each part, using the Coulomb force law and the definition of $\mathbf{E} = \mathbf{F}/q$, where q is a positive test charge.

Exercise:

Problem:

Is the direction of the magnetic field shown in [\[link\]](#) (a) consistent with the right-hand rule for current (RHR-2) in the direction shown in the figure?

Exercise:

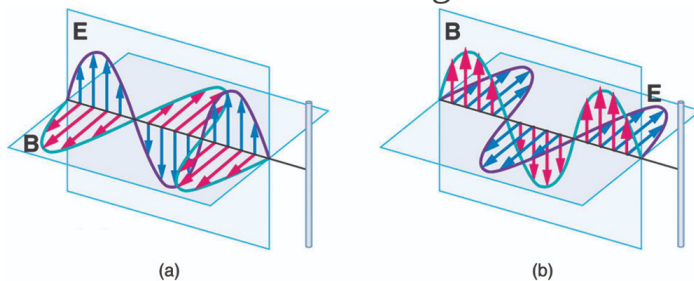
Problem:

Why is the direction of the current shown in each part of [\[link\]](#) opposite to the electric field produced by the wire's charge separation?

Exercise:

Problem:

In which situation shown in [\[link\]](#) will the electromagnetic wave be more successful in inducing a current in the wire? Explain.

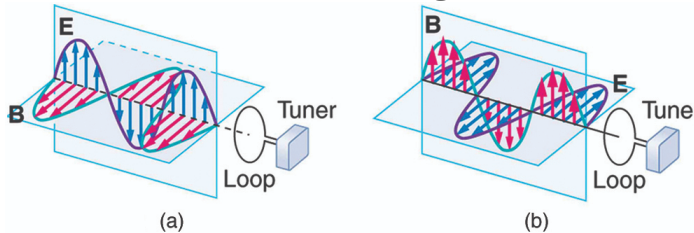


Electromagnetic waves approaching long straight wires.

Exercise:

Problem:

In which situation shown in [\[link\]](#) will the electromagnetic wave be more successful in inducing a current in the loop? Explain.



Electromagnetic waves approaching a wire loop.

Exercise:

Problem:

Should the straight wire antenna of a radio be vertical or horizontal to best receive radio waves broadcast by a vertical transmitter antenna? How should a loop antenna be aligned to best receive the signals? (Note that the direction of the loop that produces the best reception can be used to determine the location of the source. It is used for that purpose in tracking tagged animals in nature studies, for example.)

Exercise:

Problem:

Under what conditions might wires in a DC circuit emit electromagnetic waves?

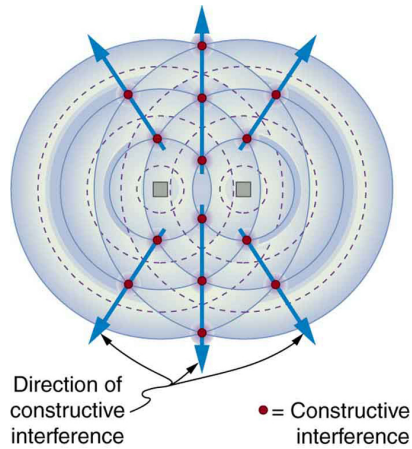
Exercise:

Problem: Give an example of interference of electromagnetic waves.

Exercise:

Problem:

[\[link\]](#) shows the interference pattern of two radio antennas broadcasting the same signal. Explain how this is analogous to the interference pattern for sound produced by two speakers. Could this be used to make a directional antenna system that broadcasts preferentially in certain directions? Explain.



An overhead view
of two radio
broadcast antennas
sending the same
signal, and the
interference pattern
they produce.

Exercise:

Problem: Can an antenna be any length? Explain your answer.

Problems & Exercises

Exercise:

Problem:

What is the maximum electric field strength in an electromagnetic wave that has a maximum magnetic field strength of $5.00 \times 10^{-4} \text{ T}$ (about 10 times the Earth's)?

Solution:

150 kV/m

Exercise:

Problem:

The maximum magnetic field strength of an electromagnetic field is 5×10^{-6} T. Calculate the maximum electric field strength if the wave is traveling in a medium in which the speed of the wave is $0.75c$.

Exercise:

Problem:

Verify the units obtained for magnetic field strength B in [\[link\]](#) (using the equation $B = \frac{E}{c}$) are in fact teslas (T).

Glossary

electric field

a vector quantity (**E**); the lines of electric force per unit charge, moving radially outward from a positive charge and in toward a negative charge

electric field strength

the magnitude of the electric field, denoted E -field

magnetic field

a vector quantity (**B**); can be used to determine the magnetic force on a moving charged particle

magnetic field strength

the magnitude of the magnetic field, denoted B -field

transverse wave

a wave, such as an electromagnetic wave, which oscillates perpendicular to the axis along the line of travel

standing wave

a wave that oscillates in place, with nodes where no motion happens

wavelength

the distance from one peak to the next in a wave

amplitude

the height, or magnitude, of an electromagnetic wave

frequency

the number of complete wave cycles (up-down-up) passing a given point within one second (cycles/second)

resonant

a system that displays enhanced oscillation when subjected to a periodic disturbance of the same frequency as its natural frequency

oscillate

to fluctuate back and forth in a steady beat

Energy in Electromagnetic Waves

- Explain how the energy and amplitude of an electromagnetic wave are related.
- Given its power output and the heating area, calculate the intensity of a microwave oven's electromagnetic field, as well as its peak electric and magnetic field strengths

Anyone who has used a microwave oven knows there is energy in **electromagnetic waves**. Sometimes this energy is obvious, such as in the warmth of the summer sun. Other times it is subtle, such as the unfelt energy of gamma rays, which can destroy living cells.

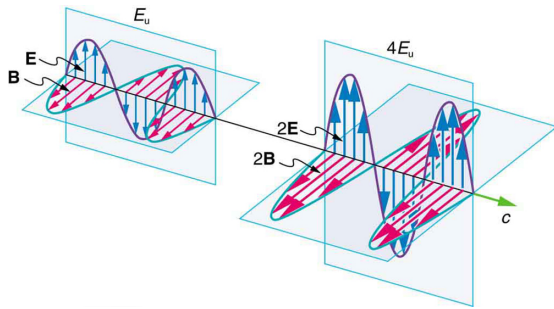
Electromagnetic waves can bring energy into a system by virtue of their **electric and magnetic fields**. These fields can exert forces and move charges in the system and, thus, do work on them. If the frequency of the electromagnetic wave is the same as the natural frequencies of the system (such as microwaves at the resonant frequency of water molecules), the transfer of energy is much more efficient.

Note:

Connections: Waves and Particles

The behavior of electromagnetic radiation clearly exhibits wave characteristics. But we shall find in later modules that at high frequencies, electromagnetic radiation also exhibits particle characteristics. These particle characteristics will be used to explain more of the properties of the electromagnetic spectrum and to introduce the formal study of modern physics.

Another startling discovery of modern physics is that particles, such as electrons and protons, exhibit wave characteristics. This simultaneous sharing of wave and particle properties for all submicroscopic entities is one of the great symmetries in nature.



Energy carried by a wave is proportional to its amplitude squared. With electromagnetic waves, larger E -fields and B -fields exert larger forces and can do more work.

But there is energy in an electromagnetic wave, whether it is absorbed or not. Once created, the fields carry energy away from a source. If absorbed, the field strengths are diminished and anything left travels on. Clearly, the larger the strength of the electric and magnetic fields, the more work they can do and the greater the energy the electromagnetic wave carries.

A wave's energy is proportional to its **amplitude** squared (E^2 or B^2). This is true for waves on guitar strings, for water waves, and for sound waves, where amplitude is proportional to pressure. In electromagnetic waves, the amplitude is the **maximum field strength** of the electric and magnetic fields. (See [\[link\]](#).)

Thus the energy carried and the **intensity** I of an electromagnetic wave is proportional to E^2 and B^2 . In fact, for a continuous sinusoidal electromagnetic wave, the average intensity I_{ave} is given by

Equation:

$$I_{\text{ave}} = \frac{c\epsilon_0 E_0^2}{2},$$

where c is the speed of light, ϵ_0 is the permittivity of free space, and E_0 is the maximum electric field strength; intensity, as always, is power per unit area (here in W/m^2).

The average intensity of an electromagnetic wave I_{ave} can also be expressed in terms of the magnetic field strength by using the relationship $B = E/c$, and the fact that $\epsilon_0 = 1/\mu_0 c^2$, where μ_0 is the permeability of free space. Algebraic manipulation produces the relationship

Equation:

$$I_{\text{ave}} = \frac{cB_0^2}{2\mu_0},$$

where B_0 is the maximum magnetic field strength.

One more expression for I_{ave} in terms of both electric and magnetic field strengths is useful. Substituting the fact that $c \cdot B_0 = E_0$, the previous expression becomes

Equation:

$$I_{\text{ave}} = \frac{E_0 B_0}{2\mu_0}.$$

Whichever of the three preceding equations is most convenient can be used, since they are really just different versions of the same principle: Energy in a wave is related to amplitude squared. Furthermore, since these equations are based on the assumption that the electromagnetic waves are sinusoidal, peak intensity is twice the average; that is, $I_0 = 2I_{\text{ave}}$.

Example:

Calculate Microwave Intensities and Fields

On its highest power setting, a certain microwave oven projects 1.00 kW of microwaves onto a 30.0 by 40.0 cm area. (a) What is the intensity in

W/m²? (b) Calculate the peak electric field strength E_0 in these waves.
(c) What is the peak magnetic field strength B_0 ?

Strategy

In part (a), we can find intensity from its definition as power per unit area. Once the intensity is known, we can use the equations below to find the field strengths asked for in parts (b) and (c).

Solution for (a)

Entering the given power into the definition of intensity, and noting the area is 0.300 by 0.400 m, yields

Equation:

$$I = \frac{P}{A} = \frac{1.00 \text{ kW}}{0.300 \text{ m} \times 0.400 \text{ m}}.$$

Here $I = I_{\text{ave}}$, so that

Equation:

$$I_{\text{ave}} = \frac{1000 \text{ W}}{0.120 \text{ m}^2} = 8.33 \times 10^3 \text{ W/m}^2.$$

Note that the peak intensity is twice the average:

Equation:

$$I_0 = 2I_{\text{ave}} = 1.67 \times 10^4 \text{ W/m}^2.$$

Solution for (b)

To find E_0 , we can rearrange the first equation given above for I_{ave} to give

Equation:

$$E_0 = \left(\frac{2I_{\text{ave}}}{c\epsilon_0} \right)^{1/2}.$$

Entering known values gives

Equation:

$$\begin{aligned}
 E_0 &= \sqrt{\frac{2(8.33 \times 10^3 \text{ W/m}^2)}{(3.00 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)}} \\
 &= 2.51 \times 10^3 \text{ V/m}.
 \end{aligned}$$

Solution for (c)

Perhaps the easiest way to find magnetic field strength, now that the electric field strength is known, is to use the relationship given by

Equation:

$$B_0 = \frac{E_0}{c}.$$

Entering known values gives

Equation:

$$\begin{aligned}
 B_0 &= \frac{2.51 \times 10^3 \text{ V/m}}{3.0 \times 10^8 \text{ m/s}} \\
 &= 8.35 \times 10^{-6} \text{ T}.
 \end{aligned}$$

Discussion

As before, a relatively strong electric field is accompanied by a relatively weak magnetic field in an electromagnetic wave, since $B = E/c$, and c is a large number.

Section Summary

- The energy carried by any wave is proportional to its amplitude squared. For electromagnetic waves, this means intensity can be expressed as

Equation:

$$I_{\text{ave}} = \frac{c\epsilon_0 E_0^2}{2},$$

where I_{ave} is the average intensity in W/m^2 , and E_0 is the maximum electric field strength of a continuous sinusoidal wave.

- This can also be expressed in terms of the maximum magnetic field strength B_0 as

Equation:

$$I_{\text{ave}} = \frac{cB_0^2}{2\mu_0}$$

and in terms of both electric and magnetic fields as

Equation:

$$I_{\text{ave}} = \frac{E_0 B_0}{2\mu_0}.$$

- The three expressions for I_{ave} are all equivalent.

Problems & Exercises

Exercise:

Problem:

What is the intensity of an electromagnetic wave with a peak electric field strength of 125 V/m?

Solution:

Equation:

$$\begin{aligned} I &= \frac{c\varepsilon_0 E_0^2}{2} \\ &= \frac{(3.00 \times 10^8 \text{ m/s})(8.85 \times 10^{-12} \text{ C}^2/\text{N}\cdot\text{m}^2)(125 \text{ V/m})^2}{2} \\ &= 20.7 \text{ W/m}^2 \end{aligned}$$

Exercise:

Problem:

Find the intensity of an electromagnetic wave having a peak magnetic field strength of $4.00 \times 10^{-9} \text{ T}$.

Exercise:**Problem:**

Assume the helium-neon lasers commonly used in student physics laboratories have power outputs of 0.250 mW. (a) If such a laser beam is projected onto a circular spot 1.00 mm in diameter, what is its intensity? (b) Find the peak magnetic field strength. (c) Find the peak electric field strength.

Solution:

$$(a) \ I = \frac{P}{A} = \frac{P}{\pi r^2} = \frac{0.250 \times 10^{-3} \text{ W}}{\pi (0.500 \times 10^{-3} \text{ m})^2} = 318 \text{ W/m}^2$$

$$\begin{aligned} I_{\text{ave}} &= \frac{cB_0^2}{2\mu_0} \Rightarrow B_0 = \left(\frac{2\mu_0 I}{c} \right)^{1/2} \\ (b) \quad &= \left(\frac{2(4\pi \times 10^{-7} \text{ T}\cdot\text{m/A})(318.3 \text{ W/m}^2)}{3.00 \times 10^8 \text{ m/s}} \right)^{1/2} \\ &= 1.63 \times 10^{-6} \text{ T} \end{aligned}$$

$$\begin{aligned} (c) \quad E_0 &= cB_0 = (3.00 \times 10^8 \text{ m/s})(1.633 \times 10^{-6} \text{ T}) \\ &= 4.90 \times 10^2 \text{ V/m} \end{aligned}$$

Exercise:

Problem:

An AM radio transmitter broadcasts 50.0 kW of power uniformly in all directions. (a) Assuming all of the radio waves that strike the ground are completely absorbed, and that there is no absorption by the atmosphere or other objects, what is the intensity 30.0 km away? (Hint: Half the power will be spread over the area of a hemisphere.) (b) What is the maximum electric field strength at this distance?

Exercise:**Problem:**

Suppose the maximum safe intensity of microwaves for human exposure is taken to be 1.00 W/m^2 . (a) If a radar unit leaks 10.0 W of microwaves (other than those sent by its antenna) uniformly in all directions, how far away must you be to be exposed to an intensity considered to be safe? Assume that the power spreads uniformly over the area of a sphere with no complications from absorption or reflection. (b) What is the maximum electric field strength at the safe intensity? (Note that early radar units leaked more than modern ones do. This caused identifiable health problems, such as cataracts, for people who worked near them.)

Solution:

(a) 89.2 cm

(b) 27.4 V/m

Exercise:

Problem:

A 2.50-m-diameter university communications satellite dish receives TV signals that have a maximum electric field strength (for one channel) of $7.50\ \mu\text{V}/\text{m}$. (See [\[link\]](#).) (a) What is the intensity of this wave? (b) What is the power received by the antenna? (c) If the orbiting satellite broadcasts uniformly over an area of $1.50 \times 10^{13}\ \text{m}^2$ (a large fraction of North America), how much power does it radiate?



Satellite dishes receive TV signals sent from orbit. Although the signals are quite weak, the receiver can detect them by being tuned to resonate at their frequency.

Exercise:

Problem:

Lasers can be constructed that produce an extremely high intensity electromagnetic wave for a brief time—called pulsed lasers. They are used to ignite nuclear fusion, for example. Such a laser may produce an electromagnetic wave with a maximum electric field strength of $1.00 \times 10^{11} \text{ V/m}$ for a time of 1.00 ns. (a) What is the maximum magnetic field strength in the wave? (b) What is the intensity of the beam? (c) What energy does it deliver on a 1.00-mm^2 area?

Solution:

(a) 333 T

(b) $1.33 \times 10^{19} \text{ W/m}^2$

(c) 13.3 kJ

Exercise:**Problem:**

Show that for a continuous sinusoidal electromagnetic wave, the peak intensity is twice the average intensity ($I_0 = 2I_{\text{ave}}$), using either the fact that $E_0 = \sqrt{2}E_{\text{rms}}$, or $B_0 = \sqrt{2}B_{\text{rms}}$, where rms means average (actually root mean square, a type of average).

Exercise:**Problem:**

Suppose a source of electromagnetic waves radiates uniformly in all directions in empty space where there are no absorption or interference effects. (a) Show that the intensity is inversely proportional to r^2 , the distance from the source squared. (b) Show that the magnitudes of the electric and magnetic fields are inversely proportional to r .

Solution:

$$(a) I = \frac{P}{A} = \frac{P}{4\pi r^2} \propto \frac{1}{r^2}$$

$$(b) I \propto E_0^2, B_0^2 \Rightarrow E_0^2, B_0^2 \propto \frac{1}{r^2} \Rightarrow E_0, B_0 \propto \frac{1}{r}$$

Exercise:

Problem: Integrated Concepts

An LC circuit with a 5.00-pF capacitor oscillates in such a manner as to radiate at a wavelength of 3.30 m. (a) What is the resonant frequency? (b) What inductance is in series with the capacitor?

Exercise:

Problem: Integrated Concepts

What capacitance is needed in series with an $800\text{ } \mu\text{H}$ inductor to form a circuit that radiates a wavelength of 196 m?

Solution:

13.5 pF

Exercise:

Problem: Integrated Concepts

Police radar determines the speed of motor vehicles using the same Doppler-shift technique employed for ultrasound in medical diagnostics. Beats are produced by mixing the double Doppler-shifted echo with the original frequency. If 1.50×10^9 -Hz microwaves are used and a beat frequency of 150 Hz is produced, what is the speed of the vehicle? (Assume the same Doppler-shift formulas are valid with the speed of sound replaced by the speed of light.)

Exercise:

Problem: Integrated Concepts

Assume the mostly infrared radiation from a heat lamp acts like a continuous wave with wavelength $1.50\ \mu\text{m}$. (a) If the lamp's 200-W output is focused on a person's shoulder, over a circular area 25.0 cm in diameter, what is the intensity in W/m^2 ? (b) What is the peak electric field strength? (c) Find the peak magnetic field strength. (d) How long will it take to increase the temperature of the 4.00-kg shoulder by 2.00°C , assuming no other heat transfer and given that its specific heat is $3.47 \times 10^3\ \text{J}/\text{kg}\cdot^\circ\text{C}$?

Solution:

(a) $4.07\ \text{kW}/\text{m}^2$

(b) $1.75\ \text{kV}/\text{m}$

(c) $5.84\ \mu\text{T}$

(d) 2 min 19 s

Exercise:

Problem: Integrated Concepts

On its highest power setting, a microwave oven increases the temperature of 0.400 kg of spaghetti by 45.0°C in 120 s. (a) What was the rate of power absorption by the spaghetti, given that its specific heat is $3.76 \times 10^3\ \text{J}/\text{kg}\cdot^\circ\text{C}$? (b) Find the average intensity of the microwaves, given that they are absorbed over a circular area 20.0 cm in diameter. (c) What is the peak electric field strength of the microwave? (d) What is its peak magnetic field strength?

Exercise:

Problem: Integrated Concepts

Electromagnetic radiation from a 5.00-mW laser is concentrated on a 1.00-mm^2 area. (a) What is the intensity in W/m^2 ? (b) Suppose a 2.00-nC static charge is in the beam. What is the maximum electric

force it experiences? (c) If the static charge moves at 400 m/s, what maximum magnetic force can it feel?

Solution:

(a) $5.00 \times 10^3 \text{ W/m}^2$

(b) $3.88 \times 10^{-6} \text{ N}$

(c) $5.18 \times 10^{-12} \text{ N}$

Exercise:

Problem: Integrated Concepts

A 200-turn flat coil of wire 30.0 cm in diameter acts as an antenna for FM radio at a frequency of 100 MHz. The magnetic field of the incoming electromagnetic wave is perpendicular to the coil and has a maximum strength of $1.00 \times 10^{-12} \text{ T}$. (a) What power is incident on the coil? (b) What average emf is induced in the coil over one-fourth of a cycle? (c) If the radio receiver has an inductance of $2.50 \mu\text{H}$, what capacitance must it have to resonate at 100 MHz?

Exercise:

Problem: Integrated Concepts

If electric and magnetic field strengths vary sinusoidally in time, being zero at $t = 0$, then $E = E_0 \sin 2\pi ft$ and $B = B_0 \sin 2\pi ft$. Let $f = 1.00 \text{ GHz}$ here. (a) When are the field strengths first zero? (b) When do they reach their most negative value? (c) How much time is needed for them to complete one cycle?

Solution:

(a) $t = 0$

(b) $7.50 \times 10^{-10} \text{ s}$

(c) $1.00 \times 10^{-9} \text{ s}$

Exercise:

Problem: Unreasonable Results

A researcher measures the wavelength of a 1.20-GHz electromagnetic wave to be 0.500 m. (a) Calculate the speed at which this wave propagates. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Exercise:

Problem: Unreasonable Results

The peak magnetic field strength in a residential microwave oven is $9.20 \times 10^{-5} \text{ T}$. (a) What is the intensity of the microwave? (b) What is unreasonable about this result? (c) What is wrong about the premise?

Solution:

(a) $1.01 \times 10^6 \text{ W/m}^2$

(b) Much too great for an oven.

(c) The assumed magnetic field is unreasonably large.

Exercise:

Problem: Unreasonable Results

An LC circuit containing a 2.00-H inductor oscillates at such a frequency that it radiates at a 1.00-m wavelength. (a) What is the capacitance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Exercise:

Problem: Unreasonable Results

An LC circuit containing a 1.00-pF capacitor oscillates at such a frequency that it radiates at a 300-nm wavelength. (a) What is the inductance of the circuit? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $2.53 \times 10^{-20} \text{ H}$

(b) L is much too small.

(c) The wavelength is unreasonably small.

Exercise:

Problem: Create Your Own Problem

Consider electromagnetic fields produced by high voltage power lines. Construct a problem in which you calculate the intensity of this electromagnetic radiation in W/m^2 based on the measured magnetic field strength of the radiation in a home near the power lines. Assume these magnetic field strengths are known to average less than a μT . The intensity is small enough that it is difficult to imagine mechanisms for biological damage due to it. Discuss how much energy may be radiating from a section of power line several hundred meters long and compare this to the power likely to be carried by the lines. An idea of how much power this is can be obtained by calculating the approximate current responsible for μT fields at distances of tens of meters.

Exercise:

Problem: Create Your Own Problem

Consider the most recent generation of residential satellite dishes that are a little less than half a meter in diameter. Construct a problem in which you calculate the power received by the dish and the maximum electric field strength of the microwave signals for a single channel

received by the dish. Among the things to be considered are the power broadcast by the satellite and the area over which the power is spread, as well as the area of the receiving dish.

Glossary

maximum field strength

the maximum amplitude an electromagnetic wave can reach, representing the maximum amount of electric force and/or magnetic flux that the wave can exert

intensity

the power of an electric or magnetic field per unit area, for example, Watts per square meter

Introduction to Fluid Statics

class="introduction"

The fluid
essential
to all life
has a
beauty of
its own.

It also
helps
support
the
weight of
this
swimmer
. (credit:
12019,
Pixabay)

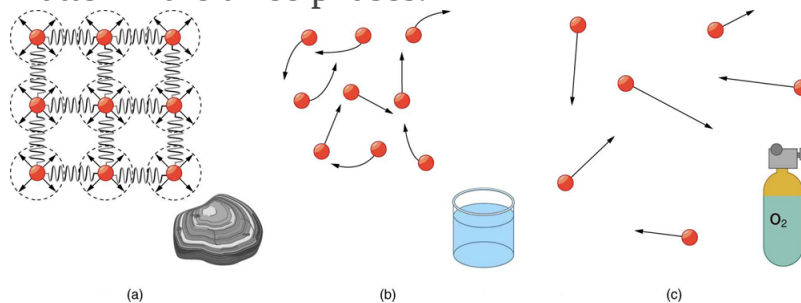


Much of what we value in life is fluid: a breath of fresh winter air; the hot blue flame in our gas cooker; the water we drink, swim in, and bathe in; the blood in our veins. What exactly is a fluid? Can we understand fluids with the laws already presented, or will new laws emerge from their study? The physical characteristics of static or stationary fluids and some of the laws that govern their behavior are the topics of this chapter. [Fluid Dynamics and Its Biological and Medical Applications](#) explores aspects of fluid flow.

What Is a Fluid?

- State the common phases of matter.
- Explain the physical characteristics of solids, liquids, and gases.
- Describe the arrangement of atoms in solids, liquids, and gases.

Matter most commonly exists as a solid, liquid, or gas; these states are known as the three common *phases of matter*. Solids have a definite shape and a specific volume, liquids have a definite volume but their shape changes depending on the container in which they are held, and gases have neither a definite shape nor a specific volume as their molecules move to fill the container in which they are held. (See [\[link\]](#).) Liquids and gases are considered to be fluids because they yield to shearing forces, whereas solids resist them. Note that the extent to which fluids yield to shearing forces (and hence flow easily and quickly) depends on a quantity called the viscosity which is discussed in detail in [Viscosity and Laminar Flow; Poiseuille's Law](#). We can understand the phases of matter and what constitutes a fluid by considering the forces between atoms that make up matter in the three phases.



(a) Atoms in a solid always have the same neighbors, held near home by forces represented here by springs. These atoms are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms together. (b) Atoms in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact.

Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its atoms. (c) Atoms in a gas are separated by distances that are considerably larger than the size of the atoms themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out freely.

Atoms in *solids* are in close contact, with forces between them that allow the atoms to vibrate but not to change positions with neighboring atoms. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.) Thus a solid *resists* all types of stress. A solid cannot be easily deformed because the atoms that make up the solid are not able to move about freely. Solids also resist compression, because their atoms form part of a lattice structure in which the atoms are a relatively fixed distance apart. Under compression, the atoms would be forced into one another. Most of the examples we have studied so far have involved solid objects which deform very little when stressed.

Note:

Connections: Submicroscopic Explanation of Solids and Liquids

Atomic and molecular characteristics explain and underlie the macroscopic characteristics of solids and fluids. This submicroscopic explanation is one theme of this text and is highlighted in the Things Great and Small features in [Conservation of Momentum](#). See, for example, microscopic description of collisions and momentum or microscopic description of pressure in a gas. This present section is devoted entirely to the submicroscopic explanation of solids and liquids.

In contrast, *liquids* deform easily when stressed and do not spring back to their original shape once the force is removed because the atoms are free to slide about and change neighbors—that is, they *flow* (so they are a type of fluid), with the molecules held together by their mutual attraction. When a liquid is placed in a container with no lid on, it remains in the container (providing the container has no holes below the surface of the liquid!). Because the atoms are closely packed, liquids, like solids, resist compression.

Atoms in *gases* are separated by distances that are large compared with the size of the atoms. The forces between gas atoms are therefore very weak, except when the atoms collide with one another. Gases thus not only flow (and are therefore considered to be fluids) but they are relatively easy to compress because there is much space and little force between atoms. When placed in an open container gases, unlike liquids, will escape. The major distinction is that gases are easily compressed, whereas liquids are not. We shall generally refer to both gases and liquids simply as **fluids**, and make a distinction between them only when they behave differently.

Note:

PhET Explorations: States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.

https://phet.colorado.edu/sims/html/states-of-matter-basics/latest/states-of-matter-basics_en.html

Section Summary

- A fluid is a state of matter that yields to sideways or shearing forces. Liquids and gases are both fluids. Fluid statics is the physics of stationary fluids.

Conceptual Questions

Exercise:

Problem:

What physical characteristic distinguishes a fluid from a solid?

Exercise:

Problem:

Which of the following substances are fluids at room temperature: air, mercury, water, glass?

Exercise:

Problem: Why are gases easier to compress than liquids and solids?

Exercise:

Problem: How do gases differ from liquids?

Glossary

fluids

liquids and gases; a fluid is a state of matter that yields to shearing forces

Density

- Define density.
- Calculate the mass of a reservoir from its density.
- Compare and contrast the densities of various substances.

Which weighs more, a ton of feathers or a ton of bricks? This old riddle plays with the distinction between mass and density. A ton is a ton, of course; but bricks have much greater density than feathers, and so we are tempted to think of them as heavier. (See [\[link\]](#).)

Density, as you will see, is an important characteristic of substances. It is crucial, for example, in determining whether an object sinks or floats in a fluid. Density is the mass per unit volume of a substance or object. In equation form, density is defined as

Equation:

$$\rho = \frac{m}{V},$$

where the Greek letter ρ (rho) is the symbol for density, m is the mass, and V is the volume occupied by the substance.

Note:
Density
Density is mass per unit volume.
Equation:

$$\rho = \frac{m}{V},$$

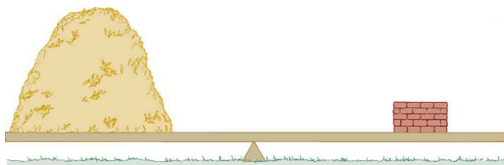
where ρ is the symbol for density, m is the mass, and V is the volume occupied by the substance.

In the riddle regarding the feathers and bricks, the masses are the same, but the volume occupied by the feathers is much greater, since their density is much lower. The SI unit of density is kg/m^3 , representative values are given in [\[link\]](#). The metric system was originally devised so that water would have a density of 1 g/cm^3 , equivalent to 10^3 kg/m^3 . Thus the basic mass unit, the kilogram, was first devised to be the mass of 1000 mL of water, which has a volume of 1000 cm^3 .

Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$
Solids		Liquids		Gases	
Aluminum	2.7	Water (4°C)	1.000	Air	

Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$	Substance	$\rho(10^3 \text{ kg/m}^3 \text{ or g/mL})$
Brass	8.44	Blood	1.05	Carbon dioxide	
Copper (average)	8.8	Sea water	1.025	Carbon monoxide	
Gold	19.32	Mercury	13.6	Hydrogen	
Iron or steel	7.8	Ethyl alcohol	0.79	Helium	
Lead	11.3	Petrol	0.68	Methane	
Polystyrene	0.10	Glycerin	1.26	Nitrogen	
Tungsten	19.30	Olive oil	0.92	Nitrous oxide	
Uranium	18.70			Oxygen	
Concrete	2.30–3.0			Steam (100° C)	
Cork	0.24				
Glass, common (average)	2.6				
Granite	2.7				
Earth's crust	3.3				
Wood	0.3–0.9				
Ice (0°C)	0.917				
Bone	1.7–2.0				

Densities of Various Substances



A ton of feathers and a ton of bricks have the same mass, but the feathers make a much bigger pile because they have a much lower density.

As you can see by examining [\[link\]](#), the density of an object may help identify its composition. The density of gold, for example, is about 2.5 times the density of iron, which is about 2.5 times the density of aluminum. Density also reveals something about the phase of the matter and its substructure. Notice that the densities of liquids and solids are roughly comparable, consistent with the fact that their atoms are in close contact. The densities of gases are much less than those of liquids and solids, because the atoms in gases are separated by large amounts of empty space.

Note:

Take-Home Experiment Sugar and Salt

A pile of sugar and a pile of salt look pretty similar, but which weighs more? If the volumes of both piles are the same, any difference in mass is due to their different densities (including the air space between crystals). Which do you think has the greater density? What values did you find? What method did you use to determine these values?

Example:

Calculating the Mass of a Reservoir From Its Volume

A reservoir has a surface area of 50.0 km^2 and an average depth of 40.0 m . What mass of water is held behind the dam? (See [\[link\]](#) for a view of a large reservoir—the Three Gorges Dam site on the Yangtze River in central China.)

Strategy

We can calculate the volume V of the reservoir from its dimensions, and find the density of water ρ in [\[link\]](#). Then the mass m can be found from the definition of density

Equation:

$$\rho = \frac{m}{V}.$$

Solution

Solving equation $\rho = m/V$ for m gives $m = \rho V$.

The volume V of the reservoir is its surface area A times its average depth h :

Equation:

$$\begin{aligned} V &= Ah = (50.0 \text{ km}^2)(40.0 \text{ m}) \\ &= \left[(50.0 \text{ km}^2) \left(\frac{10^3 \text{ m}}{1 \text{ km}} \right)^2 \right] (40.0 \text{ m}) = 2.00 \times 10^9 \text{ m}^3 \end{aligned}$$

The density of water ρ from [\[link\]](#) is $1.000 \times 10^3 \text{ kg/m}^3$. Substituting V and ρ into the expression for mass gives

Equation:

$$\begin{aligned}
 m &= (1.00 \times 10^3 \text{ kg/m}^3)(2.00 \times 10^9 \text{ m}^3) \\
 &= 2.00 \times 10^{12} \text{ kg}.
 \end{aligned}$$

Discussion

A large reservoir contains a very large mass of water. In this example, the weight of the water in the reservoir is $mg = 1.96 \times 10^{13} \text{ N}$, where g is the acceleration due to the Earth's gravity (about 9.80 m/s^2). It is reasonable to ask whether the dam must supply a force equal to this tremendous weight. The answer is no. As we shall see in the following sections, the force the dam must supply can be much smaller than the weight of the water it holds back.



Three Gorges Dam in central China. When completed in 2008, this became the world's largest hydroelectric plant, generating power equivalent to that generated by 22 average-sized nuclear power plants. The concrete dam is 181 m high and 2.3 km across. The reservoir made by this dam is 660 km long. Over 1 million people were displaced by the creation of the reservoir. (credit: Le Grand Portage)

Section Summary

- Density is the mass per unit volume of a substance or object. In equation form, density is defined as
Equation:

$$\rho = \frac{m}{V}.$$

- The SI unit of density is kg/m^3 .

Conceptual Questions

Exercise:

Problem: Approximately how does the density of air vary with altitude?

Exercise:

Problem:

Give an example in which density is used to identify the substance composing an object. Would information in addition to average density be needed to identify the substances in an object composed of more than one material?

Exercise:**Problem:**

[\[link\]](#) shows a glass of ice water filled to the brim. Will the water overflow when the ice melts? Explain your answer.

**Problems & Exercises****Exercise:**

Problem: Gold is sold by the troy ounce (31.103 g). What is the volume of 1 troy ounce of pure gold?

Solution:

1.610 cm³

Exercise:**Problem:**

Mercury is commonly supplied in flasks containing 34.5 kg (about 76 lb). What is the volume in liters of this much mercury?

Exercise:**Problem:**

(a) What is the mass of a deep breath of air having a volume of 2.00 L? (b) Discuss the effect taking such a breath has on your body's volume and density.

Solution:

(a) 2.58 g

(b) The volume of your body increases by the volume of air you inhale. The average density of your body decreases when you take a deep breath, because the density of air is substantially smaller than the average density of the body before you took the deep breath.

Exercise:

Problem:

A straightforward method of finding the density of an object is to measure its mass and then measure its volume by submerging it in a graduated cylinder. What is the density of a 240-g rock that displaces 89.0 cm³ of water? (Note that the accuracy and practical applications of this technique are more limited than a variety of others that are based on Archimedes' principle.)

Solution:

$$2.70 \text{ g/cm}^3$$

Exercise:**Problem:**

Suppose you have a coffee mug with a circular cross section and vertical sides (uniform radius). What is its inside radius if it holds 375 g of coffee when filled to a depth of 7.50 cm? Assume coffee has the same density as water.

Exercise:**Problem:**

(a) A rectangular gasoline tank can hold 50.0 kg of gasoline when full. What is the depth of the tank if it is 0.500-m wide by 0.900-m long? (b) Discuss whether this gas tank has a reasonable volume for a passenger car.

Solution:

(a) 0.163 m

(b) Equivalent to 19.4 gallons, which is reasonable

Exercise:**Problem:**

A trash compactor can reduce the volume of its contents to 0.350 their original value. Neglecting the mass of air expelled, by what factor is the density of the rubbish increased?

Exercise:**Problem:**

A 2.50-kg steel gasoline can holds 20.0 L of gasoline when full. What is the average density of the full gas can, taking into account the volume occupied by steel as well as by gasoline?

Solution:

$$7.9 \times 10^2 \text{ kg/m}^3$$

Exercise:**Problem:**

What is the density of 18.0-karat gold that is a mixture of 18 parts gold, 5 parts silver, and 1 part copper? (These values are parts by mass, not volume.) Assume that this is a simple mixture having an average density equal to the weighted densities of its constituents.

Solution:

$$15.6 \text{ g/cm}^3$$

Exercise:**Problem:**

There is relatively little empty space between atoms in solids and liquids, so that the average density of an atom is about the same as matter on a macroscopic scale—approximately 10^3 kg/m^3 . The nucleus of an atom has a radius about 10^{-5} that of the atom and contains nearly all the mass of the entire atom. (a) What is the approximate density of a nucleus? (b) One remnant of a supernova, called a neutron star, can have the density of a nucleus. What would be the radius of a neutron star with a mass 10 times that of our Sun (the radius of the Sun is $7 \times 10^8 \text{ m}$)?

Solution:

(a) 10^{18} kg/m^3

(b) $2 \times 10^4 \text{ m}$

Glossary

density

the mass per unit volume of a substance or object

Pressure

- Define pressure.
- Explain the relationship between pressure and force.
- Calculate force given pressure and area.

You have no doubt heard the word **pressure** being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids. Pressure P is defined as

Equation:

$$P = \frac{F}{A}$$

where F is a force applied to an area A that is perpendicular to the force.

Note:

Pressure

Pressure is defined as the force divided by the area perpendicular to the force over which the force is applied, or

Equation:

$$P = \frac{F}{A}.$$

A given force can have a significantly different effect depending on the area over which the force is exerted, as shown in [\[link\]](#). The SI unit for pressure is the *pascal*, where

Equation:

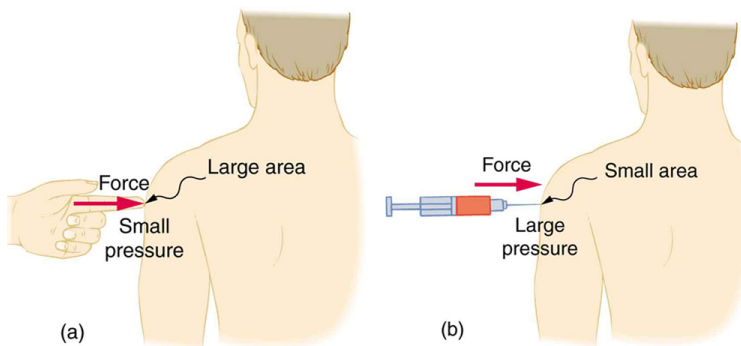
$$1 \text{ Pa} = 1 \text{ N/m}^2.$$

In addition to the pascal, there are many other units for pressure that are in common use. In meteorology, atmospheric pressure is often described in units of millibar (mb), where

Equation:

$$100 \text{ mb} = 1 \times 10^4 \text{ Pa} .$$

Pounds per square inch (lb/in^2 or psi) is still sometimes used as a measure of tire pressure, and millimeters of mercury (mm Hg) is still often used in the measurement of blood pressure. Pressure is defined for all states of matter but is particularly important when discussing fluids.



(a) While the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

Example:

Calculating Force Exerted by the Air: What Force Does a Pressure Exert?

An astronaut is working outside the International Space Station where the atmospheric pressure is essentially zero. The pressure gauge on her air tank

reads 6.90×10^6 Pa. What force does the air inside the tank exert on the flat end of the cylindrical tank, a disk 0.150 m in diameter?

Strategy

We can find the force exerted from the definition of pressure given in $P = \frac{F}{A}$, provided we can find the area A acted upon.

Solution

By rearranging the definition of pressure to solve for force, we see that

Equation:

$$F = PA.$$

Here, the pressure P is given, as is the area of the end of the cylinder A , given by $A = \pi r^2$. Thus,

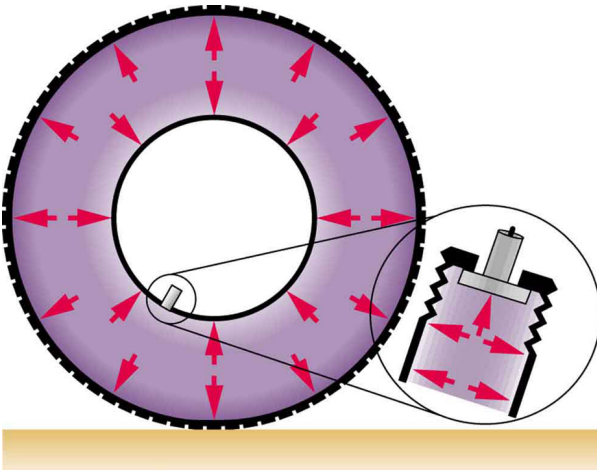
Equation:

$$\begin{aligned} F &= (6.90 \times 10^6 \text{ N/m}^2)(3.14)(0.0750 \text{ m})^2 \\ &= 1.22 \times 10^5 \text{ N.} \end{aligned}$$

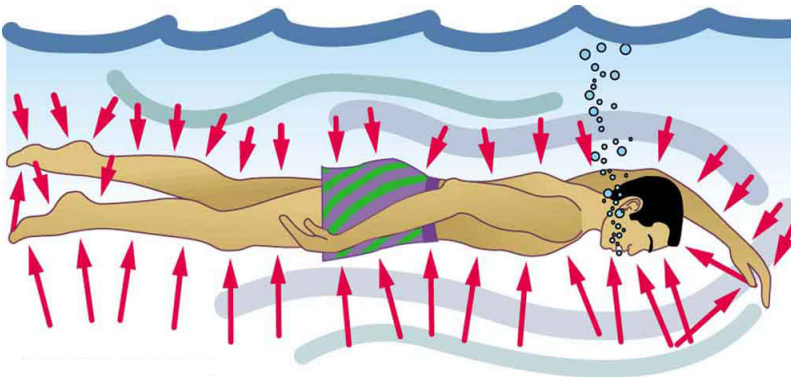
Discussion

Wow! No wonder the tank must be strong. Since we found $F = PA$, we see that the force exerted by a pressure is directly proportional to the area acted upon as well as the pressure itself.

The force exerted on the end of the tank is perpendicular to its inside surface. This direction is because the force is exerted by a static or stationary fluid. We have already seen that fluids cannot *withstand* shearing (sideways) forces; they cannot *exert* shearing forces, either. Fluid pressure has no direction, being a scalar quantity. The forces due to pressure have well-defined directions: they are always exerted perpendicular to any surface. (See the tire in [\[link\]](#), for example.) Finally, note that pressure is exerted on all surfaces. Swimmers, as well as the tire, feel pressure on all sides. (See [\[link\]](#).)



Pressure inside this tire exerts forces perpendicular to all surfaces it contacts. The arrows give representative directions and magnitudes of the forces exerted at various points. Note that static fluids do not exert shearing forces.



Pressure is exerted on all sides of this swimmer, since the water would flow into the space he occupies if he were not there.

The arrows represent the directions and magnitudes of the forces exerted at various points on the swimmer. Note that the forces are larger underneath, due to greater depth,

giving a net upward or buoyant force that is balanced by the weight of the swimmer.

Note:

PhET Explorations: Gas Properties

Pump gas molecules to a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.

[Gas](#)
[Properties](#)
[s](#)

Section Summary

- Pressure is the force per unit perpendicular area over which the force is applied. In equation form, pressure is defined as

Equation:

$$P = \frac{F}{A}.$$

- The SI unit of pressure is pascal and $1 \text{ Pa} = 1 \text{ N/m}^2$.

Conceptual Questions

Exercise:

Problem:

How is pressure related to the sharpness of a knife and its ability to cut?

Exercise:**Problem:**

Why does a dull hypodermic needle hurt more than a sharp one?

Exercise:**Problem:**

The outward force on one end of an air tank was calculated in [\[link\]](#). How is this force balanced? (The tank does not accelerate, so the force must be balanced.)

Exercise:**Problem:**

Why is force exerted by static fluids always perpendicular to a surface?

Exercise:**Problem:**

In a remote location near the North Pole, an iceberg floats in a lake. Next to the lake (assume it is not frozen) sits a comparably sized glacier sitting on land. If both chunks of ice should melt due to rising global temperatures (and the melted ice all goes into the lake), which ice chunk would give the greatest increase in the level of the lake water, if any?

Exercise:**Problem:**

How do jogging on soft ground and wearing padded shoes reduce the pressures to which the feet and legs are subjected?

Exercise:**Problem:**

Toe dancing (as in ballet) is much harder on toes than normal dancing or walking. Explain in terms of pressure.

Exercise:**Problem:**

How do you convert pressure units like millimeters of mercury, centimeters of water, and inches of mercury into units like newtons per meter squared without resorting to a table of pressure conversion factors?

Problems & Exercises**Exercise:****Problem:**

As a woman walks, her entire weight is momentarily placed on one heel of her high-heeled shoes. Calculate the pressure exerted on the floor by the heel if it has an area of 1.50 cm^2 and the woman's mass is 55.0 kg . Express the pressure in Pa. (In the early days of commercial flight, women were not allowed to wear high-heeled shoes because aircraft floors were too thin to withstand such large pressures.)

Solution:

$$3.59 \times 10^6 \text{ Pa; or } 521 \text{ lb/in}^2$$

Exercise:

Problem:

The pressure exerted by a phonograph needle on a record is surprisingly large. If the equivalent of 1.00 g is supported by a needle, the tip of which is a circle 0.200 mm in radius, what pressure is exerted on the record in N/m^2 ?

Exercise:**Problem:**

Nail tips exert tremendous pressures when they are hit by hammers because they exert a large force over a small area. What force must be exerted on a nail with a circular tip of 1.00 mm diameter to create a pressure of $3.00 \times 10^9 \text{ N/m}^2$? (This high pressure is possible because the hammer striking the nail is brought to rest in such a short distance.)

Solution:

$$2.36 \times 10^3 \text{ N}$$

Glossary**pressure**

the force per unit area perpendicular to the force, over which the force acts

Variation of Pressure with Depth in a Fluid

- Define pressure in terms of weight.
- Explain the variation of pressure with depth in a fluid.
- Calculate density given pressure and altitude.

If your ears have ever popped on a plane flight or ached during a deep dive in a swimming pool, you have experienced the effect of depth on pressure in a fluid. At the Earth's surface, the air pressure exerted on you is a result of the weight of air above you. This pressure is reduced as you climb up in altitude and the weight of air above you decreases. Under water, the pressure exerted on you increases with increasing depth. In this case, the pressure being exerted upon you is a result of both the weight of water above you *and* that of the atmosphere above you. You may notice an air pressure change on an elevator ride that transports you many stories, but you need only dive a meter or so below the surface of a pool to feel a pressure increase. The difference is that water is much denser than air, about 775 times as dense.

Consider the container in [\[link\]](#). Its bottom supports the weight of the fluid in it. Let us calculate the pressure exerted on the bottom by the weight of the fluid. That **pressure** is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):

Equation:

$$P = \frac{mg}{A}.$$

We can find the mass of the fluid from its volume and density:

Equation:

$$m = \rho V.$$

The volume of the fluid V is related to the dimensions of the container. It is

Equation:

$$V = Ah,$$

where A is the cross-sectional area and h is the depth. Combining the last two equations gives

Equation:

$$m = \rho Ah.$$

If we enter this into the expression for pressure, we obtain

Equation:

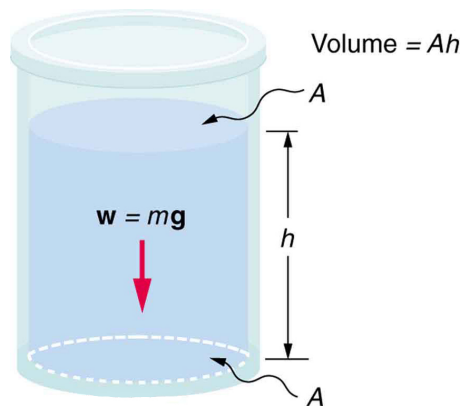
$$P = \frac{(\rho Ah)g}{A}.$$

The area cancels, and rearranging the variables yields

Equation:

$$P = h\rho g.$$

This value is the *pressure due to the weight of a fluid*. The equation has general validity beyond the special conditions under which it is derived here. Even if the container were not there, the surrounding fluid would still exert this pressure, keeping the fluid static. Thus the equation $P = h\rho g$ represents the pressure due to the weight of any fluid of *average density* ρ at any depth h below its surface. For liquids, which are nearly incompressible, this equation holds to great depths. For gases, which are quite compressible, one can apply this equation as long as the density changes are small over the depth considered. [\[link\]](#) illustrates this situation.



The bottom of this container supports the entire weight of the fluid in it. The vertical sides cannot exert an upward force on the fluid (since it cannot withstand a shearing force), and so the bottom must support it all.

Example:

Calculating the Average Pressure and Force Exerted: What Force Must a Dam Withstand?

In [\[link\]](#), we calculated the mass of water in a large reservoir. We will now consider the pressure and force acting on the dam retaining water. (See [\[link\]](#).) The dam is 500 m wide, and the water is 80.0 m deep at the dam.

(a) What is the average pressure on the dam due to the water? (b) Calculate the force exerted against the dam and compare it with the weight of water in the dam (previously found to be 1.96×10^{13} N).

Strategy for (a)

The average pressure $\bar{P}$ due to the weight of the water is the pressure at the average depth $\bar{h}$ of 40.0 m, since pressure increases linearly with depth.

Solution for (a)

The average pressure due to the weight of a fluid is

Equation:

$$\bar{P} = \bar{h}\rho g.$$

Entering the density of water from [\[link\]](#) and taking $\bar{h}$ to be the average depth of 40.0 m, we obtain

Equation:

$$\begin{aligned}\bar{P} &= (40.0 \text{ m}) \left(10^3 \frac{\text{kg}}{\text{m}^3} \right) \left(9.80 \frac{\text{m}}{\text{s}^2} \right) \\ &= 3.92 \times 10^5 \frac{\text{N}}{\text{m}^2} = 392 \text{ kPa}.\end{aligned}$$

Strategy for (b)

The force exerted on the dam by the water is the average pressure times the area of contact:

Equation:

$$F = \bar{P}A.$$

Solution for (b)

We have already found the value for $\bar{P}$. The area of the dam is $A = 80.0 \text{ m} \times 500 \text{ m} = 4.00 \times 10^4 \text{ m}^2$, so that

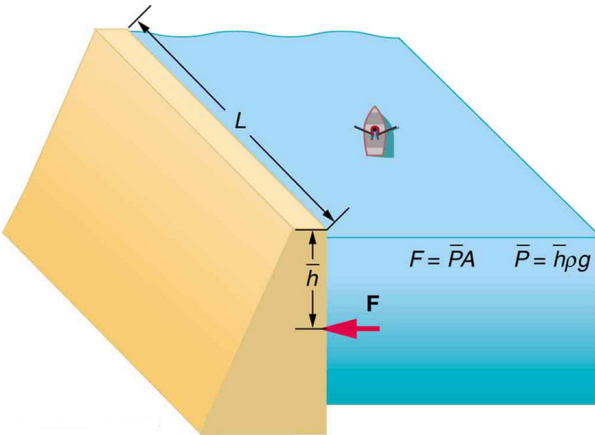
Equation:

$$\begin{aligned}F &= (3.92 \times 10^5 \text{ N/m}^2)(4.00 \times 10^4 \text{ m}^2) \\ &= 1.57 \times 10^{10} \text{ N}.\end{aligned}$$

Discussion

Although this force seems large, it is small compared with the $1.96 \times 10^{13} \text{ N}$ weight of the water in the reservoir—in fact, it is only 0.0800% of the weight. Note that the pressure found in part (a) is completely independent of the width and length of the lake—it depends only on its average depth at the dam. Thus the force depends only on the

water's average depth and the dimensions of the dam, *not* on the horizontal extent of the reservoir. In the diagram, the thickness of the dam increases with depth to balance the increasing force due to the increasing pressure. epth to balance the increasing force due to the increasing pressure.



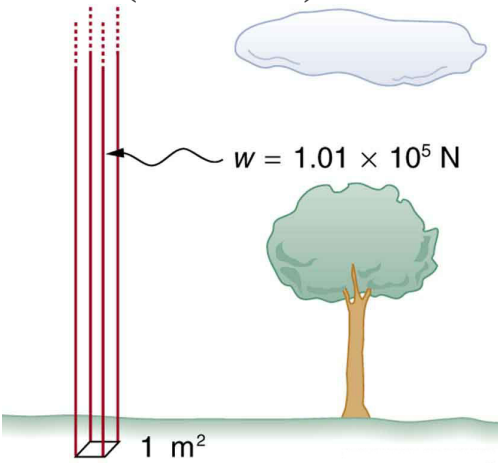
The dam must withstand the force exerted against it by the water it retains. This force is small compared with the weight of the water behind the dam.

Atmospheric pressure is another example of pressure due to the weight of a fluid, in this case due to the weight of *air* above a given height. The atmospheric pressure at the Earth's surface varies a little due to the large-scale flow of the atmosphere induced by the Earth's rotation (this creates weather "highs" and "lows"). However, the average pressure at sea level is given by the *standard atmospheric pressure* P_{atm} , measured to be

Equation:

$$1 \text{ atmosphere (atm)} = P_{\text{atm}} = 1.01 \times 10^5 \text{ N/m}^2 = 101 \text{ kPa}.$$

This relationship means that, on average, at sea level, a column of air above 1.00 m^2 of the Earth's surface has a weight of $1.01 \times 10^5 \text{ N}$, equivalent to 1 atm. (See [\[link\]](#).)



Atmospheric pressure at sea level averages $1.01 \times 10^5 \text{ Pa}$ (equivalent to 1 atm), since the column of air over this 1 m^2 , extending to the top of the atmosphere, weighs $1.01 \times 10^5 \text{ N}$.

Example:

Calculating Average Density: How Dense Is the Air?

Calculate the average density of the atmosphere, given that it extends to an altitude of 120 km. Compare this density with that of air listed in [\[link\]](#).

Strategy

If we solve $P = h\rho g$ for density, we see that

Equation:

$$\bar{\rho} = \frac{P}{hg}.$$

We then take P to be atmospheric pressure, h is given, and g is known, and so we can use this to calculate $\bar{\rho}$.

Solution

Entering known values into the expression for $\bar{\rho}$ yields

Equation:

$$\bar{\rho} = \frac{1.01 \times 10^5 \text{ N/m}^2}{(120 \times 10^3 \text{ m})(9.80 \text{ m/s}^2)} = 8.59 \times 10^{-2} \text{ kg/m}^3.$$

Discussion

This result is the average density of air between the Earth's surface and the top of the Earth's atmosphere, which essentially ends at 120 km. The density of air at sea level is given in [\[link\]](#) as 1.29 kg/m^3 —about 15 times its average value. Because air is so compressible, its density has its highest value near the Earth's surface and declines rapidly with altitude.

Example:

Calculating Depth Below the Surface of Water: What Depth of Water Creates the Same Pressure as the Entire Atmosphere?

Calculate the depth below the surface of water at which the pressure due to the weight of the water equals 1.00 atm.

Strategy

We begin by solving the equation $P = h\rho g$ for depth h :

Equation:

$$h = \frac{P}{\rho g}.$$

Then we take P to be 1.00 atm and ρ to be the density of the water that creates the pressure.

Solution

Entering the known values into the expression for h gives

Equation:

$$h = \frac{1.01 \times 10^5 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 10.3 \text{ m.}$$

Discussion

Just 10.3 m of water creates the same pressure as 120 km of air. Since water is nearly incompressible, we can neglect any change in its density over this depth.

What do you suppose is the *total* pressure at a depth of 10.3 m in a swimming pool? Does the atmospheric pressure on the water's surface affect the pressure below? The answer is yes. This seems only logical, since both the water's weight and the atmosphere's weight must be supported. So the *total* pressure at a depth of 10.3 m is 2 atm—half from the water above and half from the air above. We shall see in [Pascal's Principle](#) that fluid pressures always add in this way.

Section Summary

- Pressure is the weight of the fluid mg divided by the area A supporting it (the area of the bottom of the container):

Equation:

$$P = \frac{mg}{A}.$$

- Pressure due to the weight of a liquid is given by

Equation:

$$P = h\rho g,$$

where P is the pressure, h is the height of the liquid, ρ is the density of the liquid, and g is the acceleration due to gravity.

Conceptual Questions

Exercise:

Problem:

Atmospheric pressure exerts a large force (equal to the weight of the atmosphere above your body—about 10 tons) on the top of your body when you are lying on the beach sunbathing. Why are you able to get up?

Exercise:

Problem:

Why does atmospheric pressure decrease more rapidly than linearly with altitude?

Exercise:

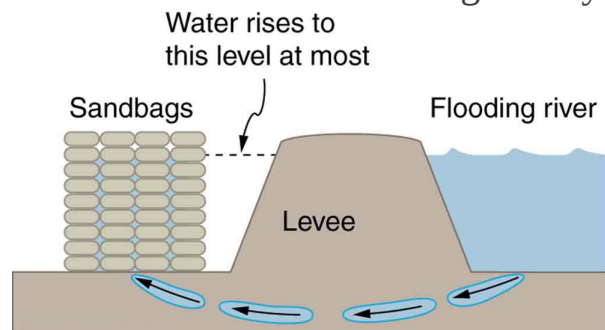
Problem:

What are two reasons why mercury rather than water is used in barometers?

Exercise:

Problem:

[\[link\]](#) shows how sandbags placed around a leak outside a river levee can effectively stop the flow of water under the levee. Explain how the small amount of water inside the column formed by the sandbags is able to balance the much larger body of water behind the levee.



Because the river level is very high, it has started to leak under the levee. Sandbags are placed around the leak, and the water held by them rises until it is the same level as the river, at which point the water there stops rising.

Exercise:

Problem:

Why is it difficult to swim under water in the Great Salt Lake?

Exercise:

Problem:

Is there a net force on a dam due to atmospheric pressure? Explain your answer.

Exercise:

Problem:

Does atmospheric pressure add to the gas pressure in a rigid tank? In a toy balloon? When, in general, does atmospheric pressure *not* affect the total pressure in a fluid?

Exercise:

Problem:

You can break a strong wine bottle by pounding a cork into it with your fist, but the cork must press directly against the liquid filling the bottle—there can be no air between the cork and liquid. Explain why the bottle breaks, and why it will not if there is air between the cork and liquid.

Problems & Exercises

Exercise:

Problem: What depth of mercury creates a pressure of 1.00 atm?

Solution:

0.760 m

Exercise:

Problem:

The greatest ocean depths on the Earth are found in the Marianas Trench near the Philippines. Calculate the pressure due to the ocean at the bottom of this trench, given its depth is 11.0 km and assuming the density of seawater is constant all the way down.

Exercise:

Problem: Verify that the SI unit of $h\rho g$ is N/m^2 .

Solution:

Equation:

$$\begin{aligned}(h\rho g)_{\text{units}} &= (\text{m})\left(\text{kg}/\text{m}^3\right)\left(\text{m}/\text{s}^2\right) = (\text{kg} \cdot \text{m}^2)/(\text{m}^3 \cdot \text{s}^2) \\ &= \left(\text{kg} \cdot \text{m}/\text{s}^2\right)\left(1/\text{m}^2\right) \\ &= \text{N}/\text{m}^2\end{aligned}$$

Exercise:

Problem:

Water towers store water above the level of consumers for times of heavy use, eliminating the need for high-speed pumps. How high above a user must the water level be to create a gauge pressure of $3.00 \times 10^5 \text{ N/m}^2$?

Exercise:**Problem:**

The aqueous humor in a person's eye is exerting a force of 0.300 N on the 1.10-cm^2 area of the cornea. (a) What pressure is this in mm Hg? (b) Is this value within the normal range for pressures in the eye?

Solution:

(a) 20.5 mm Hg

(b) The range of pressures in the eye is 12–24 mm Hg, so the result in part (a) is within that range

Exercise:**Problem:**

How much force is exerted on one side of an 8.50 cm by 11.0 cm sheet of paper by the atmosphere? How can the paper withstand such a force?

Exercise:**Problem:**

What pressure is exerted on the bottom of a 0.500-m-wide by 0.900-m-long gas tank that can hold 50.0 kg of gasoline by the weight of the gasoline in it when it is full?

Solution:

$1.09 \times 10^3 \text{ N/m}^2$

Exercise:**Problem:**

Calculate the average pressure exerted on the palm of a shot-putter's hand by the shot if the area of contact is 50.0 cm^2 and he exerts a force of 800 N on it. Express the pressure in N/m^2 and compare it with the $1.00 \times 10^6 \text{ Pa}$ pressures sometimes encountered in the skeletal system.

Exercise:**Problem:**

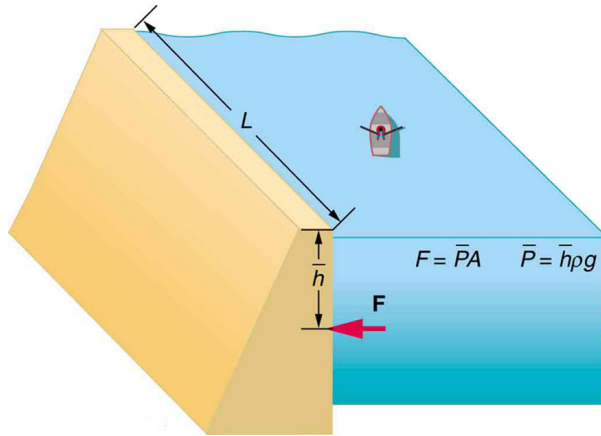
The left side of the heart creates a pressure of 120 mm Hg by exerting a force directly on the blood over an effective area of 15.0 cm^2 . What force does it exert to accomplish this?

Solution:

24.0 N

Exercise:**Problem:**

Show that the total force on a rectangular dam due to the water behind it increases with the *square* of the water depth. In particular, show that this force is given by $F = \rho g h^2 L / 2$, where ρ is the density of water, h is its depth at the dam, and L is the length of the dam. You may assume the face of the dam is vertical. (Hint: Calculate the average pressure exerted and multiply this by the area in contact with the water. (See [\[link\]](#).)



Glossary

pressure

the weight of the fluid divided by the area supporting it

Pascal's Principle

- Define pressure.
- State Pascal's principle.
- Understand applications of Pascal's principle.
- Derive relationships between forces in a hydraulic system.

Pressure is defined as force per unit area. Can pressure be increased in a fluid by pushing directly on the fluid? Yes, but it is much easier if the fluid is enclosed. The heart, for example, increases blood pressure by pushing directly on the blood in an enclosed system (valves closed in a chamber). If you try to push on a fluid in an open system, such as a river, the fluid flows away. An enclosed fluid cannot flow away, and so pressure is more easily increased by an applied force.

What happens to a pressure in an enclosed fluid? Since atoms in a fluid are free to move about, they transmit the pressure to all parts of the fluid and to the walls of the container. Remarkably, the pressure is transmitted *undiminished*. This phenomenon is called **Pascal's principle**, because it was first clearly stated by the French philosopher and scientist Blaise Pascal (1623–1662): A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

Note:

Pascal's Principle

A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.

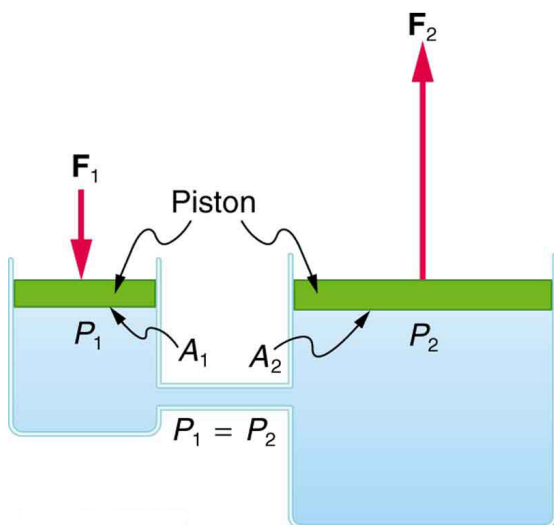
Pascal's principle, an experimentally verified fact, is what makes pressure so important in fluids. Since a change in pressure is transmitted undiminished in an enclosed fluid, we often know more about pressure than other physical quantities in fluids. Moreover, Pascal's principle implies that

the total pressure in a fluid is the sum of the pressures from different sources. We shall find this fact—that pressures add—very useful.

Blaise Pascal had an interesting life in that he was home-schooled by his father who removed all of the mathematics textbooks from his house and forbade him to study mathematics until the age of 15. This, of course, raised the boy's curiosity, and by the age of 12, he started to teach himself geometry. Despite this early deprivation, Pascal went on to make major contributions in the mathematical fields of probability theory, number theory, and geometry. He is also well known for being the inventor of the first mechanical digital calculator, in addition to his contributions in the field of fluid statics.

Application of Pascal's Principle

One of the most important technological applications of Pascal's principle is found in a *hydraulic system*, which is an enclosed fluid system used to exert forces. The most common hydraulic systems are those that operate car brakes. Let us first consider the simple hydraulic system shown in [\[link\]](#).



A typical hydraulic system
with two fluid-filled
cylinders, capped with

pistons and connected by a tube called a hydraulic line. A downward force $\mathbf{F}_1$ on the left piston creates a pressure that is transmitted undiminished to all parts of the enclosed fluid. This results in an upward force $\mathbf{F}_2$ on the right piston that is larger than $\mathbf{F}_1$ because the right piston has a larger area.

Relationship Between Forces in a Hydraulic System

We can derive a relationship between the forces in the simple hydraulic system shown in [\[link\]](#) by applying Pascal's principle. Note first that the two pistons in the system are at the same height, and so there will be no difference in pressure due to a difference in depth. Now the pressure due to F_1 acting on area A_1 is simply $P_1 = \frac{F_1}{A_1}$, as defined by $P = \frac{F}{A}$. According to Pascal's principle, this pressure is transmitted undiminished throughout the fluid and to all walls of the container. Thus, a pressure P_2 is felt at the other piston that is equal to P_1 . That is $P_1 = P_2$.

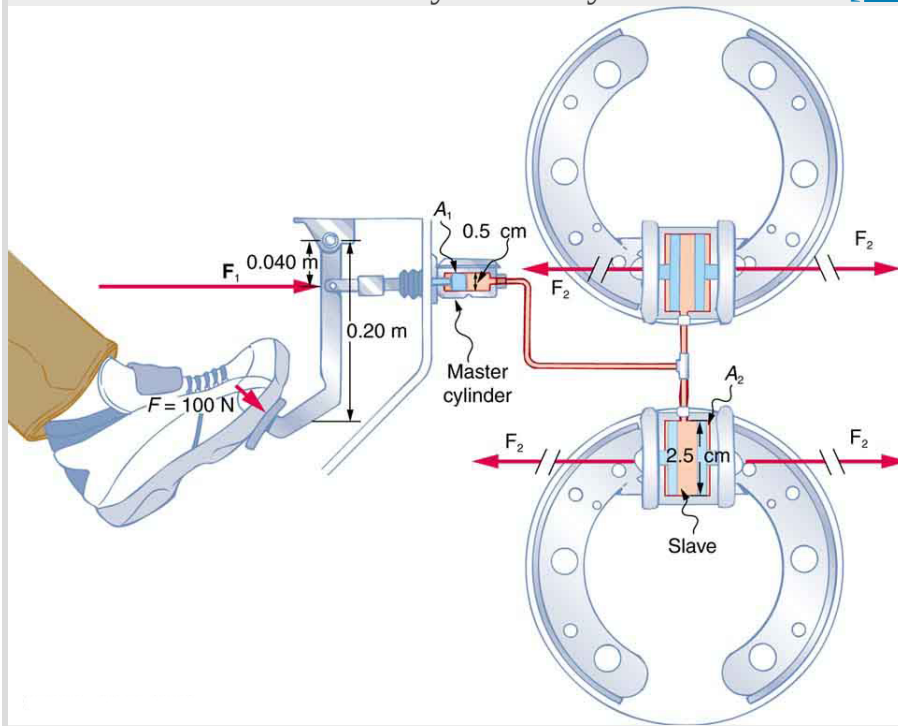
But since $P_2 = \frac{F_2}{A_2}$, we see that $\frac{F_1}{A_1} = \frac{F_2}{A_2}$.

This equation relates the ratios of force to area in any hydraulic system, providing the pistons are at the same vertical height and that friction in the system is negligible. Hydraulic systems can increase or decrease the force applied to them. To make the force larger, the pressure is applied to a larger area. For example, if a 100-N force is applied to the left cylinder in [\[link\]](#) and the right one has an area five times greater, then the force out is 500 N. Hydraulic systems are analogous to simple levers, but they have the advantage that pressure can be sent through tortuously curved lines to several places at once.

Example:

Calculating Force of Slave Cylinders: Pascal Puts on the Brakes

Consider the automobile hydraulic system shown in [\[link\]](#).



Hydraulic brakes use Pascal's principle. The driver exerts a force of 100 N on the brake pedal. This force is increased by the simple lever and again by the hydraulic system. Each of the identical slave cylinders receives the same pressure and, therefore, creates the same force output F_2 . The circular cross-sectional areas of the master and slave cylinders are represented by A_1 and A_2 , respectively

A force of 100 N is applied to the brake pedal, which acts on the cylinder—called the master—through a lever. A force of 500 N is exerted on the master cylinder. (The reader can verify that the force is 500 N using techniques of statics from [Applications of Statics, Including Problem-Solving Strategies](#).) Pressure created in the master cylinder is transmitted to four so-called slave cylinders. The master cylinder has a diameter of

0.500 cm, and each slave cylinder has a diameter of 2.50 cm. Calculate the force F_2 created at each of the slave cylinders.

Strategy

We are given the force F_1 that is applied to the master cylinder. The cross-sectional areas A_1 and A_2 can be calculated from their given diameters. Then $\frac{F_1}{A_1} = \frac{F_2}{A_2}$ can be used to find the force F_2 . Manipulate this algebraically to get F_2 on one side and substitute known values:

Solution

Pascal's principle applied to hydraulic systems is given by $\frac{F_1}{A_1} = \frac{F_2}{A_2}$:

Equation:

$$F_2 = \frac{A_2}{A_1} F_1 = \frac{\pi r_2^2}{\pi r_1^2} F_1 = \frac{(1.25 \text{ cm})^2}{(0.250 \text{ cm})^2} \times 500 \text{ N} = 1.25 \times 10^4 \text{ N}.$$

Discussion

This value is the force exerted by each of the four slave cylinders. Note that we can add as many slave cylinders as we wish. If each has a 2.50-cm diameter, each will exert $1.25 \times 10^4 \text{ N}$.

A simple hydraulic system, such as a simple machine, can increase force but cannot do more work than done on it. Work is force times distance moved, and the slave cylinder moves through a smaller distance than the master cylinder. Furthermore, the more slaves added, the smaller the distance each moves. Many hydraulic systems—such as power brakes and those in bulldozers—have a motorized pump that actually does most of the work in the system. The movement of the legs of a spider is achieved partly by hydraulics. Using hydraulics, a jumping spider can create a force that makes it capable of jumping 25 times its length!

Note:

Making Connections: Conservation of Energy

Conservation of energy applied to a hydraulic system tells us that the system cannot do more work than is done on it. Work transfers energy, and so the work output cannot exceed the work input. Power brakes and other similar hydraulic systems use pumps to supply extra energy when needed.

Section Summary

- Pressure is force per unit area.
- A change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container.
- A hydraulic system is an enclosed fluid system used to exert forces.

Conceptual Questions

Exercise:

Problem:

Suppose the master cylinder in a hydraulic system is at a greater height than the slave cylinder. Explain how this will affect the force produced at the slave cylinder.

Problems & Exercises

Exercise:

Problem:

How much pressure is transmitted in the hydraulic system considered in [\[link\]](#)? Express your answer in pascals and in atmospheres.

Solution:

$2.55 \times 10^7 \text{ Pa}$; or 251 atm

Exercise:**Problem:**

What force must be exerted on the master cylinder of a hydraulic lift to support the weight of a 2000-kg car (a large car) resting on the slave cylinder? The master cylinder has a 2.00-cm diameter and the slave has a 24.0-cm diameter.

Exercise:**Problem:**

A crass host pours the remnants of several bottles of wine into a jug after a party. He then inserts a cork with a 2.00-cm diameter into the bottle, placing it in direct contact with the wine. He is amazed when he pounds the cork into place and the bottom of the jug (with a 14.0-cm diameter) breaks away. Calculate the extra force exerted against the bottom if he pounded the cork with a 120-N force.

Solution:

5.76×10^3 N extra force

Exercise:**Problem:**

A certain hydraulic system is designed to exert a force 100 times as large as the one put into it. (a) What must be the ratio of the area of the slave cylinder to the area of the master cylinder? (b) What must be the ratio of their diameters? (c) By what factor is the distance through which the output force moves reduced relative to the distance through which the input force moves? Assume no losses to friction.

Exercise:

Problem:

(a) Verify that work input equals work output for a hydraulic system assuming no losses to friction. Do this by showing that the distance the output force moves is reduced by the same factor that the output force is increased. Assume the volume of the fluid is constant. (b) What effect would friction within the fluid and between components in the system have on the output force? How would this depend on whether or not the fluid is moving?

Solution:

$$(a) V = d_i A_i = d_o A_o \Rightarrow d_o = d_i \frac{A_i}{A_o} .$$

Now, using equation:

Equation:

$$\frac{F_1}{A_1} = \frac{F_2}{A_2} \Rightarrow F_o = F_i \frac{A_o}{A_i} .$$

Finally,

Equation:

$$W_o = F_o d_o = \frac{F_i A_o}{A_i} \frac{d_i A_i}{A_o} = F_i d_i = W_i .$$

In other words, the work output equals the work input.

(b) If the system is not moving, friction would not play a role. With friction, we know there are losses, so that $W_{\text{out}} = W_{\text{in}} - W_f$; therefore, the work output is less than the work input. In other words, with friction, you need to push harder on the input piston than was calculated for the nonfriction case.

Glossary

Pascal's Principle

a change in pressure applied to an enclosed fluid is transmitted undiminished to all portions of the fluid and to the walls of its container

Gauge Pressure, Absolute Pressure, and Pressure Measurement

- Define gauge pressure and absolute pressure.
- Understand the working of aneroid and open-tube barometers.

If you limp into a gas station with a nearly flat tire, you will notice the tire gauge on the airline reads nearly zero when you begin to fill it. In fact, if there were a gaping hole in your tire, the gauge would read zero, even though atmospheric pressure exists in the tire. Why does the gauge read zero? There is no mystery here. Tire gauges are simply designed to read zero at atmospheric pressure and positive when pressure is greater than atmospheric.

Similarly, atmospheric pressure adds to blood pressure in every part of the circulatory system. (As noted in [Pascal's Principle](#), the total pressure in a fluid is the sum of the pressures from different sources—here, the heart and the atmosphere.) But atmospheric pressure has no net effect on blood flow since it adds to the pressure coming out of the heart and going back into it, too. What is important is how much *greater* blood pressure is than atmospheric pressure. Blood pressure measurements, like tire pressures, are thus made relative to atmospheric pressure.

In brief, it is very common for pressure gauges to ignore atmospheric pressure—that is, to read zero at atmospheric pressure. We therefore define **gauge pressure** to be the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

Note:

Gauge Pressure

Gauge pressure is the pressure relative to atmospheric pressure. Gauge pressure is positive for pressures above atmospheric pressure, and negative for pressures below it.

In fact, atmospheric pressure does add to the pressure in any fluid not enclosed in a rigid container. This happens because of Pascal's principle. The total pressure, or **absolute pressure**, is thus the sum of gauge pressure and atmospheric pressure: $P_{\text{abs}} = P_{\text{g}} + P_{\text{atm}}$ where P_{abs} is absolute pressure, P_{g} is gauge pressure, and P_{atm} is atmospheric pressure. For example, if your tire gauge reads 34 psi

(pounds per square inch), then the absolute pressure is 34 psi plus 14.7 psi (P_{atm} in psi), or 48.7 psi (equivalent to 336 kPa).

Note:

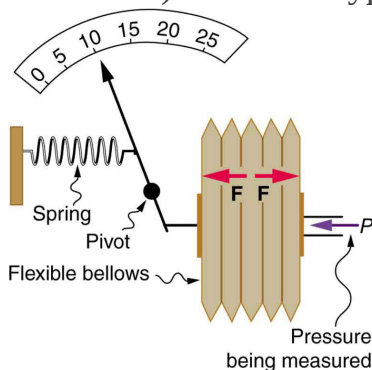
Absolute Pressure

Absolute pressure is the sum of gauge pressure and atmospheric pressure.

For reasons we will explore later, in most cases the absolute pressure in fluids cannot be negative. Fluids push rather than pull, so the smallest absolute pressure is zero. (A negative absolute pressure is a pull.) Thus the smallest possible gauge pressure is $P_g = -P_{\text{atm}}$ (this makes P_{abs} zero). There is no theoretical limit to how large a gauge pressure can be.

There are a host of devices for measuring pressure, ranging from tire gauges to blood pressure cuffs. Pascal's principle is of major importance in these devices. The undiminished transmission of pressure through a fluid allows precise remote sensing of pressures. Remote sensing is often more convenient than putting a measuring device into a system, such as a person's artery.

[\[link\]](#) shows one of the many types of mechanical pressure gauges in use today. In all mechanical pressure gauges, pressure results in a force that is converted (or transduced) into some type of readout.

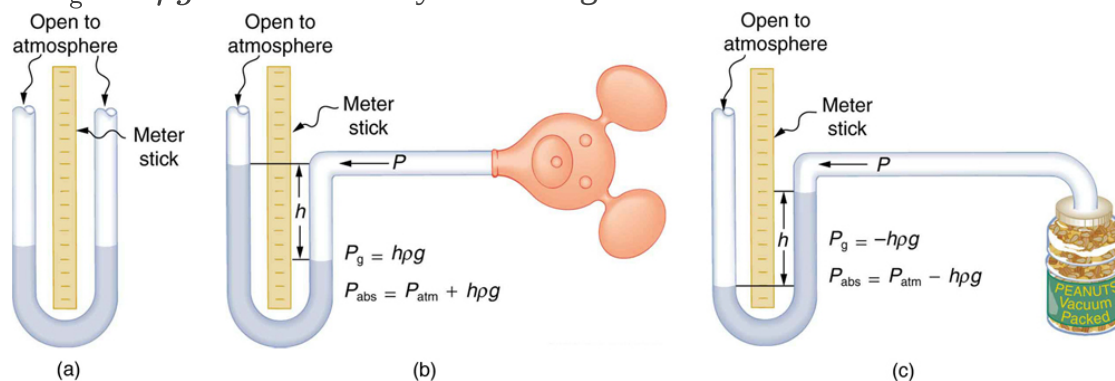


This aneroid gauge
utilizes flexible
bellows connected
to a mechanical

indicator to
measure pressure.

An entire class of gauges uses the property that pressure due to the weight of a fluid is given by $P = h\rho g$. Consider the U-shaped tube shown in [link], for example. This simple tube is called a *manometer*. In [link](a), both sides of the tube are open to the atmosphere. Atmospheric pressure therefore pushes down on each side equally so its effect cancels. If the fluid is deeper on one side, there is a greater pressure on the deeper side, and the fluid flows away from that side until the depths are equal.

Let us examine how a manometer is used to measure pressure. Suppose one side of the U-tube is connected to some source of pressure P_{abs} such as the toy balloon in [link](b) or the vacuum-packed peanut jar shown in [link](c). Pressure is transmitted undiminished to the manometer, and the fluid levels are no longer equal. In [link](b), P_{abs} is greater than atmospheric pressure, whereas in [link](c), P_{abs} is less than atmospheric pressure. In both cases, P_{abs} differs from atmospheric pressure by an amount $h\rho g$, where ρ is the density of the fluid in the manometer. In [link](b), P_{abs} can support a column of fluid of height h , and so it must exert a pressure $h\rho g$ greater than atmospheric pressure (the gauge pressure P_g is positive). In [link](c), atmospheric pressure can support a column of fluid of height h , and so P_{abs} is less than atmospheric pressure by an amount $h\rho g$ (the gauge pressure P_g is negative). A manometer with one side open to the atmosphere is an ideal device for measuring gauge pressures. The gauge pressure is $P_g = h\rho g$ and is found by measuring h .



An open-tube manometer has one side open to the atmosphere. (a) Fluid depth must be the same on both sides, or the pressure each side exerts at the bottom will be unequal and there will be flow from the

- deeper side. (b) A positive gauge pressure $P_g = h\rho g$ transmitted to one side of the manometer can support a column of fluid of height h . (c) Similarly, atmospheric pressure is greater than a negative gauge pressure P_g by an amount $h\rho g$. The jar's rigidity prevents atmospheric pressure from being transmitted to the peanuts.

Mercury manometers are often used to measure arterial blood pressure. An inflatable cuff is placed on the upper arm as shown in [\[link\]](#). By squeezing the bulb, the person making the measurement exerts pressure, which is transmitted undiminished to both the main artery in the arm and the manometer. When this applied pressure exceeds blood pressure, blood flow below the cuff is cut off. The person making the measurement then slowly lowers the applied pressure and listens for blood flow to resume. Blood pressure pulsates because of the pumping action of the heart, reaching a maximum, called **systolic pressure**, and a minimum, called **diastolic pressure**, with each heartbeat. Systolic pressure is measured by noting the value of h when blood flow first begins as cuff pressure is lowered. Diastolic pressure is measured by noting h when blood flows without interruption. The typical blood pressure of a young adult raises the mercury to a height of 120 mm at systolic and 80 mm at diastolic. This is commonly quoted as 120 over 80, or 120/80. The first pressure is representative of the maximum output of the heart; the second is due to the elasticity of the arteries in maintaining the pressure between beats. The density of the mercury fluid in the manometer is 13.6 times greater than water, so the height of the fluid will be 1/13.6 of that in a water manometer. This reduced height can make measurements difficult, so mercury manometers are used to measure larger pressures, such as blood pressure. The density of mercury is such that $1.0 \text{ mm Hg} = 133 \text{ Pa}$.

Note:

Systolic Pressure

Systolic pressure is the maximum blood pressure.

Note:

Diastolic Pressure

Diastolic pressure is the minimum blood pressure.



In routine blood pressure measurements, an inflatable cuff is placed on the upper arm at the same level as the heart.

Blood flow is detected just below the cuff, and corresponding pressures are transmitted to a mercury-filled manometer. (credit: U.S. Army photo by Spc. Micah E. Clare\4TH BCT)

Example:
Calculating Height of IV Bag: Blood Pressure and Intravenous Infusions

Intravenous infusions are usually made with the help of the gravitational force. Assuming that the density of the fluid being administered is 1.00 g/ml, at what height should the IV bag be placed above the entry point so that the fluid just enters the vein if the blood pressure in the vein is 18 mm Hg above atmospheric pressure? Assume that the IV bag is collapsible.

Strategy for (a)

For the fluid to just enter the vein, its pressure at entry must exceed the blood pressure in the vein (18 mm Hg above atmospheric pressure). We therefore need to find the height of fluid that corresponds to this gauge pressure.

Solution

We first need to convert the pressure into SI units. Since 1.0 mm Hg = 133 Pa,

Equation:

$$P = 18 \text{ mm Hg} \times \frac{133 \text{ Pa}}{1.0 \text{ mm Hg}} = 2400 \text{ Pa}.$$

Rearranging $P_g = h\rho g$ for h gives $h = \frac{P_g}{\rho g}$. Substituting known values into this equation gives

Equation:

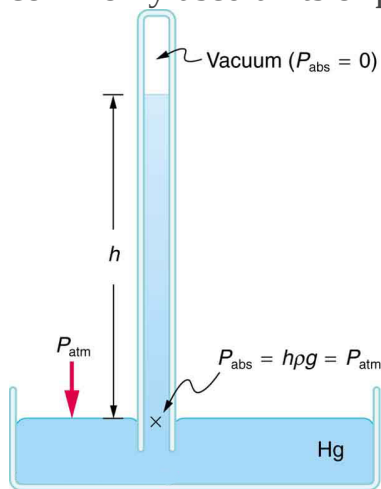
$$\begin{aligned} h &= \frac{2400 \text{ N/m}^2}{(1.0 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} \\ &= 0.24 \text{ m}. \end{aligned}$$

Discussion

The IV bag must be placed at 0.24 m above the entry point into the arm for the fluid to just enter the arm. Generally, IV bags are placed higher than this. You may have noticed that the bags used for blood collection are placed below the donor to allow blood to flow easily from the arm to the bag, which is the opposite direction of flow than required in the example presented here.

A *barometer* is a device that measures atmospheric pressure. A mercury barometer is shown in [\[link\]](#). This device measures atmospheric pressure, rather than gauge pressure, because there is a nearly pure vacuum above the mercury in the tube. The height of the mercury is such that $h\rho g = P_{\text{atm}}$. When atmospheric pressure varies, the mercury rises or falls, giving important clues to weather forecasters. The barometer can also be used as an altimeter, since average atmospheric pressure varies with altitude. Mercury barometers and manometers

are so common that units of mm Hg are often quoted for atmospheric pressure and blood pressures. [\[link\]](#) gives conversion factors for some of the more commonly used units of pressure.



A mercury barometer measures atmospheric pressure. The pressure due to the mercury's weight, $h\rho g$, equals atmospheric pressure. The atmosphere is able to force mercury in the tube to a height h because the pressure above the mercury is zero.

Conversion to N/m ² (Pa)	Conversion from atm
1.0 atm = 1.013×10^5 N/m ²	1.0 atm = 1.013×10^5 N/m ²
1.0 dyne/cm ² = 0.10 N/m ²	1.0 atm = 1.013×10^6 dyne/cm ²
1.0 kg/cm ² = 9.8×10^4 N/m ²	1.0 atm = 1.013 kg/cm ²
1.0 lb/in. ² = 6.90×10^3 N/m ²	1.0 atm = 14.7 lb/in. ²
1.0 mm Hg = 133 N/m ²	1.0 atm = 760 mm Hg
1.0 cm Hg = 1.33×10^3 N/m ²	1.0 atm = 76.0 cm Hg
1.0 cm water = 98.1 N/m ²	1.0 atm = 1.03×10^3 cm water
1.0 bar = 1.000×10^5 N/m ²	1.0 atm = 1.013 bar
1.0 millibar = 1.000×10^2 N/m ²	1.0 atm = 1013 millibar

Conversion Factors for Various Pressure Units

Section Summary

- Gauge pressure is the pressure relative to atmospheric pressure.
- Absolute pressure is the sum of gauge pressure and atmospheric pressure.
- Aneroid gauge measures pressure using a bellows-and-spring arrangement connected to the pointer of a calibrated scale.
- Open-tube manometers have U-shaped tubes and one end is always open. It is used to measure pressure.
- A mercury barometer is a device that measures atmospheric pressure.

Conceptual Questions

Exercise:

Problem:

Explain why the fluid reaches equal levels on either side of a manometer if both sides are open to the atmosphere, even if the tubes are of different diameters.

Exercise:

Problem:

[\[link\]](#) shows how a common measurement of arterial blood pressure is made. Is there any effect on the measured pressure if the manometer is lowered? What is the effect of raising the arm above the shoulder? What is the effect of placing the cuff on the upper leg with the person standing? Explain your answers in terms of pressure created by the weight of a fluid.

Exercise:

Problem:

Considering the magnitude of typical arterial blood pressures, why are mercury rather than water manometers used for these measurements?

Problems & Exercises

Exercise:

Problem:

Find the gauge and absolute pressures in the balloon and peanut jar shown in [\[link\]](#), assuming the manometer connected to the balloon uses water whereas the manometer connected to the jar contains mercury. Express in units of centimeters of water for the balloon and millimeters of mercury for the jar, taking $h = 0.0500$ m for each.

Solution:

Balloon:

$$\begin{aligned}P_g &= 5.00 \text{ cm H}_2\text{O}, \\P_{\text{abs}} &= 1.035 \times 10^3 \text{ cm H}_2\text{O}.\end{aligned}$$

Jar:

$$\begin{aligned}P_g &= -50.0 \text{ mm Hg}, \\P_{\text{abs}} &= 710 \text{ mm Hg}.\end{aligned}$$

Exercise:**Problem:**

(a) Convert normal blood pressure readings of 120 over 80 mm Hg to newtons per meter squared using the relationship for pressure due to the weight of a fluid ($P = h\rho g$) rather than a conversion factor. (b) Discuss why blood pressures for an infant could be smaller than those for an adult. Specifically, consider the smaller height to which blood must be pumped.

Exercise:**Problem:**

How tall must a water-filled manometer be to measure blood pressures as high as 300 mm Hg?

Solution:

4.08 m

Exercise:

Problem:

Pressure cookers have been around for more than 300 years, although their use has strongly declined in recent years (early models had a nasty habit of exploding). How much force must the latches holding the lid onto a pressure cooker be able to withstand if the circular lid is 25.0 cm in diameter and the gauge pressure inside is 300 atm? Neglect the weight of the lid.

Exercise:**Problem:**

Suppose you measure a standing person's blood pressure by placing the cuff on his leg 0.500 m below the heart. Calculate the pressure you would observe (in units of mm Hg) if the pressure at the heart were 120 over 80 mm Hg. Assume that there is no loss of pressure due to resistance in the circulatory system (a reasonable assumption, since major arteries are large).

Solution:

$$\Delta P = 38.7 \text{ mm Hg,}$$

$$\text{Leg blood pressure} = \frac{159}{119}.$$

Exercise:**Problem:**

A submarine is stranded on the bottom of the ocean with its hatch 25.0 m below the surface. Calculate the force needed to open the hatch from the inside, given it is circular and 0.450 m in diameter. Air pressure inside the submarine is 1.00 atm.

Exercise:**Problem:**

Assuming bicycle tires are perfectly flexible and support the weight of bicycle and rider by pressure alone, calculate the total area of the tires in contact with the ground. The bicycle plus rider has a mass of 80.0 kg, and the gauge pressure in the tires is $3.50 \times 10^5 \text{ Pa}$.

Solution:

22.4 cm²

Glossary

absolute pressure

the sum of gauge pressure and atmospheric pressure

diastolic pressure

the minimum blood pressure in the artery

gauge pressure

the pressure relative to atmospheric pressure

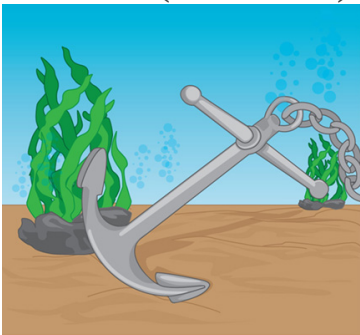
systolic pressure

the maximum blood pressure in the artery

Archimedes' Principle

- Define buoyant force.
- State Archimedes' principle.
- Understand why objects float or sink.
- Understand the relationship between density and Archimedes' principle.

When you rise from lounging in a warm bath, your arms feel strangely heavy. This is because you no longer have the buoyant support of the water. Where does this buoyant force come from? Why is it that some things float and others do not? Do objects that sink get any support at all from the fluid? Is your body buoyed by the atmosphere, or are only helium balloons affected? (See [\[link\]](#).)



(a)



(b)



(c)

(a) Even objects that sink, like this anchor, are partly supported by water when submerged. (b) Submarines have adjustable density (ballast tanks) so that they may float or sink as desired. (credit: Allied Navy) (c) Helium-filled balloons tug upward on their strings, demonstrating air's buoyant effect. (credit: Crystl)

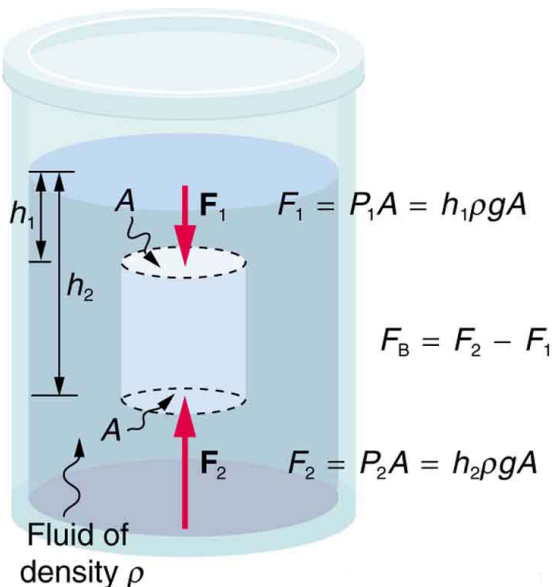
Answers to all these questions, and many others, are based on the fact that pressure increases with depth in a fluid. This means that the upward force on the bottom of an object in a fluid is greater than the downward force on the top of the object. There is a net upward, or **buoyant force** on any object in any fluid. (See [\[link\]](#).) If the buoyant force is greater than the object's

weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.

Note:

Buoyant Force

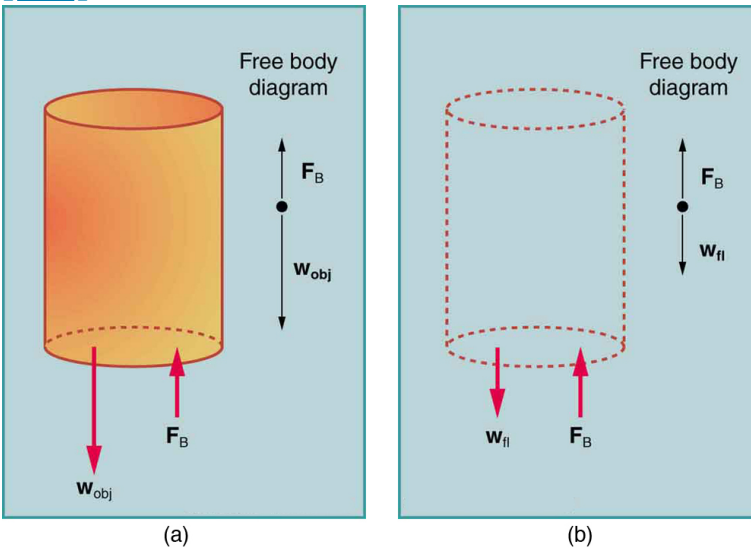
The buoyant force is the net upward force on any object in any fluid.



Pressure due to the weight of a fluid increases with depth since $P = h\rho g$. This pressure and associated upward force on the bottom of the cylinder are greater than the downward force on the top of the cylinder. Their difference is the buoyant

force $\mathbf{F}_B$. (Horizontal forces cancel.)

Just how great is this buoyant force? To answer this question, think about what happens when a submerged object is removed from a fluid, as in [\[link\]](#).



- (a) An object submerged in a fluid experiences a buoyant force F_B . If F_B is greater than the weight of the object, the object will rise. If F_B is less than the weight of the object, the object will sink.
- (b) If the object is removed, it is replaced by fluid having weight w_{fl} . Since this weight is supported by surrounding fluid, the buoyant force must equal the weight of the fluid displaced. That is, $F_B = w_{fl}$, a statement of Archimedes' principle.

The space it occupied is filled by fluid having a weight w_{fl} . This weight is supported by the surrounding fluid, and so the buoyant force must equal w_{fl} , the weight of the fluid displaced by the object. It is a tribute to the genius

of the Greek mathematician and inventor Archimedes (ca. 287–212 B.C.) that he stated this principle long before concepts of force were well established. Stated in words, **Archimedes' principle** is as follows: The buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

Equation:

$$F_B = w_{\text{fl}},$$

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object. Archimedes' principle is valid in general, for any object in any fluid, whether partially or totally submerged.

Note:

Archimedes' Principle

According to this principle the buoyant force on an object equals the weight of the fluid it displaces. In equation form, Archimedes' principle is

Equation:

$$F_B = w_{\text{fl}},$$

where F_B is the buoyant force and w_{fl} is the weight of the fluid displaced by the object.

Humm ... High-tech body swimsuits were introduced in 2008 in preparation for the Beijing Olympics. One concern (and international rule) was that these suits should not provide any buoyancy advantage. How do you think that this rule could be verified?

Note:

Making Connections: Take-Home Investigation

The density of aluminum foil is 2.7 times the density of water. Take a piece of foil, roll it up into a ball and drop it into water. Does it sink? Why or why not? Can you make it sink?

Floating and Sinking

Drop a lump of clay in water. It will sink. Then mold the lump of clay into the shape of a boat, and it will float. Because of its shape, the boat displaces more water than the lump and experiences a greater buoyant force. The same is true of steel ships.

Example:

Calculating buoyant force: dependency on shape

(a) Calculate the buoyant force on 10,000 metric tons (1.00×10^7 kg) of solid steel completely submerged in water, and compare this with the steel's weight. (b) What is the maximum buoyant force that water could exert on this same steel if it were shaped into a boat that could displace 1.00×10^5 m³ of water?

Strategy for (a)

To find the buoyant force, we must find the weight of water displaced. We can do this by using the densities of water and steel given in [\[link\]](#). We note that, since the steel is completely submerged, its volume and the water's volume are the same. Once we know the volume of water, we can find its mass and weight.

Solution for (a)

First, we use the definition of density $\rho = \frac{m}{V}$ to find the steel's volume, and then we substitute values for mass and density. This gives

Equation:

$$V_{\text{st}} = \frac{m_{\text{st}}}{\rho_{\text{st}}} = \frac{1.00 \times 10^7 \text{ kg}}{7.8 \times 10^3 \text{ kg/m}^3} = 1.28 \times 10^3 \text{ m}^3.$$

Because the steel is completely submerged, this is also the volume of water displaced, V_w . We can now find the mass of water displaced from the relationship between its volume and density, both of which are known.

This gives

Equation:

$$\begin{aligned} m_w &= \rho_w V_w = (1.000 \times 10^3 \text{ kg/m}^3)(1.28 \times 10^3 \text{ m}^3) \\ &= 1.28 \times 10^6 \text{ kg.} \end{aligned}$$

By Archimedes' principle, the weight of water displaced is $m_w g$, so the buoyant force is

Equation:

$$\begin{aligned} F_B &= w_w = m_w g = (1.28 \times 10^6 \text{ kg})(9.80 \text{ m/s}^2) \\ &= 1.3 \times 10^7 \text{ N.} \end{aligned}$$

The steel's weight is $m_w g = 9.80 \times 10^7 \text{ N}$, which is much greater than the buoyant force, so the steel will remain submerged. Note that the buoyant force is rounded to two digits because the density of steel is given to only two digits.

Strategy for (b)

Here we are given the maximum volume of water the steel boat can displace. The buoyant force is the weight of this volume of water.

Solution for (b)

The mass of water displaced is found from its relationship to density and volume, both of which are known. That is,

Equation:

$$\begin{aligned} m_w &= \rho_w V_w = (1.000 \times 10^3 \text{ kg/m}^3)(1.00 \times 10^5 \text{ m}^3) \\ &= 1.00 \times 10^8 \text{ kg.} \end{aligned}$$

The maximum buoyant force is the weight of this much water, or

Equation:

$$\begin{aligned} F_B &= w_w = m_w g = (1.00 \times 10^8 \text{ kg}) (9.80 \text{ m/s}^2) \\ &= 9.80 \times 10^8 \text{ N.} \end{aligned}$$

Discussion

The maximum buoyant force is ten times the weight of the steel, meaning the ship can carry a load nine times its own weight without sinking.

Note:**Making Connections: Take-Home Investigation**

A piece of household aluminum foil is 0.016 mm thick. Use a piece of foil that measures 10 cm by 15 cm. (a) What is the mass of this amount of foil? (b) If the foil is folded to give it four sides, and paper clips or washers are added to this “boat,” what shape of the boat would allow it to hold the most “cargo” when placed in water? Test your prediction.

Density and Archimedes’ Principle

Density plays a crucial role in Archimedes’ principle. The average density of an object is what ultimately determines whether it floats. If its average density is less than that of the surrounding fluid, it will float. This is because the fluid, having a higher density, contains more mass and hence more weight in the same volume. The buoyant force, which equals the weight of the fluid displaced, is thus greater than the weight of the object. Likewise, an object denser than the fluid will sink.

The extent to which a floating object is submerged depends on how the object’s density is related to that of the fluid. In [\[link\]](#), for example, the unloaded ship has a lower density and less of it is submerged compared with the same ship loaded. We can derive a quantitative expression for the fraction submerged by considering density. The fraction submerged is the ratio of the volume submerged to the volume of the object, or

Equation:

$$\text{fraction submerged} = \frac{V_{\text{sub}}}{V_{\text{obj}}} = \frac{V_{\text{fl}}}{V_{\text{obj}}}.$$

The volume submerged equals the volume of fluid displaced, which we call V_{fl} . Now we can obtain the relationship between the densities by substituting $\rho = \frac{m}{V}$ into the expression. This gives

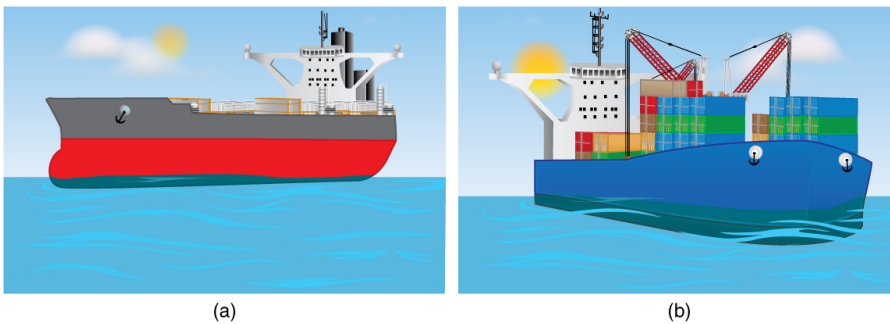
Equation:

$$\frac{V_{\text{fl}}}{V_{\text{obj}}} = \frac{m_{\text{fl}}/\rho_{\text{fl}}}{m_{\text{obj}}/\rho_{\text{obj}}},$$

where ρ_{obj} is the average density of the object and ρ_{fl} is the density of the fluid. Since the object floats, its mass and that of the displaced fluid are equal, and so they cancel from the equation, leaving

Equation:

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}.$$



An unloaded ship (a) floats higher in the water than a loaded ship (b).

We use this last relationship to measure densities. This is done by measuring the fraction of a floating object that is submerged—for example, with a hydrometer. It is useful to define the ratio of the density of an object to a fluid (usually water) as **specific gravity**:

Equation:

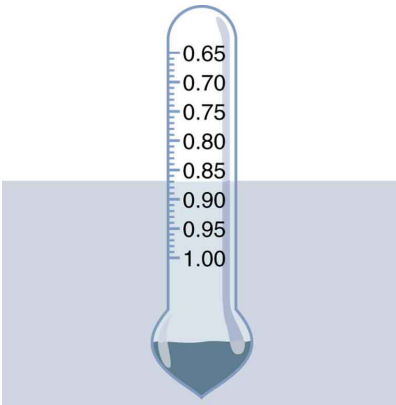
$$\text{specific gravity} = \frac{\rho}{\rho_{\text{w}}},$$

where ρ is the average density of the object or substance and ρ_{w} is the density of water at 4.00°C. Specific gravity is dimensionless, independent of whatever units are used for ρ . If an object floats, its specific gravity is less than one. If it sinks, its specific gravity is greater than one. Moreover, the fraction of a floating object that is submerged equals its specific gravity. If an object's specific gravity is exactly 1, then it will remain suspended in the fluid, neither sinking nor floating. Scuba divers try to obtain this state so that they can hover in the water. We measure the specific gravity of fluids, such as battery acid, radiator fluid, and urine, as an indicator of their condition. One device for measuring specific gravity is shown in [\[link\]](#).

Note:

Specific Gravity

Specific gravity is the ratio of the density of an object to a fluid (usually water).



This hydrometer is floating in a fluid of specific gravity 0.87. The glass hydrometer is filled with air and weighted with lead at the bottom. It floats highest in the densest fluids and has been calibrated and labeled so that specific gravity can be read from it directly.

Example:**Calculating Average Density: Floating Woman**

Suppose a 60.0-kg woman floats in freshwater with 97.0% of her volume submerged when her lungs are full of air. What is her average density?

Strategy

We can find the woman's density by solving the equation

Equation:

$$\text{fraction submerged} = \frac{\rho_{\text{obj}}}{\rho_{\text{fl}}}$$

for the density of the object. This yields

Equation:

$$\rho_{\text{obj}} = \rho_{\text{person}} = (\text{fraction submerged}) \cdot \rho_{\text{fl}}.$$

We know both the fraction submerged and the density of water, and so we can calculate the woman's density.

Solution

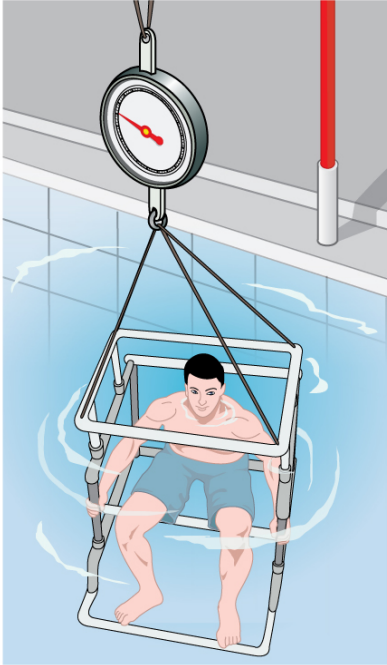
Entering the known values into the expression for her density, we obtain

Equation:

$$\rho_{\text{person}} = 0.970 \cdot \left(10^3 \frac{\text{kg}}{\text{m}^3} \right) = 970 \frac{\text{kg}}{\text{m}^3}.$$

Discussion

Her density is less than the fluid density. We expect this because she floats. Body density is one indicator of a person's percent body fat, of interest in medical diagnostics and athletic training. (See [\[link\]](#).)



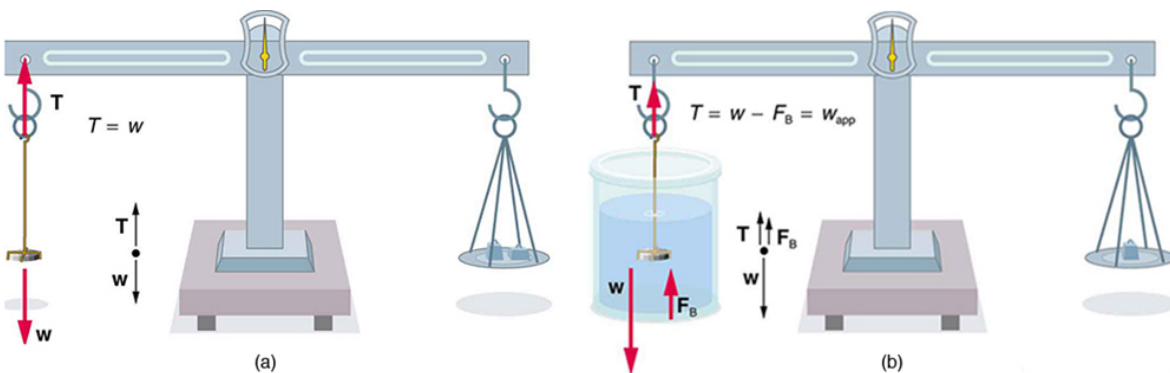
Subject in a “fat tank,” where he is weighed while completely submerged as part of a body density determination. The subject must completely empty his lungs and hold a metal weight in order to sink. Corrections are made for the residual air in his lungs (measured separately) and the metal weight. His corrected submerged weight, his weight in air,

and pinch tests of strategic fatty areas are used to calculate his percent body fat.

There are many obvious examples of lower-density objects or substances floating in higher-density fluids—oil on water, a hot-air balloon, a bit of cork in wine, an iceberg, and hot wax in a “lava lamp,” to name a few. Less obvious examples include lava rising in a volcano and mountain ranges floating on the higher-density crust and mantle beneath them. Even seemingly solid Earth has fluid characteristics.

More Density Measurements

One of the most common techniques for determining density is shown in [\[link\]](#).



(a) A coin is weighed in air. (b) The apparent weight of the coin is determined while it is completely submerged in a fluid of known density. These two measurements are used to calculate the density of the coin.

An object, here a coin, is weighed in air and then weighed again while submerged in a liquid. The density of the coin, an indication of its authenticity, can be calculated if the fluid density is known. This same technique can also be used to determine the density of the fluid if the density of the coin is known. All of these calculations are based on Archimedes' principle.

Archimedes' principle states that the buoyant force on the object equals the weight of the fluid displaced. This, in turn, means that the object *appears* to weigh less when submerged; we call this measurement the object's *apparent weight*. The object suffers an *apparent weight loss* equal to the weight of the fluid displaced. Alternatively, on balances that measure mass, the object suffers an *apparent mass loss* equal to the mass of fluid displaced. That is

Equation:

$$\text{apparent weight loss} = \text{weight of fluid displaced}$$

or

Equation:

$$\text{apparent mass loss} = \text{mass of fluid displaced.}$$

The next example illustrates the use of this technique.

Example:

Calculating Density: Is the Coin Authentic?

The mass of an ancient Greek coin is determined in air to be 8.630 g. When the coin is submerged in water as shown in [\[link\]](#), its apparent mass is 7.800 g. Calculate its density, given that water has a density of 1.000 g/cm^3 and that effects caused by the wire suspending the coin are negligible.

Strategy

To calculate the coin's density, we need its mass (which is given) and its volume. The volume of the coin equals the volume of water displaced. The volume of water displaced V_w can be found by solving the equation for density $\rho = \frac{m}{V}$ for V .

Solution

The volume of water is $V_w = \frac{m_w}{\rho_w}$ where m_w is the mass of water displaced. As noted, the mass of the water displaced equals the apparent mass loss, which is $m_w = 8.630 \text{ g} - 7.800 \text{ g} = 0.830 \text{ g}$. Thus the volume of water is $V_w = \frac{0.830 \text{ g}}{1.000 \text{ g/cm}^3} = 0.830 \text{ cm}^3$. This is also the volume of the coin, since it is completely submerged. We can now find the density of the coin using the definition of density:

Equation:

$$\rho_c = \frac{m_c}{V_c} = \frac{8.630 \text{ g}}{0.830 \text{ cm}^3} = 10.4 \text{ g/cm}^3.$$

Discussion

You can see from [\[link\]](#) that this density is very close to that of pure silver, appropriate for this type of ancient coin. Most modern counterfeits are not pure silver.

This brings us back to Archimedes' principle and how it came into being. As the story goes, the king of Syracuse gave Archimedes the task of determining whether the royal crown maker was supplying a crown of pure gold. The purity of gold is difficult to determine by color (it can be diluted with other metals and still look as yellow as pure gold), and other analytical techniques had not yet been conceived. Even ancient peoples, however, realized that the density of gold was greater than that of any other then-known substance. Archimedes purportedly agonized over his task and had his inspiration one day while at the public baths, pondering the support the water gave his body. He came up with his now-famous principle, saw how to apply it to determine density, and ran naked down the streets of Syracuse crying "Eureka!" (Greek for "I have found it"). Similar behavior can be observed in contemporary physicists from time to time!

Note:**PhET Explorations: Buoyancy**

When will objects float and when will they sink? Learn how buoyancy works with blocks. Arrows show the applied forces, and you can modify the properties of the blocks and the fluid.

https://phet.colorado.edu/sims/density-and-buoyancy/buoyancy_en.html

Section Summary

- Buoyant force is the net upward force on any object in any fluid. If the buoyant force is greater than the object's weight, the object will rise to the surface and float. If the buoyant force is less than the object's weight, the object will sink. If the buoyant force equals the object's weight, the object will remain suspended at that depth. The buoyant force is always present whether the object floats, sinks, or is suspended in a fluid.
- Archimedes' principle states that the buoyant force on an object equals the weight of the fluid it displaces.
- Specific gravity is the ratio of the density of an object to a fluid (usually water).

Conceptual Questions

Exercise:**Problem:**

More force is required to pull the plug in a full bathtub than when it is empty. Does this contradict Archimedes' principle? Explain your answer.

Exercise:**Problem:**

Do fluids exert buoyant forces in a "weightless" environment, such as in the space shuttle? Explain your answer.

Exercise:**Problem:**

Will the same ship float higher in salt water than in freshwater?
Explain your answer.

Exercise:**Problem:**

Marbles dropped into a partially filled bathtub sink to the bottom. Part of their weight is supported by buoyant force, yet the downward force on the bottom of the tub increases by exactly the weight of the marbles. Explain why.

Problem Exercises**Exercise:****Problem:**

What fraction of ice is submerged when it floats in freshwater, given the density of water at 0°C is very close to 1000 kg/m³?

Solution:

91.7%

Exercise:**Problem:**

Logs sometimes float vertically in a lake because one end has become water-logged and denser than the other. What is the average density of a uniform-diameter log that floats with 20.0% of its length above water?

Exercise:

Problem:

Find the density of a fluid in which a hydrometer having a density of 0.750 g/mL floats with 92.0% of its volume submerged.

Solution:

$$815 \text{ kg/m}^3$$

Exercise:**Problem:**

If your body has a density of 995 kg/m^3 , what fraction of you will be submerged when floating gently in: (a) freshwater? (b) salt water, which has a density of 1027 kg/m^3 ?

Exercise:**Problem:**

Bird bones have air pockets in them to reduce their weight—this also gives them an average density significantly less than that of the bones of other animals. Suppose an ornithologist weighs a bird bone in air and in water and finds its mass is 45.0 g and its apparent mass when submerged is 3.60 g (the bone is watertight). (a) What mass of water is displaced? (b) What is the volume of the bone? (c) What is its average density?

Solution:

(a) 41.4 g

(b) 41.4 cm^3

(c) 1.09 g/cm^3

Exercise:

Problem:

A rock with a mass of 540 g in air is found to have an apparent mass of 342 g when submerged in water. (a) What mass of water is displaced? (b) What is the volume of the rock? (c) What is its average density? Is this consistent with the value for granite?

Exercise:**Problem:**

Archimedes' principle can be used to calculate the density of a fluid as well as that of a solid. Suppose a chunk of iron with a mass of 390.0 g in air is found to have an apparent mass of 350.5 g when completely submerged in an unknown liquid. (a) What mass of fluid does the iron displace? (b) What is the volume of iron, using its density as given in [\[link\]](#) (c) Calculate the fluid's density and identify it.

Solution:

(a) 39.5 g

(b) 50 cm³

(c) 0.79 g/cm³

It is ethyl alcohol.

Exercise:**Problem:**

In an immersion measurement of a woman's density, she is found to have a mass of 62.0 kg in air and an apparent mass of 0.0850 kg when completely submerged with lungs empty. (a) What mass of water does she displace? (b) What is her volume? (c) Calculate her density. (d) If her lung capacity is 1.75 L, is she able to float without treading water with her lungs filled with air?

Exercise:

Problem:

Some fish have a density slightly less than that of water and must exert a force (swim) to stay submerged. What force must an 85.0-kg grouper exert to stay submerged in salt water if its body density is 1015 kg/m^3 ?

Solution:

8.21 N

Exercise:**Problem:**

(a) Calculate the buoyant force on a 2.00-L helium balloon. (b) Given the mass of the rubber in the balloon is 1.50 g, what is the net vertical force on the balloon if it is let go? You can neglect the volume of the rubber.

Exercise:**Problem:**

(a) What is the density of a woman who floats in freshwater with 4.00% of her volume above the surface? This could be measured by placing her in a tank with marks on the side to measure how much water she displaces when floating and when held under water (briefly). (b) What percent of her volume is above the surface when she floats in seawater?

Solution:

(a) 960 kg/m^3

(b) 6.34%

She indeed floats more in seawater.

Exercise:

Problem:

A certain man has a mass of 80 kg and a density of 955 kg/m^3 (excluding the air in his lungs). (a) Calculate his volume. (b) Find the buoyant force air exerts on him. (c) What is the ratio of the buoyant force to his weight?

Exercise:**Problem:**

A simple compass can be made by placing a small bar magnet on a cork floating in water. (a) What fraction of a plain cork will be submerged when floating in water? (b) If the cork has a mass of 10.0 g and a 20.0-g magnet is placed on it, what fraction of the cork will be submerged? (c) Will the bar magnet and cork float in ethyl alcohol?

Solution:

(a) 0.24

(b) 0.68

(c) Yes, the cork will float because

$$\rho_{\text{obj}} < \rho_{\text{ethyl alcohol}} (0.678 \text{ g/cm}^3 < 0.79 \text{ g/cm}^3)$$

Exercise:**Problem:**

What fraction of an iron anchor's weight will be supported by buoyant force when submerged in saltwater?

Exercise:

Problem:

Scurrilous con artists have been known to represent gold-plated tungsten ingots as pure gold and sell them to the greedy at prices much below gold value but deservedly far above the cost of tungsten. With what accuracy must you be able to measure the mass of such an ingot in and out of water to tell that it is almost pure tungsten rather than pure gold?

Solution:

The difference is 0.006%.

Exercise:**Problem:**

A twin-sized air mattress used for camping has dimensions of 100 cm by 200 cm by 15 cm when blown up. The weight of the mattress is 2 kg. How heavy a person could the air mattress hold if it is placed in freshwater?

Exercise:**Problem:**

Referring to [\[link\]](#), prove that the buoyant force on the cylinder is equal to the weight of the fluid displaced (Archimedes' principle). You may assume that the buoyant force is $F_2 - F_1$ and that the ends of the cylinder have equal areas A . Note that the volume of the cylinder (and that of the fluid it displaces) equals $(h_2 - h_1)A$.

Solution:

$$\begin{aligned} F_{\text{net}} &= F_2 - F_1 = P_2 A - P_1 A = (P_2 - P_1) A \\ &= (h_2 \rho_{\text{fl}} g - h_1 \rho_{\text{fl}} g) A \\ &= (h_2 - h_1) \rho_{\text{fl}} g A \end{aligned}$$

where ρ_{fl} = density of fluid. Therefore,

$$F_{\text{net}} = (h_2 - h_1)A\rho_{\text{fl}}g = V_{\text{fl}}\rho_{\text{fl}}g = m_{\text{fl}}g = w_{\text{fl}}$$

where is w_{fl} the weight of the fluid displaced.

Exercise:

Problem:

(a) A 75.0-kg man floats in freshwater with 3.00% of his volume above water when his lungs are empty, and 5.00% of his volume above water when his lungs are full. Calculate the volume of air he inhales—called his lung capacity—in liters. (b) Does this lung volume seem reasonable?

Glossary

Archimedes' principle

the buoyant force on an object equals the weight of the fluid it displaces

buoyant force

the net upward force on any object in any fluid

specific gravity

the ratio of the density of an object to a fluid (usually water)

Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action

- Understand cohesive and adhesive forces.
- Define surface tension.
- Understand capillary action.

Cohesion and Adhesion in Liquids

Children blow soap bubbles and play in the spray of a sprinkler on a hot summer day. (See [\[link\]](#).) An underwater spider keeps his air supply in a shiny bubble he carries wrapped around him. A technician draws blood into a small-diameter tube just by touching it to a drop on a pricked finger. A premature infant struggles to inflate her lungs. What is the common thread? All these activities are dominated by the attractive forces between atoms and molecules in liquids—both within a liquid and between the liquid and its surroundings.

Attractive forces between molecules of the same type are called **cohesive forces**. Liquids can, for example, be held in open containers because cohesive forces hold the molecules together. Attractive forces between molecules of different types are called **adhesive forces**. Such forces cause liquid drops to cling to window panes, for example. In this section we examine effects directly attributable to cohesive and adhesive forces in liquids.

Note:

Cohesive Forces

Attractive forces between molecules of the same type are called cohesive forces.

Note:

Adhesive Forces

Attractive forces between molecules of different types are called adhesive forces.



The soap bubbles in this photograph are caused by cohesive forces among molecules in liquids. (credit: Steven Depolo, Flickr)

Surface Tension

Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called **surface tension**. Molecules on the surface are pulled inward by cohesive forces, reducing the surface area. Molecules inside the liquid experience zero net force, since they have neighbors on all sides.

Note:
Surface Tension

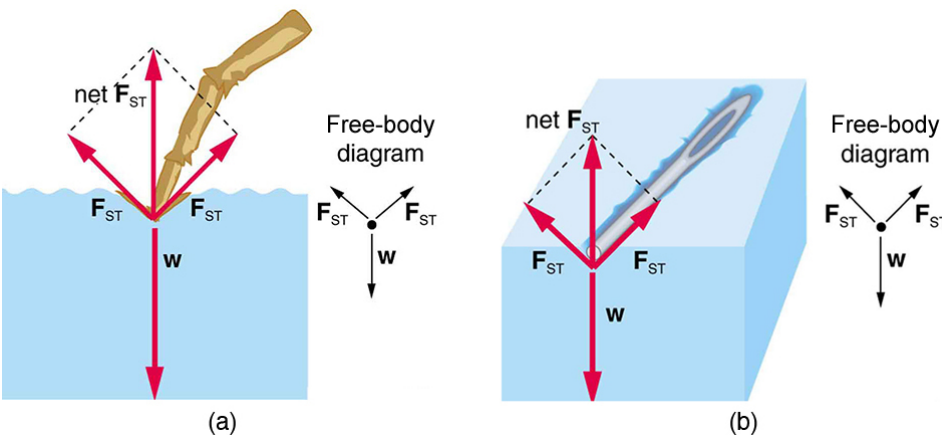
Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.

Note:

Making Connections: Surface Tension

Forces between atoms and molecules underlie the macroscopic effect called surface tension. These attractive forces pull the molecules closer together and tend to minimize the surface area. This is another example of a submicroscopic explanation for a macroscopic phenomenon.

The model of a liquid surface acting like a stretched elastic sheet can effectively explain surface tension effects. For example, some insects can walk on water (as opposed to floating in it) as we would walk on a trampoline—they dent the surface as shown in [\[link\]\(a\)](#). [\[link\]\(b\)](#) shows another example, where a needle rests on a water surface. The iron needle cannot, and does not, float, because its density is greater than that of water. Rather, its weight is supported by forces in the stretched surface that try to make the surface smaller or flatter. If the needle were placed point down on the surface, its weight acting on a smaller area would break the surface, and it would sink.



Surface tension supporting the weight of an insect and an iron needle, both of which rest on the surface without penetrating it. They are not floating; rather, they are supported by the surface of the liquid. (a) An insect leg dents the water surface. F_{ST} is a restoring force (surface tension) parallel to the surface. (b) An iron needle similarly dents a water surface until the restoring force (surface tension) grows to equal its weight.

Surface tension is proportional to the strength of the cohesive force, which varies with the type of liquid. Surface tension γ is defined to be the force F per unit length L exerted by a stretched liquid membrane:

Equation:

$$\gamma = \frac{F}{L}.$$

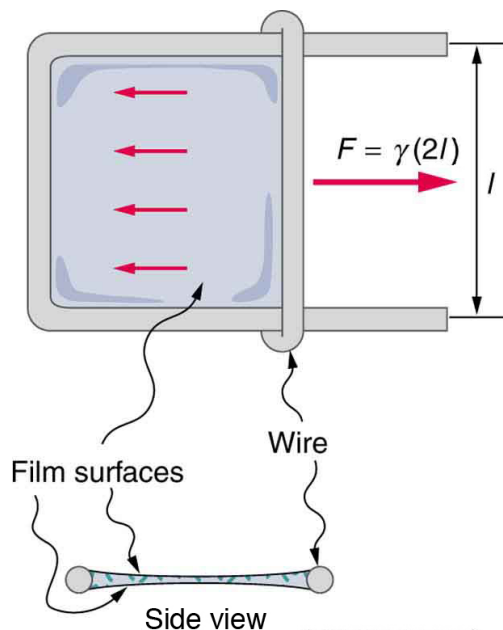
[\[link\]](#) lists values of γ for some liquids. For the insect of [\[link\]](#)(a), its weight w is supported by the upward components of the surface tension force: $w = \gamma L \sin \theta$, where L is the circumference of the insect's foot in contact with the water. [\[link\]](#) shows one way to measure surface tension. The liquid film exerts a force on the movable wire in an attempt to reduce its surface area. The magnitude of this force depends on the surface tension of the liquid and can be measured accurately.

Surface tension is the reason why liquids form bubbles and droplets. The inward surface tension force causes bubbles to be approximately spherical and raises the pressure of the gas trapped inside relative to atmospheric pressure outside. It can be shown that the gauge pressure P inside a spherical bubble is given by

Equation:

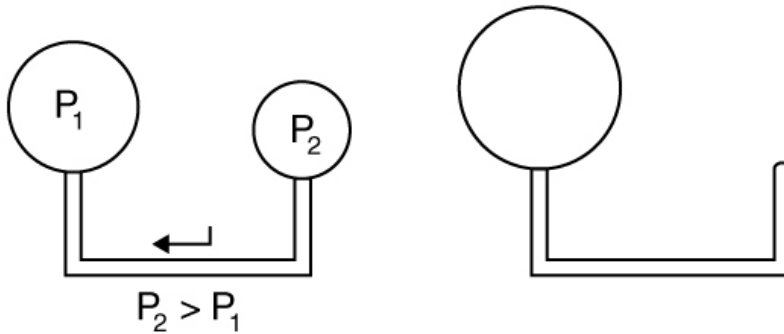
$$P = \frac{4\gamma}{r},$$

where r is the radius of the bubble. Thus the pressure inside a bubble is greatest when the bubble is the smallest. Another bit of evidence for this is illustrated in [\[link\]](#). When air is allowed to flow between two balloons of unequal size, the smaller balloon tends to collapse, filling the larger balloon.



Sliding wire device used for measuring surface tension; the device exerts a force to reduce the film's surface area. The force needed to hold the wire in place is $F = \gamma L = \gamma(2l)$, since there are *two* liquid surfaces attached to the wire. This force remains nearly constant as the

film is stretched, until the film approaches its breaking point.



With the valve closed, two balloons of different sizes are attached to each end of a tube. Upon opening the valve, the smaller balloon decreases in size with the air moving to fill the larger balloon. The pressure in a spherical balloon is inversely proportional to its radius, so that the smaller balloon has a greater internal pressure than the larger balloon, resulting in this flow.

Liquid	Surface tension γ (N/m)
Water at 0°C	0.0756

Liquid	Surface tension γ (N/m)
Water at 20°C	0.0728
Water at 100°C	0.0589
Soapy water (typical)	0.0370
Ethyl alcohol	0.0223
Glycerin	0.0631
Mercury	0.465
Olive oil	0.032
Tissue fluids (typical)	0.050
Blood, whole at 37°C	0.058
Blood plasma at 37°C	0.073
Gold at 1070°C	1.000
Oxygen at -193°C	0.0157
Helium at -269°C	0.00012

Surface Tension of Some Liquids[\[footnote\]](#)

At 20°C unless otherwise stated.

Example:

Surface Tension: Pressure Inside a Bubble

Calculate the gauge pressure inside a soap bubble 2.00×10^{-4} m in radius using the surface tension for soapy water in [\[link\]](#). Convert this pressure to

mm Hg.

Strategy

The radius is given and the surface tension can be found in [\[link\]](#), and so P can be found directly from the equation $P = \frac{4\gamma}{r}$.

Solution

Substituting r and γ into the equation $P = \frac{4\gamma}{r}$, we obtain

Equation:

$$P = \frac{4\gamma}{r} = \frac{4(0.037 \text{ N/m})}{2.00 \times 10^{-4} \text{ m}} = 740 \text{ N/m}^2 = 740 \text{ Pa}.$$

We use a conversion factor to get this into units of mm Hg:

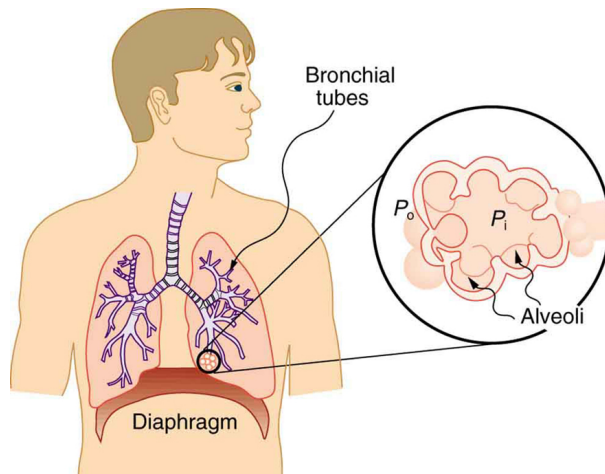
Equation:

$$P = (740 \text{ N/m}^2) \frac{1.00 \text{ mm Hg}}{133 \text{ N/m}^2} = 5.56 \text{ mm Hg}.$$

Discussion

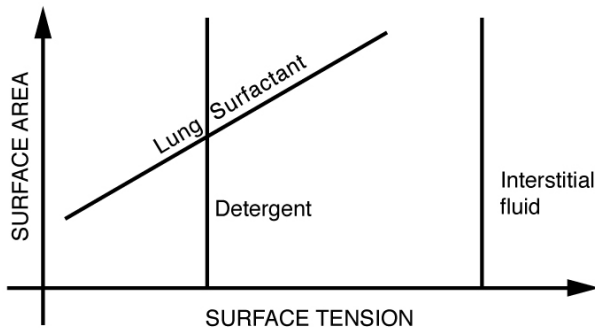
Note that if a hole were to be made in the bubble, the air would be forced out, the bubble would decrease in radius, and the pressure inside would *increase* to atmospheric pressure (760 mm Hg).

Our lungs contain hundreds of millions of mucus-lined sacs called *alveoli*, which are very similar in size, and about 0.1 mm in diameter. (See [\[link\]](#).) You can exhale without muscle action by allowing surface tension to contract these sacs. Medical patients whose breathing is aided by a positive pressure respirator have air blown into the lungs, but are generally allowed to exhale on their own. Even if there is paralysis, surface tension in the alveoli will expel air from the lungs. Since pressure increases as the radii of the alveoli decrease, an occasional deep cleansing breath is needed to fully reinflate the alveoli. Respirators are programmed to do this and we find it natural, as do our companion dogs and cats, to take a cleansing breath before settling into a nap.



Bronchial tubes in the lungs branch into ever-smaller structures, finally ending in alveoli. The alveoli act like tiny bubbles. The surface tension of their mucous lining aids in exhalation and can prevent inhalation if too great.

The tension in the walls of the alveoli results from the membrane tissue and a liquid on the walls of the alveoli containing a long lipoprotein that acts as a surfactant (a surface-tension reducing substance). The need for the surfactant results from the tendency of small alveoli to collapse and the air to fill into the larger alveoli making them even larger (as demonstrated in [\[link\]](#)). During inhalation, the lipoprotein molecules are pulled apart and the wall tension increases as the radius increases (increased surface tension). During exhalation, the molecules slide back together and the surface tension decreases, helping to prevent a collapse of the alveoli. The surfactant therefore serves to change the wall tension so that small alveoli don't collapse and large alveoli are prevented from expanding too much. This tension change is a unique property of these surfactants, and is not shared by detergents (which simply lower surface tension). (See [\[link\]](#).)



Surface tension as a function of surface area. The surface tension for lung surfactant decreases with decreasing area. This ensures that small alveoli don't collapse and large alveoli are not able to over expand.

If water gets into the lungs, the surface tension is too great and you cannot inhale. This is a severe problem in resuscitating drowning victims. A similar problem occurs in newborn infants who are born without this surfactant—their lungs are very difficult to inflate. This condition is known as *hyaline membrane disease* and is a leading cause of death for infants, particularly in premature births. Some success has been achieved in treating hyaline membrane disease by spraying a surfactant into the infant's breathing passages. Emphysema produces the opposite problem with alveoli. Alveolar walls of emphysema victims deteriorate, and the sacs combine to form larger sacs. Because pressure produced by surface tension decreases with increasing radius, these larger sacs produce smaller pressure, reducing the ability of emphysema victims to exhale. A common test for emphysema is to measure the pressure and volume of air that can be exhaled.

Note:

Making Connections: Take-Home Investigation

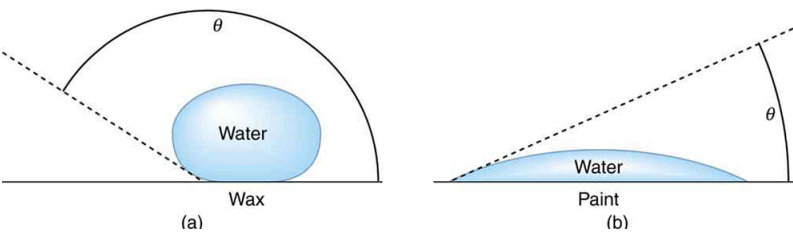
(1) Try floating a sewing needle on water. In order for this activity to work, the needle needs to be very clean as even the oil from your fingers can be sufficient to affect the surface properties of the needle. (2) Place the bristles of a paint brush into water. Pull the brush out and notice that for a short while, the bristles will stick together. The surface tension of the water surrounding the bristles is sufficient to hold the bristles together. As the bristles dry out, the surface tension effect dissipates. (3) Place a loop of thread on the surface of still water in such a way that all of the thread is in contact with the water. Note the shape of the loop. Now place a drop of detergent into the middle of the loop. What happens to the shape of the loop? Why? (4) Sprinkle pepper onto the surface of water. Add a drop of detergent. What happens? Why? (5) Float two matches parallel to each other and add a drop of detergent between them. What happens? Note: For each new experiment, the water needs to be replaced and the bowl washed to free it of any residual detergent.

Adhesion and Capillary Action

Why is it that water beads up on a waxed car but does not on bare paint? The answer is that the adhesive forces between water and wax are much smaller than those between water and paint. Competition between the forces of adhesion and cohesion are important in the macroscopic behavior of liquids. An important factor in studying the roles of these two forces is the angle θ between the tangent to the liquid surface and the surface. (See [\[link\]](#).) The **contact angle** θ is directly related to the relative strength of the cohesive and adhesive forces. The larger the strength of the cohesive force relative to the adhesive force, the larger θ is, and the more the liquid tends to form a droplet. The smaller θ is, the smaller the relative strength, so that the adhesive force is able to flatten the drop. [\[link\]](#) lists contact angles for several combinations of liquids and solids.

Note:
Contact Angle

The angle θ between the tangent to the liquid surface and the surface is called the contact angle.



In the photograph, water beads on the waxed car paint and flattens on the unwaxed paint.

(a) Water forms beads on the waxed surface because the cohesive forces responsible for surface tension are larger than the adhesive forces, which tend to flatten the drop. (b)

Water beads on bare paint are flattened considerably because the adhesive forces

between water and paint are strong, overcoming surface tension. The contact angle θ is directly related to the relative strengths of the cohesive and adhesive forces. The larger θ is, the larger the ratio of cohesive to adhesive forces. (credit: P. P.

Urone)

One important phenomenon related to the relative strength of cohesive and adhesive forces is **capillary action**—the tendency of a fluid to be raised or

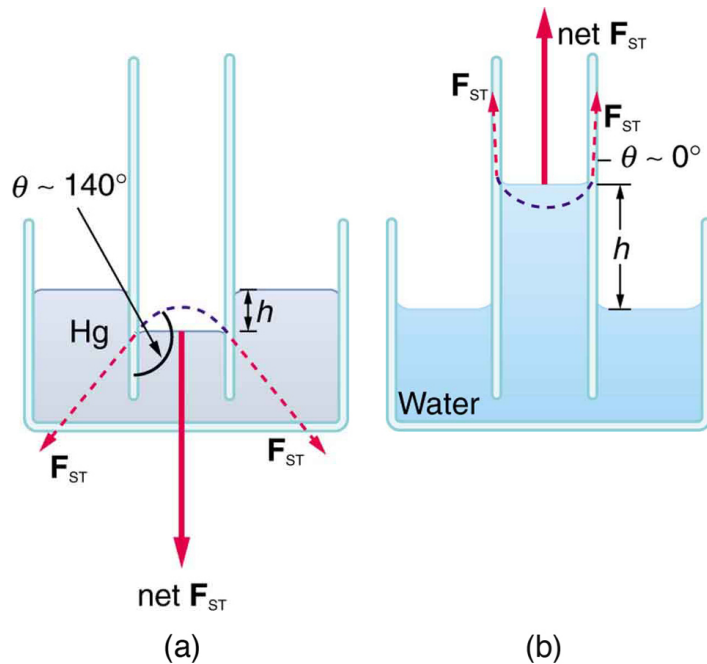
suppressed in a narrow tube, or *capillary tube*. This action causes blood to be drawn into a small-diameter tube when the tube touches a drop.

Note:

Capillary Action

The tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube, is called capillary action.

If a capillary tube is placed vertically into a liquid, as shown in [\[link\]](#), capillary action will raise or suppress the liquid inside the tube depending on the combination of substances. The actual effect depends on the relative strength of the cohesive and adhesive forces and, thus, the contact angle θ given in the table. If θ is less than 90° , then the fluid will be raised; if θ is greater than 90° , it will be suppressed. Mercury, for example, has a very large surface tension and a large contact angle with glass. When placed in a tube, the surface of a column of mercury curves downward, somewhat like a drop. The curved surface of a fluid in a tube is called a **meniscus**. The tendency of surface tension is always to reduce the surface area. Surface tension thus flattens the curved liquid surface in a capillary tube. This results in a downward force in mercury and an upward force in water, as seen in [\[link\]](#).



(a) Mercury is suppressed in a glass tube because its contact angle is greater than 90° . Surface tension exerts a downward force as it flattens the mercury, suppressing it in the tube. The dashed line shows the shape the mercury surface would have without the flattening effect of surface tension.

(b) Water is raised in a glass tube because its contact angle is nearly 0° . Surface tension therefore exerts an upward force when it flattens the surface to reduce its area.

Interface	Contact angle θ
Mercury–glass	140°
Water–glass	0°
Water–paraffin	107°
Water–silver	90°
Organic liquids (most)–glass	0°
Ethyl alcohol–glass	0°
Kerosene–glass	26°

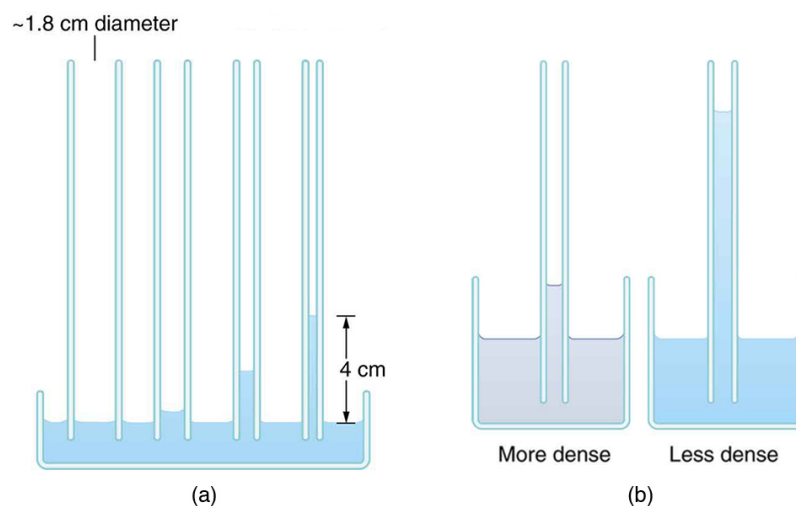
Contact Angles of Some Substances

Capillary action can move liquids horizontally over very large distances, but the height to which it can raise or suppress a liquid in a tube is limited by its weight. It can be shown that this height h is given by

Equation:

$$h = \frac{2\gamma \cos \theta}{\rho g r}.$$

If we look at the different factors in this expression, we might see how it makes good sense. The height is directly proportional to the surface tension γ , which is its direct cause. Furthermore, the height is inversely proportional to tube radius—the smaller the radius r , the higher the fluid can be raised, since a smaller tube holds less mass. The height is also inversely proportional to fluid density ρ , since a larger density means a greater mass in the same volume. (See [\[link\]](#).)



(a) Capillary action depends on the radius of a tube. The smaller the tube, the greater the height reached. The height is negligible for large-radius tubes. (b) A denser fluid in the same tube rises to a smaller height, all other factors being the same.

Example:

Calculating Radius of a Capillary Tube: Capillary Action: Tree Sap

Can capillary action be solely responsible for sap rising in trees? To answer this question, calculate the radius of a capillary tube that would raise sap 100 m to the top of a giant redwood, assuming that sap's density is 1050 kg/m^3 , its contact angle is zero, and its surface tension is the same as that of water at 20.0° C .

Strategy

The height to which a liquid will rise as a result of capillary action is given by $h = \frac{2\gamma \cos \theta}{\rho g r}$, and every quantity is known except for r .

Solution

Solving for r and substituting known values produces

Equation:

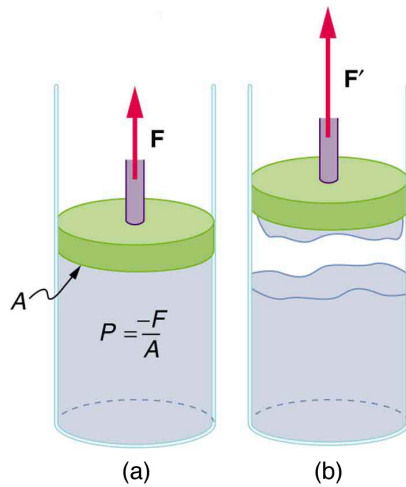
$$\begin{aligned} r &= \frac{2\gamma \cos \theta}{\rho g h} = \frac{2(0.0728 \text{ N/m})\cos(0^\circ)}{(1050 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(100 \text{ m})} \\ &= 1.41 \times 10^{-7} \text{ m.} \end{aligned}$$

Discussion

This result is unreasonable. Sap in trees moves through the *xylem*, which forms tubes with radii as small as $2.5 \times 10^{-5} \text{ m}$. This value is about 180 times as large as the radius found necessary here to raise sap 100 m. This means that capillary action alone cannot be solely responsible for sap getting to the tops of trees.

How *does* sap get to the tops of tall trees? (Recall that a column of water can only rise to a height of 10 m when there is a vacuum at the top—see [\[link\]](#).) The question has not been completely resolved, but it appears that it is pulled up like a chain held together by cohesive forces. As each molecule of sap enters a leaf and evaporates (a process called transpiration), the entire chain is pulled up a notch. So a negative pressure created by water evaporation must be present to pull the sap up through the xylem vessels. In most situations, *fluids can push but can exert only negligible pull*, because the cohesive forces seem to be too small to hold the molecules tightly together. But in this case, the cohesive force of water molecules provides a very strong pull. [\[link\]](#) shows one device for studying negative pressure.

Some experiments have demonstrated that negative pressures sufficient to pull sap to the tops of the tallest trees *can* be achieved.



(a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure $P = -F/A$.

(b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

Section Summary

- Attractive forces between molecules of the same type are called cohesive forces.
- Attractive forces between molecules of different types are called adhesive forces.
- Cohesive forces between molecules cause the surface of a liquid to contract to the smallest possible surface area. This general effect is called surface tension.
- Capillary action is the tendency of a fluid to be raised or suppressed in a narrow tube, or capillary tube which is due to the relative strength of cohesive and adhesive forces.

Conceptual Questions

Exercise:

Problem:

The density of oil is less than that of water, yet a loaded oil tanker sits lower in the water than an empty one. Why?

Exercise:

Problem:

Is surface tension due to cohesive or adhesive forces, or both?

Exercise:

Problem:

Is capillary action due to cohesive or adhesive forces, or both?

Exercise:

Problem:

Birds such as ducks, geese, and swans have greater densities than water, yet they are able to sit on its surface. Explain this ability, noting that water does not wet their feathers and that they cannot sit on soapy water.

Exercise:

Problem:

Water beads up on an oily sunbather, but not on her neighbor, whose skin is not oiled. Explain in terms of cohesive and adhesive forces.

Exercise:**Problem:**

Could capillary action be used to move fluids in a “weightless” environment, such as in an orbiting space probe?

Exercise:**Problem:**

What effect does capillary action have on the reading of a manometer with uniform diameter? Explain your answer.

Exercise:**Problem:**

Pressure between the inside chest wall and the outside of the lungs normally remains negative. Explain how pressure inside the lungs can become positive (to cause exhalation) without muscle action.

Problems & Exercises**Exercise:****Problem:**

What is the pressure inside an alveolus having a radius of 2.50×10^{-4} m if the surface tension of the fluid-lined wall is the same as for soapy water? You may assume the pressure is the same as that created by a spherical bubble.

Solution:

$$592 \text{ N/m}^2$$

Exercise:**Problem:**

(a) The pressure inside an alveolus with a 2.00×10^{-4} -m radius is 1.40×10^3 Pa, due to its fluid-lined walls. Assuming the alveolus acts like a spherical bubble, what is the surface tension of the fluid? (b) Identify the likely fluid. (You may need to extrapolate between values in [\[link\]](#).)

Exercise:**Problem:**

What is the gauge pressure in millimeters of mercury inside a soap bubble 0.100 m in diameter?

Solution:

$$2.23 \times 10^{-2} \text{ mm Hg}$$

Exercise:**Problem:**

Calculate the force on the slide wire in [\[link\]](#) if it is 3.50 cm long and the fluid is ethyl alcohol.

Exercise:**Problem:**

[\[link\]](#)(a) shows the effect of tube radius on the height to which capillary action can raise a fluid. (a) Calculate the height h for water in a glass tube with a radius of 0.900 cm—a rather large tube like the one on the left. (b) What is the radius of the glass tube on the right if it raises water to 4.00 cm?

Solution:

(a) $1.65 \times 10^{-3} \text{ m}$

(b) $3.71 \times 10^{-4} \text{ m}$

Exercise:

Problem:

We stated in [\[link\]](#) that a xylem tube is of radius $2.50 \times 10^{-5} \text{ m}$. Verify that such a tube raises sap less than a meter by finding h for it, making the same assumptions that sap's density is 1050 kg/m^3 , its contact angle is zero, and its surface tension is the same as that of water at 20.0° C .

Exercise:

Problem:

What fluid is in the device shown in [\[link\]](#) if the force is $3.16 \times 10^{-3} \text{ N}$ and the length of the wire is 2.50 cm ? Calculate the surface tension γ and find a likely match from [\[link\]](#).

Solution:

$$6.32 \times 10^{-2} \text{ N/m}$$

Based on the values in table, the fluid is probably glycerin.

Exercise:

Problem:

If the gauge pressure inside a rubber balloon with a 10.0-cm radius is 1.50 cm of water, what is the effective surface tension of the balloon?

Exercise:

Problem:

Calculate the gauge pressures inside 2.00-cm -radius bubbles of water, alcohol, and soapy water. Which liquid forms the most stable bubbles, neglecting any effects of evaporation?

Solution:

$$P_w = 14.6 \text{ N/m}^2,$$

$$P_a = 4.46 \text{ N/m}^2,$$

$$P_{sw} = 7.40 \text{ N/m}^2.$$

Alcohol forms the most stable bubble, since the absolute pressure inside is closest to atmospheric pressure.

Exercise:**Problem:**

Suppose water is raised by capillary action to a height of 5.00 cm in a glass tube. (a) To what height will it be raised in a paraffin tube of the same radius? (b) In a silver tube of the same radius?

Exercise:**Problem:**

Calculate the contact angle θ for olive oil if capillary action raises it to a height of 7.07 cm in a glass tube with a radius of 0.100 mm. Is this value consistent with that for most organic liquids?

Solution:

$$5.1^\circ$$

This is near the value of $\theta = 0^\circ$ for most organic liquids.

Exercise:**Problem:**

When two soap bubbles touch, the larger is inflated by the smaller until they form a single bubble. (a) What is the gauge pressure inside a soap bubble with a 1.50-cm radius? (b) Inside a 4.00-cm-radius soap bubble? (c) Inside the single bubble they form if no air is lost when they touch?

Exercise:**Problem:**

Calculate the ratio of the heights to which water and mercury are raised by capillary action in the same glass tube.

Solution:

−2.78

The ratio is negative because water is raised whereas mercury is lowered.

Exercise:**Problem:**

What is the ratio of heights to which ethyl alcohol and water are raised by capillary action in the same glass tube?

Glossary

adhesive forces

the attractive forces between molecules of different types

capillary action

the tendency of a fluid to be raised or lowered in a narrow tube

cohesive forces

the attractive forces between molecules of the same type

contact angle

the angle θ between the tangent to the liquid surface and the surface

surface tension

the cohesive forces between molecules which cause the surface of a liquid to contract to the smallest possible surface area

Pressures in the Body

- Explain the concept of pressure the in human body.
- Explain systolic and diastolic blood pressures.
- Describe pressures in the eye, lungs, spinal column, bladder, and skeletal system.

Pressure in the Body

Next to taking a person’s temperature and weight, measuring blood pressure is the most common of all medical examinations. Control of high blood pressure is largely responsible for the significant decreases in heart attack and stroke fatalities achieved in the last three decades. The pressures in various parts of the body can be measured and often provide valuable medical indicators. In this section, we consider a few examples together with some of the physics that accompanies them.

[\[link\]](#) lists some of the measured pressures in mm Hg, the units most commonly quoted.

Body system	Gauge pressure in mm Hg
Blood pressures in large arteries (resting)	
<i>Maximum (systolic)</i>	100–140
<i>Minimum (diastolic)</i>	60–90
Blood pressure in large veins	4–15
Eye	12–24
Brain and spinal fluid (lying down)	5–12
Bladder	
<i>While filling</i>	0–25
<i>When full</i>	100–150
Chest cavity between lungs and ribs	–8 to –4
Inside lungs	–2 to +3
Digestive tract	

Body system	Gauge pressure in mm Hg
<i>Esophagus</i>	−2
<i>Stomach</i>	0–20
<i>Intestines</i>	10–20
Middle ear	<1

Typical Pressures in Humans

Blood Pressure

Common arterial blood pressure measurements typically produce values of 120 mm Hg and 80 mm Hg, respectively, for systolic and diastolic pressures. Both pressures have health implications. When systolic pressure is chronically high, the risk of stroke and heart attack is increased. If, however, it is too low, fainting is a problem. **Systolic pressure** increases dramatically during exercise to increase blood flow and returns to normal afterward. This change produces no ill effects and, in fact, may be beneficial to the tone of the circulatory system.

Diastolic pressure can be an indicator of fluid balance. When low, it may indicate that a person is hemorrhaging internally and needs a transfusion. Conversely, high diastolic pressure indicates a ballooning of the blood vessels, which may be due to the transfusion of too much fluid into the circulatory system. High diastolic pressure is also an indication that blood vessels are not dilating properly to pass blood through. This can seriously strain the heart in its attempt to pump blood.

Blood leaves the heart at about 120 mm Hg but its pressure continues to decrease (to almost 0) as it goes from the aorta to smaller arteries to small veins (see [link](#)). The pressure differences in the circulation system are caused by blood flow through the system as well as the position of the person. For a person standing up, the pressure in the feet will be larger than at the heart due to the weight of the blood ($P = h\rho g$). If we assume that the distance between the heart and the feet of a person in an upright position is 1.4 m, then the increase in pressure in the feet relative to that in the heart (for a static column of blood) is given by

Equation:

$$\Delta P = \Delta h\rho g = (1.4 \text{ m})\left(1050 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right) = 1.4 \times 10^4 \text{ Pa} = 108 \text{ mm Hg}.$$

Note:

Increase in Pressure in the Feet of a Person

Equation:

$$\Delta P = \Delta h\rho g = (1.4 \text{ m})\left(1050 \text{ kg/m}^3\right)\left(9.80 \text{ m/s}^2\right) = 1.4 \times 10^4 \text{ Pa} = 108 \text{ mm Hg}.$$

Standing a long time can lead to an accumulation of blood in the legs and swelling. This is the reason why soldiers who are required to stand still for long periods of time have been known to faint. Elastic bandages around the calf can help prevent this accumulation and can also help provide increased pressure to enable the veins to send blood back up to the heart. For similar reasons, doctors recommend tight stockings for long-haul flights.

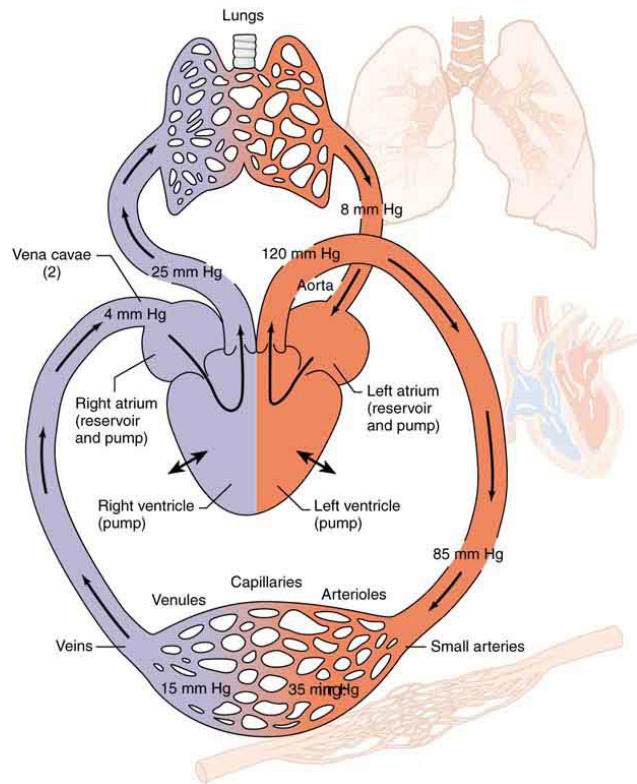
Blood pressure may also be measured in the major veins, the heart chambers, arteries to the brain, and the lungs. But these pressures are usually only monitored during surgery or for patients in intensive care since the measurements are invasive. To obtain these pressure measurements, qualified health care workers thread thin tubes, called catheters, into appropriate locations to transmit pressures to external measuring devices.

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body ([link](#)). Right-heart failure, for example, results in a rise in the pressure in the vena cavae and a drop in pressure in the arteries to the lungs. Left-heart failure results in a rise in the pressure entering the left side of the heart and a drop in aortal pressure. Implications of these and other pressures on flow in the circulatory system will be discussed in more detail in [Fluid Dynamics and Its Biological and Medical Applications](#).

Note:

Two Pumps of the Heart

The heart consists of two pumps—the right side forcing blood through the lungs and the left causing blood to flow through the rest of the body.



Schematic of the circulatory system showing typical pressures. The two pumps in the heart increase pressure and that pressure is reduced as the blood flows through the body. Long-term deviations from these pressures have medical implications discussed in some detail in the [Fluid Dynamics and Its Biological and Medical Applications](#). Only aortal or arterial blood pressure can be measured noninvasively.

Pressure in the Eye

The shape of the eye is maintained by fluid pressure, called **intraocular pressure**, which is normally in the range of 12.0 to 24.0 mm Hg. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called **glaucoma**. The net pressure can become as great as 85.0 mm Hg, an abnormally large pressure that can permanently damage the optic nerve. To get an idea of the force involved, suppose the back of the eye has an area of 6.0 cm^2 , and the net pressure is 85.0 mm Hg. Force is given by $F = PA$. To get F in newtons, we convert the area to m^2 ($1 \text{ m}^2 = 10^4 \text{ cm}^2$). Then we calculate as follows:

Equation:

$$F = h\rho gA = (85.0 \times 10^{-3} \text{ m}) (13.6 \times 10^3 \text{ kg/m}^3) (9.80 \text{ m/s}^2) (6.0 \times 10^{-4} \text{ m}^2) = 6.8 \text{ N}.$$

Note:

Eye Pressure

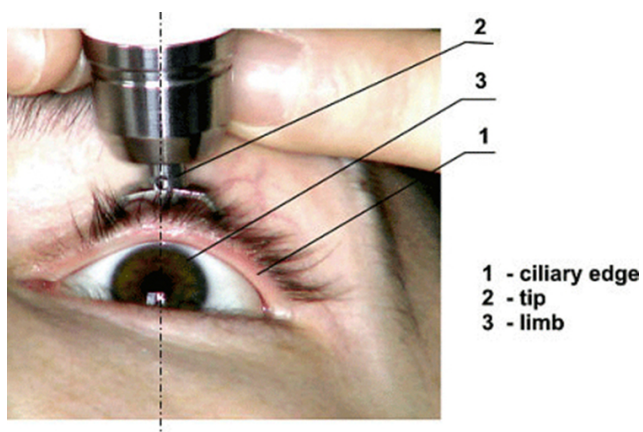
The shape of the eye is maintained by fluid pressure, called intraocular pressure. When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma. The force is calculated as

Equation:

$$F = h\rho gA = (85.0 \times 10^{-3} \text{ m}) (13.6 \times 10^3 \text{ kg/m}^3) (9.80 \text{ m/s}^2) (6.0 \times 10^{-4} \text{ m}^2) = 6.8 \text{ N}.$$

This force is the weight of about a 680-g mass. A mass of 680 g resting on the eye (imagine 1.5 lb resting on your eye) would be sufficient to cause it damage. (A normal force here would be the weight of about 120 g, less than one-quarter of our initial value.)

People over 40 years of age are at greatest risk of developing glaucoma and should have their intraocular pressure tested routinely. Most measurements involve exerting a force on the (anesthetized) eye over some area (a pressure) and observing the eye's response. A noncontact approach uses a puff of air and a measurement is made of the force needed to indent the eye ([link](#)). If the intraocular pressure is high, the eye will deform less and rebound more vigorously than normal. Excessive intraocular pressures can be detected reliably and sometimes controlled effectively.



The intraocular eye pressure can be read with a tonometer. (credit: DevelopAll at the Wikipedia Project.)

Example:**Calculating Gauge Pressure and Depth: Damage to the Eardrum**

Suppose a 3.00-N force can rupture an eardrum. (a) If the eardrum has an area of 1.00 cm^2 , calculate the maximum tolerable gauge pressure on the eardrum in newtons per meter squared and convert it to millimeters of mercury. (b) At what depth in freshwater would this person's eardrum rupture, assuming the gauge pressure in the middle ear is zero?

Strategy for (a)

The pressure can be found directly from its definition since we know the force and area. We are looking for the gauge pressure.

Solution for (a)**Equation:**

$$P_g = F/A = 3.00 \text{ N}/(1.00 \times 10^{-4} \text{ m}^2) = 3.00 \times 10^4 \text{ N/m}^2.$$

We now need to convert this to units of mm Hg:

Equation:

$$P_g = 3.0 \times 10^4 \text{ N/m}^2 \left(\frac{1.0 \text{ mm Hg}}{133 \text{ N/m}^2} \right) = 226 \text{ mm Hg}.$$

Strategy for (b)

Here we will use the fact that the water pressure varies linearly with depth h below the surface.

Solution for (b)

$P = h\rho g$ and therefore $h = P/\rho g$. Using the value above for P , we have

Equation:

$$h = \frac{3.0 \times 10^4 \text{ N/m}^2}{(1.00 \times 10^3 \text{ kg/m}^3)(9.80 \text{ m/s}^2)} = 3.06 \text{ m}.$$

Discussion

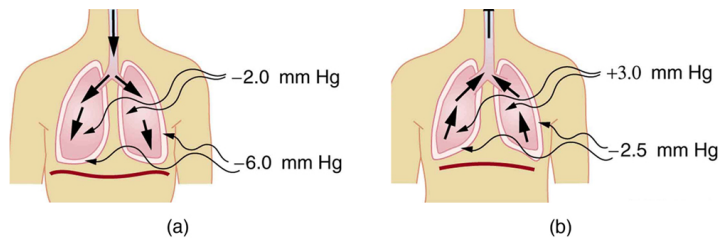
Similarly, increased pressure exerted upon the eardrum from the middle ear can arise when an infection causes a fluid buildup.

Pressure Associated with the Lungs

The pressure inside the lungs increases and decreases with each breath. The pressure drops to below atmospheric pressure (negative gauge pressure) when you inhale, causing air to flow into the lungs. It increases above atmospheric pressure (positive gauge pressure) when you exhale, forcing air out.

Lung pressure is controlled by several mechanisms. Muscle action in the diaphragm and rib cage is necessary for inhalation; this muscle action increases the volume of the lungs thereby reducing the pressure within them [\[link\]](#). Surface tension in the alveoli creates a positive pressure opposing inhalation. (See [Cohesion and Adhesion in Liquids: Surface Tension and Capillary Action](#).) You can exhale without muscle action by letting surface tension in the alveoli create its own positive pressure. Muscle action can add to this positive pressure to produce forced exhalation, such as when you blow up a balloon, blow out a candle, or cough.

The lungs, in fact, would collapse due to the surface tension in the alveoli, if they were not attached to the inside of the chest wall by liquid adhesion. The gauge pressure in the liquid attaching the lungs to the inside of the chest wall is thus negative, ranging from -4 to -8 mm Hg during exhalation and inhalation, respectively. If air is allowed to enter the chest cavity, it breaks the attachment, and one or both lungs may collapse. Suction is applied to the chest cavity of surgery patients and trauma victims to reestablish negative pressure and inflate the lungs.



(a) During inhalation, muscles expand the chest, and the diaphragm moves downward, reducing pressure inside the lungs to less than atmospheric (negative gauge pressure). Pressure between the lungs and chest wall is even lower to overcome the positive pressure created by surface tension in the lungs. (b) During gentle exhalation, the muscles simply relax and surface tension in the alveoli creates a positive pressure inside the lungs, forcing air out. Pressure between the chest wall and lungs remains negative to keep them attached to the chest wall, but it is less negative than during inhalation.

Other Pressures in the Body

Spinal Column and Skull

Normally, there is a 5- to 12-mm Hg pressure in the fluid surrounding the brain and filling the spinal column. This cerebrospinal fluid serves many purposes, one of which is to supply flotation to the brain. The buoyant force supplied by the fluid nearly equals the weight of the brain, since their densities are nearly equal. If there is a loss of fluid, the brain rests on the inside of the skull, causing severe headaches, constricted blood flow, and serious damage. Spinal fluid pressure is measured by means of a needle inserted between vertebrae that transmits the pressure to a suitable measuring device.

Bladder Pressure

This bodily pressure is one of which we are often aware. In fact, there is a relationship between our awareness of this pressure and a subsequent increase in it. Bladder pressure climbs steadily from zero to about 25 mm Hg as the bladder fills to its normal capacity of 500 cm³. This pressure triggers the **micturition reflex**, which stimulates the feeling of needing to urinate. What is more, it also causes muscles around the bladder to contract, raising the pressure to over 100 mm Hg, accentuating the sensation. Coughing, straining, tensing in cold weather, wearing tight clothes, and experiencing simple nervous tension all can increase bladder pressure and trigger this reflex. So can the weight of a pregnant woman's fetus, especially if it is kicking vigorously or pushing down with its head! Bladder pressure can be measured by a catheter or by inserting a needle through the bladder wall and transmitting the pressure to an appropriate measuring device. One hazard of high bladder pressure (sometimes created by an obstruction), is that such pressure can force urine back into the kidneys, causing potentially severe damage.

Pressures in the Skeletal System

These pressures are the largest in the body, due both to the high values of initial force, and the small areas to which this force is applied, such as in the joints.. For example, when a person lifts an object improperly, a force of 5000 N may be created between vertebrae in the spine, and this may be applied to an area as small as 10 cm². The pressure created is $P = F/A = (5000 \text{ N})/(10^{-3} \text{ m}^2) = 5.0 \times 10^6 \text{ N/m}^2$ or about 50 atm! This pressure can damage both the spinal discs (the cartilage between vertebrae), as well as the bony vertebrae themselves. Even under normal circumstances, forces between vertebrae in the spine are large enough to create pressures of several atmospheres. Most causes of excessive pressure in the skeletal system can be avoided by lifting properly and avoiding extreme physical activity. (See [Forces and Torques in Muscles and Joints.](#))

There are many other interesting and medically significant pressures in the body. For example, pressure caused by various muscle actions drives food and waste through the digestive system. Stomach pressure behaves much like bladder pressure and is tied to the sensation of hunger. Pressure in the relaxed esophagus is normally negative because pressure in the chest cavity is normally negative. Positive pressure in the stomach may thus force acid into the esophagus, causing "heartburn." Pressure in the middle ear can result in significant force on the eardrum if it differs greatly from atmospheric pressure, such as while scuba diving. The decrease in external pressure is also noticeable during plane flights (due to a decrease in the weight of air above

relative to that at the Earth's surface). The Eustachian tubes connect the middle ear to the throat and allow us to equalize pressure in the middle ear to avoid an imbalance of force on the eardrum.

Many pressures in the human body are associated with the flow of fluids. Fluid flow will be discussed in detail in the [Fluid Dynamics and Its Biological and Medical Applications](#).

Section Summary

- Measuring blood pressure is among the most common of all medical examinations.
- The pressures in various parts of the body can be measured and often provide valuable medical indicators.
- The shape of the eye is maintained by fluid pressure, called intraocular pressure.
- When the circulation of fluid in the eye is blocked, it can lead to a buildup in pressure, a condition called glaucoma.
- Some of the other pressures in the body are spinal and skull pressures, bladder pressure, pressures in the skeletal system.

Problems & Exercises

Exercise:

Problem:

During forced exhalation, such as when blowing up a balloon, the diaphragm and chest muscles create a pressure of 60.0 mm Hg between the lungs and chest wall. What force in newtons does this pressure create on the 600 cm² surface area of the diaphragm?

Solution:

479 N

Exercise:

Problem:

You can chew through very tough objects with your incisors because they exert a large force on the small area of a pointed tooth. What pressure in pascals can you create by exerting a force of 500 N with your tooth on an area of 1.00 mm²?

Exercise:

Problem:

One way to force air into an unconscious person's lungs is to squeeze on a balloon appropriately connected to the subject. What force must you exert on the balloon with your hands to create a gauge pressure of 4.00 cm water, assuming you squeeze on an effective area of 50.0 cm²?

Solution:

1.96 N

Exercise:

Problem:

Heroes in movies hide beneath water and breathe through a hollow reed (villains never catch on to this trick). In practice, you cannot inhale in this manner if your lungs are more than 60.0 cm below the surface. What is the maximum negative gauge pressure you can create in your lungs on dry land, assuming you can achieve -3.00 cm water pressure with your lungs 60.0 cm below the surface?

Solution:

-63.0 cm H_2O

Exercise:

Problem:

Gauge pressure in the fluid surrounding an infant's brain may rise as high as 85.0 mm Hg (5 to 12 mm Hg is normal), creating an outward force large enough to make the skull grow abnormally large. (a) Calculate this outward force in newtons on each side of an infant's skull if the effective area of each side is 70.0 cm^2 . (b) What is the net force acting on the skull?

Exercise:

Problem:

A full-term fetus typically has a mass of 3.50 kg. (a) What pressure does the weight of such a fetus create if it rests on the mother's bladder, supported on an area of 90.0 cm^2 ? (b) Convert this pressure to millimeters of mercury and determine if it alone is great enough to trigger the micturition reflex (it will add to any pressure already existing in the bladder).

Solution:

(a) $3.81 \times 10^3 \text{ N/m}^2$

(b) 28.7 mm Hg, which is sufficient to trigger micturition reflex

Exercise:

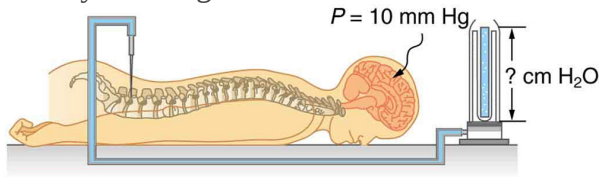
Problem:

If the pressure in the esophagus is -2.00 mm Hg while that in the stomach is $+20.0$ mm Hg, to what height could stomach fluid rise in the esophagus, assuming a density of 1.10 g/mL ? (This movement will not occur if the muscle closing the lower end of the esophagus is working properly.)

Exercise:

Problem:

Pressure in the spinal fluid is measured as shown in [\[link\]](#). If the pressure in the spinal fluid is 10.0 mm Hg: (a) What is the reading of the water manometer in cm water? (b) What is the reading if the person sits up, placing the top of the fluid 60 cm above the tap? The fluid density is 1.05 g/mL.



A water manometer used to measure pressure in the spinal fluid. The height of the fluid in the manometer is measured relative to the spinal column, and the manometer is open to the atmosphere.

The measured pressure will be considerably greater if the person sits up.

Solution:

(a) 13.6 m water

(b) 76.5 cm water

Exercise:**Problem:**

Calculate the maximum force in newtons exerted by the blood on an aneurysm, or ballooning, in a major artery, given the maximum blood pressure for this person is 150 mm Hg and the effective area of the aneurysm is 20.0 cm². Note that this force is great enough to cause further enlargement and subsequently greater force on the ever-thinner vessel wall.

Exercise:**Problem:**

During heavy lifting, a disk between spinal vertebrae is subjected to a 5000-N compressional force. (a) What pressure is created, assuming that the disk has a uniform circular cross section 2.00 cm in radius? (b) What deformation is produced if the disk is 0.800 cm thick and has a Young's modulus of $1.5 \times 10^9 \text{ N/m}^2$?

Solution:

(a) $3.98 \times 10^6 \text{ Pa}$

(b) 2.1×10^{-3} cm

Exercise:

Problem:

When a person sits erect, increasing the vertical position of their brain by 36.0 cm, the heart must continue to pump blood to the brain at the same rate. (a) What is the gain in gravitational potential energy for 100 mL of blood raised 36.0 cm? (b) What is the drop in pressure, neglecting any losses due to friction? (c) Discuss how the gain in gravitational potential energy and the decrease in pressure are related.

Exercise:

Problem:

(a) How high will water rise in a glass capillary tube with a 0.500-mm radius? (b) How much gravitational potential energy does the water gain? (c) Discuss possible sources of this energy.

Solution:

(a) 2.97 cm

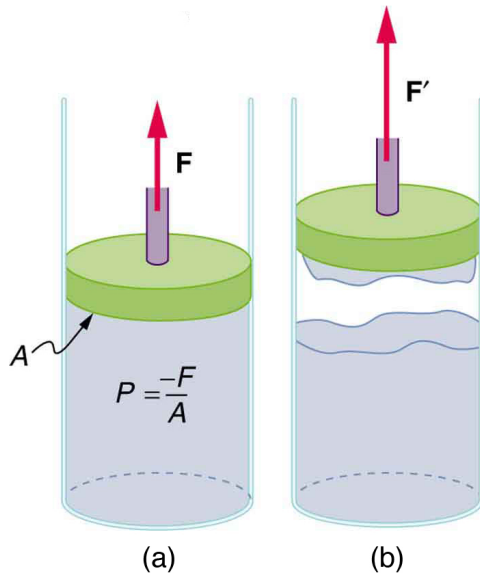
(b) 3.39×10^{-6} J

(c) Work is done by the surface tension force through an effective distance $h/2$ to raise the column of water.

Exercise:

Problem:

A negative pressure of 25.0 atm can sometimes be achieved with the device in [\[link\]](#) before the water separates. (a) To what height could such a negative gauge pressure raise water? (b) How much would a steel wire of the same diameter and length as this capillary stretch if suspended from above?



(a) When the piston is raised, it stretches the liquid slightly, putting it under tension and creating a negative absolute pressure $P = -F/A$ (b) The liquid eventually separates, giving an experimental limit to negative pressure in this liquid.

Exercise:

Problem:

Suppose you hit a steel nail with a 0.500-kg hammer, initially moving at 15.0 m/s and brought to rest in 2.80 mm. (a) What average force is exerted on the nail? (b) How much is the nail compressed if it is 2.50 mm in diameter and 6.00-cm long? (c) What pressure is created on the 1.00-mm-diameter tip of the nail?

Solution:

(a) $2.01 \times 10^4 \text{ N}$

(b) $1.17 \times 10^{-3} \text{ m}$

(c) $2.56 \times 10^{10} \text{ N/m}^2$

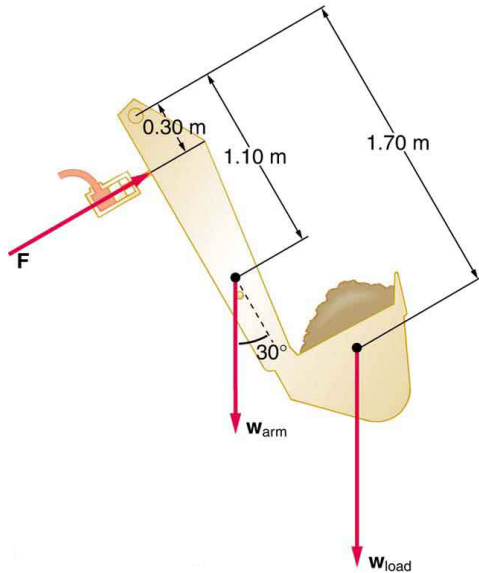
Exercise:

Problem:

Calculate the pressure due to the ocean at the bottom of the Marianas Trench near the Philippines, given its depth is 11.0 km and assuming the density of sea water is constant all the way down. (b) Calculate the percent decrease in volume of sea water due to such a pressure, assuming its bulk modulus is the same as water and is constant. (c) What would be the percent increase in its density? Is the assumption of constant density valid? Will the actual pressure be greater or smaller than that calculated under this assumption?

Exercise:**Problem:**

The hydraulic system of a backhoe is used to lift a load as shown in [\[link\]](#). (a) Calculate the force F the slave cylinder must exert to support the 400-kg load and the 150-kg brace and shovel. (b) What is the pressure in the hydraulic fluid if the slave cylinder is 2.50 cm in diameter? (c) What force would you have to exert on a lever with a mechanical advantage of 5.00 acting on a master cylinder 0.800 cm in diameter to create this pressure?



Hydraulic and mechanical lever systems are used in heavy machinery such as this backhoe.

Solution:

(a) $1.38 \times 10^4 \text{ N}$

(b) $2.81 \times 10^7 \text{ N/m}^2$

(c) 283 N

Exercise:

Problem:

Some miners wish to remove water from a mine shaft. A pipe is lowered to the water 90 m below, and a negative pressure is applied to raise the water. (a) Calculate the pressure needed to raise the water. (b) What is unreasonable about this pressure? (c) What is unreasonable about the premise?

Exercise:

Problem:

You are pumping up a bicycle tire with a hand pump, the piston of which has a 2.00-cm radius.

(a) What force in newtons must you exert to create a pressure of 6.90×10^5 Pa (b) What is unreasonable about this (a) result? (c) Which premises are unreasonable or inconsistent?

Solution:

(a) 867 N

(b) This is too much force to exert with a hand pump.

(c) The assumed radius of the pump is too large; it would be nearly two inches in diameter—too large for a pump or even a master cylinder. The pressure is reasonable for bicycle tires.

Exercise:

Problem:

Consider a group of people trying to stay afloat after their boat strikes a log in a lake. Construct a problem in which you calculate the number of people that can cling to the log and keep their heads out of the water. Among the variables to be considered are the size and density of the log, and what is needed to keep a person's head and arms above water without swimming or treading water.

Exercise:

Problem:

The alveoli in emphysema victims are damaged and effectively form larger sacs. Construct a problem in which you calculate the loss of pressure due to surface tension in the alveoli because of their larger average diameters. (Part of the lung's ability to expel air results from pressure created by surface tension in the alveoli.) Among the things to consider are the normal surface tension of the fluid lining the alveoli, the average alveolar radius in normal individuals and its average in emphysema sufferers.

Glossary

diastolic pressure

minimum arterial blood pressure; indicator for the fluid balance

glaucoma

condition caused by the buildup of fluid pressure in the eye

intraocular pressure

fluid pressure in the eye

micturition reflex

stimulates the feeling of needing to urinate, triggered by bladder pressure

systolic pressure

maximum arterial blood pressure; indicator for the blood flow

Introduction to Fluid Dynamics and Its Biological and Medical Applications

class="introduction"

Many fluids are flowing in this scene.

Water from the hose and smoke from the fire are visible flows.

Less visible are the flow of air and the flow of fluids on the ground and within the people fighting the fire.

Explore all types of flow, such as visible, implied, turbulent, laminar, and so on,

present in
this scene.

Make a
list and
discuss
the
relative
energies
involved
in the
various
flows,
including
the level
of
confidence
in your
estimates.

(credit:
Andrew
Magill,
Flickr)



We have dealt with many situations in which fluids are static. But by their very definition, fluids flow. Examples come easily—a column of smoke rises from a camp fire, water streams from a fire hose, blood courses through your veins. Why does rising smoke curl and twist? How does a nozzle increase the speed of water emerging from a hose? How does the body regulate blood flow? The physics of fluids in motion—**fluid dynamics**—allows us to answer these and many other questions.

Glossary

fluid dynamics

the physics of fluids in motion

Flow Rate and Its Relation to Velocity

- Calculate flow rate.
- Define units of volume.
- Describe incompressible fluids.
- Explain the consequences of the equation of continuity.

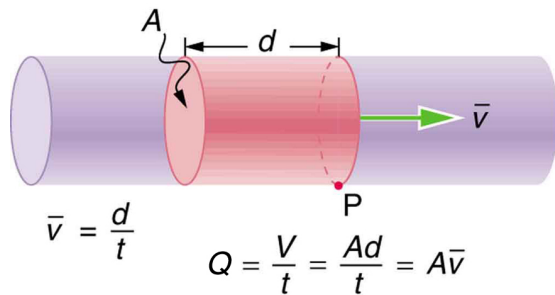
Flow rate Q is defined to be the volume of fluid passing by some location through an area during a period of time, as seen in [\[link\]](#). In symbols, this can be written as

Equation:

$$Q = \frac{V}{t},$$

where V is the volume and t is the elapsed time.

The SI unit for flow rate is m^3/s , but a number of other units for Q are in common use. For example, the heart of a resting adult pumps blood at a rate of 5.00 liters per minute (L/min). Note that a **liter** (L) is 1/1000 of a cubic meter or 1000 cubic centimeters (10^{-3} m^3 or 10^3 cm^3). In this text we shall use whatever metric units are most convenient for a given situation.



Flow rate is the volume of fluid per unit time flowing past a point through the area A . Here the shaded cylinder of fluid flows past point P in a uniform pipe in time t . The volume of the cylinder is Ad

and the average velocity is
 $v = d/t$ so that the flow rate
is $Q = Ad/t = Av$.

Example:

Calculating Volume from Flow Rate: The Heart Pumps a Lot of Blood in a Lifetime

How many cubic meters of blood does the heart pump in a 75-year lifetime, assuming the average flow rate is 5.00 L/min?

Strategy

Time and flow rate Q are given, and so the volume V can be calculated from the definition of flow rate.

Solution

Solving $Q = V/t$ for volume gives

Equation:

$$V = Qt.$$

Substituting known values yields

Equation:

$$\begin{aligned} V &= \left(\frac{5.00 \text{ L}}{1 \text{ min}} \right) (75 \text{ y}) \left(\frac{1 \text{ m}^3}{10^3 \text{ L}} \right) \left(5.26 \times 10^5 \frac{\text{min}}{\text{y}} \right) \\ &= 2.0 \times 10^5 \text{ m}^3. \end{aligned}$$

Discussion

This amount is about 200,000 tons of blood. For comparison, this value is equivalent to about 200 times the volume of water contained in a 6-lane 50-m lap pool.

Flow rate and velocity are related, but quite different, physical quantities. To make the distinction clear, think about the flow rate of a river. The

greater the velocity of the water, the greater the flow rate of the river. But flow rate also depends on the size of the river. A rapid mountain stream carries far less water than the Amazon River in Brazil, for example. The precise relationship between flow rate Q and velocity v is

Equation:

$$Q = Av,$$

where A is the cross-sectional area and v is the average velocity. This equation seems logical enough. The relationship tells us that flow rate is directly proportional to both the magnitude of the average velocity (hereafter referred to as the speed) and the size of a river, pipe, or other conduit. The larger the conduit, the greater its cross-sectional area. [\[link\]](#) illustrates how this relationship is obtained. The shaded cylinder has a volume

Equation:

$$V = Ad,$$

which flows past the point P in a time t . Dividing both sides of this relationship by t gives

Equation:

$$\frac{V}{t} = \frac{Ad}{t}.$$

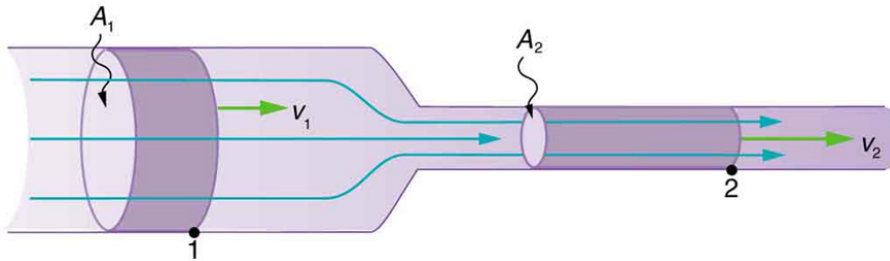
We note that $Q = V/t$ and the average speed is $v = d/t$. Thus the equation becomes $Q = Av$.

[\[link\]](#) shows an incompressible fluid flowing along a pipe of decreasing radius. Because the fluid is incompressible, the same amount of fluid must flow past any point in the tube in a given time to ensure continuity of flow. In this case, because the cross-sectional area of the pipe decreases, the velocity must necessarily increase. This logic can be extended to say that the flow rate must be the same at all points along the pipe. In particular, for points 1 and 2,

Equation:

$$\left. \begin{array}{l} Q_1 = Q_2 \\ A_1 v_1 = A_2 v_2 \end{array} \right\}.$$

This is called the equation of continuity and is valid for any incompressible fluid. The consequences of the equation of continuity can be observed when water flows from a hose into a narrow spray nozzle: it emerges with a large speed—that is the purpose of the nozzle. Conversely, when a river empties into one end of a reservoir, the water slows considerably, perhaps picking up speed again when it leaves the other end of the reservoir. In other words, speed increases when cross-sectional area decreases, and speed decreases when cross-sectional area increases.



When a tube narrows, the same volume occupies a greater length. For the same volume to pass points 1 and 2 in a given time, the speed must be greater at point 2. The process is exactly reversible. If the fluid flows in the opposite direction, its speed will decrease when the tube widens. (Note that the relative volumes of the two cylinders and the corresponding velocity vector arrows are not drawn to scale.)

Since liquids are essentially incompressible, the equation of continuity is valid for all liquids. However, gases are compressible, and so the equation must be applied with caution to gases if they are subjected to compression or expansion.

Example:**Calculating Fluid Speed: Speed Increases When a Tube Narrows**

A nozzle with a radius of 0.250 cm is attached to a garden hose with a radius of 0.900 cm. The flow rate through hose and nozzle is 0.500 L/s. Calculate the speed of the water (a) in the hose and (b) in the nozzle.

Strategy

We can use the relationship between flow rate and speed to find both velocities. We will use the subscript 1 for the hose and 2 for the nozzle.

Solution for (a)

First, we solve $Q = Av$ for v_1 and note that the cross-sectional area is $A = \pi r^2$, yielding

Equation:

$$v_1 = \frac{Q}{A_1} = \frac{Q}{\pi r_1^2}.$$

Substituting known values and making appropriate unit conversions yields

Equation:

$$v_1 = \frac{(0.500 \text{ L/s})(10^{-3} \text{ m}^3/\text{L})}{\pi(9.00 \times 10^{-3} \text{ m})^2} = 1.96 \text{ m/s}.$$

Solution for (b)

We could repeat this calculation to find the speed in the nozzle v_2 , but we will use the equation of continuity to give a somewhat different insight.

Using the equation which states

Equation:

$$A_1 v_1 = A_2 v_2,$$

solving for v_2 and substituting πr^2 for the cross-sectional area yields

Equation:

$$v_2 = \frac{A_1}{A_2} v_1 = \frac{\pi r_1^2}{\pi r_2^2} v_1 = \frac{r_1^2}{r_2^2} v_1.$$

Substituting known values,

Equation:

$$v_2 = \frac{(0.900 \text{ cm})^2}{(0.250 \text{ cm})^2} 1.96 \text{ m/s} = 25.5 \text{ m/s}.$$

Discussion

A speed of 1.96 m/s is about right for water emerging from a nozzleless hose. The nozzle produces a considerably faster stream merely by constricting the flow to a narrower tube.

The solution to the last part of the example shows that speed is inversely proportional to the *square* of the radius of the tube, making for large effects when radius varies. We can blow out a candle at quite a distance, for example, by pursing our lips, whereas blowing on a candle with our mouth wide open is quite ineffective.

In many situations, including in the cardiovascular system, branching of the flow occurs. The blood is pumped from the heart into arteries that subdivide into smaller arteries (arterioles) which branch into very fine vessels called capillaries. In this situation, continuity of flow is maintained but it is the *sum* of the flow rates in each of the branches in any portion along the tube that is maintained. The equation of continuity in a more general form becomes

Equation:

$$n_1 A_1 v_1 = n_2 A_2 v_2,$$

where n_1 and n_2 are the number of branches in each of the sections along the tube.

Example:

Calculating Flow Speed and Vessel Diameter: Branching in the Cardiovascular System

The aorta is the principal blood vessel through which blood leaves the heart in order to circulate around the body. (a) Calculate the average speed of the blood in the aorta if the flow rate is 5.0 L/min. The aorta has a radius of 10 mm. (b) Blood also flows through smaller blood vessels known as capillaries. When the rate of blood flow in the aorta is 5.0 L/min, the speed of blood in the capillaries is about 0.33 mm/s. Given that the average diameter of a capillary is 8.0 μm , calculate the number of capillaries in the blood circulatory system.

Strategy

We can use $Q = Av$ to calculate the speed of flow in the aorta and then use the general form of the equation of continuity to calculate the number of capillaries as all of the other variables are known.

Solution for (a)

The flow rate is given by $Q = Av$ or $v = \frac{Q}{\pi r^2}$ for a cylindrical vessel. Substituting the known values (converted to units of meters and seconds) gives

Equation:

$$v = \frac{(5.0 \text{ L/min})(10^{-3} \text{ m}^3/\text{L})(1 \text{ min}/60 \text{ s})}{\pi(0.010 \text{ m})^2} = 0.27 \text{ m/s}.$$

Solution for (b)

Using $n_1 A_1 v_1 = n_2 A_2 v_2$, assigning the subscript 1 to the aorta and 2 to the capillaries, and solving for n_2 (the number of capillaries) gives $n_2 = \frac{n_1 A_1 v_1}{A_2 v_2}$. Converting all quantities to units of meters and seconds and substituting into the equation above gives

Equation:

$$n_2 = \frac{(1)(\pi)(10 \times 10^{-3} \text{ m})^2(0.27 \text{ m/s})}{(\pi)(4.0 \times 10^{-6} \text{ m})^2(0.33 \times 10^{-3} \text{ m/s})} = 5.0 \times 10^9 \text{ capillaries}.$$

Discussion

Note that the speed of flow in the capillaries is considerably reduced relative to the speed in the aorta due to the significant increase in the total cross-sectional area at the capillaries. This low speed is to allow sufficient

time for effective exchange to occur although it is equally important for the flow not to become stationary in order to avoid the possibility of clotting. Does this large number of capillaries in the body seem reasonable? In active muscle, one finds about 200 capillaries per mm^3 , or about 200×10^6 per 1 kg of muscle. For 20 kg of muscle, this amounts to about 4×10^9 capillaries.

Section Summary

- Flow rate Q is defined to be the volume V flowing past a point in time t , or $Q = \frac{V}{t}$ where V is volume and t is time.
- The SI unit of volume is m^3 .
- Another common unit is the liter (L), which is 10^{-3} m^3 .
- Flow rate and velocity are related by $Q = Av$ where A is the cross-sectional area of the flow and v is its average velocity.
- For incompressible fluids, flow rate at various points is constant. That is,

Equation:

$$\begin{aligned}Q_1 &= Q_2 \\A_1v_1 &= A_2v_2 \quad . \\n_1A_1v_1 &= n_2A_2v_2\end{aligned}$$

Conceptual Questions

Exercise:

Problem:

What is the difference between flow rate and fluid velocity? How are they related?

Exercise:

Problem:

Many figures in the text show streamlines. Explain why fluid velocity is greatest where streamlines are closest together. (Hint: Consider the relationship between fluid velocity and the cross-sectional area through which it flows.)

Exercise:**Problem:**

Identify some substances that are incompressible and some that are not.

Problems & Exercises**Exercise:****Problem:**

What is the average flow rate in cm^3/s of gasoline to the engine of a car traveling at 100 km/h if it averages 10.0 km/L?

Solution:

$$2.78 \text{ cm}^3/\text{s}$$

Exercise:**Problem:**

The heart of a resting adult pumps blood at a rate of 5.00 L/min. (a) Convert this to cm^3/s . (b) What is this rate in m^3/s ?

Exercise:**Problem:**

Blood is pumped from the heart at a rate of 5.0 L/min into the aorta (of radius 1.0 cm). Determine the speed of blood through the aorta.

Solution:

27 cm/s

Exercise:**Problem:**

Blood is flowing through an artery of radius 2 mm at a rate of 40 cm/s. Determine the flow rate and the volume that passes through the artery in a period of 30 s.

Exercise:**Problem:**

The Huka Falls on the Waikato River is one of New Zealand's most visited natural tourist attractions (see [\[link\]](#)). On average the river has a flow rate of about 300,000 L/s. At the gorge, the river narrows to 20 m wide and averages 20 m deep. (a) What is the average speed of the river in the gorge? (b) What is the average speed of the water in the river downstream of the falls when it widens to 60 m and its depth increases to an average of 40 m?



The Huka Falls in Taupo,
New Zealand, demonstrate
flow rate. (credit:
RaviGogna, Flickr)

Solution:

(a) 0.75 m/s

(b) 0.13 m/s

Exercise:**Problem:**

A major artery with a cross-sectional area of 1.00 cm^2 branches into 18 smaller arteries, each with an average cross-sectional area of 0.400 cm^2 . By what factor is the average velocity of the blood reduced when it passes into these branches?

Exercise:**Problem:**

(a) As blood passes through the capillary bed in an organ, the capillaries join to form venules (small veins). If the blood speed increases by a factor of 4.00 and the total cross-sectional area of the venules is 10.0 cm^2 , what is the total cross-sectional area of the capillaries feeding these venules? (b) How many capillaries are involved if their average diameter is $10.0 \mu\text{m}$?

Solution:

(a) 40.0 cm^2

(b) 5.09×10^7

Exercise:**Problem:**

The human circulation system has approximately 1×10^9 capillary vessels. Each vessel has a diameter of about $8 \mu\text{m}$. Assuming cardiac output is 5 L/min, determine the average velocity of blood flow through each capillary vessel.

Exercise:**Problem:**

(a) Estimate the time it would take to fill a private swimming pool with a capacity of 80,000 L using a garden hose delivering 60 L/min. (b) How long would it take to fill if you could divert a moderate size river, flowing at $5000 \text{ m}^3/\text{s}$, into it?

Solution:

(a) 22 h

(b) 0.016 s

Exercise:**Problem:**

The flow rate of blood through a $2.00 \times 10^{-6}\text{-m}$ -radius capillary is $3.80 \times 10^{-9} \text{ cm}^3/\text{s}$. (a) What is the speed of the blood flow? (This small speed allows time for diffusion of materials to and from the blood.) (b) Assuming all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of $90.0 \text{ cm}^3/\text{s}$? (The large number obtained is an overestimate, but it is still reasonable.)

Exercise:**Problem:**

(a) What is the fluid speed in a fire hose with a 9.00-cm diameter carrying 80.0 L of water per second? (b) What is the flow rate in cubic meters per second? (c) Would your answers be different if salt water replaced the fresh water in the fire hose?

Solution:

(a) 12.6 m/s

(b) $0.0800 \text{ m}^3/\text{s}$

(c) No, independent of density.

Exercise:

Problem:

The main uptake air duct of a forced air gas heater is 0.300 m in diameter. What is the average speed of air in the duct if it carries a volume equal to that of the house's interior every 15 min ? The inside volume of the house is equivalent to a rectangular solid 13.0 m wide by 20.0 m long by 2.75 m high.

Exercise:

Problem:

Water is moving at a velocity of 2.00 m/s through a hose with an internal diameter of 1.60 cm . (a) What is the flow rate in liters per second? (b) The fluid velocity in this hose's nozzle is 15.0 m/s . What is the nozzle's inside diameter?

Solution:

(a) 0.402 L/s

(b) 0.584 cm

Exercise:

Problem:

Prove that the speed of an incompressible fluid through a constriction, such as in a Venturi tube, increases by a factor equal to the square of the factor by which the diameter decreases. (The converse applies for flow out of a constriction into a larger-diameter region.)

Exercise:

Problem:

Water emerges straight down from a faucet with a 1.80-cm diameter at a speed of 0.500 m/s. (Because of the construction of the faucet, there is no variation in speed across the stream.) (a) What is the flow rate in cm^3/s ? (b) What is the diameter of the stream 0.200 m below the faucet? Neglect any effects due to surface tension.

Solution:

(a) $127 \text{ cm}^3/\text{s}$

(b) 0.890 cm

Exercise:**Problem: Unreasonable Results**

A mountain stream is 10.0 m wide and averages 2.00 m in depth. During the spring runoff, the flow in the stream reaches $100,000 \text{ m}^3/\text{s}$. (a) What is the average velocity of the stream under these conditions? (b) What is unreasonable about this velocity? (c) What is unreasonable or inconsistent about the premises?

Glossary

flow rate

abbreviated Q , it is the volume V that flows past a particular point during a time t , or $Q = V/t$

liter

a unit of volume, equal to 10^{-3} m^3

Bernoulli's Equation

- Explain the terms in Bernoulli's equation.
- Explain how Bernoulli's equation is related to conservation of energy.
- Explain how to derive Bernoulli's principle from Bernoulli's equation.
- Calculate with Bernoulli's principle.
- List some applications of Bernoulli's principle.

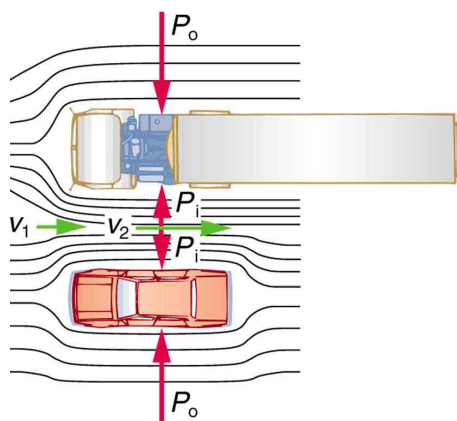
When a fluid flows into a narrower channel, its speed increases. That means its kinetic energy also increases. Where does that change in kinetic energy come from? The increased kinetic energy comes from the net work done on the fluid to push it into the channel and the work done on the fluid by the gravitational force, if the fluid changes vertical position. Recall the work-energy theorem,

Equation:

$$W_{\text{net}} = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2.$$

There is a pressure difference when the channel narrows. This pressure difference results in a net force on the fluid: recall that pressure times area equals force. The net work done increases the fluid's kinetic energy. As a result, the *pressure will drop in a rapidly-moving fluid*, whether or not the fluid is confined to a tube.

There are a number of common examples of pressure dropping in rapidly-moving fluids. Shower curtains have a disagreeable habit of bulging into the shower stall when the shower is on. The high-velocity stream of water and air creates a region of lower pressure inside the shower, and standard atmospheric pressure on the other side. The pressure difference results in a net force inward pushing the curtain in. You may also have noticed that when passing a truck on the highway, your car tends to veer toward it. The reason is the same—the high velocity of the air between the car and the truck creates a region of lower pressure, and the vehicles are pushed together by greater pressure on the outside. (See [\[link\]](#).) This effect was observed as far back as the mid-1800s, when it was found that trains passing in opposite directions tipped precariously toward one another.



An overhead view of a car passing a truck on a highway. Air passing between the vehicles flows in a narrower channel and must increase its speed (v_2 is greater than v_1), causing the pressure between them to drop (P_i is less than P_o). Greater pressure on the outside pushes the car and truck together.

Note:

Making Connections: Take-Home Investigation with a Sheet of Paper
Hold the short edge of a sheet of paper parallel to your mouth with one hand on each side of your mouth. The page should slant downward over your hands. Blow over the top of the page. Describe what happens and explain the reason for this behavior.

Bernoulli's Equation

The relationship between pressure and velocity in fluids is described quantitatively by **Bernoulli's equation**, named after its discoverer, the Swiss scientist Daniel Bernoulli (1700–1782). Bernoulli's equation states that for an incompressible, frictionless fluid, the following sum is constant:

Equation:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant},$$

where P is the absolute pressure, ρ is the fluid density, v is the velocity of the fluid, h is the height above some reference point, and g is the acceleration due to gravity. If we follow a small volume of fluid along its path, various quantities in the sum may change, but the total remains constant. Let the subscripts 1 and 2 refer to any two points along the path that the bit of fluid follows; Bernoulli's equation becomes

Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2.$$

Bernoulli's equation is a form of the conservation of energy principle. Note that the second and third terms are the kinetic and potential energy with m replaced by ρ . In fact, each term in the equation has units of energy per unit volume. We can prove this for the second term by substituting $\rho = m/V$ into it and gathering terms:

Equation:

$$\frac{1}{2}\rho v^2 = \frac{\frac{1}{2}mv^2}{V} = \frac{\text{KE}}{V}.$$

So $\frac{1}{2}\rho v^2$ is the kinetic energy per unit volume. Making the same substitution into the third term in the equation, we find

Equation:

$$\rho gh = \frac{mgh}{V} = \frac{PE_g}{V},$$

so ρgh is the gravitational potential energy per unit volume. Note that pressure P has units of energy per unit volume, too. Since $P = F/A$, its units are N/m^2 . If we multiply these by m/m , we obtain $\text{N} \cdot \text{m}/\text{m}^3 = \text{J}/\text{m}^3$, or energy per unit volume. Bernoulli's equation is, in fact, just a convenient statement of conservation of energy for an incompressible fluid in the absence of friction.

Note:

Making Connections: Conservation of Energy

Conservation of energy applied to fluid flow produces Bernoulli's equation. The net work done by the fluid's pressure results in changes in the fluid's KE and PE_g per unit volume. If other forms of energy are involved in fluid flow, Bernoulli's equation can be modified to take these forms into account. Such forms of energy include thermal energy dissipated because of fluid viscosity.

The general form of Bernoulli's equation has three terms in it, and it is broadly applicable. To understand it better, we will look at a number of specific situations that simplify and illustrate its use and meaning.

Bernoulli's Equation for Static Fluids

Let us first consider the very simple situation where the fluid is static—that is, $v_1 = v_2 = 0$. Bernoulli's equation in that case is

Equation:

$$P_1 + \rho gh_1 = P_2 + \rho gh_2.$$

We can further simplify the equation by taking $h_2 = 0$ (we can always choose some height to be zero, just as we often have done for other situations involving the gravitational force, and take all other heights to be relative to this). In that case, we get

Equation:

$$P_2 = P_1 + \rho gh_1 .$$

This equation tells us that, in static fluids, pressure increases with depth. As we go from point 1 to point 2 in the fluid, the depth increases by h_1 , and consequently, P_2 is greater than P_1 by an amount ρgh_1 . In the very simplest case, P_1 is zero at the top of the fluid, and we get the familiar relationship $P = \rho gh$. (Recall that $P = \rho gh$ and $\Delta PE_g = mgh$.)

Bernoulli's equation includes the fact that the pressure due to the weight of a fluid is ρgh . Although we introduce Bernoulli's equation for fluid flow, it includes much of what we studied for static fluids in the preceding chapter.

Bernoulli's Principle—Bernoulli's Equation at Constant Depth

Another important situation is one in which the fluid moves but its depth is constant—that is, $h_1 = h_2$. Under that condition, Bernoulli's equation becomes

Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2 .$$

Situations in which fluid flows at a constant depth are so important that this equation is often called **Bernoulli's principle**. It is Bernoulli's equation for fluids at constant depth. (Note again that this applies to a small volume of fluid as we follow it along its path.) As we have just discussed, pressure drops as speed increases in a moving fluid. We can see this from Bernoulli's principle. For example, if v_2 is greater than v_1 in the equation, then P_2 must be less than P_1 for the equality to hold.

Example:**Calculating Pressure: Pressure Drops as a Fluid Speeds Up**

In [\[link\]](#), we found that the speed of water in a hose increased from 1.96 m/s to 25.5 m/s going from the hose to the nozzle. Calculate the pressure in the hose, given that the absolute pressure in the nozzle is $1.01 \times 10^5 \text{ N/m}^2$ (atmospheric, as it must be) and assuming level, frictionless flow.

Strategy

Level flow means constant depth, so Bernoulli's principle applies. We use the subscript 1 for values in the hose and 2 for those in the nozzle. We are thus asked to find P_1 .

Solution

Solving Bernoulli's principle for P_1 yields

Equation:

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2 - \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho(v_2^2 - v_1^2).$$

Substituting known values,

Equation:

$$\begin{aligned} P_1 &= 1.01 \times 10^5 \text{ N/m}^2 \\ &\quad + \frac{1}{2}(10^3 \text{ kg/m}^3)[(25.5 \text{ m/s})^2 - (1.96 \text{ m/s})^2] \\ &= 4.24 \times 10^5 \text{ N/m}^2. \end{aligned}$$

Discussion

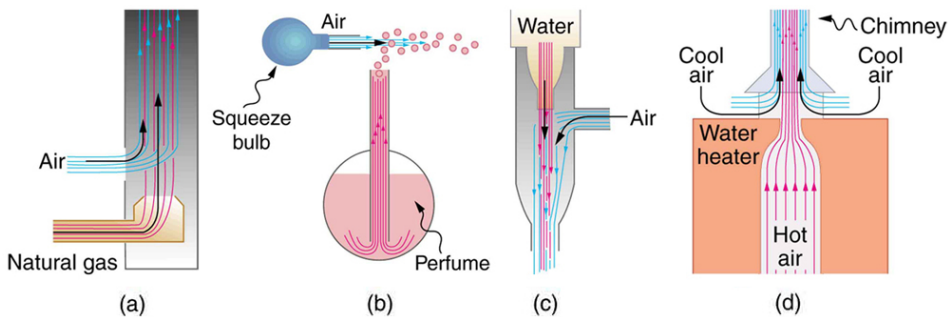
This absolute pressure in the hose is greater than in the nozzle, as expected since v is greater in the nozzle. The pressure P_2 in the nozzle must be atmospheric since it emerges into the atmosphere without other changes in conditions.

Applications of Bernoulli's Principle

There are a number of devices and situations in which fluid flows at a constant height and, thus, can be analyzed with Bernoulli's principle.

Entrainment

People have long put the Bernoulli principle to work by using reduced pressure in high-velocity fluids to move things about. With a higher pressure on the outside, the high-velocity fluid forces other fluids into the stream. This process is called *entrainment*. Entrainment devices have been in use since ancient times, particularly as pumps to raise water small heights, as in draining swamps, fields, or other low-lying areas. Some other devices that use the concept of entrainment are shown in [\[link\]](#).



Examples of entrainment devices that use increased fluid speed to create low pressures, which then entrain one fluid into another. (a) A Bunsen burner uses an adjustable gas nozzle, entraining air for proper combustion. (b) An atomizer uses a squeeze bulb to create a jet of air that entrains drops of perfume. Paint sprayers and carburetors use very similar techniques to move their respective liquids. (c) A common aspirator uses a high-speed stream of water to create a region of lower pressure. Aspirators may be used as suction pumps in dental and surgical situations or for draining a flooded basement or producing a reduced pressure in a vessel. (d) The chimney of a water heater is designed to entrain air into the pipe leading through the ceiling.

Wings and Sails

The airplane wing is a beautiful example of Bernoulli's principle in action. [\[link\]](#)(a) shows the characteristic shape of a wing. The wing is tilted upward at a small angle and the upper surface is longer, causing air to flow faster over it. The pressure on top of the wing is therefore reduced, creating a net upward force or lift. (Wings can also gain lift by pushing air downward, utilizing the conservation of momentum principle. The deflected air molecules result in an upward force on the wing — Newton's third law.) Sails also have the characteristic shape of a wing. (See [\[link\]](#)(b).) The pressure on the front side of the sail, P_{front} , is lower than the pressure on the back of the sail, P_{back} . This results in a forward force and even allows you to sail into the wind.

Note:

Making Connections: Take-Home Investigation with Two Strips of Paper

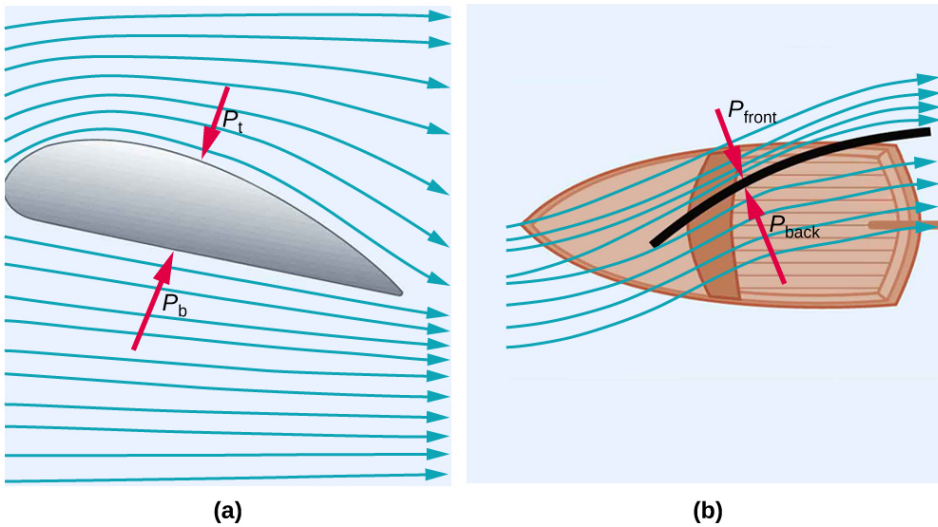
For a good illustration of Bernoulli's principle, make two strips of paper, each about 15 cm long and 4 cm wide. Hold the small end of one strip up to your lips and let it drape over your finger. Blow across the paper. What happens? Now hold two strips of paper up to your lips, separated by your fingers. Blow between the strips. What happens?

Velocity measurement

[\[link\]](#) shows two devices that measure fluid velocity based on Bernoulli's principle. The manometer in [\[link\]](#)(a) is connected to two tubes that are small enough not to appreciably disturb the flow. The tube facing the oncoming fluid creates a dead spot having zero velocity ($v_1 = 0$) in front of it, while fluid passing the other tube has velocity v_2 . This means that Bernoulli's principle as stated in $P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2$ becomes

Equation:

$$P_1 = P_2 + \frac{1}{2}\rho v_2^2.$$



(a) The Bernoulli principle helps explain lift generated by a wing. (b) Sails use the same technique to generate part of their thrust.

Thus pressure P_2 over the second opening is reduced by $\frac{1}{2}\rho v_2^2$, and so the fluid in the manometer rises by h on the side connected to the second opening, where

Equation:

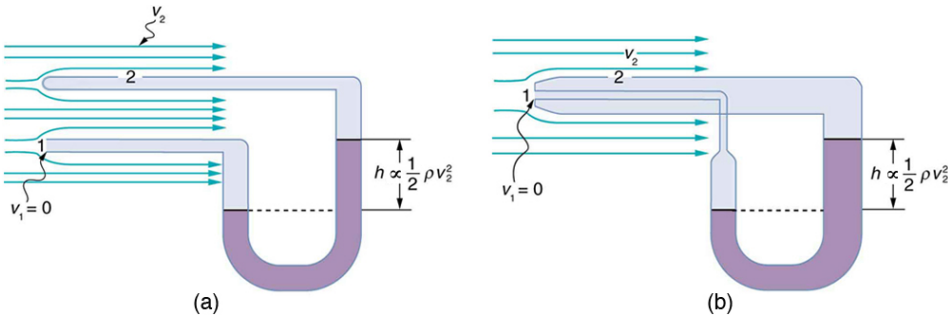
$$h \propto \frac{1}{2}\rho v_2^2.$$

(Recall that the symbol $\propto$ means “proportional to.”) Solving for v_2 , we see that

Equation:

$$v_2 \propto \sqrt{h}.$$

[\[link\]](#)(b) shows a version of this device that is in common use for measuring various fluid velocities; such devices are frequently used as air speed indicators in aircraft.



Measurement of fluid speed based on Bernoulli's principle. (a) A manometer is connected to two tubes that are close together and small enough not to disturb the flow. Tube 1 is open at the end facing the flow. A dead spot having zero speed is created there. Tube 2 has an opening on the side, and so the fluid has a speed v across the opening; thus, pressure there drops. The difference in pressure at the manometer is $\frac{1}{2} \rho v_2^2$, and so h is proportional to $\frac{1}{2} \rho v_2^2$. (b) This type of velocity measuring device is a Prandtl tube, also known as a pitot tube.

Summary

- Bernoulli's equation states that the sum on each side of the following equation is constant, or the same at any two points in an incompressible frictionless fluid:

Equation:

$$P_1 + \frac{1}{2} \rho v_1^2 + \rho g h_1 = P_2 + \frac{1}{2} \rho v_2^2 + \rho g h_2.$$

- Bernoulli's principle is Bernoulli's equation applied to situations in which depth is constant. The terms involving depth (or height h) subtract out, yielding

Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 = P_2 + \frac{1}{2}\rho v_2^2.$$

- Bernoulli's principle has many applications, including entrainment, wings and sails, and velocity measurement.

Conceptual Questions

Exercise:

Problem:

You can squirt water a considerably greater distance by placing your thumb over the end of a garden hose and then releasing, than by leaving it completely uncovered. Explain how this works.

Exercise:

Problem:

Water is shot nearly vertically upward in a decorative fountain and the stream is observed to broaden as it rises. Conversely, a stream of water falling straight down from a faucet narrows. Explain why, and discuss whether surface tension enhances or reduces the effect in each case.

Exercise:

Problem:

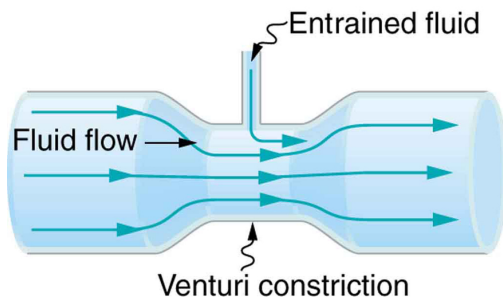
Look back to [\[link\]](#). Answer the following two questions. Why is P_o less than atmospheric? Why is P_o greater than P_i ?

Exercise:

Problem: Give an example of entrainment not mentioned in the text.

Exercise:**Problem:**

Many entrainment devices have a constriction, called a Venturi, such as shown in [\[link\]](#). How does this bolster entrainment?



A tube with a narrow segment designed to enhance entrainment is called a Venturi. These are very commonly used in carburetors and aspirators.

Exercise:**Problem:**

Some chimney pipes have a T-shape, with a crosspiece on top that helps draw up gases whenever there is even a slight breeze. Explain how this works in terms of Bernoulli's principle.

Exercise:**Problem:**

Is there a limit to the height to which an entrainment device can raise a fluid? Explain your answer.

Exercise:**Problem:**

Why is it preferable for airplanes to take off into the wind rather than with the wind?

Exercise:**Problem:**

Roofs are sometimes pushed off vertically during a tropical cyclone, and buildings sometimes explode outward when hit by a tornado. Use Bernoulli's principle to explain these phenomena.

Exercise:

Problem: Why does a sailboat need a keel?

Exercise:**Problem:**

It is dangerous to stand close to railroad tracks when a rapidly moving commuter train passes. Explain why atmospheric pressure would push you toward the moving train.

Exercise:**Problem:**

Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

Exercise:**Problem:**

A perfume bottle or atomizer sprays a fluid that is in the bottle. ([link](#).) How does the fluid rise up in the vertical tube in the bottle?



Atomizer:
perfume
bottle with
tube to carry
perfume up
through the
bottle.
(credit:
Antonia Foy,
Flickr)

Exercise:

Problem:

If you lower the window on a car while moving, an empty plastic bag can sometimes fly out the window. Why does this happen?

Problems & Exercises

Exercise:

Problem: Verify that pressure has units of energy per unit volume.

Solution:

$$\begin{aligned} P &= \frac{\text{Force}}{\text{Area}}, \\ (P)_{\text{units}} &= \text{N/m}^2 = \text{N} \cdot \text{m/m}^3 = \text{J/m}^3 \\ &= \text{energy/volume} \end{aligned}$$

Exercise:**Problem:**

Suppose you have a wind speed gauge like the pitot tube shown in [\[link\]](#)(b). By what factor must wind speed increase to double the value of h in the manometer? Is this independent of the moving fluid and the fluid in the manometer?

Exercise:**Problem:**

If the pressure reading of your pitot tube is 15.0 mm Hg at a speed of 200 km/h, what will it be at 700 km/h at the same altitude?

Solution:

184 mm Hg

Exercise:**Problem:**

Calculate the maximum height to which water could be squirted with the hose in [\[link\]](#) example if it: (a) Emerges from the nozzle. (b) Emerges with the nozzle removed, assuming the same flow rate.

Exercise:

Problem:

Every few years, winds in Boulder, Colorado, attain sustained speeds of 45.0 m/s (about 100 mi/h) when the jet stream descends during early spring. Approximately what is the force due to the Bernoulli effect on a roof having an area of 220 m²? Typical air density in Boulder is 1.14 kg/m³, and the corresponding atmospheric pressure is 8.89×10^4 N/m². (Bernoulli's principle as stated in the text assumes laminar flow. Using the principle here produces only an approximate result, because there is significant turbulence.)

Solution:

$$2.54 \times 10^5 \text{ N}$$

Exercise:**Problem:**

(a) Calculate the approximate force on a square meter of sail, given the horizontal velocity of the wind is 6.00 m/s parallel to its front surface and 3.50 m/s along its back surface. Take the density of air to be 1.29 kg/m³. (The calculation, based on Bernoulli's principle, is approximate due to the effects of turbulence.) (b) Discuss whether this force is great enough to be effective for propelling a sailboat.

Exercise:**Problem:**

(a) What is the pressure drop due to the Bernoulli effect as water goes into a 3.00-cm-diameter nozzle from a 9.00-cm-diameter fire hose while carrying a flow of 40.0 L/s? (b) To what maximum height above the nozzle can this water rise? (The actual height will be significantly smaller due to air resistance.)

Solution:

(a) $1.58 \times 10^6 \text{ N/m}^2$

(b) 163 m

Exercise:

Problem:

(a) Using Bernoulli's equation, show that the measured fluid speed v for a pitot tube, like the one in [\[link\]](#)(b), is given by

Equation:

$$v = \left(\frac{2\rho'gh}{\rho} \right)^{1/2},$$

where h is the height of the manometer fluid, ρ' is the density of the manometer fluid, ρ is the density of the moving fluid, and g is the acceleration due to gravity. (Note that v is indeed proportional to the square root of h , as stated in the text.) (b) Calculate v for moving air if a mercury manometer's h is 0.200 m.

Glossary

Bernoulli's equation

the equation resulting from applying conservation of energy to an incompressible frictionless fluid: $P + 1/2\rho v^2 + \rho gh = \text{constant}$, through the fluid

Bernoulli's principle

Bernoulli's equation applied at constant depth: $P_1 + 1/2\rho v_1^2 = P_2 + 1/2\rho v_2^2$

The Most General Applications of Bernoulli's Equation

- Calculate using Torricelli's theorem.
- Calculate power in fluid flow.

Torricelli's Theorem

[\[link\]](#) shows water gushing from a large tube through a dam. What is its speed as it emerges? Interestingly, if resistance is negligible, the speed is just what it would be if the water fell a distance h from the surface of the reservoir; the water's speed is independent of the size of the opening. Let us check this out. Bernoulli's equation must be used since the depth is not constant. We consider water flowing from the surface (point 1) to the tube's outlet (point 2). Bernoulli's equation as stated in previously is

Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2.$$

Both P_1 and P_2 equal atmospheric pressure (P_1 is atmospheric pressure because it is the pressure at the top of the reservoir. P_2 must be atmospheric pressure, since the emerging water is surrounded by the atmosphere and cannot have a pressure different from atmospheric pressure.) and subtract out of the equation, leaving

Equation:

$$\frac{1}{2}\rho v_1^2 + \rho gh_1 = \frac{1}{2}\rho v_2^2 + \rho gh_2.$$

Solving this equation for v_2^2 , noting that the density ρ cancels (because the fluid is incompressible), yields

Equation:

$$v_2^2 = v_1^2 + 2g(h_1 - h_2).$$

We let $h = h_1 - h_2$; the equation then becomes

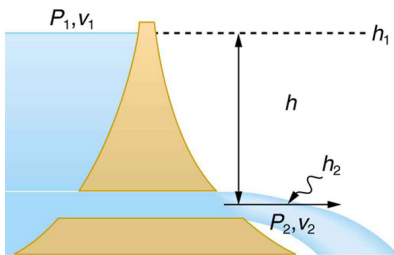
Equation:

$$v_2^2 = v_1^2 + 2gh$$

where h is the height dropped by the water. This is simply a kinematic equation for any object falling a distance h with negligible resistance. In fluids, this last equation is called *Torricelli's theorem*. Note that the result is independent of the velocity's direction, just as we found when applying conservation of energy to falling objects.



(a)



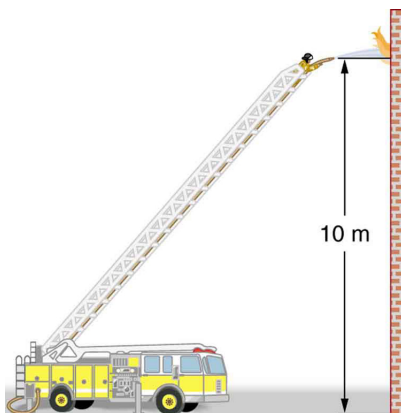
(b)

(a) Water gushes from the base of the Studen Kladenetz dam in Bulgaria.

(credit: Kiril Kapustin;

<http://www.ImagesFromBulgaria.com>

(b) In the absence of significant resistance, water flows from the reservoir with the same speed it would have if it fell the distance h without friction. This is an example of Torricelli's theorem.



Pressure in the nozzle of this fire hose is less than at ground level for two reasons: the water has to go uphill to get to the nozzle, and speed increases in the nozzle. In spite of its

lowered pressure, the water can exert a large force on anything it strikes, by virtue of its kinetic energy. Pressure in the water stream becomes equal to atmospheric pressure once it emerges into the air.

All preceding applications of Bernoulli's equation involved simplifying conditions, such as constant height or constant pressure. The next example is a more general application of Bernoulli's equation in which pressure, velocity, and height all change. (See [\[link\]](#).)

Example:

Calculating Pressure: A Fire Hose Nozzle

Fire hoses used in major structure fires have inside diameters of 6.40 cm. Suppose such a hose carries a flow of 40.0 L/s starting at a gauge pressure of $1.62 \times 10^6 \text{ N/m}^2$. The hose goes 10.0 m up a ladder to a nozzle having an inside diameter of 3.00 cm. Assuming negligible resistance, what is the pressure in the nozzle?

Strategy

Here we must use Bernoulli's equation to solve for the pressure, since depth is not constant.

Solution

Bernoulli's equation states

Equation:

$$P_1 + \frac{1}{2}\rho v_1^2 + \rho gh_1 = P_2 + \frac{1}{2}\rho v_2^2 + \rho gh_2,$$

where the subscripts 1 and 2 refer to the initial conditions at ground level and the final conditions inside the nozzle, respectively. We must first find the speeds v_1 and v_2 . Since $Q = A_1 v_1$, we get

Equation:

$$v_1 = \frac{Q}{A_1} = \frac{40.0 \times 10^{-3} \text{ m}^3/\text{s}}{\pi(3.20 \times 10^{-2} \text{ m})^2} = 12.4 \text{ m/s}.$$

Similarly, we find

Equation:

$$v_2 = 56.6 \text{ m/s}.$$

(This rather large speed is helpful in reaching the fire.) Now, taking h_1 to be zero, we solve Bernoulli's equation for P_2 :

Equation:

$$P_2 = P_1 + \frac{1}{2}\rho(v_1^2 - v_2^2) - \rho gh_2.$$

Substituting known values yields

Equation:

$$P_2 = 1.62 \times 10^6 \text{ N/m}^2 + \frac{1}{2}(1000 \text{ kg/m}^3)[(12.4 \text{ m/s})^2 - (56.6 \text{ m/s})^2] - (1000 \text{ kg/m}^3)(9.80 \text{ m/s}^2)(10.0 \text{ m})$$

Discussion

This value is a gauge pressure, since the initial pressure was given as a gauge pressure. Thus the nozzle pressure equals atmospheric pressure, as it must because the water exits into the atmosphere without changes in its conditions.

Power in Fluid Flow

Power is the *rate* at which work is done or energy in any form is used or supplied. To see the relationship of power to fluid flow, consider Bernoulli's equation:

Equation:

$$P + \frac{1}{2}\rho v^2 + \rho gh = \text{constant}.$$

All three terms have units of energy per unit volume, as discussed in the previous section. Now, considering units, if we multiply energy per unit volume by flow rate (volume per unit time), we get units of power. That is, $(E/V)(V/t) = E/t$. This means that if we multiply Bernoulli's equation by flow rate Q , we get power. In equation form, this is

Equation:

$$\left(P + \frac{1}{2}\rho v^2 + \rho gh\right)Q = \text{power}.$$

Each term has a clear physical meaning. For example, PQ is the power supplied to a fluid, perhaps by a pump, to give it its pressure P . Similarly, $\frac{1}{2}\rho v^2 Q$ is the power supplied to a fluid to give it its kinetic energy. And ρghQ is the power going to gravitational potential energy.

Note:

Making Connections: Power

Power is defined as the rate of energy transferred, or E/t . Fluid flow involves several types of power. Each type of power is identified with a specific type of energy being expended or changed in form.

Example:

Calculating Power in a Moving Fluid

Suppose the fire hose in the previous example is fed by a pump that receives water through a hose with a 6.40-cm diameter coming from a hydrant with a pressure of $0.700 \times 10^6 \text{ N/m}^2$. What power does the pump supply to the water?

Strategy

Here we must consider energy forms as well as how they relate to fluid flow. Since the input and output hoses have the same diameters and are at the same height, the pump does not change the speed of the water nor its height, and so the water's kinetic energy and gravitational potential energy are unchanged. That means the pump only supplies power to increase water pressure by $0.92 \times 10^6 \text{ N/m}^2$ (from $0.700 \times 10^6 \text{ N/m}^2$ to $1.62 \times 10^6 \text{ N/m}^2$).

Solution

As discussed above, the power associated with pressure is

Equation:

$$\begin{aligned}
 \text{power} &= PQ \\
 &= (0.920 \times 10^6 \text{ N/m}^2)(40.0 \times 10^{-3} \text{ m}^3/\text{s}). \\
 &= 3.68 \times 10^4 \text{ W} = 36.8 \text{ kW}
 \end{aligned}$$

Discussion

Such a substantial amount of power requires a large pump, such as is found on some fire trucks. (This kilowatt value converts to about 50 hp.) The pump in this example increases only the water's pressure. If a pump—such as the heart—directly increases velocity and height as well as pressure, we would have to calculate all three terms to find the power it supplies.

Summary

- Power in fluid flow is given by the equation $(P_1 + \frac{1}{2}\rho v^2 + \rho gh)Q = \text{power}$, where the first term is power associated with pressure, the second is power associated with velocity, and the third is power associated with height.

Conceptual Questions

Exercise:

Problem:

Based on Bernoulli's equation, what are three forms of energy in a fluid? (Note that these forms are conservative, unlike heat transfer and other dissipative forms not included in Bernoulli's equation.)

Exercise:

Problem:

Water that has emerged from a hose into the atmosphere has a gauge pressure of zero. Why? When you put your hand in front of the emerging stream you feel a force, yet the water's gauge pressure is zero. Explain where the force comes from in terms of energy.

Exercise:

Problem:

The old rubber boot shown in [\[link\]](#) has two leaks. To what maximum height can the water squirt from Leak 1? How does the velocity of water emerging from Leak 2 differ from that of leak 1? Explain your responses in terms of energy.



Water emerges from two leaks in an old boot.

Exercise:**Problem:**

Water pressure inside a hose nozzle can be less than atmospheric pressure due to the Bernoulli effect. Explain in terms of energy how the water can emerge from the nozzle against the opposing atmospheric pressure.

Problems & Exercises**Exercise:****Problem:**

Hoover Dam on the Colorado River is the highest dam in the United States at 221 m, with an output of 1300 MW. The dam generates electricity with water taken from a depth of 150 m and an average flow rate of $650 \text{ m}^3/\text{s}$. (a) Calculate the power in this flow. (b) What is the ratio of this power to the facility's average of 680 MW?

Solution:

(a) $9.56 \times 10^8 \text{ W}$

(b) 1.4

Exercise:**Problem:**

A frequently quoted rule of thumb in aircraft design is that wings should produce about 1000 N of lift per square meter of wing. (The fact that a wing has a top and bottom surface does not double its area.) (a) At takeoff, an aircraft travels at 60.0 m/s, so that the air speed relative to the bottom of the wing is 60.0 m/s. Given the sea level density of air to be 1.29 kg/m^3 , how fast must it move over the upper surface to create the ideal lift? (b) How fast must air move over the upper surface at a cruising speed of 245 m/s and at an altitude where air density is one-fourth that at sea level? (Note that this is not all of the aircraft's lift—some comes from the body of the plane, some from engine thrust, and so on. Furthermore, Bernoulli's principle gives an approximate answer because flow over the wing creates turbulence.)

Exercise:**Problem:**

The left ventricle of a resting adult's heart pumps blood at a flow rate of $83.0 \text{ cm}^3/\text{s}$, increasing its pressure by 110 mm Hg, its speed from zero to 30.0 cm/s, and its height by 5.00 cm. (All numbers are averaged over the entire heartbeat.) Calculate the total power output of the left ventricle. Note that most of the power is used to increase blood pressure.

Solution:

1.26 W

Exercise:**Problem:**

A sump pump (used to drain water from the basement of houses built below the water table) is draining a flooded basement at the rate of 0.750 L/s, with an output pressure of $3.00 \times 10^5 \text{ N/m}^2$. (a) The water enters a hose with a 3.00-cm inside diameter and rises 2.50 m above the pump. What is its pressure at this point? (b) The hose goes over the foundation wall, losing 0.500 m in height, and widens to 4.00 cm in diameter. What is the pressure now? You may neglect frictional losses in both parts of the problem.

Viscosity and Laminar Flow; Poiseuille's Law

- Define laminar flow and turbulent flow.
- Explain what viscosity is.
- Calculate flow and resistance with Poiseuille's law.
- Explain how pressure drops due to resistance.

Laminar Flow and Viscosity

When you pour yourself a glass of juice, the liquid flows freely and quickly. But when you pour syrup on your pancakes, that liquid flows slowly and sticks to the pitcher. The difference is fluid friction, both within the fluid itself and between the fluid and its surroundings. We call this property of fluids *viscosity*. Juice has low viscosity, whereas syrup has high viscosity. In the previous sections we have considered ideal fluids with little or no viscosity. In this section, we will investigate what factors, including viscosity, affect the rate of fluid flow.

The precise definition of viscosity is based on *laminar*, or nonturbulent, flow. Before we can define viscosity, then, we need to define laminar flow and turbulent flow.

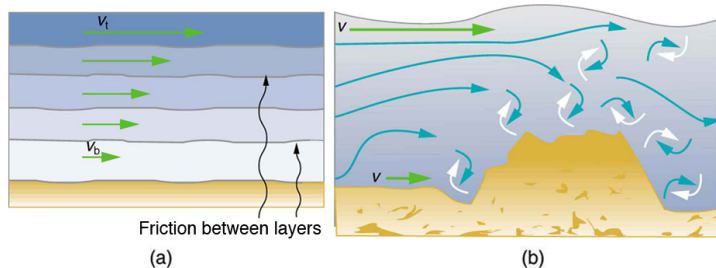
[\[link\]](#) shows both types of flow. **Laminar** flow is characterized by the smooth flow of the fluid in layers that do not mix. Turbulent flow, or **turbulence**, is characterized by eddies and swirls that mix layers of fluid together.



Smoke rises
smoothly for a
while and then

begins to form
swirls and eddies.
The smooth flow is
called laminar flow,
whereas the swirls
and eddies typify
turbulent flow. If
you watch the
smoke (being
careful not to
breathe on it), you
will notice that it
rises more rapidly
when flowing
smoothly than after
it becomes
turbulent, implying
that turbulence
poses more
resistance to flow.
(credit:
Creativity103)

[\[link\]](#) shows schematically how laminar and turbulent flow differ. Layers flow without mixing when flow is laminar. When there is turbulence, the layers mix, and there are significant velocities in directions other than the overall direction of flow. The lines that are shown in many illustrations are the paths followed by small volumes of fluids. These are called *streamlines*. Streamlines are smooth and continuous when flow is laminar, but break up and mix when flow is turbulent. Turbulence has two main causes. First, any obstruction or sharp corner, such as in a faucet, creates turbulence by imparting velocities perpendicular to the flow. Second, high speeds cause turbulence. The drag both between adjacent layers of fluid and between the fluid and its surroundings forms swirls and eddies, if the speed is great enough. We shall concentrate on laminar flow for the remainder of this section, leaving certain aspects of turbulence for later sections.



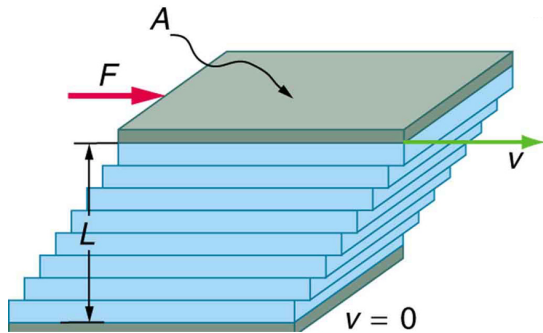
(a) Laminar flow occurs in layers without mixing. Notice that viscosity causes drag between layers as well as with the fixed surface. (b) An obstruction in the vessel produces turbulence. Turbulent flow mixes the fluid. There is more interaction, greater heating, and more resistance than in laminar flow.

Note:

Making Connections: Take-Home Experiment: Go Down to the River

Try dropping simultaneously two sticks into a flowing river, one near the edge of the river and one near the middle. Which one travels faster? Why?

[\[link\]](#) shows how viscosity is measured for a fluid. Two parallel plates have the specific fluid between them. The bottom plate is held fixed, while the top plate is moved to the right, dragging fluid with it. The layer (or lamina) of fluid in contact with either plate does not move relative to the plate, and so the top layer moves at v while the bottom layer remains at rest. Each successive layer from the top down exerts a force on the one below it, trying to drag it along, producing a continuous variation in speed from v to 0 as shown. Care is taken to insure that the flow is laminar; that is, the layers do not mix. The motion in [\[link\]](#) is like a continuous shearing motion. Fluids have zero shear strength, but the *rate* at which they are sheared is related to the same geometrical factors A and L as is shear deformation for solids.



The graphic shows laminar flow of fluid between two plates of area A . The bottom plate is fixed. When the top plate is pushed to the right, it drags the fluid along with it.

A force F is required to keep the top plate in [link](#) moving at a constant velocity v , and experiments have shown that this force depends on four factors. First, F is directly proportional to v (until the speed is so high that turbulence occurs—then a much larger force is needed, and it has a more complicated dependence on v). Second, F is proportional to the area A of the plate. This relationship seems reasonable, since A is directly proportional to the amount of fluid being moved. Third, F is inversely proportional to the distance between the plates L . This relationship is also reasonable; L is like a lever arm, and the greater the lever arm, the less force that is needed. Fourth, F is directly proportional to the *coefficient of viscosity*, η . The greater the viscosity, the greater the force required. These dependencies are combined into the equation

Equation:

$$F = \eta \frac{vA}{L},$$

which gives us a working definition of fluid **viscosity** η . Solving for η gives

Equation:

$$\eta = \frac{FL}{vA},$$

which defines viscosity in terms of how it is measured. The SI unit of viscosity is $\text{N} \cdot \text{m}/[(\text{m}/\text{s})\text{m}^2] = (\text{N}/\text{m}^2)\text{s}$ or $\text{Pa} \cdot \text{s}$. [\[link\]](#) lists the coefficients of viscosity for various fluids.

Viscosity varies from one fluid to another by several orders of magnitude. As you might expect, the viscosities of gases are much less than those of liquids, and these viscosities are often temperature dependent. The viscosity of blood can be reduced by aspirin consumption, allowing it to flow more easily around the body. (When used over the long term in low doses, aspirin can help prevent heart attacks, and reduce the risk of blood clotting.)

Laminar Flow Confined to Tubes—Poiseuille’s Law

What causes flow? The answer, not surprisingly, is pressure difference. In fact, there is a very simple relationship between horizontal flow and pressure. Flow rate Q is in the direction from high to low pressure. The greater the pressure differential between two points, the greater the flow rate. This relationship can be stated as

Equation:

$$Q = \frac{P_2 - P_1}{R},$$

where P_1 and P_2 are the pressures at two points, such as at either end of a tube, and R is the resistance to flow. The resistance R includes everything, except pressure, that affects flow rate. For example, R is greater for a long tube than for a short one. The greater the viscosity of a fluid, the greater the value of R . Turbulence greatly increases R , whereas increasing the diameter of a tube decreases R .

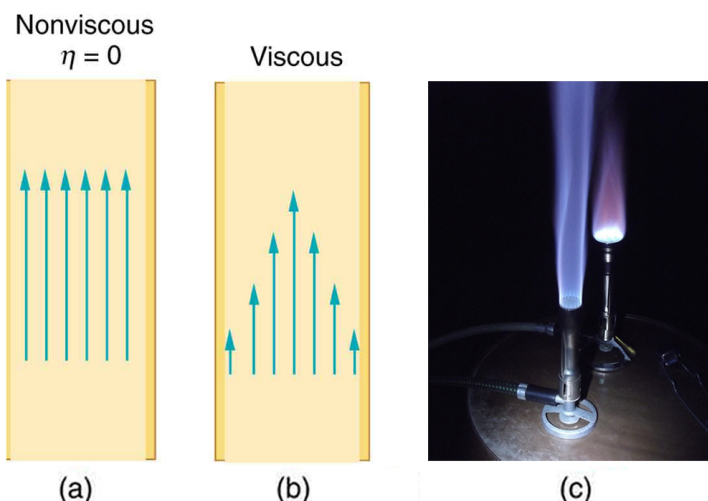
If viscosity is zero, the fluid is frictionless and the resistance to flow is also zero. Comparing frictionless flow in a tube to viscous flow, as in [\[link\]](#), we see that for a viscous fluid, speed is greatest at midstream because of drag at the boundaries. We can see the effect of viscosity in a Bunsen burner flame, even though the viscosity of natural gas is small.

The resistance R to laminar flow of an incompressible fluid having viscosity η through a horizontal tube of uniform radius r and length l , such as the one in [\[link\]](#), is given by

Equation:

$$R = \frac{8\eta l}{\pi r^4}.$$

This equation is called **Poiseuille's law for resistance** after the French scientist J. L. Poiseuille (1799–1869), who derived it in an attempt to understand the flow of blood, an often turbulent fluid.



(a) If fluid flow in a tube has negligible resistance, the speed is the same all across the tube. (b) When a viscous fluid flows through a tube, its speed at the walls is zero, increasing steadily to its maximum at the center of the tube. (c) The shape of the Bunsen burner flame is due to the velocity profile across the tube. (credit: Jason Woodhead)

Let us examine Poiseuille's expression for R to see if it makes good intuitive sense. We see that resistance is directly proportional to both fluid viscosity η and the length l of a tube. After all, both of these directly affect the amount of friction encountered—the greater either is, the greater the resistance and the smaller the flow. The radius r of a tube affects the resistance, which again makes sense, because the greater the radius, the greater the flow (all other factors remaining the same). But it is surprising that r is raised to the *fourth* power in Poiseuille's law. This exponent means that any change in the radius of a tube has a very large effect on resistance. For example, doubling the radius of a tube decreases resistance by a factor of $2^4 = 16$.

Taken together, $Q = \frac{P_2 - P_1}{R}$ and $R = \frac{8\eta l}{\pi r^4}$ give the following expression for flow rate:

Equation:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

This equation describes laminar flow through a tube. It is sometimes called Poiseuille's law for laminar flow, or simply **Poiseuille's law**.

Example:

Using Flow Rate: Plaque Deposits Reduce Blood Flow

Suppose the flow rate of blood in a coronary artery has been reduced to half its normal value by plaque deposits. By what factor has the radius of the artery been reduced, assuming no turbulence occurs?

Strategy

Assuming laminar flow, Poiseuille's law states that

Equation:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

We need to compare the artery radius before and after the flow rate reduction.

Solution

With a constant pressure difference assumed and the same length and viscosity, along the artery we have

Equation:

$$\frac{Q_1}{r_1^4} = \frac{Q_2}{r_2^4}.$$

So, given that $Q_2 = 0.5Q_1$, we find that $r_2^4 = 0.5r_1^4$.

Therefore, $r_2 = (0.5)^{0.25}r_1 = 0.841r_1$, a decrease in the artery radius of 16%.

Discussion

This decrease in radius is surprisingly small for this situation. To restore the blood flow in spite of this buildup would require an increase in the pressure difference ($P_2 - P_1$) of a factor of two, with subsequent strain on the heart.

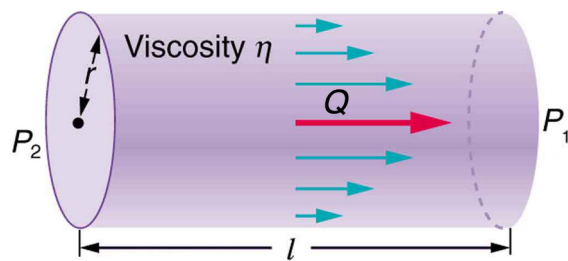
Fluid	Temperature (°C)	Viscosity η (mPa·s)
<i>Gases</i>		
Air	0	0.0171
	20	0.0181
	40	0.0190
	100	0.0218
Ammonia	20	0.00974
Carbon dioxide	20	0.0147
Helium	20	0.0196
Hydrogen	0	0.0090
Mercury	20	0.0450
Oxygen	20	0.0203
Steam	100	0.0130
<i>Liquids</i>		
Water	0	1.792
	20	1.002
	37	0.6947
	40	0.653
	100	0.282
Whole blood [footnote]	20	3.015

Fluid	Temperature (°C)	Viscosity η (mPa·s)
The ratios of the viscosities of blood to water are nearly constant between 0°C and 37°C.	37	2.084
Blood plasma [footnote] See note on Whole Blood.	20	1.810
	37	1.257
Ethyl alcohol	20	1.20
Methanol	20	0.584
Oil (heavy machine)	20	660
Oil (motor, SAE 10)	30	200
Oil (olive)	20	138
Glycerin	20	1500
Honey	20	2000–10000
Maple Syrup	20	2000–3000
Milk	20	3.0
Oil (Corn)	20	65

Coefficients of Viscosity of Various Fluids

The circulatory system provides many examples of Poiseuille's law in action—with blood flow regulated by changes in vessel size and blood pressure. Blood vessels are not rigid but elastic. Adjustments to blood flow are primarily made by varying the size of the vessels, since the resistance is so sensitive to the radius. During vigorous exercise, blood vessels are selectively dilated to important muscles and organs and blood pressure increases. This creates both greater overall blood flow and increased flow to specific areas. Conversely, decreases in vessel radii, perhaps from plaques in

the arteries, can greatly reduce blood flow. If a vessel's radius is reduced by only 5% (to 0.95 of its original value), the flow rate is reduced to about $(0.95)^4 = 0.81$ of its original value. A 19% decrease in flow is caused by a 5% decrease in radius. The body may compensate by increasing blood pressure by 19%, but this presents hazards to the heart and any vessel that has weakened walls. Another example comes from automobile engine oil. If you have a car with an oil pressure gauge, you may notice that oil pressure is high when the engine is cold. Motor oil has greater viscosity when cold than when warm, and so pressure must be greater to pump the same amount of cold oil.



Poiseuille's law applies to laminar flow of an incompressible fluid of viscosity η through a tube of length l and radius r . The direction of flow is from greater to lower pressure.

Flow rate Q is directly proportional to the pressure difference $P_2 - P_1$, and inversely proportional to the length l of the tube and viscosity η of the fluid. Flow rate increases with r^4 , the fourth power of the radius.

Example:
What Pressure Produces This Flow Rate?

An intravenous (IV) system is supplying saline solution to a patient at the rate of $0.120 \text{ cm}^3/\text{s}$ through a needle of radius 0.150 mm and length 2.50 cm . What pressure is needed at the entrance of the needle to cause this flow, assuming the viscosity of the saline solution to be the same as that of water? The gauge pressure of the blood in the patient's vein is 8.00 mm Hg . (Assume that the temperature is 20°C .)

Strategy

Assuming laminar flow, Poiseuille's law applies. This is given by

Equation:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l},$$

where P_2 is the pressure at the entrance of the needle and P_1 is the pressure in the vein. The only unknown is P_2 .

Solution

Solving for P_2 yields

Equation:

$$P_2 = \frac{8\eta l}{\pi r^4} Q + P_1.$$

P_1 is given as 8.00 mm Hg , which converts to $1.066 \times 10^3 \text{ N/m}^2$. Substituting this and the other known values yields

Equation:

$$\begin{aligned} P_2 &= \left[\frac{8(1.00 \times 10^{-3} \text{ N}\cdot\text{s/m}^2)(2.50 \times 10^{-2} \text{ m})}{\pi(0.150 \times 10^{-3} \text{ m})^4} \right] (1.20 \times 10^{-7} \text{ m}^3/\text{s}) + 1.066 \times 10^3 \text{ N/m}^2 \\ &= 1.62 \times 10^4 \text{ N/m}^2. \end{aligned}$$

Discussion

This pressure could be supplied by an IV bottle with the surface of the saline solution 1.61 m above the entrance to the needle (this is left for you to solve in this chapter's Problems and Exercises), assuming that there is negligible pressure drop in the tubing leading to the needle.

Flow and Resistance as Causes of Pressure Drops

You may have noticed that water pressure in your home might be lower than normal on hot summer days when there is more use. This pressure drop occurs in the water

main before it reaches your home. Let us consider flow through the water main as illustrated in [\[link\]](#). We can understand why the pressure P_1 to the home drops during times of heavy use by rearranging

Equation:

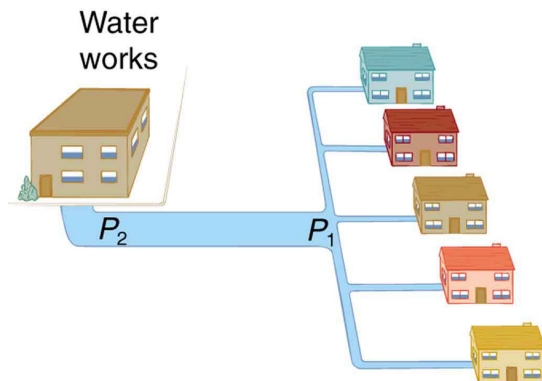
$$Q = \frac{P_2 - P_1}{R}$$

to

Equation:

$$P_2 - P_1 = RQ,$$

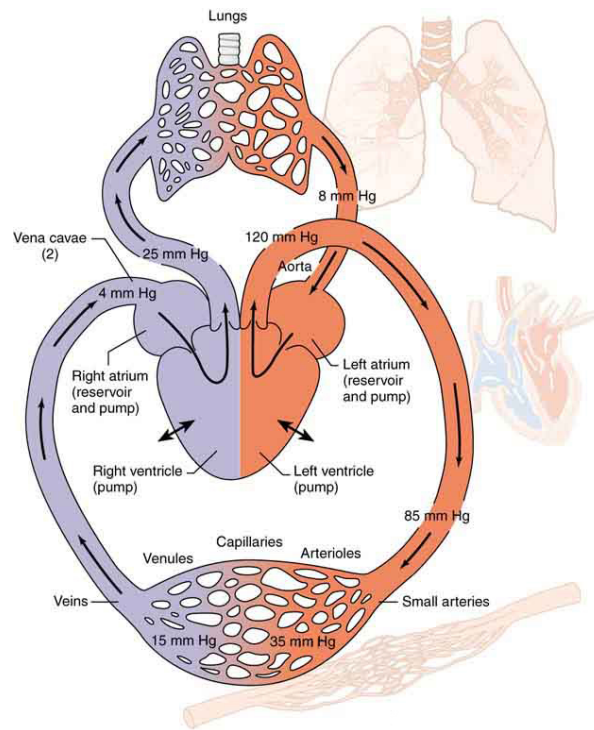
where, in this case, P_2 is the pressure at the water works and R is the resistance of the water main. During times of heavy use, the flow rate Q is large. This means that $P_2 - P_1$ must also be large. Thus P_1 must decrease. It is correct to think of flow and resistance as causing the pressure to drop from P_2 to P_1 . $P_2 - P_1 = RQ$ is valid for both laminar and turbulent flows.



During times of heavy use, there is a significant pressure drop in a water main, and P_1 supplied to users is significantly less than P_2 created at the water works. If the flow is very small, then the pressure drop is negligible, and $P_2 \approx P_1$.

We can use $P_2 - P_1 = RQ$ to analyze pressure drops occurring in more complex systems in which the tube radius is not the same everywhere. Resistance will be much greater in narrow places, such as an obstructed coronary artery. For a given flow rate Q , the pressure drop will be greatest where the tube is most narrow. This is how water faucets control flow. Additionally, R is greatly increased by turbulence, and a constriction that creates turbulence greatly reduces the pressure downstream. Plaque in an artery reduces pressure and hence flow, both by its resistance and by the turbulence it creates.

[\[link\]](#) is a schematic of the human circulatory system, showing average blood pressures in its major parts for an adult at rest. Pressure created by the heart's two pumps, the right and left ventricles, is reduced by the resistance of the blood vessels as the blood flows through them. The left ventricle increases arterial blood pressure that drives the flow of blood through all parts of the body except the lungs. The right ventricle receives the lower pressure blood from two major veins and pumps it through the lungs for gas exchange with atmospheric gases – the disposal of carbon dioxide from the blood and the replenishment of oxygen. Only one major organ is shown schematically, with typical branching of arteries to ever smaller vessels, the smallest of which are the capillaries, and rejoining of small veins into larger ones. Similar branching takes place in a variety of organs in the body, and the circulatory system has considerable flexibility in flow regulation to these organs by the dilation and constriction of the arteries leading to them and the capillaries within them. The sensitivity of flow to tube radius makes this flexibility possible over a large range of flow rates.



Schematic of the circulatory system.

Pressure difference is created by the two pumps in the heart and is reduced by resistance in the vessels. Branching of vessels into capillaries allows blood to reach individual cells and exchange substances, such as oxygen and waste products, with them. The system has an impressive ability to regulate flow to individual organs, accomplished largely by varying vessel diameters.

Each branching of larger vessels into smaller vessels increases the total cross-sectional area of the tubes through which the blood flows. For example, an artery with a cross section of 1 cm^2 may branch into 20 smaller arteries, each with cross sections of 0.5 cm^2 , with a total of 10 cm^2 . In that manner, the resistance of the branchings is reduced so that pressure is not entirely lost. Moreover, because $Q = Av$ and A increases through branching, the average velocity of the blood in the smaller vessels is reduced. The blood velocity in the aorta (diameter = 1 cm) is about 25 cm/s , while in the capillaries ($20 \mu\text{m}$ in diameter) the velocity is about 1

mm/s. This reduced velocity allows the blood to exchange substances with the cells in the capillaries and alveoli in particular.

Section Summary

- Laminar flow is characterized by smooth flow of the fluid in layers that do not mix.
- Turbulence is characterized by eddies and swirls that mix layers of fluid together.
- Fluid viscosity η is due to friction within a fluid. Representative values are given in [\[link\]](#). Viscosity has units of $(\text{N}/\text{m}^2)\text{s}$ or $\text{Pa} \cdot \text{s}$.
- Flow is proportional to pressure difference and inversely proportional to resistance:

Equation:

$$Q = \frac{P_2 - P_1}{R}.$$

- For laminar flow in a tube, Poiseuille's law for resistance states that

Equation:

$$R = \frac{8\eta l}{\pi r^4}.$$

- Poiseuille's law for flow in a tube is

Equation:

$$Q = \frac{(P_2 - P_1)\pi r^4}{8\eta l}.$$

- The pressure drop caused by flow and resistance is given by

Equation:

$$P_2 - P_1 = RQ.$$

Conceptual Questions

Exercise:

Problem:

Explain why the viscosity of a liquid decreases with temperature—that is, how might increased temperature reduce the effects of cohesive forces in a liquid? Also explain why the viscosity of a gas increases with temperature—that is, how does increased gas temperature create more collisions between atoms and molecules?

Exercise:

Problem:

When paddling a canoe upstream, it is wisest to travel as near to the shore as possible. When canoeing downstream, it may be best to stay near the middle. Explain why.

Exercise:

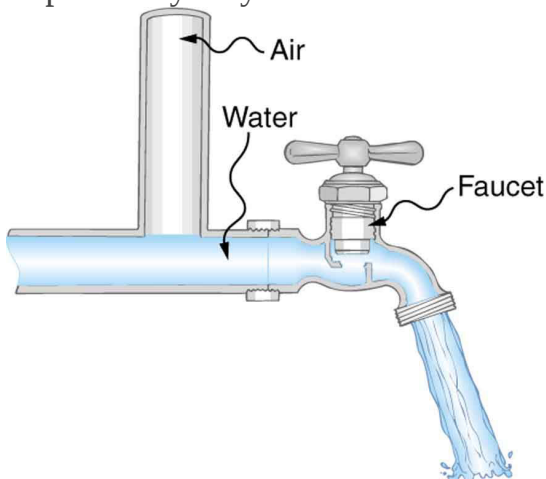
Problem:

Why does flow decrease in your shower when someone flushes the toilet?

Exercise:

Problem:

Plumbing usually includes air-filled tubes near water faucets, as shown in [\[link\]](#). Explain why they are needed and how they work.



The vertical tube near the water tap remains full of air and serves a useful purpose.

Problems & Exercises

Exercise:

Problem:

(a) Calculate the retarding force due to the viscosity of the air layer between a cart and a level air track given the following information—air temperature is 20°C , the cart is moving at 0.400 m/s , its surface area is $2.50 \times 10^{-2}\text{ m}^2$, and the thickness of the air layer is $6.00 \times 10^{-5}\text{ m}$. (b) What is the ratio of this force to the weight of the 0.300-kg cart?

Solution:

(a) $3.02 \times 10^{-3}\text{ N}$

(b) 1.03×10^{-3}

Exercise:

Problem:

What force is needed to pull one microscope slide over another at a speed of 1.00 cm/s , if there is a 0.500-mm -thick layer of 20°C water between them and the contact area is 8.00 cm^2 ?

Exercise:

Problem:

A glucose solution being administered with an IV has a flow rate of $4.00\text{ cm}^3/\text{min}$. What will the new flow rate be if the glucose is replaced by whole blood having the same density but a viscosity 2.50 times that of the glucose? All other factors remain constant.

Solution:

$1.60\text{ cm}^3/\text{min}$

Exercise:

Problem:

The pressure drop along a length of artery is 100 Pa, the radius is 10 mm, and the flow is laminar. The average speed of the blood is 15 mm/s. (a) What is the net force on the blood in this section of artery? (b) What is the power expended maintaining the flow?

Exercise:**Problem:**

A small artery has a length of 1.1×10^{-3} m and a radius of 2.5×10^{-5} m. If the pressure drop across the artery is 1.3 kPa, what is the flow rate through the artery? (Assume that the temperature is 37° C.)

Solution:

$$8.7 \times 10^{-11} \text{ m}^3/\text{s}$$

Exercise:**Problem:**

Fluid originally flows through a tube at a rate of 100 cm³/s. To illustrate the sensitivity of flow rate to various factors, calculate the new flow rate for the following changes with all other factors remaining the same as in the original conditions. (a) Pressure difference increases by a factor of 1.50. (b) A new fluid with 3.00 times greater viscosity is substituted. (c) The tube is replaced by one having 4.00 times the length. (d) Another tube is used with a radius 0.100 times the original. (e) Yet another tube is substituted with a radius 0.100 times the original and half the length, *and* the pressure difference is increased by a factor of 1.50.

Exercise:**Problem:**

The arterioles (small arteries) leading to an organ, constrict in order to decrease flow to the organ. To shut down an organ, blood flow is reduced naturally to 1.00% of its original value. By what factor did the radii of the arterioles constrict? Penguins do this when they stand on ice to reduce the blood flow to their feet.

Solution:

0.316

Exercise:

Problem:

Angioplasty is a technique in which arteries partially blocked with plaque are dilated to increase blood flow. By what factor must the radius of an artery be increased in order to increase blood flow by a factor of 10?

Exercise:

Problem:

(a) Suppose a blood vessel's radius is decreased to 90.0% of its original value by plaque deposits and the body compensates by increasing the pressure difference along the vessel to keep the flow rate constant. By what factor must the pressure difference increase? (b) If turbulence is created by the obstruction, what additional effect would it have on the flow rate?

Solution:

(a) 1.52

(b) Turbulence will decrease the flow rate of the blood, which would require an even larger increase in the pressure difference, leading to higher blood pressure.

Exercise:

Problem:

A spherical particle falling at a terminal speed in a liquid must have the gravitational force balanced by the drag force and the buoyant force. The buoyant force is equal to the weight of the displaced fluid, while the drag force is assumed to be given by Stokes Law, $F_s = 6\pi r\eta v$. Show that the terminal speed is given by

Equation:

$$v = \frac{2R^2g}{9\eta}(\rho_s - \rho_1),$$

where R is the radius of the sphere, ρ_s is its density, and ρ_1 is the density of the fluid and η the coefficient of viscosity.

Exercise:

Problem:

Using the equation of the previous problem, find the viscosity of motor oil in which a steel ball of radius 0.8 mm falls with a terminal speed of 4.32 cm/s. The densities of the ball and the oil are 7.86 and 0.88 g/mL, respectively.

Solution:**Equation:**

$$225 \text{ mPa} \cdot \text{s}$$

Exercise:**Problem:**

A skydiver will reach a terminal velocity when the air drag equals their weight. For a skydiver with high speed and a large body, turbulence is a factor. The drag force then is approximately proportional to the square of the velocity. Taking the drag force to be $F_D = \frac{1}{2} \rho A v^2$ and setting this equal to the person's weight, find the terminal speed for a person falling "spread eagle." Find both a formula and a number for v_t , with assumptions as to size.

Exercise:**Problem:**

A layer of oil 1.50 mm thick is placed between two microscope slides. Researchers find that a force of $5.50 \times 10^{-4} \text{ N}$ is required to glide one over the other at a speed of 1.00 cm/s when their contact area is 6.00 cm^2 . What is the oil's viscosity? What type of oil might it be?

Solution:**Equation:**

$$0.138 \text{ Pa} \cdot \text{s},$$

or

Olive oil.

Exercise:

Problem:

(a) Verify that a 19.0% decrease in laminar flow through a tube is caused by a 5.00% decrease in radius, assuming that all other factors remain constant, as stated in the text. (b) What increase in flow is obtained from a 5.00% increase in radius, again assuming all other factors remain constant?

Exercise:**Problem:**

[\[link\]](#) dealt with the flow of saline solution in an IV system. (a) Verify that a pressure of $1.62 \times 10^4 \text{ N/m}^2$ is created at a depth of 1.61 m in a saline solution, assuming its density to be that of sea water. (b) Calculate the new flow rate if the height of the saline solution is decreased to 1.50 m. (c) At what height would the direction of flow be reversed? (This reversal can be a problem when patients stand up.)

Solution:

(a) $1.62 \times 10^4 \text{ N/m}^2$

(b) $0.111 \text{ cm}^3/\text{s}$

(c) 10.6 cm

Exercise:**Problem:**

When physicians diagnose arterial blockages, they quote the reduction in flow rate. If the flow rate in an artery has been reduced to 10.0% of its normal value by a blood clot and the average pressure difference has increased by 20.0%, by what factor has the clot reduced the radius of the artery?

Exercise:**Problem:**

During a marathon race, a runner's blood flow increases to 10.0 times her resting rate. Her blood's viscosity has dropped to 95.0% of its normal value, and the blood pressure difference across the circulatory system has increased by 50.0%. By what factor has the average radii of her blood vessels increased?

Solution:

Exercise:**Problem:**

Water supplied to a house by a water main has a pressure of $3.00 \times 10^5 \text{ N/m}^2$ early on a summer day when neighborhood use is low. This pressure produces a flow of 20.0 L/min through a garden hose. Later in the day, pressure at the exit of the water main and entrance to the house drops, and a flow of only 8.00 L/min is obtained through the same hose. (a) What pressure is now being supplied to the house, assuming resistance is constant? (b) By what factor did the flow rate in the water main increase in order to cause this decrease in delivered pressure? The pressure at the entrance of the water main is $5.00 \times 10^5 \text{ N/m}^2$, and the original flow rate was 200 L/min. (c) How many more users are there, assuming each would consume 20.0 L/min in the morning?

Exercise:**Problem:**

An oil gusher shoots crude oil 25.0 m into the air through a pipe with a 0.100-m diameter. Neglecting air resistance but not the resistance of the pipe, and assuming laminar flow, calculate the gauge pressure at the entrance of the 50.0-m-long vertical pipe. Take the density of the oil to be 900 kg/m^3 and its viscosity to be $1.00 (\text{N/m}^2) \cdot \text{s}$ (or $1.00 \text{ Pa} \cdot \text{s}$). Note that you must take into account the pressure due to the 50.0-m column of oil in the pipe.

Solution:

$$2.95 \times 10^6 \text{ N/m}^2 (\text{gauge pressure})$$

Exercise:**Problem:**

Concrete is pumped from a cement mixer to the place it is being laid, instead of being carried in wheelbarrows. The flow rate is 200.0 L/min through a 50.0-m-long, 8.00-cm-diameter hose, and the pressure at the pump is $8.00 \times 10^6 \text{ N/m}^2$. (a) Calculate the resistance of the hose. (b) What is the viscosity of the concrete, assuming the flow is laminar? (c) How much power is being supplied, assuming the point of use is at the same level as the pump? You may neglect the power supplied to increase the concrete's velocity.

Exercise:

Problem: Construct Your Own Problem

Consider a coronary artery constricted by arteriosclerosis. Construct a problem in which you calculate the amount by which the diameter of the artery is decreased, based on an assessment of the decrease in flow rate.

Exercise:

Problem:

Consider a river that spreads out in a delta region on its way to the sea. Construct a problem in which you calculate the average speed at which water moves in the delta region, based on the speed at which it was moving up river. Among the things to consider are the size and flow rate of the river before it spreads out and its size once it has spread out. You can construct the problem for the river spreading out into one large river or into multiple smaller rivers.

Glossary

laminar

a type of fluid flow in which layers do not mix

turbulence

fluid flow in which layers mix together via eddies and swirls

viscosity

the friction in a fluid, defined in terms of the friction between layers

Poiseuille's law for resistance

the resistance to laminar flow of an incompressible fluid in a tube: $R = 8\eta l / \pi r^4$

Poiseuille's law

the rate of laminar flow of an incompressible fluid in a tube: $Q = (P_2 - P_1) \pi r^4 / 8\eta l$

Motion of an Object in a Viscous Fluid

- Calculate the Reynolds number for an object moving through a fluid.
- Explain whether the Reynolds number indicates laminar or turbulent flow.
- Describe the conditions under which an object has a terminal speed.

A moving object in a viscous fluid is equivalent to a stationary object in a flowing fluid stream. (For example, when you ride a bicycle at 10 m/s in still air, you feel the air in your face exactly as if you were stationary in a 10-m/s wind.) Flow of the stationary fluid around a moving object may be laminar, turbulent, or a combination of the two. Just as with flow in tubes, it is possible to predict when a moving object creates turbulence. We use another form of the Reynolds number N'_R , defined for an object moving in a fluid to be

Equation:

$$N'_R = \frac{\rho v L}{\eta} (\text{object in fluid}),$$

where L is a characteristic length of the object (a sphere's diameter, for example), ρ the fluid density, η its viscosity, and v the object's speed in the fluid. If N'_R is less than about 1, flow around the object can be laminar, particularly if the object has a smooth shape. The transition to turbulent flow occurs for N'_R between 1 and about 10, depending on surface roughness and so on. Depending on the surface, there can be a *turbulent wake* behind the object with some laminar flow over its surface. For an N'_R between 10 and 10^6 , the flow may be either laminar or turbulent and may oscillate between the two. For N'_R greater than about 10^6 , the flow is entirely turbulent, even at the surface of the object. (See [\[link\]](#).) Laminar flow occurs mostly when the objects in the fluid are small, such as raindrops, pollen, and blood cells in plasma.

Example:

Does a Ball Have a Turbulent Wake?

Calculate the Reynolds number N'_R for a ball with a 7.40-cm diameter thrown at 40.0 m/s.

Strategy

We can use $N'_R = \frac{\rho v L}{\eta}$ to calculate N'_R , since all values in it are either given or can be found in tables of density and viscosity.

Solution

Substituting values into the equation for N'_R yields

Equation:

$$\begin{aligned} N'_R &= \frac{\rho v L}{\eta} = \frac{(1.29 \text{ kg/m}^3)(40.0 \text{ m/s})(0.0740 \text{ m})}{1.81 \times 10^{-5} \text{ Pa}\cdot\text{s}} \\ &= 2.11 \times 10^5. \end{aligned}$$

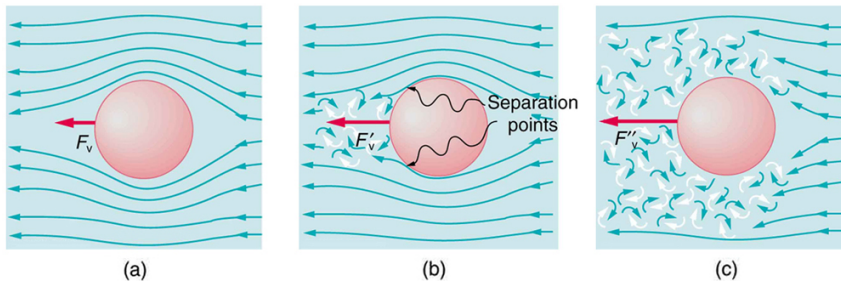
Discussion

This value is sufficiently high to imply a turbulent wake. Most large objects, such as airplanes and sailboats, create significant turbulence as they move. As noted before, the Bernoulli principle gives only qualitatively-correct results in such situations.

One of the consequences of viscosity is a resistance force called **viscous drag** F_V that is exerted on a moving object. This force typically depends on the object's speed (in contrast with simple friction). Experiments have shown that for laminar flow (N'_R less than about one) viscous drag is proportional to speed, whereas for N'_R between about 10 and 10^6 , viscous drag is proportional to speed squared. (This relationship is a strong dependence and is pertinent to bicycle racing, where even a small headwind causes significantly increased drag on the racer. Cyclists take turns being the leader in the pack for this reason.) For N'_R greater than 10^6 , drag increases dramatically and behaves with greater complexity. For laminar flow around a sphere, F_V is proportional to fluid viscosity η , the object's characteristic size L , and its speed v . All of which makes sense—the more viscous the fluid and the larger the object, the more drag we expect. Recall Stoke's law $F_S = 6\pi r\eta v$. For the special case of a small sphere of radius R moving slowly in a fluid of viscosity η , the drag force F_S is given by

Equation:

$$F_S = 6\pi R\eta v.$$



(a) Motion of this sphere to the right is equivalent to fluid flow to the left. Here the flow is laminar with N/R less than 1. There is a force, called viscous drag F_V , to the left on the ball due to the fluid's viscosity. (b) At a higher speed, the flow becomes partially turbulent, creating a wake starting where the flow lines separate from the surface. Pressure in the wake is less than in front of the sphere, because fluid speed is less, creating a net force to the left F'_V that is significantly greater than for laminar flow. Here N/R is greater than 10. (c) At much higher speeds, where N/R is greater than 10^6 , flow becomes turbulent everywhere on the surface and behind the sphere. Drag increases dramatically.

An interesting consequence of the increase in F_V with speed is that an object falling through a fluid will not continue to accelerate indefinitely (as it would if we neglect air resistance, for example). Instead, viscous drag increases, slowing acceleration, until a critical speed, called the **terminal speed**, is reached and the acceleration of the object becomes zero. Once this happens, the object continues to fall at constant speed (the terminal speed). This is the case for particles of sand falling in the ocean, cells falling in a centrifuge, and sky divers falling through the air. [\[link\]](#) shows some of the factors that affect terminal speed. There is a viscous drag on the object that

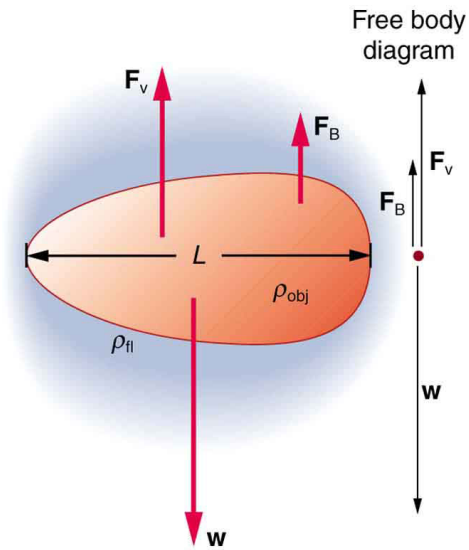
depends on the viscosity of the fluid and the size of the object. But there is also a buoyant force that depends on the density of the object relative to the fluid. Terminal speed will be greatest for low-viscosity fluids and objects with high densities and small sizes. Thus a skydiver falls more slowly with outspread limbs than when they are in a pike position—head first with hands at their side and legs together.

Note:

Take-Home Experiment: Don't Lose Your Marbles

By measuring the terminal speed of a slowly moving sphere in a viscous fluid, one can find the viscosity of that fluid (at that temperature). It can be difficult to find small ball bearings around the house, but a small marble will do. Gather two or three fluids (syrup, motor oil, honey, olive oil, etc.) and a thick, tall clear glass or vase. Drop the marble into the center of the fluid and time its fall (after letting it drop a little to reach its terminal speed). Compare your values for the terminal speed and see if they are inversely proportional to the viscosities as listed in [\[link\]](#). Does it make a difference if the marble is dropped near the side of the glass?

Knowledge of terminal speed is useful for estimating sedimentation rates of small particles. We know from watching mud settle out of dirty water that sedimentation is usually a slow process. Centrifuges are used to speed sedimentation by creating accelerated frames in which gravitational acceleration is replaced by centripetal acceleration, which can be much greater, increasing the terminal speed.



There are three forces acting on an object falling through a viscous fluid: its weight w , the viscous drag F_V , and the buoyant force F_B .

Section Summary

- When an object moves in a fluid, there is a different form of the Reynolds number $N'_R = \frac{\rho v L}{\eta}$ (object in fluid), which indicates whether flow is laminar or turbulent.
- For N'_R less than about one, flow is laminar.
- For N'_R greater than 10^6 , flow is entirely turbulent.

Conceptual Questions

Exercise:

Problem:

What direction will a helium balloon move inside a car that is slowing down—toward the front or back? Explain your answer.

Exercise:**Problem:**

Will identical raindrops fall more rapidly in 5°C air or 25°C air, neglecting any differences in air density? Explain your answer.

Exercise:**Problem:**

If you took two marbles of different sizes, what would you expect to observe about the relative magnitudes of their terminal velocities?

Glossary

viscous drag

a resistance force exerted on a moving object, with a nontrivial dependence on velocity

terminal speed

the speed at which the viscous drag of an object falling in a viscous fluid is equal to the other forces acting on the object (such as gravity), so that the acceleration of the object is zero

Molecular Transport Phenomena: Diffusion, Osmosis, and Related Processes

- Define diffusion, osmosis, dialysis, and active transport.
- Calculate diffusion rates.

Diffusion

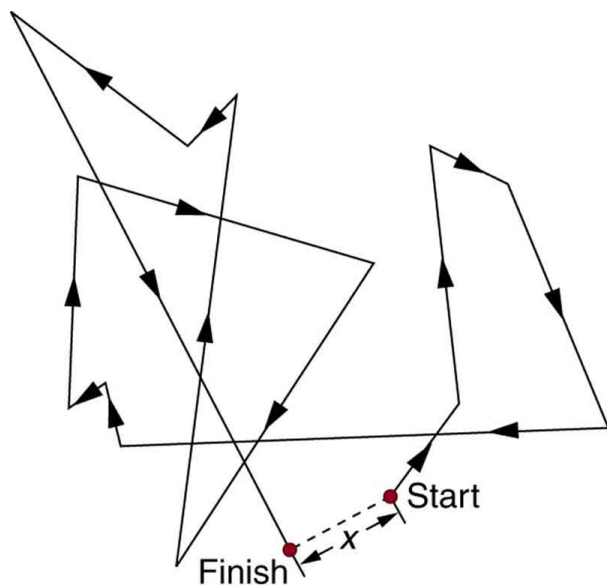
There is something fishy about the ice cube from your freezer—how did it pick up those food odors? How does soaking a sprained ankle in Epsom salt reduce swelling? The answer to these questions are related to atomic and molecular transport phenomena—another mode of fluid motion. Atoms and molecules are in constant motion at any temperature. In fluids they move about randomly even in the absence of macroscopic flow. This motion is called a random walk and is illustrated in [\[link\]](#). **Diffusion** is the movement of substances due to random thermal molecular motion. Fluids, like fish fumes or odors entering ice cubes, can even diffuse through solids.

Diffusion is a slow process over macroscopic distances. The densities of common materials are great enough that molecules cannot travel very far before having a collision that can scatter them in any direction, including straight backward. It can be shown that the average distance x_{rms} that a molecule travels is proportional to the square root of time:

Equation:

$$x_{\text{rms}} = \sqrt{2Dt},$$

where x_{rms} stands for the **root-mean-square distance** and is the statistical average for the process. The quantity D is the diffusion constant for the particular molecule in a specific medium. [\[link\]](#) lists representative values of D for various substances, in units of m^2/s .



The random thermal motion of a molecule in a fluid in time t . This type of motion is called a random walk.

Diffusing molecule	Medium	$D \text{ (m}^2\text{/s)}$
Hydrogen (H_2)	Air	6.4×10^{-5}
Oxygen (O_2)	Air	1.8×10^{-5}
Oxygen (O_2)	Water	1.0×10^{-9}
Glucose ($\text{C}_6\text{H}_{12}\text{O}_6$)	Water	6.7×10^{-10}
Hemoglobin	Water	6.9×10^{-11}

Diffusing molecule	Medium	D (m ² /s)
DNA	Water	1.3×10^{-12}

Diffusion Constants for Various Molecules[\[footnote\]](#)

At 20°C and 1 atm

Note that D gets progressively smaller for more massive molecules. This decrease is because the average molecular speed at a given temperature is inversely proportional to molecular mass. Thus the more massive molecules diffuse more slowly. Another interesting point is that D for oxygen in air is much greater than D for oxygen in water. In water, an oxygen molecule makes many more collisions in its random walk and is slowed considerably. In water, an oxygen molecule moves only about 40 μm in 1 s. (Each molecule actually collides about 10^{10} times per second!). Finally, note that diffusion constants increase with temperature, because average molecular speed increases with temperature. This is because the average kinetic energy of molecules, $\frac{1}{2}mv^2$, is proportional to absolute temperature.

Example:

Calculating Diffusion: How Long Does Glucose Diffusion Take?

Calculate the average time it takes a glucose molecule to move 1.0 cm in water.

Strategy

We can use $x_{\text{rms}} = \sqrt{2Dt}$, the expression for the average distance moved in time t , and solve it for t . All other quantities are known.

Solution

Solving for t and substituting known values yields

Equation:

$$\begin{aligned}
 t &= \frac{x_{\text{rms}}^2}{2D} = \frac{(0.010 \text{ m})^2}{2(6.7 \times 10^{-10} \text{ m}^2/\text{s})} \\
 &= 7.5 \times 10^4 \text{ s} = 21 \text{ h.}
 \end{aligned}$$

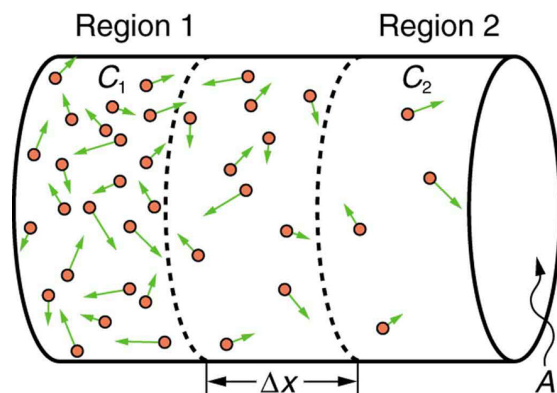
Discussion

This is a remarkably long time for glucose to move a mere centimeter! For this reason, we stir sugar into water rather than waiting for it to diffuse.

Because diffusion is typically very slow, its most important effects occur over small distances. For example, the cornea of the eye gets most of its oxygen by diffusion through the thin tear layer covering it.

The Rate and Direction of Diffusion

If you very carefully place a drop of food coloring in a still glass of water, it will slowly diffuse into the colorless surroundings until its concentration is the same everywhere. This type of diffusion is called free diffusion, because there are no barriers inhibiting it. Let us examine its direction and rate. Molecular motion is random in direction, and so simple chance dictates that more molecules will move out of a region of high concentration than into it. The net rate of diffusion is higher initially than after the process is partially completed. (See [\[link\]](#).)



Diffusion proceeds from a region of higher concentration to a lower one. The net rate of movement is proportional to the difference in concentration.

The net rate of diffusion is proportional to the concentration difference. Many more molecules will leave a region of high concentration than will enter it from a region of low concentration. In fact, if the concentrations were the same, there would be *no* net movement. The net rate of diffusion is also proportional to the diffusion constant D , which is determined experimentally. The farther a molecule can diffuse in a given time, the more likely it is to leave the region of high concentration. Many of the factors that affect the rate are hidden in the diffusion constant D . For example, temperature and cohesive and adhesive forces all affect values of D .

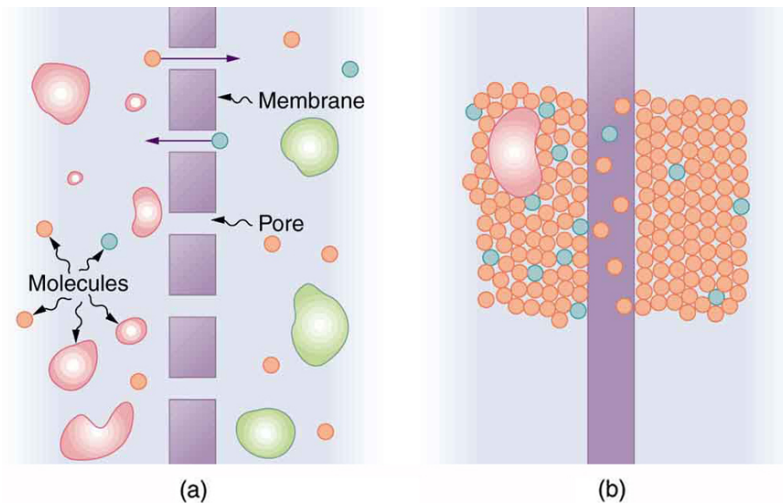
Diffusion is the dominant mechanism by which the exchange of nutrients and waste products occur between the blood and tissue, and between air and blood in the lungs. In the evolutionary process, as organisms became larger, they needed quicker methods of transportation than net diffusion, because of the larger distances involved in the transport, leading to the development of circulatory systems. Less sophisticated, single-celled organisms still rely totally on diffusion for the removal of waste products and the uptake of nutrients.

Osmosis and Dialysis—Diffusion across Membranes

Some of the most interesting examples of diffusion occur through barriers that affect the rates of diffusion. For example, when you soak a swollen ankle in Epsom salt, water diffuses through your skin. Many substances regularly move through cell membranes; oxygen moves in, carbon dioxide moves out, nutrients go in, and wastes go out, for example. Because membranes are thin structures (typically 6.5×10^{-9} to 10×10^{-9} m across) diffusion rates through them can be high. Diffusion through membranes is an important method of transport.

Membranes are generally selectively permeable, or **semipermeable**. (See [\[link\]](#).) One type of semipermeable membrane has small pores that allow only small molecules to pass through. In other types of membranes, the molecules may actually dissolve in the membrane or react with molecules

in the membrane while moving across. Membrane function, in fact, is the subject of much current research, involving not only physiology but also chemistry and physics.

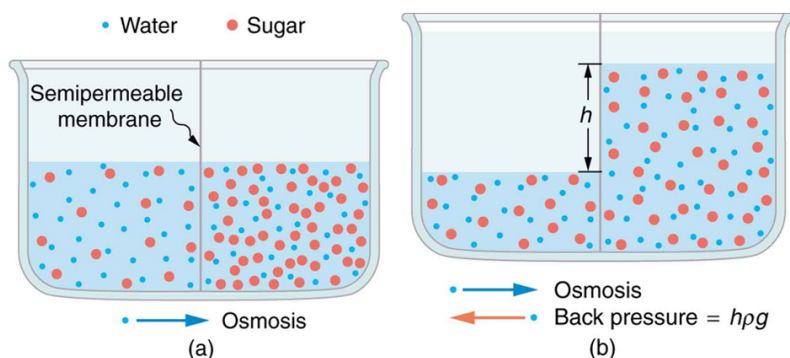


(a) A semipermeable membrane with small pores that allow only small molecules to pass through. (b) Certain molecules dissolve in this membrane and diffuse across it.

Osmosis is the transport of water through a semipermeable membrane from a region of high concentration to a region of low concentration. Osmosis is driven by the imbalance in water concentration. For example, water is more concentrated in your body than in Epsom salt. When you soak a swollen ankle in Epsom salt, the water moves out of your body into the lower-concentration region in the salt. Similarly, **dialysis** is the transport of any other molecule through a semipermeable membrane due to its concentration difference. Both osmosis and dialysis are used by the kidneys to cleanse the blood.

Osmosis can create a substantial pressure. Consider what happens if osmosis continues for some time, as illustrated in [\[link\]](#). Water moves by osmosis from the left into the region on the right, where it is less

concentrated, causing the solution on the right to rise. This movement will continue until the pressure ρgh created by the extra height of fluid on the right is large enough to stop further osmosis. This pressure is called a *back pressure*. The back pressure ρgh that stops osmosis is also called the **relative osmotic pressure** if neither solution is pure water, and it is called the **osmotic pressure** if one solution is pure water. Osmotic pressure can be large, depending on the size of the concentration difference. For example, if pure water and sea water are separated by a semipermeable membrane that passes no salt, osmotic pressure will be 25.9 atm. This value means that water will diffuse through the membrane until the salt water surface rises 268 m above the pure-water surface! One example of pressure created by osmosis is turgor in plants (many wilt when too dry). Turgor describes the condition of a plant in which the fluid in a cell exerts a pressure against the cell wall. This pressure gives the plant support. Dialysis can similarly cause substantial pressures.



- (a) Two sugar-water solutions of different concentrations, separated by a semipermeable membrane that passes water but not sugar. Osmosis will be to the right, since water is less concentrated there. (b) The fluid level rises until the back pressure ρgh equals the relative osmotic pressure; then, the net transfer of water is zero.

Reverse osmosis and **reverse dialysis** (also called filtration) are processes that occur when back pressure is sufficient to reverse the normal direction of substances through membranes. Back pressure can be created naturally as on the right side of [\[link\]](#). (A piston can also create this pressure.) Reverse osmosis can be used to desalinate water by simply forcing it through a membrane that will not pass salt. Similarly, reverse dialysis can be used to filter out any substance that a given membrane will not pass.

One further example of the movement of substances through membranes deserves mention. We sometimes find that substances pass in the direction opposite to what we expect. Cypress tree roots, for example, extract pure water from salt water, although osmosis would move it in the opposite direction. This is not reverse osmosis, because there is no back pressure to cause it. What is happening is called **active transport**, a process in which a living membrane expends energy to move substances across it. Many living membranes move water and other substances by active transport. The kidneys, for example, not only use osmosis and dialysis—they also employ significant active transport to move substances into and out of blood. In fact, it is estimated that at least 25% of the body's energy is expended on active transport of substances at the cellular level. The study of active transport carries us into the realms of microbiology, biophysics, and biochemistry and it is a fascinating application of the laws of nature to living structures.

Section Summary

- Diffusion is the movement of substances due to random thermal molecular motion.
- The average distance x_{rms} a molecule travels by diffusion in a given amount of time is given by

Equation:

$$x_{\text{rms}} = \sqrt{2Dt},$$

where D is the diffusion constant, representative values of which are found in [\[link\]](#).

- Osmosis is the transport of water through a semipermeable membrane from a region of high concentration to a region of low concentration.
- Dialysis is the transport of any other molecule through a semipermeable membrane due to its concentration difference.
- Both processes can be reversed by back pressure.
- Active transport is a process in which a living membrane expends energy to move substances across it.

Conceptual Questions

Exercise:

Problem:

Why would you expect the rate of diffusion to increase with temperature? Can you give an example, such as the fact that you can dissolve sugar more rapidly in hot water?

Exercise:

Problem: How are osmosis and dialysis similar? How do they differ?

Problem Exercises

Exercise:

Problem:

You can smell perfume very shortly after opening the bottle. To show that it is not reaching your nose by diffusion, calculate the average distance a perfume molecule moves in one second in air, given its diffusion constant D to be $1.00 \times 10^{-6} \text{ m}^2/\text{s}$.

Solution:

$$1.41 \times 10^{-3} \text{ m}$$

Exercise:

Problem:

What is the ratio of the average distances that oxygen will diffuse in a given time in air and water? Why is this distance less in water (equivalently, why is D less in water)?

Exercise:**Problem:**

Oxygen reaches the veinless cornea of the eye by diffusing through its tear layer, which is 0.500-mm thick. How long does it take the average oxygen molecule to do this?

Solution:

$$1.3 \times 10^2 \text{ s}$$

Exercise:**Problem:**

(a) Find the average time required for an oxygen molecule to diffuse through a 0.200-mm-thick tear layer on the cornea. (b) How much time is required to diffuse 0.500 cm^3 of oxygen to the cornea if its surface area is 1.00 cm^2 ?

Exercise:**Problem:**

Suppose hydrogen and oxygen are diffusing through air. A small amount of each is released simultaneously. How much time passes before the hydrogen is 1.00 s ahead of the oxygen? Such differences in arrival times are used as an analytical tool in gas chromatography.

Solution:

$$0.391 \text{ s}$$

Glossary

diffusion

the movement of substances due to random thermal molecular motion

semipermeable

a type of membrane that allows only certain small molecules to pass through

osmosis

the transport of water through a semipermeable membrane from a region of high concentration to one of low concentration

dialysis

the transport of any molecule other than water through a semipermeable membrane from a region of high concentration to one of low concentration

relative osmotic pressure

the back pressure which stops the osmotic process if neither solution is pure water

osmotic pressure

the back pressure which stops the osmotic process if one solution is pure water

reverse osmosis

the process that occurs when back pressure is sufficient to reverse the normal direction of osmosis through membranes

reverse dialysis

the process that occurs when back pressure is sufficient to reverse the normal direction of dialysis through membranes

active transport

the process in which a living membrane expends energy to move substances across

Introduction to Temperature, Kinetic Theory, and the Gas Laws

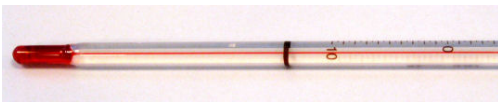
class="introduction"

The welder's
gloves and
helmet
protect him
from the
electric arc
that transfers
enough
thermal
energy to
melt the rod,
spray sparks,
and burn the
retina of an
unprotected
eye. The
thermal
energy can
be felt on
exposed skin
a few meters
away, and its
light can be
seen for
kilometers.
(credit:
Kevin S.
O'Brien/U.S
. Navy)



Heat is something familiar to each of us. We feel the warmth of the summer Sun, the chill of a clear summer night, the heat of coffee after a winter stroll, and the cooling effect of our sweat. Heat transfer is maintained by temperature differences. Manifestations of **heat transfer**—the movement of heat energy from one place or material to another—are apparent throughout the universe. Heat from beneath Earth's surface is brought to the surface in flows of incandescent lava. The Sun warms Earth's surface and is the source of much of the energy we find on it. Rising levels of atmospheric carbon dioxide threaten to trap more of the Sun's energy, perhaps fundamentally altering the ecosphere. In space, supernovas explode, briefly radiating more heat than an entire galaxy does.

What is heat? How do we define it? How is it related to temperature? What are heat's effects? How is it related to other forms of energy and to work? We will find that, in spite of the richness of the phenomena, there is a small set of underlying physical principles that unite the subjects and tie them to other fields.



In a typical thermometer like this one, the alcohol, with a red dye, expands

more rapidly than the glass containing it. When the thermometer's temperature increases, the liquid from the bulb is forced into the narrow tube, producing a large change in the length of the column for a small change in temperature.

(credit: Chemical Engineer, Wikimedia Commons)

Temperature

- Define temperature.
- Convert temperatures between the Celsius, Fahrenheit, and Kelvin scales.
- Define thermal equilibrium.
- State the zeroth law of thermodynamics.

The concept of temperature has evolved from the common concepts of hot and cold. Human perception of what feels hot or cold is a relative one. For example, if you place one hand in hot water and the other in cold water, and then place both hands in tepid water, the tepid water will feel cool to the hand that was in hot water, and warm to the one that was in cold water. The scientific definition of temperature is less ambiguous than your senses of hot and cold. **Temperature** is operationally defined to be what we measure with a thermometer. (Many physical quantities are defined solely in terms of how they are measured. We shall see later how temperature is related to the kinetic energies of atoms and molecules, a more physical explanation.) Two accurate thermometers, one placed in hot water and the other in cold water, will show the hot water to have a higher temperature. If they are then placed in the tepid water, both will give identical readings (within measurement uncertainties). In this section, we discuss temperature, its measurement by thermometers, and its relationship to thermal equilibrium. Again, temperature is the quantity measured by a thermometer.

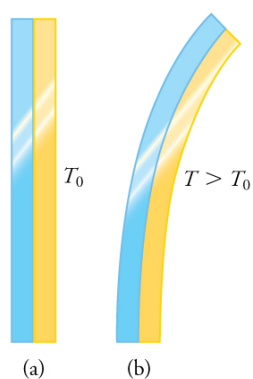
Note:

Misconception Alert: Human Perception vs. Reality

On a cold winter morning, the wood on a porch feels warmer than the metal of your bike. The wood and bicycle are in thermal equilibrium with the outside air, and are thus the same temperature. They *feel* different because of the difference in the way that they conduct heat away from your skin. The metal conducts heat away from your body faster than the wood does (see more about conductivity in [Conduction](#)). This is just one example demonstrating that the human sense of hot and cold is not determined by temperature alone.

Another factor that affects our perception of temperature is humidity. Most people feel much hotter on hot, humid days than on hot, dry days. This is because on humid days, sweat does not evaporate from the skin as efficiently as it does on dry days. It is the evaporation of sweat (or water from a sprinkler or pool) that cools us off.

Any physical property that depends on temperature, and whose response to temperature is reproducible, can be used as the basis of a thermometer. Because many physical properties depend on temperature, the variety of thermometers is remarkable. For example, volume increases with temperature for most substances. This property is the basis for the common alcohol thermometer, the old mercury thermometer, and the bimetallic strip ([link](#)). Other properties used to measure temperature include electrical resistance and color, as shown in [link](#), and the emission of infrared radiation, as shown in [link](#).



The curvature of a bimetallic strip depends on

temperature.
(a) The strip is straight at the starting temperature, where its two components have the same length.

(b) At a higher temperature, this strip bends to the right, because the metal on the left has expanded more than the metal on the right.



Each of the six squares on this plastic (liquid crystal)

thermometer contains a film of a different heat-sensitive liquid crystal material. Below 95°F, all six squares are black.

When the plastic thermometer is exposed to temperature that increases to 95°F, the first liquid crystal square changes color. When the temperature increases above 96.8°F the second liquid crystal square also changes color, and so forth. (credit: Arkrishna, Wikimedia Commons)



Fireman Jason Ormand uses a pyrometer to check the temperature of an aircraft carrier's ventilation system. Infrared radiation (whose emission varies with temperature)

from the vent is measured and a temperature readout is quickly produced.

Infrared measurements are also frequently used as a measure of body temperature.

These modern thermometers, placed in the ear canal, are more accurate than alcohol thermometers placed under the tongue or in the armpit. (credit: Lamel J. Hinton/U.S. Navy)

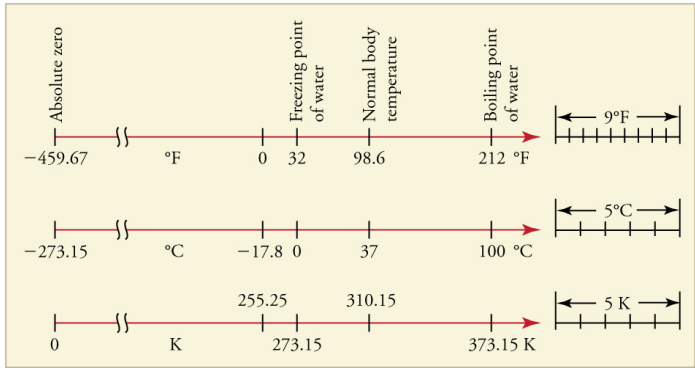
Temperature Scales

Thermometers are used to measure temperature according to well-defined scales of measurement, which use pre-defined reference points to help compare quantities. The three most common temperature scales are the Fahrenheit, Celsius, and Kelvin scales. A temperature scale can be created by identifying two easily reproducible temperatures. The freezing and boiling temperatures of water at standard atmospheric pressure are commonly used.

The **Celsius** scale (which replaced the slightly different *centigrade* scale) has the freezing point of water at 0°C and the boiling point at 100°C . Its unit is the **degree Celsius** ($^{\circ}\text{C}$). On the **Fahrenheit** scale (still the most frequently used in the United States), the freezing point of water is at 32°F and the boiling point is at 212°F . The unit of temperature on this scale is the **degree Fahrenheit** ($^{\circ}\text{F}$). Note that a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Only 100 Celsius degrees

span the same range as 180 Fahrenheit degrees, thus one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale $180/100 = 9/5$.

The **Kelvin** scale is the temperature scale that is commonly used in science. It is an *absolute temperature* scale defined to have 0 K at the lowest possible temperature, called **absolute zero**. The official temperature unit on this scale is the *kelvin*, which is abbreviated K, and is not accompanied by a degree sign. The freezing and boiling points of water are 273.15 K and 373.15 K, respectively. Thus, the magnitude of temperature differences is the same in units of kelvins and degrees Celsius. Unlike other temperature scales, the Kelvin scale is an absolute scale. It is used extensively in scientific work because a number of physical quantities, such as the volume of an ideal gas, are directly related to absolute temperature. The kelvin is the SI unit used in scientific work.



Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The relationships between the three common temperature scales is shown in [\[link\]](#). Temperatures on these scales can be converted using the equations in [\[link\]](#).

To convert from ...	Use this equation ...	Also written as ...
---------------------	-----------------------	---------------------

To convert from ...	Use this equation ...	Also written as ...
Celsius to Fahrenheit	$T(^{\circ}\text{F}) = \frac{9}{5}T(^{\circ}\text{C}) + 32$	$T_{\text{F}} = \frac{9}{5}T_{\text{C}} + 32$
Fahrenheit to Celsius	$T(^{\circ}\text{C}) = \frac{5}{9}(T(^{\circ}\text{F}) - 32)$	$T_{\text{C}} = \frac{5}{9}(T_{\text{F}} - 32)$
Celsius to Kelvin	$T(\text{K}) = T(^{\circ}\text{C}) + 273.15$	$T_{\text{K}} = T_{\text{C}} + 273.15$
Kelvin to Celsius	$T(^{\circ}\text{C}) = T(\text{K}) - 273.15$	$T_{\text{C}} = T_{\text{K}} - 273.15$
Fahrenheit to Kelvin	$T(\text{K}) = \frac{5}{9}(T(^{\circ}\text{F}) - 32) + 273.15$	$T_{\text{K}} = \frac{5}{9}(T_{\text{F}} - 32) + 273.15$
Kelvin to Fahrenheit	$T(^{\circ}\text{F}) = \frac{9}{5}(T(\text{K}) - 273.15) + 32$	$T_{\text{F}} = \frac{9}{5}(T_{\text{K}} - 273.15) + 32$

Temperature Conversions

Notice that the conversions between Fahrenheit and Kelvin look quite complicated. In fact, they are simple combinations of the conversions between Fahrenheit and Celsius, and the conversions between Celsius and Kelvin.

Example:

Converting between Temperature Scales: Room Temperature

“Room temperature” is generally defined to be 25°C . (a) What is room temperature in $^{\circ}\text{F}$?

(b) What is it in K?

Strategy

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from °C to °F, use the equation

Equation:

$$T_{\text{°F}} = \frac{9}{5}T_{\text{°C}} + 32.$$

2. Plug the known value into the equation and solve:

Equation:

$$T_{\text{°F}} = \frac{9}{5}25^{\circ}\text{C} + 32 = 77^{\circ}\text{F}.$$

Solution for (b)

1. Choose the right equation. To convert from °C to K, use the equation

Equation:

$$T_{\text{K}} = T_{\text{°C}} + 273.15.$$

2. Plug the known value into the equation and solve:

Equation:

$$T_{\text{K}} = 25^{\circ}\text{C} + 273.15 = 298 \text{ K}.$$

Example:**Converting between Temperature Scales: the Reaumur Scale**

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is 0°R and the boiling temperature is 80°R. If “room temperature” is 25°C on the Celsius scale, what is it on the Reaumur scale?

Strategy

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is 80°R. On the Celsius scale it is 100°C. Therefore $100^{\circ}\text{C} = 80^{\circ}\text{R}$. Both scales start at 0° for freezing, so we can derive a simple formula to convert between temperatures on the two scales.

Solution

1. Derive a formula to convert from one scale to the other:

Equation:

$$T_{\text{°R}} = \frac{0.8^{\circ}\text{R}}{^{\circ}\text{C}} \times T_{\text{°C}}.$$

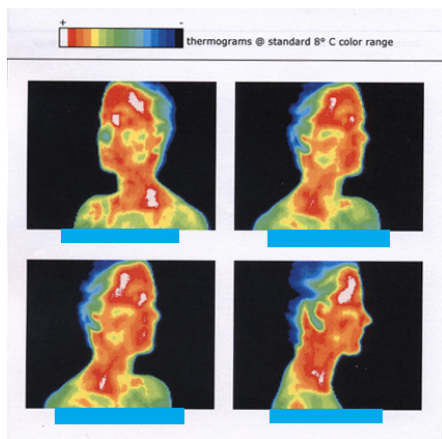
2. Plug the known value into the equation and solve:

Equation:

$$T_{\text{R}} = \frac{0.8^{\circ}\text{R}}{^{\circ}\text{C}} \times 25^{\circ}\text{C} = 20^{\circ}\text{R}.$$

Temperature Ranges in the Universe

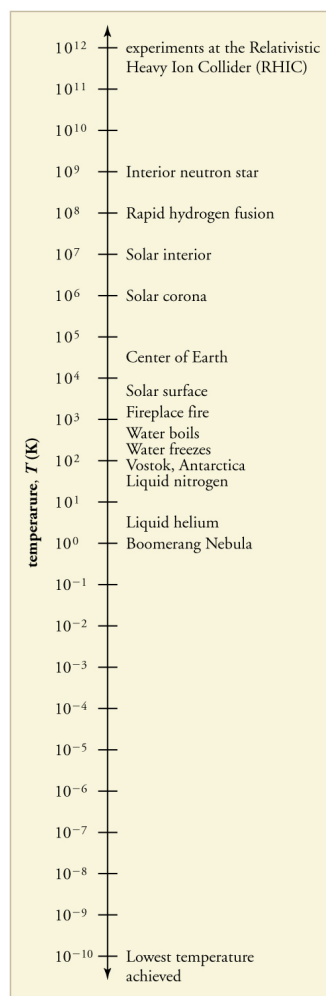
[\[link\]](#) shows the wide range of temperatures found in the universe. Human beings have been known to survive with body temperatures within a small range, from 24°C to 44°C (75°F to 111°F). The average normal body temperature is usually given as 37.0°C (98.6°F), and variations in this temperature can indicate a medical condition: a fever, an infection, a tumor, or circulatory problems (see [\[link\]](#)).



This image of radiation from a person's body (an infrared thermograph) shows the location of temperature abnormalities in the upper body. Dark blue corresponds to cold areas and red to white corresponds to hot areas. An elevated temperature might be an indication of malignant tissue (a cancerous tumor in the breast, for example), while a depressed temperature

might be due to a decline in blood flow from a clot. In this case, the abnormalities are caused by a condition called hyperhidrosis.
(credit: Porcelina81, Wikimedia Commons)

The lowest temperatures ever recorded have been measured during laboratory experiments: 4.5×10^{-10} K at the Massachusetts Institute of Technology (USA), and 1.0×10^{-10} K at Helsinki University of Technology (Finland). In comparison, the coldest recorded place on Earth's surface is Vostok, Antarctica at 183 K (-89°C), and the coldest place (outside the lab) known in the universe is the Boomerang Nebula, with a temperature of 1 K.

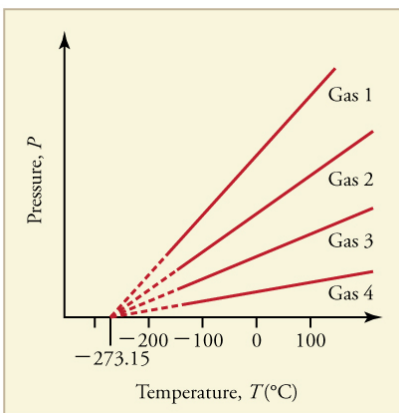


Each increment on this logarithmic scale indicates an increase by a factor of ten, and thus illustrates the tremendous range of temperatures in nature. Note that zero on a logarithmic scale would occur off the bottom of the page at infinity.

Note:

Making Connections: Absolute Zero

What is absolute zero? Absolute zero is the temperature at which all molecular motion has ceased. The concept of absolute zero arises from the behavior of gases. [\[link\]](#) shows how the pressure of gases at a constant volume decreases as temperature decreases. Various scientists have noted that the pressures of gases extrapolate to zero at the same temperature, -273.15°C . This extrapolation implies that there is a lowest temperature. This temperature is called *absolute zero*. Today we know that most gases first liquefy and then freeze, and it is not actually possible to reach absolute zero. The numerical value of absolute zero temperature is -273.15°C or 0 K.



Graph of pressure versus temperature for various

gases kept at a constant volume. Note that all of the graphs extrapolate to zero pressure at the same temperature.

Thermal Equilibrium and the Zeroth Law of Thermodynamics

Thermometers actually take their *own* temperature, not the temperature of the object they are measuring. This raises the question of how we can be certain that a thermometer measures the temperature of the object with which it is in contact. It is based on the fact that any two systems placed in *thermal contact* (meaning heat transfer can occur between them) will reach the same temperature. That is, heat will flow from the hotter object to the cooler one until they have exactly the same temperature. The objects are then in **thermal equilibrium**, and no further changes will occur. The systems interact and change because their temperatures differ, and the changes stop once their temperatures are the same. Thus, if enough time is allowed for this transfer of heat to run its course, the temperature a thermometer registers *does* represent the system with which it is in thermal equilibrium. Thermal equilibrium is established when two bodies are in contact with each other and can freely exchange energy.

Furthermore, experimentation has shown that if two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system C, then A is also in thermal equilibrium with C. This conclusion may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

Note:

The Zeroth Law of Thermodynamics

If two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

This law was postulated in the 1930s, after the first and second laws of thermodynamics had been developed and named. It is called the *zeroth law* because it comes logically before the first and second laws (discussed in [Thermodynamics](#)). An example of this law in action is seen in babies in incubators: babies in incubators normally have very few clothes on, so to an observer they look as if they may not be warm enough. However, the temperature of the air, the cot, and the baby is the same, because they are in thermal equilibrium, which is accomplished by maintaining air temperature to keep the baby comfortable.

Exercise:
Check Your Understanding

Problem: Does the temperature of a body depend on its size?

Solution:

No, the system can be divided into smaller parts each of which is at the same temperature. We say that the temperature is an *intensive* quantity. Intensive quantities are independent of size.

Section Summary

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted to temperatures on another scale using the following equations:

Equation:

$$T_{\text{°F}} = \frac{9}{5}T_{\text{°C}} + 32$$

Equation:

$$T_{\text{°C}} = \frac{5}{9}(T_{\text{°F}} - 32)$$

Equation:

$$T_{\text{K}} = T_{\text{°C}} + 273.15$$

Equation:

$$T_{\text{°C}} = T_{\text{K}} - 273.15$$

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

Conceptual Questions

Exercise:

Problem: What does it mean to say that two systems are in thermal equilibrium?

Exercise:

Problem:

Give an example of a physical property that varies with temperature and describe how it is used to measure temperature.

Exercise:

Problem:

When a cold alcohol thermometer is placed in a hot liquid, the column of alcohol goes *down* slightly before going up. Explain why.

Exercise:

Problem:

If you add boiling water to a cup at room temperature, what would you expect the final equilibrium temperature of the unit to be? You will need to include the surroundings as part of the system. Consider the zeroth law of thermodynamics.

Problems & Exercises

Exercise:

Problem: What is the Fahrenheit temperature of a person with a 39.0°C fever?

Solution:

102°F

Exercise:

Problem:

Frost damage to most plants occurs at temperatures of 28.0°F or lower. What is this temperature on the Kelvin scale?

Exercise:

Problem:

To conserve energy, room temperatures are kept at 68.0°F in the winter and 78.0°F in the summer. What are these temperatures on the Celsius scale?

Solution:

20.0°C and 25.6°C

Exercise:**Problem:**

A tungsten light bulb filament may operate at 2900 K . What is its Fahrenheit temperature? What is this on the Celsius scale?

Exercise:**Problem:**

The surface temperature of the Sun is about 5750 K . What is this temperature on the Fahrenheit scale?

Solution:

9890°F

Exercise:**Problem:**

One of the hottest temperatures ever recorded on the surface of Earth was 134°F in Death Valley, CA. What is this temperature in Celsius degrees? What is this temperature in Kelvin?

Exercise:**Problem:**

(a) Suppose a cold front blows into your locale and drops the temperature by 40.0 Fahrenheit degrees. How many degrees Celsius does the temperature decrease when there is a 40.0°F decrease in temperature? (b) Show that any change in temperature in Fahrenheit degrees is nine-fifths the change in Celsius degrees.

Solution:

(a) 22.2°C

$$\begin{aligned}
 \Delta T(^{\circ}\text{F}) &= T_2(^{\circ}\text{F}) - T_1(^{\circ}\text{F}) \\
 \text{(b)} \quad &= \frac{9}{5}T_2(^{\circ}\text{C}) + 32.0^{\circ} - \left(\frac{9}{5}T_1(^{\circ}\text{C}) + 32.0^{\circ}\right) \\
 &= \frac{9}{5}(T_2(^{\circ}\text{C}) - T_1(^{\circ}\text{C})) = \frac{9}{5}\Delta T(^{\circ}\text{C})
 \end{aligned}$$

Exercise:

Problem:

(a) At what temperature do the Fahrenheit and Celsius scales have the same numerical value? (b) At what temperature do the Fahrenheit and Kelvin scales have the same numerical value?

Glossary

temperature

the quantity measured by a thermometer

Celsius scale

temperature scale in which the freezing point of water is 0°C and the boiling point of water is 100°C

degree Celsius

unit on the Celsius temperature scale

Fahrenheit scale

temperature scale in which the freezing point of water is 32°F and the boiling point of water is 212°F

degree Fahrenheit

unit on the Fahrenheit temperature scale

Kelvin scale

temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

absolute zero

the lowest possible temperature; the temperature at which all molecular motion ceases

thermal equilibrium

the condition in which heat no longer flows between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics

law that states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

Thermal Expansion of Solids and Liquids

- Define and describe thermal expansion.
- Calculate the linear expansion of an object given its initial length, change in temperature, and coefficient of linear expansion.
- Calculate the volume expansion of an object given its initial volume, change in temperature, and coefficient of volume expansion.
- Calculate thermal stress on an object given its original volume, temperature change, volume change, and bulk modulus.



Thermal expansion joints like these in the Auckland Harbour Bridge in New Zealand allow bridges to change length without buckling. (credit: Ingolfson, Wikimedia Commons)

The expansion of alcohol in a thermometer is one of many commonly encountered examples of **thermal expansion**, the change in size or volume of a given mass with temperature. Hot air rises because its volume increases, which causes the hot air's density to be smaller than the density of surrounding air, causing a buoyant (upward) force on the hot air. The same happens in all liquids and gases, driving natural heat transfer upwards in homes, oceans, and weather systems. Solids also undergo thermal expansion. Railroad tracks and bridges, for example, have expansion joints to allow them to freely expand and contract with temperature changes.

What are the basic properties of thermal expansion? First, thermal expansion is clearly related to temperature change. The greater the temperature change, the more a bimetallic strip will bend. Second, it depends on the material. In a thermometer, for example, the expansion of alcohol is much greater than the expansion of the glass containing it.

What is the underlying cause of thermal expansion? As is discussed in [Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature](#), an increase in temperature implies an increase in the kinetic energy of the individual atoms. In a solid, unlike in a gas, the atoms or molecules are closely packed together, but their kinetic energy (in the form of small, rapid vibrations) pushes neighboring atoms or molecules apart from each other. This neighbor-to-neighbor pushing results in a slightly greater distance, on average, between neighbors, and adds up to a larger size for the whole body. For most substances under ordinary conditions, there is no preferred direction, and an increase in temperature will increase the solid's size by a certain fraction in each dimension.

Note:

Linear Thermal Expansion—Thermal Expansion in One Dimension

The change in length ΔL is proportional to length L . The dependence of thermal expansion on temperature, substance, and length is summarized in the equation

Equation:

$$\Delta L = \alpha L \Delta T,$$

where ΔL is the change in length L , ΔT is the change in temperature, and α is the **coefficient of linear expansion**, which varies slightly with temperature.

[\[link\]](#) lists representative values of the coefficient of linear expansion, which may have units of $1/^{\circ}\text{C}$ or $1/\text{K}$. Because the size of a kelvin and a degree Celsius are the same, both α and ΔT can be expressed in units of kelvins or degrees Celsius. The equation $\Delta L = \alpha L \Delta T$ is accurate for small changes in temperature and can be used for large changes in temperature if an average value of α is used.

Material	Coefficient of linear expansion $\alpha(1/^{\circ}\text{C})$	Coefficient of volume expansion $\beta(1/^{\circ}\text{C})$
Solids		
Aluminum	25×10^{-6}	75×10^{-6}
Brass	19×10^{-6}	56×10^{-6}
Copper	17×10^{-6}	51×10^{-6}

Material	Coefficient of linear expansion $\alpha(1/^{\circ}\text{C})$	Coefficient of volume expansion $\beta(1/^{\circ}\text{C})$
Gold	14×10^{-6}	42×10^{-6}
Iron or Steel	12×10^{-6}	35×10^{-6}
Invar (Nickel-iron alloy)	0.9×10^{-6}	2.7×10^{-6}
Lead	29×10^{-6}	87×10^{-6}
Silver	18×10^{-6}	54×10^{-6}
Glass (ordinary)	9×10^{-6}	27×10^{-6}
Glass (Pyrex®)	3×10^{-6}	9×10^{-6}
Quartz	0.4×10^{-6}	1×10^{-6}

Material	Coefficient of linear expansion	Coefficient of volume expansion
	$\alpha(1/^{\circ}\text{C})$	$\beta(1/^{\circ}\text{C})$
Concrete, Brick	$\sim 12 \times 10^{-6}$	$\sim 36 \times 10^{-6}$
Marble (average)	7×10^{-6}	2.1×10^{-5}
Liquids		
Ether		1650×10^{-6}
Ethyl alcohol		1100×10^{-6}
Petrol		950×10^{-6}
Glycerin		500×10^{-6}
Mercury		180×10^{-6}

Material	Coefficient of linear expansion $\alpha(1/^{\circ}\text{C})$	Coefficient of volume expansion $\beta(1/^{\circ}\text{C})$
Water		210×10^{-6}
Gases		
Air and most other gases at atmospheric pressure		3400×10^{-6}

Thermal Expansion Coefficients at 20°C[\[footnote\]](#)

Values for liquids and gases are approximate.

Example:

Calculating Linear Thermal Expansion: The Golden Gate Bridge

The main span of San Francisco's Golden Gate Bridge is 1275 m long at its coldest. The bridge is exposed to temperatures ranging from -15°C to 40°C . What is its change in length between these temperatures? Assume that the bridge is made entirely of steel.

Strategy

Use the equation for linear thermal expansion $\Delta L = \alpha L \Delta T$ to calculate the change in length, ΔL . Use the coefficient of linear expansion, α , for steel from [\[link\]](#), and note that the change in temperature, ΔT , is 55°C .

Solution

Plug all of the known values into the equation to solve for ΔL .

Equation:

$$\Delta L = \alpha L \Delta T = \left(\frac{12 \times 10^{-6}}{^{\circ}\text{C}} \right) (1275 \text{ m})(55^{\circ}\text{C}) = 0.84 \text{ m}.$$

Discussion

Although not large compared with the length of the bridge, this change in length is observable. It is generally spread over many expansion joints so that the expansion at each joint is small.

Thermal Expansion in Two and Three Dimensions

Objects expand in all dimensions, as illustrated in [\[link\]](#). That is, their areas and volumes, as well as their lengths, increase with temperature. Holes also get larger with temperature. If you cut a hole in a metal plate, the remaining material will expand exactly as it would if the plug was still in place. The plug would get bigger, and so the hole must get bigger too. (Think of the ring of neighboring atoms or molecules on the wall of the hole as pushing each other farther apart as temperature increases. Obviously, the ring of neighbors must get slightly larger, so the hole gets slightly larger).

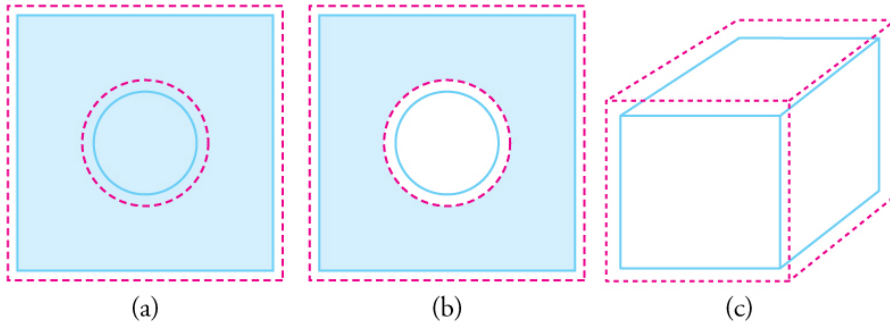
Note:**Thermal Expansion in Two Dimensions**

For small temperature changes, the change in area ΔA is given by

Equation:

$$\Delta A = 2\alpha A \Delta T,$$

where ΔA is the change in area A , ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.



In general, objects expand in all directions as temperature increases. In these drawings, the original boundaries of the objects are shown with solid lines, and the expanded boundaries with dashed lines. (a) Area increases because both length and width increase. The area of a circular plug also increases. (b) If the plug is removed, the hole it leaves becomes larger with increasing temperature, just as if the expanding plug were still in place. (c) Volume also increases, because all three dimensions increase.

Note:

Thermal Expansion in Three Dimensions

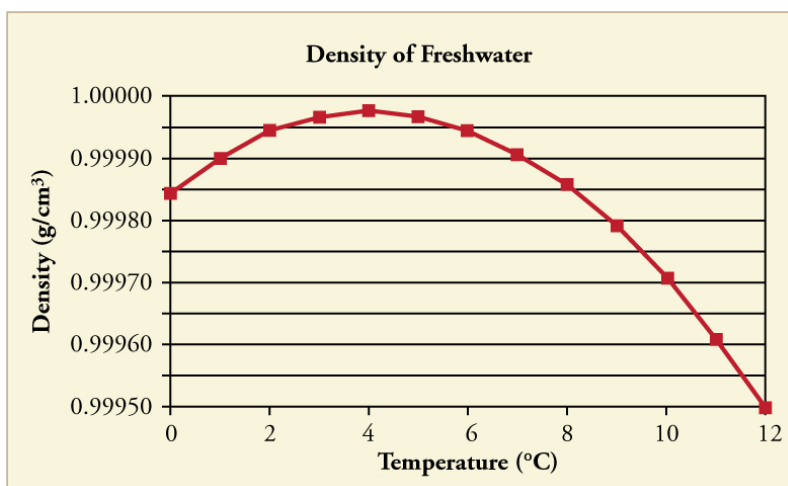
The change in volume ΔV is very nearly $\Delta V = 3\alpha V \Delta T$. This equation is usually written as

Equation:

$$\Delta V = \beta V \Delta T,$$

where β is the **coefficient of volume expansion** and $\beta \approx 3\alpha$. Note that the values of β in [\[link\]](#) are almost exactly equal to 3α .

In general, objects will expand with increasing temperature. Water is the most important exception to this rule. Water expands with increasing temperature (its density *decreases*) when it is at temperatures greater than 4°C (40°F). However, it expands with *decreasing* temperature when it is between $+4^{\circ}\text{C}$ and 0°C (40°F to 32°F). Water is densest at $+4^{\circ}\text{C}$. (See [\[link\]](#).) Perhaps the most striking effect of this phenomenon is the freezing of water in a pond. When water near the surface cools down to 4°C it is denser than the remaining water and thus will sink to the bottom. This “turnover” results in a layer of warmer water near the surface, which is then cooled. Eventually the pond has a uniform temperature of 4°C . If the temperature in the surface layer drops below 4°C , the water is less dense than the water below, and thus stays near the top. As a result, the pond surface can completely freeze over. The ice on top of liquid water provides an insulating layer from winter’s harsh exterior air temperatures. Fish and other aquatic life can survive in 4°C water beneath ice, due to this unusual characteristic of water. It also produces circulation of water in the pond that is necessary for a healthy ecosystem of the body of water.



The density of water as a function of temperature. Note that the thermal expansion is actually very small. The maximum density at $+4^{\circ}\text{C}$ is only 0.0075% greater than the density at 2°C , and 0.012% greater than that at 0°C .

Note:**Making Connections: Real-World Connections—Filling the Tank**

Differences in the thermal expansion of materials can lead to interesting effects at the gas station. One example is the dripping of gasoline from a freshly filled tank on a hot day. Gasoline starts out at the temperature of the ground under the gas station, which is cooler than the air temperature above. The gasoline cools the steel tank when it is filled. Both gasoline and steel tank expand as they warm to air temperature, but gasoline expands much more than steel, and so it may overflow.

This difference in expansion can also cause problems when interpreting the gasoline gauge. The actual amount (mass) of gasoline left in the tank when the gauge hits “empty” is a lot less in the summer than in the winter. The gasoline has the same volume as it does in the winter when the “add fuel” light goes on, but because the gasoline has expanded, there is less mass. If you are used to getting another 40 miles on “empty” in the winter, beware—you will probably run out much more quickly in the summer.



Because the gas expands more than the gas tank with increasing temperature, you can't drive as many miles on “empty” in the summer as you can in the winter.

(credit: Hector Alejandro,
Flickr)

Example:

Calculating Thermal Expansion: Gas vs. Gas Tank

Suppose your 60.0-L (15.9-gal) steel gasoline tank is full of gas, so both the tank and the gasoline have a temperature of 15.0°C. How much gasoline has spilled by the time they warm to 35.0°C?

Strategy

The tank and gasoline increase in volume, but the gasoline increases more, so the amount spilled is the difference in their volume changes. (The gasoline tank can be treated as solid steel.) We can use the equation for volume expansion to calculate the change in volume of the gasoline and of the tank.

Solution

1. Use the equation for volume expansion to calculate the increase in volume of the steel tank:

Equation:

$$\Delta V_s = \beta_s V_s \Delta T.$$

2. The increase in volume of the gasoline is given by this equation:

Equation:

$$\Delta V_{\text{gas}} = \beta_{\text{gas}} V_{\text{gas}} \Delta T.$$

3. Find the difference in volume to determine the amount spilled as

Equation:

$$V_{\text{spill}} = \Delta V_{\text{gas}} - \Delta V_s.$$

Alternatively, we can combine these three equations into a single equation. (Note that the original volumes are equal.)

Equation:

$$\begin{aligned}
 V_{\text{spill}} &= (\beta_{\text{gas}} - \beta_{\text{s}})V\Delta T \\
 &= [(950 - 35) \times 10^{-6} / ^\circ\text{C}] (60.0 \text{ L})(20.0^\circ\text{C}) \\
 &= 1.10 \text{ L.}
 \end{aligned}$$

Discussion

This amount is significant, particularly for a 60.0-L tank. The effect is so striking because the gasoline and steel expand quickly. The rate of change in thermal properties is discussed in [Heat and Heat Transfer Methods](#).

If you try to cap the tank tightly to prevent overflow, you will find that it leaks anyway, either around the cap or by bursting the tank. Tightly constricting the expanding gas is equivalent to compressing it, and both liquids and solids resist being compressed with extremely large forces. To avoid rupturing rigid containers, these containers have air gaps, which allow them to expand and contract without stressing them.

Thermal Stress

Thermal stress is created by thermal expansion or contraction (see [Elasticity: Stress and Strain](#) for a discussion of stress and strain). Thermal stress can be destructive, such as when expanding gasoline ruptures a tank. It can also be useful, for example, when two parts are joined together by heating one in manufacturing, then slipping it over the other and allowing the combination to cool. Thermal stress can explain many phenomena, such as the weathering of rocks and pavement by the expansion of ice when it freezes.

Example:

Calculating Thermal Stress: Gas Pressure

What pressure would be created in the gasoline tank considered in [\[link\]](#), if the gasoline increases in temperature from 15.0°C to 35.0°C without being allowed to expand? Assume that the bulk modulus B for gasoline is $1.00 \times 10^9 \text{ N/m}^2$. (For more on bulk modulus, see [Elasticity: Stress and Strain](#).)

Strategy

To solve this problem, we must use the following equation, which relates a change in volume ΔV to pressure:

Equation:

$$\Delta V = \frac{1}{B} \frac{F}{A} V_0,$$

where F/A is pressure, V_0 is the original volume, and B is the bulk modulus of the material involved. We will use the amount spilled in [\[link\]](#) as the change in volume, ΔV .

Solution

1. Rearrange the equation for calculating pressure:

Equation:

$$P = \frac{F}{A} = \frac{\Delta V}{V_0} B.$$

2. Insert the known values. The bulk modulus for gasoline is $B = 1.00 \times 10^9 \text{ N/m}^2$. In the previous example, the change in volume $\Delta V = 1.10 \text{ L}$ is the amount that would spill. Here, $V_0 = 60.0 \text{ L}$ is the original volume of the gasoline. Substituting these values into the equation, we obtain

Equation:

$$P = \frac{1.10 \text{ L}}{60.0 \text{ L}} (1.00 \times 10^9 \text{ Pa}) = 1.83 \times 10^7 \text{ Pa}.$$

Discussion

This pressure is about 2500 lb/in^2 , *much* more than a gasoline tank can handle.

Forces and pressures created by thermal stress are typically as great as that in the example above. Railroad tracks and roadways can buckle on hot days if they lack sufficient expansion joints. (See [\[link\]](#).) Power lines sag more in the summer than in the winter, and will snap in cold weather if there is

insufficient slack. Cracks open and close in plaster walls as a house warms and cools. Glass cooking pans will crack if cooled rapidly or unevenly, because of differential contraction and the stresses it creates. (Pyrex® is less susceptible because of its small coefficient of thermal expansion.) Nuclear reactor pressure vessels are threatened by overly rapid cooling, and although none have failed, several have been cooled faster than considered desirable. Biological cells are ruptured when foods are frozen, detracting from their taste. Repeated thawing and freezing accentuate the damage. Even the oceans can be affected. A significant portion of the rise in sea level that is resulting from global warming is due to the thermal expansion of sea water.



Thermal stress contributes to the formation of potholes.
(credit: Editor5807, Wikimedia Commons)

Metal is regularly used in the human body for hip and knee implants. Most implants need to be replaced over time because, among other things, metal does not bond with bone. Researchers are trying to find better metal coatings that would allow metal-to-bone bonding. One challenge is to find a coating that has an expansion coefficient similar to that of metal. If the

expansion coefficients are too different, the thermal stresses during the manufacturing process lead to cracks at the coating-metal interface.

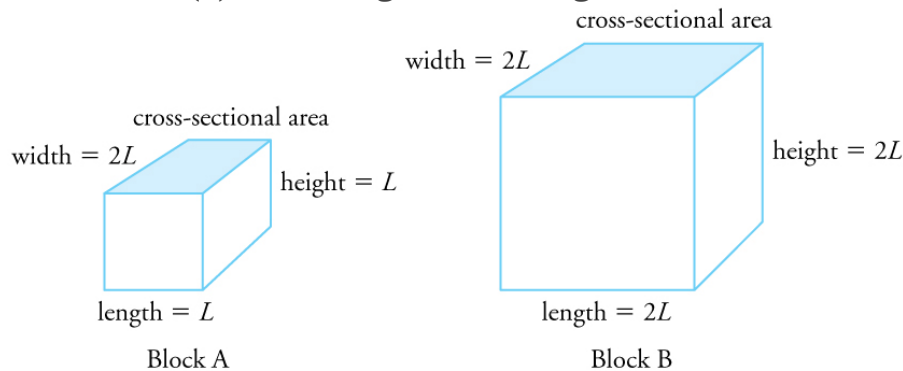
Another example of thermal stress is found in the mouth. Dental fillings can expand differently from tooth enamel. It can give pain when eating ice cream or having a hot drink. Cracks might occur in the filling. Metal fillings (gold, silver, etc.) are being replaced by composite fillings (porcelain), which have smaller coefficients of expansion, and are closer to those of teeth.

Exercise:

Check Your Understanding

Problem:

Two blocks, A and B, are made of the same material. Block A has dimensions $l \times w \times h = L \times 2L \times L$ and Block B has dimensions $2L \times 2L \times 2L$. If the temperature changes, what is (a) the change in the volume of the two blocks, (b) the change in the cross-sectional area $l \times w$, and (c) the change in the height h of the two blocks?



Solution:

(a) The change in volume is proportional to the original volume. Block A has a volume of $L \times 2L \times L = 2L^3$. Block B has a volume of $2L \times 2L \times 2L = 8L^3$, which is 4 times that of Block A. Thus the change in volume of Block B should be 4 times the change in volume of Block A.

(b) The change in area is proportional to the area. The cross-sectional area of Block A is $L \times 2L = 2L^2$, while that of Block B is

$2L \times 2L = 4L^2$. Because cross-sectional area of Block B is twice that of Block A, the change in the cross-sectional area of Block B is twice that of Block A.

(c) The change in height is proportional to the original height. Because the original height of Block B is twice that of A, the change in the height of Block B is twice that of Block A.

Section Summary

- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- Thermal expansion is large for gases, and relatively small, but not negligible, for liquids and solids.
- Linear thermal expansion is

Equation:

$$\Delta L = \alpha L \Delta T,$$

where ΔL is the change in length L , ΔT is the change in temperature, and α is the coefficient of linear expansion, which varies slightly with temperature.

- The change in area due to thermal expansion is

Equation:

$$\Delta A = 2\alpha A \Delta T,$$

where ΔA is the change in area.

- The change in volume due to thermal expansion is

Equation:

$$\Delta V = \beta V \Delta T,$$

where β is the coefficient of volume expansion and $\beta \approx 3\alpha$. Thermal stress is created when thermal expansion is constrained.

Conceptual Questions

Exercise:**Problem:**

Thermal stresses caused by uneven cooling can easily break glass cookware. Explain why Pyrex®, a glass with a small coefficient of linear expansion, is less susceptible.

Exercise:**Problem:**

Water expands significantly when it freezes: a volume increase of about 9% occurs. As a result of this expansion and because of the formation and growth of crystals as water freezes, anywhere from 10% to 30% of biological cells are burst when animal or plant material is frozen. Discuss the implications of this cell damage for the prospect of preserving human bodies by freezing so that they can be thawed at some future date when it is hoped that all diseases are curable.

Exercise:**Problem:**

One method of getting a tight fit, say of a metal peg in a hole in a metal block, is to manufacture the peg slightly larger than the hole. The peg is then inserted when at a different temperature than the block. Should the block be hotter or colder than the peg during insertion? Explain your answer.

Exercise:**Problem:**

Does it really help to run hot water over a tight metal lid on a glass jar before trying to open it? Explain your answer.

Exercise:

Problem:

Liquids and solids expand with increasing temperature, because the kinetic energy of a body's atoms and molecules increases. Explain why some materials *shrink* with increasing temperature.

Problems & Exercises**Exercise:****Problem:**

The height of the Washington Monument is measured to be 170 m on a day when the temperature is 35.0°C . What will its height be on a day when the temperature falls to -10.0°C ? Although the monument is made of limestone, assume that its thermal coefficient of expansion is the same as marble's.

Solution:

169.98 m

Exercise:**Problem:**

How much taller does the Eiffel Tower become at the end of a day when the temperature has increased by 15°C ? Its original height is 321 m and you can assume it is made of steel.

Exercise:**Problem:**

What is the change in length of a 3.00-cm-long column of mercury if its temperature changes from 37.0°C to 40.0°C , assuming the mercury is unconstrained?

Solution:

$$5.4 \times 10^{-6} \text{ m}$$

Exercise:**Problem:**

How large an expansion gap should be left between steel railroad rails if they may reach a maximum temperature 35.0°C greater than when they were laid? Their original length is 10.0 m.

Exercise:**Problem:**

You are looking to purchase a small piece of land in Hong Kong. The price is “only” \$60,000 per square meter! The land title says the dimensions are $20 \text{ m} \times 30 \text{ m}$. By how much would the total price change if you measured the parcel with a steel tape measure on a day when the temperature was 20°C above normal?

Solution:

Because the area gets smaller, the price of the land DECREASES by ~\$17,000.

Exercise:**Problem:**

Global warming will produce rising sea levels partly due to melting ice caps but also due to the expansion of water as average ocean temperatures rise. To get some idea of the size of this effect, calculate the change in length of a column of water 1.00 km high for a temperature increase of 1.00°C . Note that this calculation is only approximate because ocean warming is not uniform with depth.

Exercise:**Problem:**

Show that 60.0 L of gasoline originally at 15.0°C will expand to 61.1 L when it warms to 35.0°C , as claimed in [\[link\]](#).

Solution:

Equation:

$$\begin{aligned} V &= V_0 + \Delta V = V_0(1 + \beta\Delta T) \\ &= (60.00 \text{ L})[1 + (950 \times 10^{-6}/^{\circ}\text{C})(35.0^{\circ}\text{C} - 15.0^{\circ}\text{C})] \\ &= 61.1 \text{ L} \end{aligned}$$

Exercise:

Problem:

(a) Suppose a meter stick made of steel and one made of invar (an alloy of iron and nickel) are the same length at 0°C . What is their difference in length at 22.0°C ? (b) Repeat the calculation for two 30.0-m-long surveyor's tapes.

Exercise:

Problem:

(a) If a 500-mL glass beaker is filled to the brim with ethyl alcohol at a temperature of 5.00°C , how much will overflow when its temperature reaches 22.0°C ? (b) How much less water would overflow under the same conditions?

Solution:

(a) 9.35 mL

(b) 7.56 mL

Exercise:

Problem:

Most automobiles have a coolant reservoir to catch radiator fluid that may overflow when the engine is hot. A radiator is made of copper and is filled to its 16.0-L capacity when at 10.0°C. What volume of radiator fluid will overflow when the radiator and fluid reach their 95.0°C operating temperature, given that the fluid's volume coefficient of expansion is $\beta = 400 \times 10^{-6} / ^\circ\text{C}$? Note that this coefficient is approximate, because most car radiators have operating temperatures of greater than 95.0°C.

Exercise:**Problem:**

A physicist makes a cup of instant coffee and notices that, as the coffee cools, its level drops 3.00 mm in the glass cup. Show that this decrease cannot be due to thermal contraction by calculating the decrease in level if the 350 cm³ of coffee is in a 7.00-cm-diameter cup and decreases in temperature from 95.0°C to 45.0°C. (Most of the drop in level is actually due to escaping bubbles of air.)

Solution:

0.832 mm

Exercise:**Problem:**

(a) The density of water at 0°C is very nearly 1000 kg/m³ (it is actually 999.84 kg/m³), whereas the density of ice at 0°C is 917 kg/m³. Calculate the pressure necessary to keep ice from expanding when it freezes, neglecting the effect such a large pressure would have on the freezing temperature. (This problem gives you only an indication of how large the forces associated with freezing water might be.) (b) What are the implications of this result for biological cells that are frozen?

Exercise:**Problem:**

Show that $\beta \approx 3\alpha$, by calculating the change in volume ΔV of a cube with sides of length L .

Solution:

We know how the length changes with temperature: $\Delta L = \alpha L_0 \Delta T$. Also we know that the volume of a cube is related to its length by $V = L^3$, so the final volume is then $V = V_0 + \Delta V = (L_0 + \Delta L)^3$. Substituting for ΔL gives

Equation:

$$V = (L_0 + \alpha L_0 \Delta T)^3 = L_0^3 (1 + \alpha \Delta T)^3.$$

Now, because $\alpha \Delta T$ is small, we can use the binomial expansion:

Equation:

$$V \approx L_0^3 (1 + 3\alpha \Delta T) = L_0^3 + 3\alpha L_0^3 \Delta T.$$

So writing the length terms in terms of volumes gives $V = V_0 + \Delta V \approx V_0 + 3\alpha V_0 \Delta T$, and so

Equation:

$$\Delta V = \beta V_0 \Delta T \approx 3\alpha V_0 \Delta T, \text{ or } \beta \approx 3\alpha.$$

Glossary

thermal expansion

the change in size or volume of an object with change in temperature

coefficient of linear expansion

α , the change in length, per unit length, per 1°C change in temperature; a constant used in the calculation of linear expansion; the coefficient of linear expansion depends on the material and to some degree on the temperature of the material

coefficient of volume expansion

β , the change in volume, per unit volume, per 1°C change in temperature

thermal stress

stress caused by thermal expansion or contraction

The Ideal Gas Law

- State the ideal gas law in terms of molecules and in terms of moles.
- Use the ideal gas law to calculate pressure change, temperature change, volume change, or the number of molecules or moles in a given volume.
- Use Avogadro's number to convert between number of molecules and number of moles.

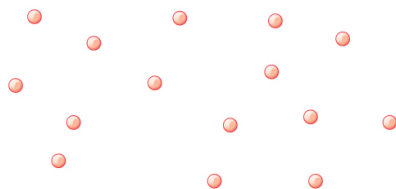


The air inside this hot air balloon flying over Putrajaya, Malaysia, is hotter than the ambient air. As a result, the balloon experiences a buoyant force pushing it upward.
(credit: Kevin Poh, Flickr)

In this section, we continue to explore the thermal behavior of gases. In particular, we examine the characteristics of atoms and molecules that compose gases. (Most gases, for example nitrogen, N_2 , and oxygen, O_2 , are composed of two or more atoms. We will primarily use the term “molecule” in discussing a gas because the term can also be applied to monatomic gases, such as helium.)

Gases are easily compressed. We can see evidence of this in [\[link\]](#), where you will note that gases have the *largest* coefficients of volume expansion. The large coefficients mean that gases expand and contract very rapidly with temperature changes. In addition, you will note that most gases expand at the *same* rate, or have the same β . This raises the question as to why gases should all act in nearly the same way, when liquids and solids have widely varying expansion rates.

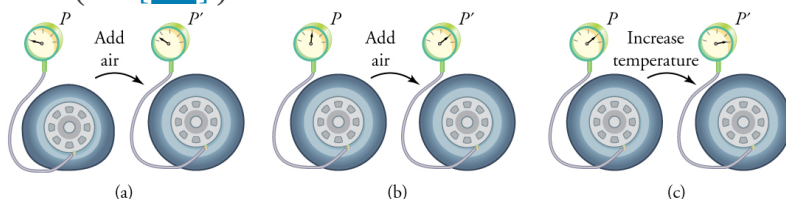
The answer lies in the large separation of atoms and molecules in gases, compared to their sizes, as illustrated in [\[link\]](#). Because atoms and molecules have large separations, forces between them can be ignored, except when they collide with each other during collisions. The motion of atoms and molecules (at temperatures well above the boiling temperature) is fast, such that the gas occupies all of the accessible volume and the expansion of gases is rapid. In contrast, in liquids and solids, atoms and molecules are closer together and are quite sensitive to the forces between them.



Atoms and molecules in a gas are typically widely separated, as shown.

Because the forces between them are quite weak at these distances, the properties of a gas depend more on the number of atoms per unit volume and on temperature than on the type of atom.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into an initially deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If we continue to pump air into it, the pressure increases. The pressure will further increase when the car is driven and the tires move. Most manufacturers specify optimal tire pressure for cold tires. (See [\[link\]](#).)



(a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion and the pressure increases with

more air. (c) Once the tire is inflated, its pressure increases with temperature.

At room temperatures, collisions between atoms and molecules can be ignored. In this case, the gas is called an ideal gas, in which case the relationship between the pressure, volume, and temperature is given by the equation of state called the ideal gas law.

Note:

Ideal Gas Law

The **ideal gas law** states that

Equation:

$$PV = NkT,$$

where P is the absolute pressure of a gas, V is the volume it occupies, N is the number of atoms and molecules in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** in honor of Austrian physicist Ludwig Boltzmann (1844–1906) and has the value

Equation:

$$k = 1.38 \times 10^{-23} \text{ J/K}.$$

The ideal gas law can be derived from basic principles, but was originally deduced from experimental measurements of Charles' law (that volume occupied by a gas is proportional to temperature at a fixed pressure) and from Boyle's law (that for a fixed temperature, the product PV is a constant). In the ideal gas model, the volume occupied by its atoms and molecules is a negligible fraction of V . The ideal gas law describes the behavior of real gases under most conditions. (Note, for example, that N is the total number of atoms and molecules, independent of the type of gas.)

Let us see how the ideal gas law is consistent with the behavior of filling the tire when it is pumped slowly and the temperature is constant. At first, the pressure P is essentially equal to atmospheric pressure, and the volume V increases in direct proportion to the number of atoms and molecules N put into the tire. Once the volume of the tire is constant, the equation $PV = NkT$ predicts that the pressure should increase in proportion to *the number N of atoms and molecules*.

Example:

Calculating Pressure Changes Due to Temperature Changes: Tire Pressure

Suppose your bicycle tire is fully inflated, with an absolute pressure of 7.00×10^5 Pa (a gauge pressure of just under 90.0 lb/in²) at a temperature of 18.0°C. What is the pressure after its temperature has risen to 35.0°C? Assume that there are no appreciable leaks or changes in volume.

Strategy

The pressure in the tire is changing only because of changes in temperature. First we need to identify what we know and what we want to know, and then identify an equation to solve for the unknown.

We know the initial pressure $P_0 = 7.00 \times 10^5$ Pa, the initial temperature $T_0 = 18.0^\circ\text{C}$, and the final temperature $T_f = 35.0^\circ\text{C}$. We must find the final pressure P_f . How can we use the equation $PV = NkT$? At first, it may seem that not enough information is given, because the volume V and number of atoms N are not specified. What we can do is use the equation twice: $P_0V_0 = NkT_0$ and $P_fV_f = NkT_f$. If we divide P_fV_f by P_0V_0 we can come up with an equation that allows us to solve for P_f .

Equation:

$$\frac{P_f V_f}{P_0 V_0} = \frac{N_f k T_f}{N_0 k T_0}$$

Since the volume is constant, V_f and V_0 are the same and they cancel out. The same is true for N_f and N_0 , and k , which is a constant. Therefore,

Equation:

$$\frac{P_f}{P_0} = \frac{T_f}{T_0}.$$

We can then rearrange this to solve for P_f :

Equation:

$$P_f = P_0 \frac{T_f}{T_0},$$

where the temperature must be in units of kelvins, because T_0 and T_f are absolute temperatures.

Solution

1. Convert temperatures from Celsius to Kelvin.

Equation:

$$\begin{aligned} T_0 &= (18.0 + 273)\text{K} = 291 \text{ K} \\ T_f &= (35.0 + 273)\text{K} = 308 \text{ K} \end{aligned}$$

2. Substitute the known values into the equation.

Equation:

$$P_f = P_0 \frac{T_f}{T_0} = 7.00 \times 10^5 \text{ Pa} \left(\frac{308 \text{ K}}{291 \text{ K}} \right) = 7.41 \times 10^5 \text{ Pa}$$

Discussion

The final temperature is about 6% greater than the original temperature, so the final pressure is about 6% greater as well. Note that *absolute* pressure and *absolute* temperature must be used in the ideal gas law.

Note:

Making Connections: Take-Home Experiment—Refrigerating a Balloon

Inflate a balloon at room temperature. Leave the inflated balloon in the refrigerator overnight. What happens to the balloon, and why?

Example:

Calculating the Number of Molecules in a Cubic Meter of Gas

How many molecules are in a typical object, such as gas in a tire or water in a drink? We can use the ideal gas law to give us an idea of how large N typically is.

Calculate the number of molecules in a cubic meter of gas at standard temperature and pressure (STP), which is defined to be 0°C and atmospheric pressure.

Strategy

Because pressure, volume, and temperature are all specified, we can use the ideal gas law $PV = NkT$, to find N .

Solution

1. Identify the knowns.

Equation:

$$T = 0^\circ\text{C} = 273 \text{ K}$$

$$P = 1.01 \times 10^5 \text{ Pa}$$

$$V = 1.00 \text{ m}^3$$

$$k = 1.38 \times 10^{-23} \text{ J/K}$$

2. Identify the unknown: number of molecules, N .

3. Rearrange the ideal gas law to solve for N .

Equation:

$$PV = NkT$$

$$N = \frac{PV}{kT}$$

4. Substitute the known values into the equation and solve for N .

Equation:

$$N = \frac{PV}{kT} = \frac{(1.01 \times 10^5 \text{ Pa})(1.00 \text{ m}^3)}{(1.38 \times 10^{-23} \text{ J/K})(273 \text{ K})} = 2.68 \times 10^{25} \text{ molecules}$$

Discussion

This number is undeniably large, considering that a gas is mostly empty space. N is huge, even in small volumes. For example, 1 cm^3 of a gas at STP has 2.68×10^{19} molecules in it. Once again, note that N is the same for all types or mixtures of gases.

Moles and Avogadro's Number

It is sometimes convenient to work with a unit other than molecules when measuring the amount of substance. A **mole** (abbreviated mol) is defined to be the amount of a substance that contains as many atoms or molecules as there are atoms in exactly 12 grams (0.012 kg) of carbon-12. The actual number of atoms or molecules in one mole is called **Avogadro's number** (N_A), in recognition of Italian scientist Amedeo Avogadro (1776–1856). He developed the concept of the mole, based on the hypothesis that equal volumes of gas, at the same pressure and temperature, contain equal numbers of molecules. That is, the number is independent of the type of gas. This hypothesis has been confirmed, and the value of Avogadro's number is

Equation:

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}.$$

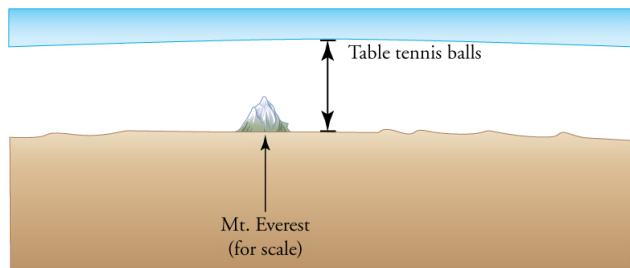
Note:

Avogadro's Number

One mole always contains 6.02×10^{23} particles (atoms or molecules), independent of the element or substance. A mole of any substance has a mass in grams equal to its molecular mass, which can be calculated from the atomic masses given in the periodic table of elements.

Equation:

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}$$



How big is a mole? On a macroscopic level, one mole of table tennis balls would cover the Earth to a depth of about 40 km.

Exercise:
Check Your Understanding

Problem:

The active ingredient in a Tylenol pill is 325 mg of acetaminophen ($\text{C}_8\text{H}_9\text{NO}_2$). Find the number of active molecules of acetaminophen in a single pill.

Solution:

We first need to calculate the molar mass (the mass of one mole) of acetaminophen. To do this, we need to multiply the number of atoms of each element by the element's atomic mass.

Equation:

$$(8 \text{ moles of carbon})(12 \text{ grams/mole}) + (9 \text{ moles hydrogen})(1 \text{ gram/mole}) \\ + (1 \text{ mole nitrogen})(14 \text{ grams/mole}) + (2 \text{ moles oxygen})(16 \text{ grams/mole}) = 151 \text{ g}$$

Then we need to calculate the number of moles in 325 mg.

Equation:

$$\left(\frac{325 \text{ mg}}{151 \text{ grams/mole}} \right) \left(\frac{1 \text{ gram}}{1000 \text{ mg}} \right) = 2.15 \times 10^{-3} \text{ moles}$$

Then use Avogadro's number to calculate the number of molecules.

Equation:

$$N = (2.15 \times 10^{-3} \text{ moles}) (6.02 \times 10^{23} \text{ molecules/mole}) = 1.30 \times 10^{21} \text{ molecules}$$

Example:

Calculating Moles per Cubic Meter and Liters per Mole

Calculate: (a) the number of moles in 1.00 m^3 of gas at STP, and (b) the number of liters of gas per mole.

Strategy and Solution

(a) We are asked to find the number of moles per cubic meter, and we know from [\[link\]](#) that the number of molecules per cubic meter at STP is 2.68×10^{25} . The number of moles can be found by dividing the number of molecules by Avogadro's number. We let n stand for the number of moles,

Equation:

$$n \text{ mol/m}^3 = \frac{N \text{ molecules/m}^3}{6.02 \times 10^{23} \text{ molecules/mol}} = \frac{2.68 \times 10^{25} \text{ molecules/m}^3}{6.02 \times 10^{23} \text{ molecules/mol}} = 44.5 \text{ mol/m}^3.$$

(b) Using the value obtained for the number of moles in a cubic meter, and converting cubic meters to liters, we obtain

Equation:

$$\frac{(10^3 \text{ L/m}^3)}{44.5 \text{ mol/m}^3} = 22.5 \text{ L/mol.}$$

Discussion

This value is very close to the accepted value of 22.4 L/mol. The slight difference is due to rounding errors caused by using three-digit input. Again this number is the same for all gases. In other words, it is independent of the gas.

The (average) molar weight of air (approximately 80% N₂ and 20% O₂) is $M = 28.8 \text{ g}$. Thus the mass of one cubic meter of air is 1.28 kg. If a living room has dimensions $5 \text{ m} \times 5 \text{ m} \times 3 \text{ m}$, the mass of air inside the room is 96 kg, which is the typical mass of a human.

Exercise:

Check Your Understanding

Problem:

The density of air at standard conditions ($P = 1 \text{ atm}$ and $T = 20^\circ\text{C}$) is 1.28 kg/m^3 . At what pressure is the density 0.64 kg/m^3 if the temperature and number of molecules are kept constant?

Solution:

The best way to approach this question is to think about what is happening. If the density drops to half its original value and no molecules are lost, then the volume must double. If we look at the equation $PV = NkT$, we see that when the temperature is constant, the pressure is inversely proportional to volume. Therefore, if the volume doubles, the pressure must drop to half its original value, and $P_f = 0.50 \text{ atm}$.

The Ideal Gas Law Restated Using Moles

A very common expression of the ideal gas law uses the number of moles, n , rather than the number of atoms and molecules, N . We start from the ideal gas law,

Equation:

$$PV = NkT,$$

and multiply and divide the equation by Avogadro's number N_A . This gives

Equation:

$$PV = \frac{N}{N_A} N_A k T.$$

Note that $n = N/N_A$ is the number of moles. We define the universal gas constant $R = N_A k$, and obtain the ideal gas law in terms of moles.

Note:

Ideal Gas Law (in terms of moles)

The ideal gas law (in terms of moles) is

Equation:

$$PV = nRT.$$

The numerical value of R in SI units is

Equation:

$$R = N_A k = (6.02 \times 10^{23} \text{ mol}^{-1})(1.38 \times 10^{-23} \text{ J/K}) = 8.31 \text{ J/mol} \cdot \text{K}.$$

In other units,

Equation:

$$R = 1.99 \text{ cal/mol} \cdot \text{K}$$

$$R = 0.0821 \text{ L} \cdot \text{atm/mol} \cdot \text{K}.$$

You can use whichever value of R is most convenient for a particular problem.

Example:

Calculating Number of Moles: Gas in a Bike Tire

How many moles of gas are in a bike tire with a volume of $2.00 \times 10^{-3} \text{ m}^3$ (2.00 L), a pressure of $7.00 \times 10^5 \text{ Pa}$ (a gauge pressure of just under 90.0 lb/in²), and at a temperature of 18.0°C?

Strategy

Identify the knowns and unknowns, and choose an equation to solve for the unknown. In this case, we solve the ideal gas law, $PV = nRT$, for the number of moles n .

Solution

1. Identify the knowns.

Equation:

$$P = 7.00 \times 10^5 \text{ Pa}$$

$$V = 2.00 \times 10^{-3} \text{ m}^3$$

$$T = 18.0^\circ\text{C} = 291 \text{ K}$$

$$R = 8.31 \text{ J/mol} \cdot \text{K}$$

2. Rearrange the equation to solve for n and substitute known values.

Equation:

$$\begin{aligned} n &= \frac{PV}{RT} = \frac{(7.00 \times 10^5 \text{ Pa})(2.00 \times 10^{-3} \text{ m}^3)}{(8.31 \text{ J/mol}\cdot\text{K})(291 \text{ K})} \\ &= 0.579 \text{ mol} \end{aligned}$$

Discussion

The most convenient choice for R in this case is $8.31 \text{ J/mol}\cdot\text{K}$, because our known quantities are in SI units. The pressure and temperature are obtained from the initial conditions in [\[link\]](#), but we would get the same answer if we used the final values.

The ideal gas law can be considered to be another manifestation of the law of conservation of energy (see [Conservation of Energy](#)). Work done on a gas results in an increase in its energy, increasing pressure and/or temperature, or decreasing volume. This increased energy can also be viewed as increased internal kinetic energy, given the gas's atoms and molecules.

The Ideal Gas Law and Energy

Let us now examine the role of energy in the behavior of gases. When you inflate a bike tire by hand, you do work by repeatedly exerting a force through a distance. This energy goes into increasing the pressure of air inside the tire and increasing the temperature of the pump and the air.

The ideal gas law is closely related to energy: the units on both sides are joules. The right-hand side of the ideal gas law in $PV = NkT$ is NkT . This term is roughly the amount of translational kinetic energy of N atoms or molecules at an absolute temperature T , as we shall see formally in [Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature](#). The left-hand side of the ideal gas law is PV , which also has the units of joules. We know from our study of fluids that pressure is one type of potential energy per unit volume, so pressure multiplied by volume is energy. The important point is that there is energy in a gas related to both its pressure and its volume. The energy can be changed when the gas is doing work as it expands—something we explore in [Heat and Heat Transfer Methods](#)—similar to what occurs in gasoline or steam engines and turbines.

Note:

Problem-Solving Strategy: The Ideal Gas Law

Step 1 Examine the situation to determine that an ideal gas is involved. Most gases are nearly ideal.

Step 2 Make a list of what quantities are given, or can be inferred from the problem as stated (identify the known quantities). Convert known values into proper SI units (K for temperature, Pa for pressure, m^3 for volume, molecules for N , and moles for n).

Step 3 Identify exactly what needs to be determined in the problem (identify the unknown quantities). A written list is useful.

Step 4 Determine whether the number of molecules or the number of moles is known, in order to decide which form of the ideal gas law to use. The first form is $PV = NkT$ and involves N , the number of atoms or molecules. The second form is $PV = nRT$ and involves n , the number of moles.

Step 5 Solve the ideal gas law for the quantity to be determined (the unknown quantity). You may need to take a ratio of final states to initial states to eliminate the unknown quantities that are kept fixed.

Step 6 Substitute the known quantities, along with their units, into the appropriate equation, and obtain numerical solutions complete with units. Be certain to use absolute temperature and absolute pressure.

Step 7 Check the answer to see if it is reasonable: Does it make sense?

Exercise:

Check Your Understanding

Problem:

Liquids and solids have densities about 1000 times greater than gases. Explain how this implies that the distances between atoms and molecules in gases are about 10 times greater than the size of their atoms and molecules.

Solution:

Atoms and molecules are close together in solids and liquids. In gases they are separated by empty space. Thus gases have lower densities than liquids and solids. Density is mass per unit volume, and volume is related to the size of a body (such as a sphere) cubed. So if the distance between atoms and molecules increases by a factor of 10, then the volume occupied increases by a factor of 1000, and the density decreases by a factor of 1000.

Section Summary

- The ideal gas law relates the pressure and volume of a gas to the number of gas molecules and the temperature of the gas.
- The ideal gas law can be written in terms of the number of molecules of gas:

Equation:

$$PV = NkT,$$

where P is pressure, V is volume, T is temperature, N is number of molecules, and k is the Boltzmann constant

Equation:

$$k = 1.38 \times 10^{-23} \text{ J/K}.$$

- A mole is the number of atoms in a 12-g sample of carbon-12.
- The number of molecules in a mole is called Avogadro's number N_A ,

Equation:

$$N_A = 6.02 \times 10^{23} \text{ mol}^{-1}.$$

- A mole of any substance has a mass in grams equal to its molecular weight, which can be determined from the periodic table of elements.
- The ideal gas law can also be written and solved in terms of the number of moles of gas:

Equation:

$$PV = nRT,$$

where n is number of moles and R is the universal gas constant,

Equation:

$$R = 8.31 \text{ J/mol} \cdot \text{K}.$$

- The ideal gas law is generally valid at temperatures well above the boiling temperature.

Conceptual Questions

Exercise:

Problem:

Find out the human population of Earth. Is there a mole of people inhabiting Earth? If the average mass of a person is 60 kg, calculate the mass of a mole of people. How does the mass of a mole of people compare with the mass of Earth?

Exercise:

Problem:

Under what circumstances would you expect a gas to behave significantly differently than predicted by the ideal gas law?

Exercise:

Problem:

A constant-volume gas thermometer contains a fixed amount of gas. What property of the gas is measured to indicate its temperature?

Problems & Exercises

Exercise:

Problem:

The gauge pressure in your car tires is $2.50 \times 10^5 \text{ N/m}^2$ at a temperature of 35.0°C when you drive it onto a ferry boat to Alaska. What is their gauge pressure later, when their temperature has dropped to -40.0°C ?

Solution:

1.62 atm

Exercise:**Problem:**

Convert an absolute pressure of $7.00 \times 10^5 \text{ N/m}^2$ to gauge pressure in lb/in^2 . (This value was stated to be just less than 90.0 lb/in^2 in [\[link\]](#). Is it?)

Exercise:**Problem:**

Suppose a gas-filled incandescent light bulb is manufactured so that the gas inside the bulb is at atmospheric pressure when the bulb has a temperature of 20.0°C . (a) Find the gauge pressure inside such a bulb when it is hot, assuming its average temperature is 60.0°C (an approximation) and neglecting any change in volume due to thermal expansion or gas leaks. (b) The actual final pressure for the light bulb will be less than calculated in part (a) because the glass bulb will expand. What will the actual final pressure be, taking this into account? Is this a negligible difference?

Solution:

(a) 0.136 atm

(b) 0.135 atm. The difference between this value and the value from part (a) is negligible.

Exercise:**Problem:**

Large helium-filled balloons are used to lift scientific equipment to high altitudes. (a) What is the pressure inside such a balloon if it starts out at sea level with a temperature of 10.0°C and rises to an altitude where its volume is twenty times the original volume and its temperature is -50.0°C ? (b) What is the gauge pressure? (Assume atmospheric pressure is constant.)

Exercise:**Problem:**

Confirm that the units of nRT are those of energy for each value of R : (a) $8.31 \text{ J/mol} \cdot \text{K}$, (b) $1.99 \text{ cal/mol} \cdot \text{K}$, and (c) $0.0821 \text{ L} \cdot \text{atm/mol} \cdot \text{K}$.

Solution:

$$(a) nRT = (\text{mol})(\text{J/mol} \cdot \text{K})(\text{K}) = \text{J}$$

$$(b) nRT = (\text{mol})(\text{cal/mol} \cdot \text{K})(\text{K}) = \text{cal}$$

$$\begin{aligned}
 nRT &= (\text{mol})(\text{L} \cdot \text{atm}/\text{mol} \cdot \text{K})(\text{K}) \\
 \text{(c)} \quad &= \text{L} \cdot \text{atm} = (\text{m}^3)(\text{N}/\text{m}^2) \\
 &= \text{N} \cdot \text{m} = \text{J}
 \end{aligned}$$

Exercise:

Problem:

In the text, it was shown that $N/V = 2.68 \times 10^{25} \text{ m}^{-3}$ for gas at STP. (a) Show that this quantity is equivalent to $N/V = 2.68 \times 10^{19} \text{ cm}^{-3}$, as stated. (b) About how many atoms are there in one μm^3 (a cubic micrometer) at STP? (c) What does your answer to part (b) imply about the separation of atoms and molecules?

Exercise:

Problem:

Calculate the number of moles in the 2.00-L volume of air in the lungs of the average person. Note that the air is at 37.0°C (body temperature).

Solution:

$$7.86 \times 10^{-2} \text{ mol}$$

Exercise:

Problem:

An airplane passenger has 100 cm^3 of air in his stomach just before the plane takes off from a sea-level airport. What volume will the air have at cruising altitude if cabin pressure drops to $7.50 \times 10^4 \text{ N}/\text{m}^2$?

Exercise:

Problem:

(a) What is the volume (in km^3) of Avogadro's number of sand grains if each grain is a cube and has sides that are 1.0 mm long? (b) How many kilometers of beaches in length would this cover if the beach averages 100 m in width and 10.0 m in depth? Neglect air spaces between grains.

Solution:

$$\text{(a)} \quad 6.02 \times 10^5 \text{ km}^3$$

$$\text{(b)} \quad 6.02 \times 10^8 \text{ km}$$

Exercise:

Problem:

An expensive vacuum system can achieve a pressure as low as $1.00 \times 10^{-7} \text{ N/m}^2$ at 20°C . How many atoms are there in a cubic centimeter at this pressure and temperature?

Exercise:**Problem:**

The number density of gas atoms at a certain location in the space above our planet is about $1.00 \times 10^{11} \text{ m}^{-3}$, and the pressure is $2.75 \times 10^{-10} \text{ N/m}^2$ in this space. What is the temperature there?

Solution:

-73.9°C

Exercise:**Problem:**

A bicycle tire has a pressure of $7.00 \times 10^5 \text{ N/m}^2$ at a temperature of 18.0°C and contains 2.00 L of gas. What will its pressure be if you let out an amount of air that has a volume of 100 cm^3 at atmospheric pressure? Assume tire temperature and volume remain constant.

Exercise:**Problem:**

A high-pressure gas cylinder contains 50.0 L of toxic gas at a pressure of $1.40 \times 10^7 \text{ N/m}^2$ and a temperature of 25.0°C . Its valve leaks after the cylinder is dropped. The cylinder is cooled to dry ice temperature (-78.5°C) to reduce the leak rate and pressure so that it can be safely repaired. (a) What is the final pressure in the tank, assuming a negligible amount of gas leaks while being cooled and that there is no phase change? (b) What is the final pressure if one-tenth of the gas escapes? (c) To what temperature must the tank be cooled to reduce the pressure to 1.00 atm (assuming the gas does not change phase and that there is no leakage during cooling)? (d) Does cooling the tank appear to be a practical solution?

Solution:

(a) $9.14 \times 10^6 \text{ N/m}^2$

(b) $8.23 \times 10^6 \text{ N/m}^2$

(c) 2.16 K

(d) No. The final temperature needed is much too low to be easily achieved for a large object.

Exercise:

Problem:

Find the number of moles in 2.00 L of gas at 35.0°C and under $7.41 \times 10^7 \text{ N/m}^2$ of pressure.

Exercise:**Problem:**

Calculate the depth to which Avogadro's number of table tennis balls would cover Earth. Each ball has a diameter of 3.75 cm. Assume the space between balls adds an extra 25.0% to their volume and assume they are not crushed by their own weight.

Solution:

41 km

Exercise:**Problem:**

(a) What is the gauge pressure in a 25.0°C car tire containing 3.60 mol of gas in a 30.0 L volume? (b) What will its gauge pressure be if you add 1.00 L of gas originally at atmospheric pressure and 25.0°C? Assume the temperature returns to 25.0°C and the volume remains constant.

Exercise:**Problem:**

(a) In the deep space between galaxies, the density of atoms is as low as 10^6 atoms/m^3 , and the temperature is a frigid 2.7 K. What is the pressure? (b) What volume (in m^3) is occupied by 1 mol of gas? (c) If this volume is a cube, what is the length of its sides in kilometers?

Solution:

(a) $3.7 \times 10^{-17} \text{ Pa}$

(b) $6.0 \times 10^{17} \text{ m}^3$

(c) $8.4 \times 10^2 \text{ km}$

Glossary

ideal gas law

the physical law that relates the pressure and volume of a gas to the number of gas molecules or number of moles of gas and the temperature of the gas

Boltzmann constant

k , a physical constant that relates energy to temperature; $k = 1.38 \times 10^{-23} \text{ J/K}$

Avogadro's number

N_A , the number of molecules or atoms in one mole of a substance; $N_A = 6.02 \times 10^{23}$
particles/mole

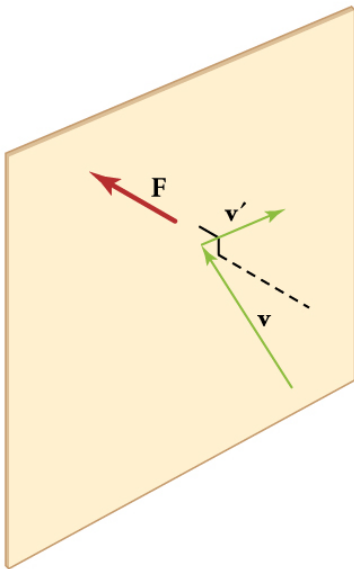
mole

the quantity of a substance whose mass (in grams) is equal to its molecular mass

Kinetic Theory: Atomic and Molecular Explanation of Pressure and Temperature

- Express the ideal gas law in terms of molecular mass and velocity.
- Define thermal energy.
- Calculate the kinetic energy of a gas molecule, given its temperature.
- Describe the relationship between the temperature of a gas and the kinetic energy of atoms and molecules.
- Describe the distribution of speeds of molecules in a gas.

We have developed macroscopic definitions of pressure and temperature. Pressure is the force divided by the area on which the force is exerted, and temperature is measured with a thermometer. We gain a better understanding of pressure and temperature from the kinetic theory of gases, which assumes that atoms and molecules are in continuous random motion.



When a molecule collides with a rigid wall, the component of its momentum perpendicular to the wall is reversed. A force is thus exerted on the wall, creating pressure.

[\[link\]](#) shows an elastic collision of a gas molecule with the wall of a container, so that it exerts a force on the wall (by Newton's third law). Because a huge number of molecules will collide with the wall in a short time, we observe an average force per unit area. These collisions are the source of pressure in a gas. As the number of molecules increases, the number of collisions and thus the pressure increase. Similarly, the gas pressure is higher if the average velocity of molecules is higher. The actual relationship is derived in the [Things Great and Small](#) feature below. The following relationship is found:

Equation:

$$PV = \frac{1}{3}Nm\overline{v^2},$$

where P is the pressure (average force per unit area), V is the volume of gas in the container, N is the number of molecules in the container, m is the mass of a molecule, and $\overline{v^2}$ is the average of the molecular speed squared.

What can we learn from this atomic and molecular version of the ideal gas law? We can derive a relationship between temperature and the average translational kinetic energy of molecules in a gas. Recall the previous expression of the ideal gas law:

Equation:

$$PV = NkT.$$

Equating the right-hand side of this equation with the right-hand side of $PV = \frac{1}{3}Nm\overline{v^2}$ gives

Equation:

$$\frac{1}{3}Nm\overline{v^2} = NkT.$$

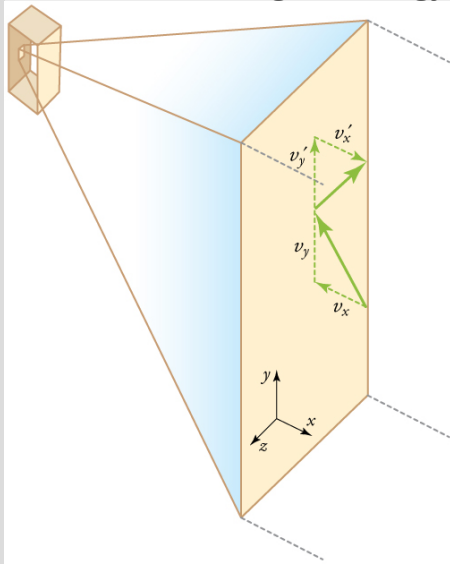
Note:

Making Connections: Things Great and Small—Atomic and Molecular Origin of Pressure in a Gas

[\[link\]](#) shows a box filled with a gas. We know from our previous discussions that putting more gas into the box produces greater pressure, and that increasing the temperature of the gas also produces a greater pressure. But why should increasing the temperature of the gas increase the pressure in the box? A look at the atomic and

molecular scale gives us some answers, and an alternative expression for the ideal gas law.

The figure shows an expanded view of an elastic collision of a gas molecule with the wall of a container. Calculating the average force exerted by such molecules will lead us to the ideal gas law, and to the connection between temperature and molecular kinetic energy. We assume that a molecule is small compared with the separation of molecules in the gas, and that its interaction with other molecules can be ignored. We also assume the wall is rigid and that the molecule's direction changes, but that its speed remains constant (and hence its kinetic energy and the magnitude of its momentum remain constant as well). This assumption is not always valid, but the same result is obtained with a more detailed description of the molecule's exchange of energy and momentum with the wall.



Gas in a box exerts an outward pressure on its walls. A molecule colliding with a rigid wall has the direction of its velocity and momentum in the x -direction reversed. This direction is perpendicular to the wall.

The components of its velocity momentum in the y - and z -directions are not changed, which means there is no force parallel to the wall.

If the molecule's velocity changes in the x -direction, its momentum changes from $-mv_x$ to $+mv_x$. Thus, its change in momentum is $\Delta mv = +mv_x - (-mv_x) = 2mv_x$. The force exerted on the molecule is given by

Equation:

$$F = \frac{\Delta p}{\Delta t} = \frac{2mv_x}{\Delta t}.$$

There is no force between the wall and the molecule until the molecule hits the wall. During the short time of the collision, the force between the molecule and wall is relatively large. We are looking for an average force; we take Δt to be the average time between collisions of the molecule with this wall. It is the time it would take the molecule to go across the box and back (a distance $2l$) at a speed of v_x . Thus $\Delta t = 2l/v_x$, and the expression for the force becomes

Equation:

$$F = \frac{2mv_x}{2l/v_x} = \frac{mv_x^2}{l}.$$

This force is due to *one* molecule. We multiply by the number of molecules N and use their average squared velocity to find the force

Equation:

$$F = N \frac{\overline{mv_x^2}}{l},$$

where the bar over a quantity means its average value. We would like to have the force in terms of the speed v , rather than the x -component of the velocity. We note that the total velocity squared is the sum of the squares of its components, so that

Equation:

$$\overline{v^2} = \overline{v_x^2} + \overline{v_y^2} + \overline{v_z^2}.$$

Because the velocities are random, their average components in all directions are the same:

Equation:

$$\overline{v_x^2} = \overline{v_y^2} = \overline{v_z^2}.$$

Thus,

Equation:

$$\overline{v^2} = 3\overline{v_x^2},$$

or

Equation:

$$\overline{v_x^2} = \frac{1}{3}\overline{v^2}.$$

Substituting $\frac{1}{3}\overline{v^2}$ into the expression for F gives

Equation:

$$F = N \frac{m\overline{v^2}}{3l}.$$

The pressure is F/A , so that we obtain

Equation:

$$P = \frac{F}{A} = N \frac{m\overline{v^2}}{3Al} = \frac{1}{3} \frac{Nm\overline{v^2}}{V},$$

where we used $V = Al$ for the volume. This gives the important result.

Equation:

$$PV = \frac{1}{3}Nm\overline{v^2}$$

This equation is another expression of the ideal gas law.

We can get the average kinetic energy of a molecule, $\frac{1}{2}mv^2$, from the right-hand side of the equation by canceling N and multiplying by $3/2$. This calculation produces the result that the average kinetic energy of a molecule is directly related to absolute temperature.

Equation:

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

The average translational kinetic energy of a molecule, $\overline{KE}$, is called **thermal energy**. The equation $\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$ is a molecular interpretation of temperature, and it has been found to be valid for gases and reasonably accurate in liquids and solids. It is another definition of temperature based on an expression of the molecular energy.

It is sometimes useful to rearrange $\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$, and solve for the average speed of molecules in a gas in terms of temperature,

Equation:

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}},$$

where v_{rms} stands for root-mean-square (rms) speed.

Example:

Calculating Kinetic Energy and Speed of a Gas Molecule

(a) What is the average kinetic energy of a gas molecule at 20.0°C (room temperature)? (b) Find the rms speed of a nitrogen molecule (N_2) at this temperature.

Strategy for (a)

The known in the equation for the average kinetic energy is the temperature.

Equation:

$$\overline{KE} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

Before substituting values into this equation, we must convert the given temperature to kelvins. This conversion gives $T = (20.0 + 273) \text{ K} = 293 \text{ K}$.

Solution for (a)

The temperature alone is sufficient to find the average translational kinetic energy. Substituting the temperature into the translational kinetic energy equation gives

Equation:

$$\overline{KE} = \frac{3}{2}kT = \frac{3}{2}(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K}) = 6.07 \times 10^{-21} \text{ J}.$$

Strategy for (b)

Finding the rms speed of a nitrogen molecule involves a straightforward calculation using the equation

Equation:

$$\sqrt{v^2} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}},$$

but we must first find the mass of a nitrogen molecule. Using the molecular mass of nitrogen N_2 from the periodic table,

Equation:

$$m = \frac{2(14.0067) \times 10^{-3} \text{ kg/mol}}{6.02 \times 10^{23} \text{ mol}^{-1}} = 4.65 \times 10^{-26} \text{ kg}.$$

Solution for (b)

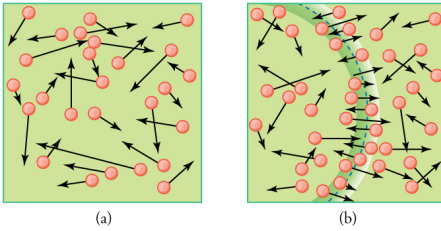
Substituting this mass and the value for k into the equation for v_{rms} yields

Equation:

$$v_{\text{rms}} = \sqrt{\frac{3kT}{m}} = \sqrt{\frac{3(1.38 \times 10^{-23} \text{ J/K})(293 \text{ K})}{4.65 \times 10^{-26} \text{ kg}}} = 511 \text{ m/s}.$$

Discussion

Note that the average kinetic energy of the molecule is independent of the type of molecule. The average translational kinetic energy depends only on absolute temperature. The kinetic energy is very small compared to macroscopic energies, so that we do not feel when an air molecule is hitting our skin. The rms velocity of the nitrogen molecule is surprisingly large. These large molecular velocities do not yield macroscopic movement of air, since the molecules move in all directions with equal likelihood. The *mean free path* (the distance a molecule can move on average between collisions) of molecules in air is very small, and so the molecules move rapidly but do not get very far in a second. The high value for rms speed is reflected in the speed of sound, however, which is about 340 m/s at room temperature. The faster the rms speed of air molecules, the faster that sound vibrations can be transferred through the air. The speed of sound increases with temperature and is greater in gases with small molecular masses, such as helium. (See [\[link\]](#).)



(a) There are many molecules moving so fast in an ordinary gas that they collide a billion times every second. (b) Individual molecules do not move very far in a small amount of time, but disturbances like sound waves are transmitted at speeds related to the molecular speeds.

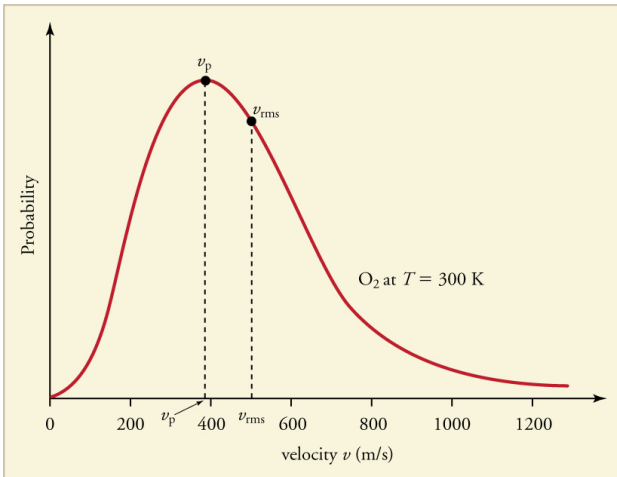
Note:

Making Connections: Historical Note—Kinetic Theory of Gases

The kinetic theory of gases was developed by Daniel Bernoulli (1700–1782), who is best known in physics for his work on fluid flow (hydrodynamics). Bernoulli's work predates the atomistic view of matter established by Dalton.

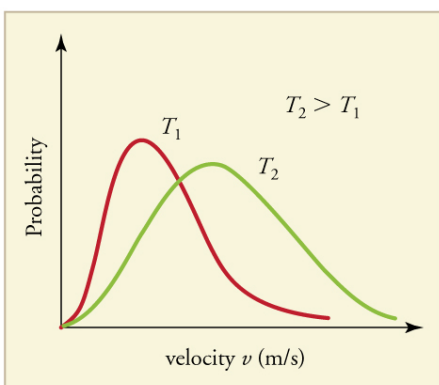
Distribution of Molecular Speeds

The motion of molecules in a gas is random in magnitude and direction for individual molecules, but a gas of many molecules has a predictable distribution of molecular speeds. This distribution is called the *Maxwell-Boltzmann distribution*, after its originators, who calculated it based on kinetic theory, and has since been confirmed experimentally. (See [\[link\]](#).) The distribution has a long tail, because a few molecules may go several times the rms speed. The most probable speed v_p is less than the rms speed v_{rms} . [\[link\]](#) shows that the curve is shifted to higher speeds at higher temperatures, with a broader range of speeds.



The Maxwell-Boltzmann distribution of molecular speeds in an ideal gas. The most likely speed v_p is less than the rms speed v_{rms} . Although very high speeds are possible, only a tiny fraction of the molecules have speeds that are an order of magnitude greater than v_{rms} .

The distribution of thermal speeds depends strongly on temperature. As temperature increases, the speeds are shifted to higher values and the distribution is broadened.



The Maxwell-Boltzmann distribution is shifted to

higher speeds and is broadened at higher temperatures.

What is the implication of the change in distribution with temperature shown in [\[link\]](#) for humans? All other things being equal, if a person has a fever, he or she is likely to lose more water molecules, particularly from linings along moist cavities such as the lungs and mouth, creating a dry sensation in the mouth.

Example:

Calculating Temperature: Escape Velocity of Helium Atoms

In order to escape Earth's gravity, an object near the top of the atmosphere (at an altitude of 100 km) must travel away from Earth at 11.1 km/s. This speed is called the *escape velocity*. At what temperature would helium atoms have an rms speed equal to the escape velocity?

Strategy

Identify the knowns and unknowns and determine which equations to use to solve the problem.

Solution

1. Identify the knowns: v is the escape velocity, 11.1 km/s.
2. Identify the unknowns: We need to solve for temperature, T . We also need to solve for the mass m of the helium atom.
3. Determine which equations are needed.

- To solve for mass m of the helium atom, we can use information from the periodic table:

Equation:

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}}.$$

- To solve for temperature T , we can rearrange either

Equation:

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

or

Equation:

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}$$

to yield

Equation:

$$T = \frac{m\overline{v^2}}{3k},$$

where k is the Boltzmann constant and m is the mass of a helium atom.

4. Plug the known values into the equations and solve for the unknowns.

Equation:

$$m = \frac{\text{molar mass}}{\text{number of atoms per mole}} = \frac{4.0026 \times 10^{-3} \text{ kg/mol}}{6.02 \times 10^{23} \text{ mol}} = 6.65 \times 10^{-27} \text{ kg}$$

Equation:

$$T = \frac{(6.65 \times 10^{-27} \text{ kg})(11.1 \times 10^3 \text{ m/s})^2}{3(1.38 \times 10^{-23} \text{ J/K})} = 1.98 \times 10^4 \text{ K}$$

Discussion

This temperature is much higher than atmospheric temperature, which is approximately 250 K (-25°C or -10°F) at high altitude. Very few helium atoms are left in the atmosphere, but there were many when the atmosphere was formed. The reason for the loss of helium atoms is that there are a small number of helium atoms with speeds higher than Earth's escape velocity even at normal temperatures. The speed of a helium atom changes from one instant to the next, so that at any instant, there is a small, but nonzero chance that the speed is greater than the escape speed and the molecule escapes from Earth's gravitational pull. Heavier molecules, such as oxygen, nitrogen, and water (very little of which reach a very high altitude), have smaller rms speeds, and so it is much less likely that any of them will have speeds greater than the escape velocity. In fact, so few have speeds above the escape velocity that billions of years are required to lose significant amounts of the atmosphere. [\[link\]](#) shows the impact of a lack of an atmosphere on the Moon. Because the gravitational pull of the Moon is much weaker, it has lost almost its

entire atmosphere. The comparison between Earth and the Moon is discussed in this chapter's Problems and Exercises.



This photograph of Apollo 17 Commander Eugene Cernan driving the lunar rover on the Moon in 1972 looks as though it was taken at night with a large spotlight. In fact, the light is coming from the Sun. Because the acceleration due to gravity on the Moon is so low (about $1/6$ that of Earth), the Moon's escape velocity is much smaller. As a result, gas molecules escape very easily from the Moon, leaving it with virtually no atmosphere. Even during the daytime, the sky is black because there is no gas to scatter sunlight. (credit: Harrison H. Schmitt/NASA)

Exercise:**Check Your Understanding****Problem:**

If you consider a very small object such as a grain of pollen, in a gas, then the number of atoms and molecules striking its surface would also be relatively small. Would the grain of pollen experience any fluctuations in pressure due to statistical fluctuations in the number of gas atoms and molecules striking it in a given amount of time?

Solution:

Yes. Such fluctuations actually occur for a body of any size in a gas, but since the numbers of atoms and molecules are immense for macroscopic bodies, the fluctuations are a tiny percentage of the number of collisions, and the averages spoken of in this section vary imperceptibly. Roughly speaking the fluctuations are proportional to the inverse square root of the number of collisions, so for small bodies they can become significant. This was actually observed in the 19th century for pollen grains in water, and is known as the Brownian effect.

Note:

PhET Explorations: Gas Properties

Pump gas molecules into a box and see what happens as you change the volume, add or remove heat, change gravity, and more. Measure the temperature and pressure, and discover how the properties of the gas vary in relation to each other.

[Gas](#)
[Propertie](#)
[s](#)

Section Summary

- Kinetic theory is the atomistic description of gases as well as liquids and solids.

- Kinetic theory models the properties of matter in terms of continuous random motion of atoms and molecules.
- The ideal gas law can also be expressed as

Equation:

$$PV = \frac{1}{3}Nm\overline{v^2},$$

where P is the pressure (average force per unit area), V is the volume of gas in the container, N is the number of molecules in the container, m is the mass of a molecule, and $\overline{v^2}$ is the average of the molecular speed squared.

- Thermal energy is defined to be the average translational kinetic energy $\overline{\text{KE}}$ of an atom or molecule.
- The temperature of gases is proportional to the average translational kinetic energy of atoms and molecules.

Equation:

$$\overline{\text{KE}} = \frac{1}{2}m\overline{v^2} = \frac{3}{2}kT$$

or

Equation:

$$\sqrt{\overline{v^2}} = v_{\text{rms}} = \sqrt{\frac{3kT}{m}}.$$

- The motion of individual molecules in a gas is random in magnitude and direction. However, a gas of many molecules has a predictable distribution of molecular speeds, known as the *Maxwell-Boltzmann distribution*.

Conceptual Questions

Exercise:

Problem:

How is momentum related to the pressure exerted by a gas? Explain on the atomic and molecular level, considering the behavior of atoms and molecules.

Problems & Exercises

Exercise:**Problem:**

Some incandescent light bulbs are filled with argon gas. What is v_{rms} for argon atoms near the filament, assuming their temperature is 2500 K?

Solution:

$$1.25 \times 10^3 \text{ m/s}$$

Exercise:**Problem:**

Average atomic and molecular speeds (v_{rms}) are large, even at low temperatures. What is v_{rms} for helium atoms at 5.00 K, just one degree above helium's liquefaction temperature?

Exercise:**Problem:**

(a) What is the average kinetic energy in joules of hydrogen atoms on the 5500°C surface of the Sun? (b) What is the average kinetic energy of helium atoms in a region of the solar corona where the temperature is $6.00 \times 10^5 \text{ K}$?

Solution:

(a) $1.20 \times 10^{-19} \text{ J}$

(b) $1.24 \times 10^{-17} \text{ J}$

Exercise:**Problem:**

The escape velocity of any object from Earth is 11.2 km/s. (a) Express this speed in m/s and km/h. (b) At what temperature would oxygen molecules (molecular mass is equal to 32.0 g/mol) have an average velocity v_{rms} equal to Earth's escape velocity of 11.1 km/s?

Exercise:

Problem:

The escape velocity from the Moon is much smaller than from Earth and is only 2.38 km/s. At what temperature would hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity v_{rms} equal to the Moon's escape velocity?

Solution:

458 K

Exercise:**Problem:**

Nuclear fusion, the energy source of the Sun, hydrogen bombs, and fusion reactors, occurs much more readily when the average kinetic energy of the atoms is high—that is, at high temperatures. Suppose you want the atoms in your fusion experiment to have average kinetic energies of 6.40×10^{-14} J. What temperature is needed?

Exercise:**Problem:**

Suppose that the average velocity (v_{rms}) of carbon dioxide molecules (molecular mass is equal to 44.0 g/mol) in a flame is found to be 1.05×10^5 m/s. What temperature does this represent?

Solution:

1.95×10^7 K

Exercise:**Problem:**

Hydrogen molecules (molecular mass is equal to 2.016 g/mol) have an average velocity v_{rms} equal to 193 m/s. What is the temperature?

Exercise:

Problem:

Much of the gas near the Sun is atomic hydrogen. Its temperature would have to be 1.5×10^7 K for the average velocity v_{rms} to equal the escape velocity from the Sun. What is that velocity?

Solution:

$$6.09 \times 10^5 \text{ m/s}$$

Exercise:**Problem:**

There are two important isotopes of uranium— ^{235}U and ^{238}U ; these isotopes are nearly identical chemically but have different atomic masses. Only ^{235}U is very useful in nuclear reactors. One of the techniques for separating them (gas diffusion) is based on the different average velocities v_{rms} of uranium hexafluoride gas, UF_6 . (a) The molecular masses for $^{235}\text{U UF}_6$ and $^{238}\text{U UF}_6$ are 349.0 g/mol and 352.0 g/mol, respectively. What is the ratio of their average velocities? (b) At what temperature would their average velocities differ by 1.00 m/s? (c) Do your answers in this problem imply that this technique may be difficult?

Glossary

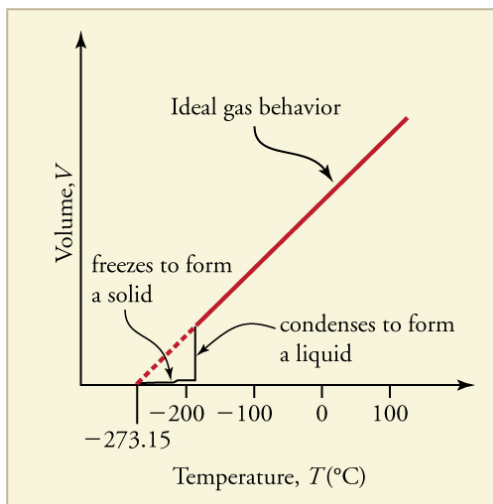
thermal energy

$\overline{\text{KE}}$, the average translational kinetic energy of a molecule

Phase Changes

- Interpret a phase diagram.
- State Dalton's law.
- Identify and describe the triple point of a gas from its phase diagram.
- Describe the state of equilibrium between a liquid and a gas, a liquid and a solid, and a gas and a solid.

Up to now, we have considered the behavior of ideal gases. Real gases are like ideal gases at high temperatures. At lower temperatures, however, the interactions between the molecules and their volumes cannot be ignored. The molecules are very close (condensation occurs) and there is a dramatic decrease in volume, as seen in [\[link\]](#). The substance changes from a gas to a liquid. When a liquid is cooled to even lower temperatures, it becomes a solid. The volume never reaches zero because of the finite volume of the molecules.



A sketch of volume versus temperature for a real gas at constant pressure. The linear (straight line) part of the graph represents ideal gas behavior—volume and temperature are directly and positively related and

the line extrapolates to zero volume at -273.15°C , or absolute zero. When the gas becomes a liquid, however, the volume actually decreases precipitously at the liquefaction point. The volume decreases slightly once the substance is solid, but it never becomes zero.

High pressure may also cause a gas to change phase to a liquid. Carbon dioxide, for example, is a gas at room temperature and atmospheric pressure, but becomes a liquid under sufficiently high pressure. If the pressure is reduced, the temperature drops and the liquid carbon dioxide solidifies into a snow-like substance at the temperature -78°C . Solid CO_2 is called “dry ice.” Another example of a gas that can be in a liquid phase is liquid nitrogen (LN_2). LN_2 is made by liquefaction of atmospheric air (through compression and cooling). It boils at 77 K (-196°C) at atmospheric pressure. LN_2 is useful as a refrigerant and allows for the preservation of blood, sperm, and other biological materials. It is also used to reduce noise in electronic sensors and equipment, and to help cool down their current-carrying wires. In dermatology, LN_2 is used to freeze and painlessly remove warts and other growths from the skin.

PV Diagrams

We can examine aspects of the behavior of a substance by plotting a graph of pressure versus volume, called a **PV diagram**. When the substance behaves like an ideal gas, the ideal gas law describes the relationship between its pressure and volume. That is,

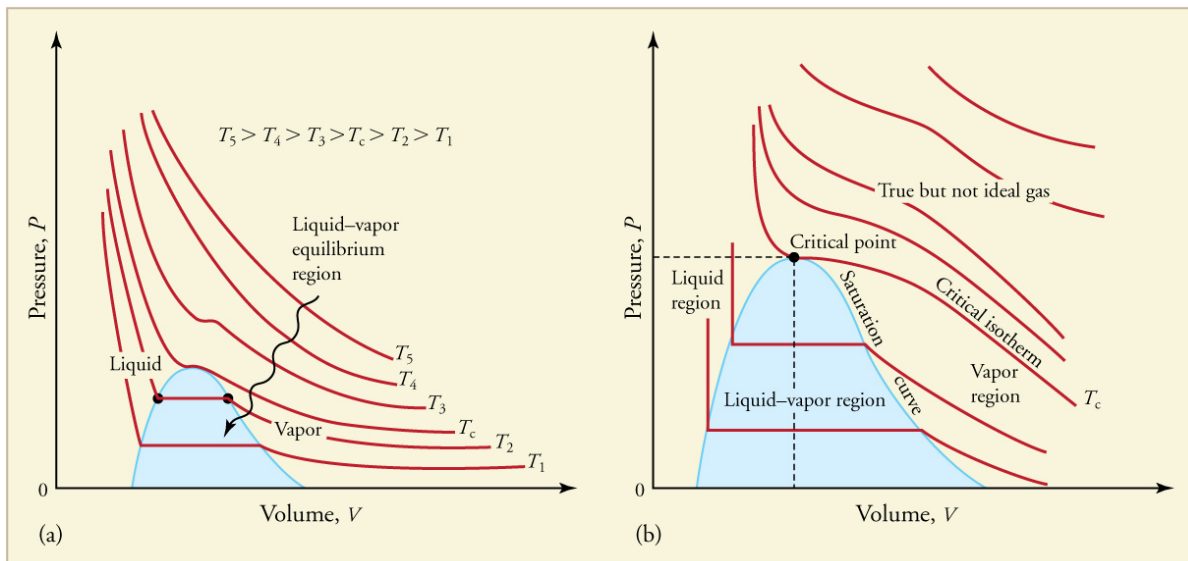
Equation:

$$PV = NkT \text{ (ideal gas).}$$

Now, assuming the number of molecules and the temperature are fixed,
Equation:

$$PV = \text{constant (ideal gas, constant temperature).}$$

For example, the volume of the gas will decrease as the pressure increases. If you plot the relationship $PV = \text{constant}$ on a PV diagram, you find a hyperbola. [\[link\]](#) shows a graph of pressure versus volume. The hyperbolas represent ideal-gas behavior at various fixed temperatures, and are called *isotherms*. At lower temperatures, the curves begin to look less like hyperbolas—the gas is not behaving ideally and may even contain liquid. There is a **critical point**—that is, a **critical temperature**—above which liquid cannot exist. At sufficiently high pressure above the critical point, the gas will have the density of a liquid but will not condense. Carbon dioxide, for example, cannot be liquefied at a temperature above 31.0°C. **Critical pressure** is the minimum pressure needed for liquid to exist at the critical temperature. [\[link\]](#) lists representative critical temperatures and pressures.



PV diagrams. (a) Each curve (isotherm) represents the relationship

between P and V at a fixed temperature; the upper curves are at higher temperatures. The lower curves are not hyperbolas, because the gas is no longer an ideal gas. (b) An expanded portion of the PV diagram for low temperatures, where the phase can change from a gas to a liquid. The term “vapor” refers to the gas phase when it exists at a temperature below the boiling temperature.

Substance	Critical temperature		Critical pressure	
	K	°C	Pa	atm
Water	647.4	374.3	22.12×10^6	219.0
Sulfur dioxide	430.7	157.6	7.88×10^6	78.0
Ammonia	405.5	132.4	11.28×10^6	111.7
Carbon dioxide	304.2	31.1	7.39×10^6	73.2

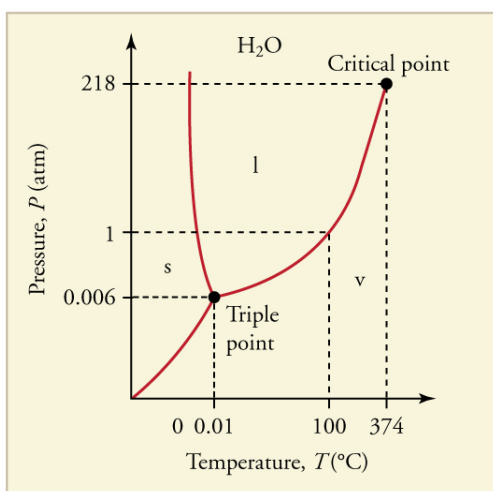
Substance	Critical temperature		Critical pressure	
	K	°C	Pa	atm
Oxygen	154.8	−118.4	5.08×10^6	50.3
Nitrogen	126.2	−146.9	3.39×10^6	33.6
Hydrogen	33.3	−239.9	1.30×10^6	12.9
Helium	5.3	−267.9	0.229×10^6	2.27

Critical Temperatures and Pressures

Phase Diagrams

The plots of pressure versus temperatures provide considerable insight into thermal properties of substances. There are well-defined regions on these graphs that correspond to various phases of matter, so PT graphs are called **phase diagrams**. [\[link\]](#) shows the phase diagram for water. Using the graph, if you know the pressure and temperature you can determine the phase of water. The solid lines—boundaries between phases—indicate temperatures and pressures at which the phases coexist (that is, they exist together in ratios, depending on pressure and temperature). For example, the boiling point of water is 100°C at 1.00 atm. As the pressure increases, the boiling temperature rises steadily to 374°C at a pressure of 218 atm. A pressure cooker (or even a covered pot) will cook food faster because the

water can exist as a liquid at temperatures greater than 100°C without all boiling away. The curve ends at a point called the *critical point*, because at higher temperatures the liquid phase does not exist at any pressure. The critical point occurs at the critical temperature, as you can see for water from [\[link\]](#). The critical temperature for oxygen is -118°C , so oxygen cannot be liquefied above this temperature.



The phase diagram (PT graph) for water. Note that the axes are nonlinear and the graph is not to scale. This graph is simplified—there are several other exotic phases of ice at higher pressures.

Similarly, the curve between the solid and liquid regions in [\[link\]](#) gives the melting temperature at various pressures. For example, the melting point is 0°C at 1.00 atm, as expected. Note that, at a fixed temperature, you can change the phase from solid (ice) to liquid (water) by increasing the pressure. Ice melts from pressure in the hands of a snowball maker. From

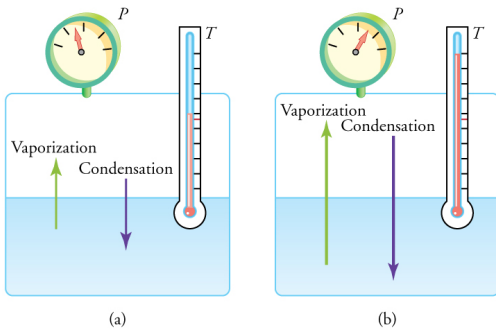
the phase diagram, we can also say that the melting temperature of ice rises with increased pressure. When a car is driven over snow, the increased pressure from the tires melts the snowflakes; afterwards the water refreezes and forms an ice layer.

At sufficiently low pressures there is no liquid phase, but the substance can exist as either gas or solid. For water, there is no liquid phase at pressures below 0.00600 atm. The phase change from solid to gas is called **sublimation**. It accounts for large losses of snow pack that never make it into a river, the routine automatic defrosting of a freezer, and the freeze-drying process applied to many foods. Carbon dioxide, on the other hand, sublimates at standard atmospheric pressure of 1 atm. (The solid form of CO_2 is known as dry ice because it does not melt. Instead, it moves directly from the solid to the gas state.)

All three curves on the phase diagram meet at a single point, the **triple point**, where all three phases exist in equilibrium. For water, the triple point occurs at 273.16 K (0.01°C), and is a more accurate calibration temperature than the melting point of water at 1.00 atm, or 273.15 K (0.0°C). See [\[link\]](#) for the triple point values of other substances.

Equilibrium

Liquid and gas phases are in equilibrium at the boiling temperature. (See [\[link\]](#).) If a substance is in a closed container at the boiling point, then the liquid is boiling and the gas is condensing at the same rate without net change in their relative amount. Molecules in the liquid escape as a gas at the same rate at which gas molecules stick to the liquid, or form droplets and become part of the liquid phase. The combination of temperature and pressure has to be “just right”; if the temperature and pressure are increased, equilibrium is maintained by the same increase of boiling and condensation rates.



Equilibrium between liquid and gas at two different boiling points inside a closed container.

(a) The rates of boiling and condensation are equal at this combination of temperature and pressure, so the liquid and

gas phases are in equilibrium. (b) At a higher temperature, the boiling rate is faster and the rates at which

molecules leave the liquid and enter the gas are also

faster. Because there are more molecules in the gas, the gas pressure is higher and the rate at which gas molecules

condense and enter the liquid is faster. As a result the gas and liquid are in equilibrium at this higher temperature.

Substance	Temperature		Pressure	
	K	°C	Pa	atm
Water	273.16	0.01	6.10×10^2	0.00600
Carbon dioxide	216.55	-56.60	5.16×10^5	5.11
Sulfur dioxide	197.68	-75.47	1.67×10^3	0.0167
Ammonia	195.40	-77.75	6.06×10^3	0.0600
Nitrogen	63.18	-210.0	1.25×10^4	0.124
Oxygen	54.36	-218.8	1.52×10^2	0.00151
Hydrogen	13.84	-259.3	7.04×10^3	0.0697

Triple Point Temperatures and Pressures

One example of equilibrium between liquid and gas is that of water and steam at 100°C and 1.00 atm. This temperature is the boiling point at that pressure, so they should exist in equilibrium. Why does an open pot of water at 100°C boil completely away? The gas surrounding an open pot is

not pure water: it is mixed with air. If pure water and steam are in a closed container at 100°C and 1.00 atm , they would coexist—but with air over the pot, there are fewer water molecules to condense, and water boils. What about water at 20.0°C and 1.00 atm ? This temperature and pressure correspond to the liquid region, yet an open glass of water at this temperature will completely evaporate. Again, the gas around it is air and not pure water vapor, so that the reduced evaporation rate is greater than the condensation rate of water from dry air. If the glass is sealed, then the liquid phase remains. We call the gas phase a **vapor** when it exists, as it does for water at 20.0°C , at a temperature below the boiling temperature.

Exercise:

Check Your Understanding

Problem:

Explain why a cup of water (or soda) with ice cubes stays at 0°C , even on a hot summer day.

Solution:

The ice and liquid water are in thermal equilibrium, so that the temperature stays at the freezing temperature as long as ice remains in the liquid. (Once all of the ice melts, the water temperature will start to rise.)

Vapor Pressure, Partial Pressure, and Dalton's Law

Vapor pressure is defined as the pressure at which a gas coexists with its solid or liquid phase. Vapor pressure is created by faster molecules that break away from the liquid or solid and enter the gas phase. The vapor pressure of a substance depends on both the substance and its temperature—an increase in temperature increases the vapor pressure.

Partial pressure is defined as the pressure a gas would create if it occupied the total volume available. In a mixture of gases, *the total pressure is the sum of partial pressures of the component gases*, assuming ideal gas behavior and no chemical reactions between the components. This law is

known as **Dalton's law of partial pressures**, after the English scientist John Dalton (1766–1844), who proposed it. Dalton's law is based on kinetic theory, where each gas creates its pressure by molecular collisions, independent of other gases present. It is consistent with the fact that pressures add according to [Pascal's Principle](#). Thus water evaporates and ice sublimates when their vapor pressures exceed the partial pressure of water vapor in the surrounding mixture of gases. If their vapor pressures are less than the partial pressure of water vapor in the surrounding gas, liquid droplets or ice crystals (frost) form.

Exercise:

Check Your Understanding

Problem:

Is energy transfer involved in a phase change? If so, will energy have to be supplied to change phase from solid to liquid and liquid to gas? What about gas to liquid and liquid to solid? Why do they spray the orange trees with water in Florida when the temperatures are near or just below freezing?

Solution:

Yes, energy transfer is involved in a phase change. We know that atoms and molecules in solids and liquids are bound to each other because we know that force is required to separate them. So in a phase change from solid to liquid and liquid to gas, a force must be exerted, perhaps by collision, to separate atoms and molecules. Force exerted through a distance is work, and energy is needed to do work to go from solid to liquid and liquid to gas. This is intuitively consistent with the need for energy to melt ice or boil water. The converse is also true. Going from gas to liquid or liquid to solid involves atoms and molecules pushing together, doing work and releasing energy.

Note:

PhET Explorations: States of Matter—Basics

Heat, cool, and compress atoms and molecules and watch as they change between solid, liquid, and gas phases.

https://phet.colorado.edu/sims/html/states-of-matter-basics/latest/states-of-matter-basics_en.html

Section Summary

- Most substances have three distinct phases: gas, liquid, and solid.
- Phase changes among the various phases of matter depend on temperature and pressure.
- The existence of the three phases with respect to pressure and temperature can be described in a phase diagram.
- Two phases coexist (i.e., they are in thermal equilibrium) at a set of pressures and temperatures. These are described as a line on a phase diagram.
- The three phases coexist at a single pressure and temperature. This is known as the triple point and is described by a single point on a phase diagram.
- A gas at a temperature below its boiling point is called a vapor.
- Vapor pressure is the pressure at which a gas coexists with its solid or liquid phase.
- Partial pressure is the pressure a gas would create if it existed alone.
- Dalton's law states that the total pressure is the sum of the partial pressures of all of the gases present.

Conceptual Questions

Exercise:

Problem:

A pressure cooker contains water and steam in equilibrium at a pressure greater than atmospheric pressure. How does this greater pressure increase cooking speed?

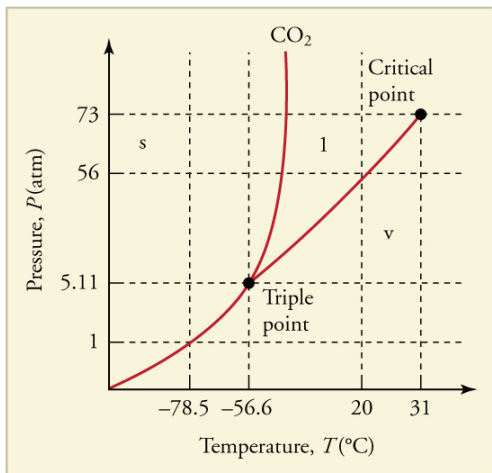
Exercise:

Problem:

Why does condensation form most rapidly on the coldest object in a room—for example, on a glass of ice water?

Exercise:**Problem:**

What is the vapor pressure of solid carbon dioxide (dry ice) at -78.5°C ?



The phase diagram for carbon dioxide. The axes are nonlinear, and the graph is not to scale. Dry ice is solid carbon dioxide and has a sublimation temperature of -78.5°C .

Exercise:**Problem:**

Can carbon dioxide be liquefied at room temperature (20°C)? If so, how? If not, why not? (See [\[link\]](#).)

Exercise:**Problem:**

Oxygen cannot be liquefied at room temperature by placing it under a large enough pressure to force its molecules together. Explain why this is.

Exercise:

Problem: What is the distinction between gas and vapor?

Glossary

PV diagram

a graph of pressure vs. volume

critical point

the temperature above which a liquid cannot exist

critical temperature

the temperature above which a liquid cannot exist

critical pressure

the minimum pressure needed for a liquid to exist at the critical temperature

vapor

a gas at a temperature below the boiling temperature

vapor pressure

the pressure at which a gas coexists with its solid or liquid phase

phase diagram

a graph of pressure vs. temperature of a particular substance, showing at which pressures and temperatures the three phases of the substance occur

triple point

the pressure and temperature at which a substance exists in equilibrium as a solid, liquid, and gas

sublimation

the phase change from solid to gas

partial pressure

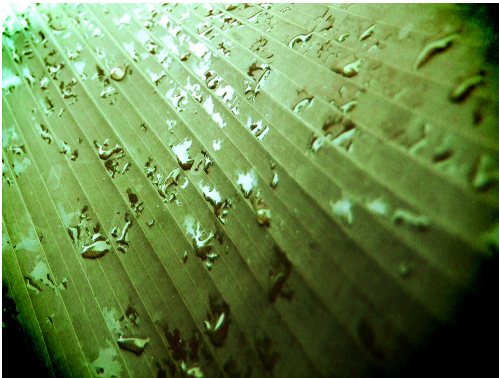
the pressure a gas would create if it occupied the total volume of space available

Dalton's law of partial pressures

the physical law that states that the total pressure of a gas is the sum of partial pressures of the component gases

Humidity, Evaporation, and Boiling

- Explain the relationship between vapor pressure of water and the capacity of air to hold water vapor.
- Explain the relationship between relative humidity and partial pressure of water vapor in the air.
- Calculate vapor density using vapor pressure.
- Calculate humidity and dew point.



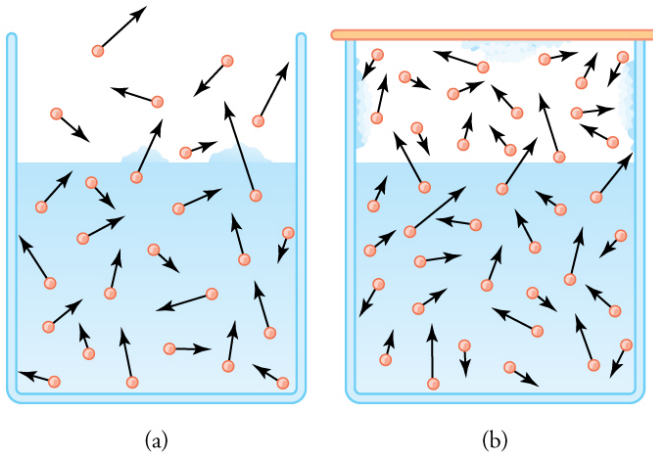
Dew drops like these, on a banana leaf photographed just after sunrise, form when the air temperature drops to or below the dew point. At the dew point, the rate at which water molecules join together is greater than the rate at which they separate, and some of the water condenses to form droplets. (credit: Aaron Escobar, Flickr)

The expression “it’s not the heat, it’s the humidity” makes a valid point. We keep cool in hot weather by evaporating sweat from our skin and water

from our breathing passages. Because evaporation is inhibited by high humidity, we feel hotter at a given temperature when the humidity is high. Low humidity, on the other hand, can cause discomfort from excessive drying of mucous membranes and can lead to an increased risk of respiratory infections.

When we say humidity, we really mean **relative humidity**. Relative humidity tells us how much water vapor is in the air compared with the maximum possible. At its maximum, denoted as **saturation**, the relative humidity is 100%, and evaporation is inhibited. The amount of water vapor in the air depends on temperature. For example, relative humidity rises in the evening, as air temperature declines, sometimes reaching the **dew point**. At the dew point temperature, relative humidity is 100%, and fog may result from the condensation of water droplets if they are small enough to stay in suspension. Conversely, if you wish to dry something (perhaps your hair), it is more effective to blow hot air over it rather than cold air, because, among other things, the increase in temperature increases the energy of the molecules, so the rate of evaporation increases.

The amount of water vapor in the air depends on the vapor pressure of water. The liquid and solid phases are continuously giving off vapor because some of the molecules have high enough speeds to enter the gas phase; see [\[link\]](#)(a). If a lid is placed over the container, as in [\[link\]](#)(b), evaporation continues, increasing the pressure, until sufficient vapor has built up for condensation to balance evaporation. Then equilibrium has been achieved, and the vapor pressure is equal to the partial pressure of water in the container. Vapor pressure increases with temperature because molecular speeds are higher as temperature increases. [\[link\]](#) gives representative values of water vapor pressure over a range of temperatures.



(a) Because of the distribution of speeds and kinetic energies, some water molecules can break away to the vapor phase even at temperatures below the ordinary boiling point. (b) If the container is sealed, evaporation will continue until there is enough vapor density for the condensation rate to equal the evaporation rate. This vapor density and the partial pressure it creates are the saturation values. They increase with temperature and are independent of the presence of other gases, such as air. They depend only on the vapor pressure of water.

Relative humidity is related to the partial pressure of water vapor in the air. At 100% humidity, the partial pressure is equal to the vapor pressure, and no more water can enter the vapor phase. If the partial pressure is less than the vapor pressure, then evaporation will take place, as humidity is less than 100%. If the partial pressure is greater than the vapor pressure, condensation takes place. In everyday language, people sometimes refer to

the capacity of air to “hold” water vapor, but this is not actually what happens. The water vapor is not held by the air. The amount of water in air is determined by the vapor pressure of water and has nothing to do with the properties of air.

Temperature (°C)	Vapor pressure (Pa)	Saturation vapor density (g/m ³)
−50	4.0	0.039
−20	1.04×10^2	0.89
−10	2.60×10^2	2.36
0	6.10×10^2	4.84
5	8.68×10^2	6.80
10	1.19×10^3	9.40

Temperature (°C)	Vapor pressure (Pa)	Saturation vapor density (g/m ³)
15	1.69×10^3	12.8
20	2.33×10^3	17.2
25	3.17×10^3	23.0
30	4.24×10^3	30.4
37	6.31×10^3	44.0
40	7.34×10^3	51.1
50	1.23×10^4	82.4
60	1.99×10^4	130
70	3.12×10^4	197

Temperature (°C)	Vapor pressure (Pa)	Saturation vapor density (g/m ³)
80	4.73×10^4	294
90	7.01×10^4	418
95	8.59×10^4	505
100	1.01×10^5	598
120	1.99×10^5	1095
150	4.76×10^5	2430
200	1.55×10^6	7090
220	2.32×10^6	10,200

Saturation Vapor Density of Water

Example:**Calculating Density Using Vapor Pressure**

[\[link\]](#) gives the vapor pressure of water at 20.0°C as 2.33×10^3 Pa. Use the ideal gas law to calculate the density of water vapor in g/m³ that would create a partial pressure equal to this vapor pressure. Compare the result with the saturation vapor density given in the table.

Strategy

To solve this problem, we need to break it down into a two steps. The partial pressure follows the ideal gas law,

Equation:

$$PV = nRT,$$

where n is the number of moles. If we solve this equation for n/V to calculate the number of moles per cubic meter, we can then convert this quantity to grams per cubic meter as requested. To do this, we need to use the molecular mass of water, which is given in the periodic table.

Solution

1. Identify the knowns and convert them to the proper units:

- temperature $T = 20^\circ\text{C} = 293$ K
- vapor pressure P of water at 20°C is 2.33×10^3 Pa
- molecular mass of water is 18.0 g/mol

2. Solve the ideal gas law for n/V .

Equation:

$$\frac{n}{V} = \frac{P}{RT}$$

3. Substitute known values into the equation and solve for n/V .

Equation:

$$\frac{n}{V} = \frac{P}{RT} = \frac{2.33 \times 10^3 \text{ Pa}}{(8.31 \text{ J/mol} \cdot \text{K})(293 \text{ K})} = 0.957 \text{ mol/m}^3$$

4. Convert the density in moles per cubic meter to grams per cubic meter.

Equation:

$$\rho = \left(0.957 \frac{\text{mol}}{\text{m}^3}\right) \left(\frac{18.0 \text{ g}}{\text{mol}}\right) = 17.2 \text{ g/m}^3$$

Discussion

The density is obtained by assuming a pressure equal to the vapor pressure of water at 20.0°C. The density found is identical to the value in [\[link\]](#), which means that a vapor density of 17.2 g/m³ at 20.0°C creates a partial pressure of 2.33×10^3 Pa, equal to the vapor pressure of water at that temperature. If the partial pressure is equal to the vapor pressure, then the liquid and vapor phases are in equilibrium, and the relative humidity is 100%. Thus, there can be no more than 17.2 g of water vapor per m³ at 20.0°C, so that this value is the saturation vapor density at that temperature. This example illustrates how water vapor behaves like an ideal gas: the pressure and density are consistent with the ideal gas law (assuming the density in the table is correct). The saturation vapor densities listed in [\[link\]](#) are the maximum amounts of water vapor that air can hold at various temperatures.

Note:**Percent Relative Humidity**

We define **percent relative humidity** as the ratio of vapor density to saturation vapor density, or

Equation:

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100$$

We can use this and the data in [\[link\]](#) to do a variety of interesting calculations, keeping in mind that relative humidity is based on the comparison of the partial pressure of water vapor in air and ice.

Example:**Calculating Humidity and Dew Point**

(a) Calculate the percent relative humidity on a day when the temperature is 25.0°C and the air contains 9.40 g of water vapor per m³. (b) At what temperature will this air reach 100% relative humidity (the saturation density)? This temperature is the dew point. (c) What is the humidity when the air temperature is 25.0°C and the dew point is – 10.0°C?

Strategy and Solution

(a) Percent relative humidity is defined as the ratio of vapor density to saturation vapor density.

Equation:

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100$$

The first is given to be 9.40 g/m³, and the second is found in [\[link\]](#) to be 23.0 g/m³. Thus,

Equation:

$$\text{percent relative humidity} = \frac{9.40 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 40.9\%$$

(b) The air contains 9.40 g/m³ of water vapor. The relative humidity will be 100% at a temperature where 9.40 g/m³ is the saturation density. Inspection of [\[link\]](#) reveals this to be the case at 10.0°C, where the relative humidity will be 100%. That temperature is called the dew point for air with this concentration of water vapor.

(c) Here, the dew point temperature is given to be – 10.0°C. Using [\[link\]](#), we see that the vapor density is 2.36 g/m³, because this value is the saturation vapor density at – 10.0°C. The saturation vapor density at 25.0°C is seen to be 23.0 g/m³. Thus, the relative humidity at 25.0°C is

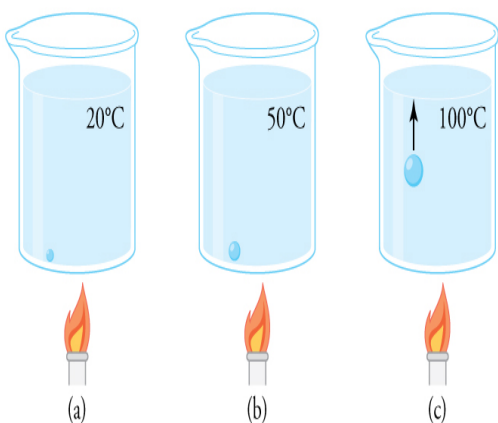
Equation:

$$\text{percent relative humidity} = \frac{2.36 \text{ g/m}^3}{23.0 \text{ g/m}^3} \times 100 = 10.3\%.$$

Discussion

The importance of dew point is that air temperature cannot drop below 10.0°C in part (b), or -10.0°C in part (c), without water vapor condensing out of the air. If condensation occurs, considerable transfer of heat occurs (discussed in [Heat and Heat Transfer Methods](#)), which prevents the temperature from further dropping. When dew points are below 0°C , freezing temperatures are a greater possibility, which explains why farmers keep track of the dew point. Low humidity in deserts means low dew-point temperatures. Thus condensation is unlikely. If the temperature drops, vapor does not condense in liquid drops. Because no heat is released into the air, the air temperature drops more rapidly compared to air with higher humidity. Likewise, at high temperatures, liquid droplets do not evaporate, so that no heat is removed from the gas to the liquid phase. This explains the large range of temperature in arid regions.

Why does water boil at 100°C ? You will note from [\[link\]](#) that the vapor pressure of water at 100°C is $1.01 \times 10^5 \text{ Pa}$, or 1.00 atm. Thus, it can evaporate without limit at this temperature and pressure. But why does it form bubbles when it boils? This is because water ordinarily contains significant amounts of dissolved air and other impurities, which are observed as small bubbles of air in a glass of water. If a bubble starts out at the bottom of the container at 20°C , it contains water vapor (about 2.30%). The pressure inside the bubble is fixed at 1.00 atm (we ignore the slight pressure exerted by the water around it). As the temperature rises, the amount of air in the bubble stays the same, but the water vapor increases; the bubble expands to keep the pressure at 1.00 atm. At 100°C , water vapor enters the bubble continuously since the partial pressure of water is equal to 1.00 atm in equilibrium. It cannot reach this pressure, however, since the bubble also contains air and total pressure is 1.00 atm. The bubble grows in size and thereby increases the buoyant force. The bubble breaks away and rises rapidly to the surface—we call this boiling! (See [\[link\]](#).)



(a) An air bubble in water starts out saturated with water vapor at 20°C. (b)

As the temperature rises, water vapor enters the bubble because its vapor pressure increases. The bubble expands to keep its pressure at 1.00 atm.

(c) At 100°C, water vapor enters the bubble continuously because water's vapor pressure exceeds its partial pressure in the bubble, which must be less than 1.00 atm. The bubble grows and rises to the surface.

Exercise:

Check Your Understanding

Problem:

Freeze drying is a process in which substances, such as foods, are dried by placing them in a vacuum chamber and lowering the atmospheric pressure around them. How does the lowered atmospheric pressure speed the drying process, and why does it cause the temperature of the food to drop?

Solution:

Decreased the atmospheric pressure results in decreased partial pressure of water, hence a lower humidity. So evaporation of water from food, for example, will be enhanced. The molecules of water most likely to break away from the food will be those with the greatest velocities. Those remaining thus have a lower average velocity and a lower temperature. This can (and does) result in the freezing and drying of the food; hence the process is aptly named freeze drying.

Note:

PhET Explorations: States of Matter

Watch different types of molecules form a solid, liquid, or gas. Add or remove heat and watch the phase change. Change the temperature or volume of a container and see a pressure-temperature diagram respond in real time. Relate the interaction potential to the forces between molecules.

https://phet.colorado.edu/sims/html/states-of-matter/latest/states-of-matter_en.html

Section Summary

- Relative humidity is the fraction of water vapor in a gas compared to the saturation value.
- The saturation vapor density can be determined from the vapor pressure for a given temperature.

- Percent relative humidity is defined to be

Equation:

$$\text{percent relative humidity} = \frac{\text{vapor density}}{\text{saturation vapor density}} \times 100.$$

- The dew point is the temperature at which air reaches 100% relative humidity.

Conceptual Questions

Exercise:

Problem:

Because humidity depends only on water's vapor pressure and temperature, are the saturation vapor densities listed in [\[link\]](#) valid in an atmosphere of helium at a pressure of $1.01 \times 10^5 \text{ N/m}^2$, rather than air? Are those values affected by altitude on Earth?

Exercise:

Problem:

Why does a beaker of 40.0°C water placed in a vacuum chamber start to boil as the chamber is evacuated (air is pumped out of the chamber)? At what pressure does the boiling begin? Would food cook any faster in such a beaker?

Exercise:

Problem:

Why does rubbing alcohol evaporate much more rapidly than water at STP (standard temperature and pressure)?

Problems & Exercises

Exercise:

Problem:

Dry air is 78.1% nitrogen. What is the partial pressure of nitrogen when the atmospheric pressure is $1.01 \times 10^5 \text{ N/m}^2$?

Solution:

$$7.89 \times 10^4 \text{ Pa}$$

Exercise:**Problem:**

(a) What is the vapor pressure of water at 20.0°C ? (b) What percentage of atmospheric pressure does this correspond to? (c) What percent of 20.0°C air is water vapor if it has 100% relative humidity? (The density of dry air at 20.0°C is 1.20 kg/m^3 .)

Exercise:**Problem:**

Pressure cookers increase cooking speed by raising the boiling temperature of water above its value at atmospheric pressure. (a) What pressure is necessary to raise the boiling point to 120.0°C ? (b) What gauge pressure does this correspond to?

Solution:

(a) $1.99 \times 10^5 \text{ Pa}$

(b) 0.97 atm

Exercise:

Problem:

(a) At what temperature does water boil at an altitude of 1500 m (about 5000 ft) on a day when atmospheric pressure is $8.59 \times 10^4 \text{ N/m}^2$? (b) What about at an altitude of 3000 m (about 10,000 ft) when atmospheric pressure is $7.00 \times 10^4 \text{ N/m}^2$?

Exercise:**Problem:**

What is the atmospheric pressure on top of Mt. Everest on a day when water boils there at a temperature of 70.0°C ?

Solution:

$$3.12 \times 10^4 \text{ Pa}$$

Exercise:**Problem:**

At a spot in the high Andes, water boils at 80.0°C , greatly reducing the cooking speed of potatoes, for example. What is atmospheric pressure at this location?

Exercise:**Problem:**

What is the relative humidity on a 25.0°C day when the air contains 18.0 g/m^3 of water vapor?

Solution:

$$78.3\%$$

Exercise:

Problem:

What is the density of water vapor in g/m^3 on a hot dry day in the desert when the temperature is 40.0°C and the relative humidity is 6.00%?

Exercise:**Problem:**

A deep-sea diver should breathe a gas mixture that has the same oxygen partial pressure as at sea level, where dry air contains 20.9% oxygen and has a total pressure of $1.01 \times 10^5 \text{ N}/\text{m}^2$. (a) What is the partial pressure of oxygen at sea level? (b) If the diver breathes a gas mixture at a pressure of $2.00 \times 10^6 \text{ N}/\text{m}^2$, what percent oxygen should it be to have the same oxygen partial pressure as at sea level?

Solution:

(a) $2.12 \times 10^4 \text{ Pa}$

(b) 1.06 %

Exercise:**Problem:**

The vapor pressure of water at 40.0°C is $7.34 \times 10^3 \text{ N}/\text{m}^2$. Using the ideal gas law, calculate the density of water vapor in g/m^3 that creates a partial pressure equal to this vapor pressure. The result should be the same as the saturation vapor density at that temperature ($51.1 \text{ g}/\text{m}^3$).

Exercise:

Problem:

Air in human lungs has a temperature of 37.0°C and a saturation vapor density of 44.0 g/m^3 . (a) If 2.00 L of air is exhaled and very dry air inhaled, what is the maximum loss of water vapor by the person? (b) Calculate the partial pressure of water vapor having this density, and compare it with the vapor pressure of $6.31 \times 10^3\text{ N/m}^2$.

Solution:

(a) $8.80 \times 10^{-2}\text{ g}$

(b) $6.30 \times 10^3\text{ Pa}$; the two values are nearly identical.

Exercise:**Problem:**

If the relative humidity is 90.0% on a muggy summer morning when the temperature is 20.0°C , what will it be later in the day when the temperature is 30.0°C , assuming the water vapor density remains constant?

Exercise:**Problem:**

Late on an autumn day, the relative humidity is 45.0% and the temperature is 20.0°C . What will the relative humidity be that evening when the temperature has dropped to 10.0°C , assuming constant water vapor density?

Solution:

82.3%

Exercise:

Problem:

Atmospheric pressure atop Mt. Everest is $3.30 \times 10^4 \text{ N/m}^2$. (a) What is the partial pressure of oxygen there if it is 20.9% of the air? (b) What percent oxygen should a mountain climber breathe so that its partial pressure is the same as at sea level, where atmospheric pressure is $1.01 \times 10^5 \text{ N/m}^2$? (c) One of the most severe problems for those climbing very high mountains is the extreme drying of breathing passages. Why does this drying occur?

Exercise:**Problem:**

What is the dew point (the temperature at which 100% relative humidity would occur) on a day when relative humidity is 39.0% at a temperature of 20.0°C ?

Solution:

4.77°C

Exercise:**Problem:**

On a certain day, the temperature is 25.0°C and the relative humidity is 90.0%. How many grams of water must condense out of each cubic meter of air if the temperature falls to 15.0°C ? Such a drop in temperature can, thus, produce heavy dew or fog.

Exercise:**Problem: Integrated Concepts**

The boiling point of water increases with depth because pressure increases with depth. At what depth will fresh water have a boiling point of 150°C , if the surface of the water is at sea level?

Solution:

38.3 m

Exercise:**Problem: Integrated Concepts**

(a) At what depth in fresh water is the critical pressure of water reached, given that the surface is at sea level? (b) At what temperature will this water boil? (c) Is a significantly higher temperature needed to boil water at a greater depth?

Exercise:**Problem: Integrated Concepts**

To get an idea of the small effect that temperature has on Archimedes' principle, calculate the fraction of a copper block's weight that is supported by the buoyant force in 0°C water and compare this fraction with the fraction supported in 95.0°C water.

Solution:

$\frac{(F_B/w_{Cu})}{(F_B/w_{Cu})'} = 1.02$. The buoyant force supports nearly the exact same amount of force on the copper block in both circumstances.

Exercise:**Problem: Integrated Concepts**

If you want to cook in water at 150°C, you need a pressure cooker that can withstand the necessary pressure. (a) What pressure is required for the boiling point of water to be this high? (b) If the lid of the pressure cooker is a disk 25.0 cm in diameter, what force must it be able to withstand at this pressure?

Exercise:

Problem: Unreasonable Results

(a) How many moles per cubic meter of an ideal gas are there at a pressure of $1.00 \times 10^{14} \text{ N/m}^2$ and at 0°C ? (b) What is unreasonable about this result? (c) Which premise or assumption is responsible?

Solution:

(a) $4.41 \times 10^{10} \text{ mol/m}^3$

(b) It's unreasonably large.

(c) At high pressures such as these, the ideal gas law can no longer be applied. As a result, unreasonable answers come up when it is used.

Exercise:**Problem: Unreasonable Results**

(a) An automobile mechanic claims that an aluminum rod fits loosely into its hole on an aluminum engine block because the engine is hot and the rod is cold. If the hole is 10.0% bigger in diameter than the 22.0°C rod, at what temperature will the rod be the same size as the hole? (b) What is unreasonable about this temperature? (c) Which premise is responsible?

Exercise:**Problem: Unreasonable Results**

The temperature inside a supernova explosion is said to be $2.00 \times 10^{13} \text{ K}$. (a) What would the average velocity v_{rms} of hydrogen atoms be? (b) What is unreasonable about this velocity? (c) Which premise or assumption is responsible?

Solution:

(a) $7.03 \times 10^8 \text{ m/s}$

(b) The velocity is too high—it's greater than the speed of light.

(c) The assumption that hydrogen inside a supernova behaves as an idea gas is responsible, because of the great temperature and density in the core of a star. Furthermore, when a velocity greater than the speed of light is obtained, classical physics must be replaced by relativity, a subject not yet covered.

Exercise:

Problem: Unreasonable Results

Suppose the relative humidity is 80% on a day when the temperature is 30.0°C . (a) What will the relative humidity be if the air cools to 25.0°C and the vapor density remains constant? (b) What is unreasonable about this result? (c) Which premise is responsible?

Glossary

dew point

the temperature at which relative humidity is 100%; the temperature at which water starts to condense out of the air

saturation

the condition of 100% relative humidity

percent relative humidity

the ratio of vapor density to saturation vapor density

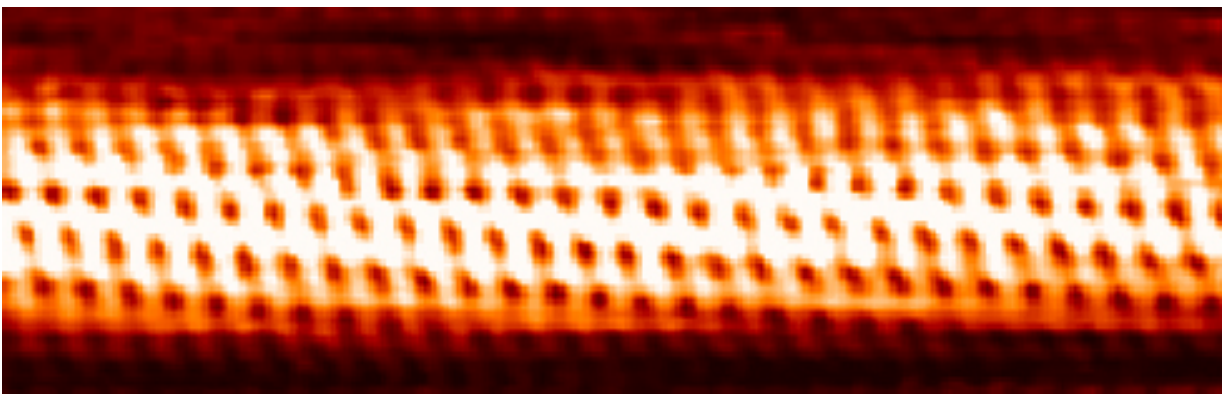
relative humidity

the amount of water in the air relative to the maximum amount the air can hold

Introduction to Atomic Physics

class="introduction"

Individual
carbon
atoms are
visible in
this image
of a carbon
nanotube
made by a
scanning
tunneling
electron
microscope
. (credit:
Taner
Yildirim,
National
Institute of
Standards
and
Technology
, via
Wikimedia
Commons)



From childhood on, we learn that atoms are a substructure of all things around us, from the air we breathe to the autumn leaves that blanket a forest trail. Invisible to the eye, the existence and properties of atoms are used to explain many phenomena—a theme found throughout this text. In this chapter, we discuss the discovery of atoms and their own substructures; we then apply quantum mechanics to the description of atoms, and their properties and interactions. Along the way, we will find, much like the scientists who made the original discoveries, that new concepts emerge with applications far beyond the boundaries of atomic physics.

Discovery of the Atom

- Describe the basic structure of the atom, the substructure of all matter.

How do we know that atoms are really there if we cannot see them with our eyes? A brief account of the progression from the proposal of atoms by the Greeks to the first direct evidence of their existence follows.

People have long speculated about the structure of matter and the existence of atoms. The earliest significant ideas to survive are due to the ancient Greeks in the fifth century BCE, especially those of the philosophers Leucippus and Democritus. (There is some evidence that philosophers in both India and China made similar speculations, at about the same time.) They considered the question of whether a substance can be divided without limit into ever smaller pieces. There are only a few possible answers to this question. One is that infinitesimally small subdivision is possible. Another is what Democritus in particular believed—that there is a smallest unit that cannot be further subdivided. Democritus called this the **atom**. We now know that atoms themselves can be subdivided, but their identity is destroyed in the process, so the Greeks were correct in a respect. The Greeks also felt that atoms were in constant motion, another correct notion.

The Greeks and others speculated about the properties of atoms, proposing that only a few types existed and that all matter was formed as various combinations of these types. The famous proposal that the basic elements were earth, air, fire, and water was brilliant, but incorrect. The Greeks had identified the most common examples of the four states of matter (solid, gas, plasma, and liquid), rather than the basic elements. More than 2000 years passed before observations could be made with equipment capable of revealing the true nature of atoms.

Over the centuries, discoveries were made regarding the properties of substances and their chemical reactions. Certain systematic features were recognized, but similarities between common and rare elements resulted in efforts to transmute them (lead into gold, in particular) for financial gain. Secrecy was endemic. Alchemists discovered and rediscovered many facts but did not make them broadly available. As the Middle Ages ended, alchemy gradually faded, and the science of chemistry arose. It was no

longer possible, nor considered desirable, to keep discoveries secret. Collective knowledge grew, and by the beginning of the 19th century, an important fact was well established—the masses of reactants in specific chemical reactions always have a particular mass ratio. This is very strong indirect evidence that there are basic units (atoms and molecules) that have these same mass ratios. The English chemist John Dalton (1766–1844) did much of this work, with significant contributions by the Italian physicist Amedeo Avogadro (1776–1856). It was Avogadro who developed the idea of a fixed number of atoms and molecules in a mole, and this special number is called Avogadro's number in his honor. The Austrian physicist Johann Josef Loschmidt was the first to measure the value of the constant in 1865 using the kinetic theory of gases.

Note:**Patterns and Systematics**

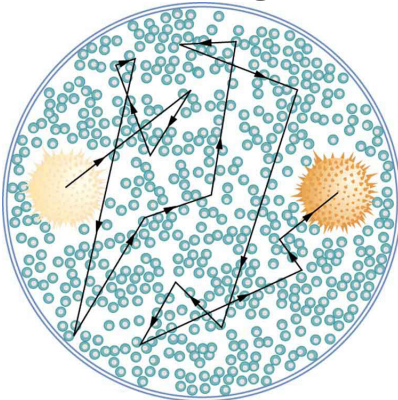
The recognition and appreciation of patterns has enabled us to make many discoveries. The periodic table of elements was proposed as an organized summary of the known elements long before all elements had been discovered, and it led to many other discoveries. We shall see in later chapters that patterns in the properties of subatomic particles led to the proposal of quarks as their underlying structure, an idea that is still bearing fruit.

Knowledge of the properties of elements and compounds grew, culminating in the mid-19th-century development of the periodic table of the elements by Dmitri Mendeleev (1834–1907), the great Russian chemist. Mendeleev proposed an ingenious array that highlighted the periodic nature of the properties of elements. Believing in the systematics of the periodic table, he also predicted the existence of then-unknown elements to complete it. Once these elements were discovered and determined to have properties predicted by Mendeleev, his periodic table became universally accepted.

Also during the 19th century, the kinetic theory of gases was developed. Kinetic theory is based on the existence of atoms and molecules in random

thermal motion and provides a microscopic explanation of the gas laws, heat transfer, and thermodynamics (see [Introduction to Temperature, Kinetic Theory, and the Gas Laws](#) and [Introduction to Laws of Thermodynamics](#)). Kinetic theory works so well that it is another strong indication of the existence of atoms. But it is still indirect evidence—individual atoms and molecules had not been observed. There were heated debates about the validity of kinetic theory until direct evidence of atoms was obtained.

The first truly direct evidence of atoms is credited to Robert Brown, a Scottish botanist. In 1827, he noticed that tiny pollen grains suspended in still water moved about in complex paths. This can be observed with a microscope for any small particles in a fluid. The motion is caused by the random thermal motions of fluid molecules colliding with particles in the fluid, and it is now called **Brownian motion**. (See [\[link\]](#).) Statistical fluctuations in the numbers of molecules striking the sides of a visible particle cause it to move first this way, then that. Although the molecules cannot be directly observed, their effects on the particle can be. By examining Brownian motion, the size of molecules can be calculated. The smaller and more numerous they are, the smaller the fluctuations in the numbers striking different sides.



The position of a
pollen grain in
water, measured
every few seconds
under a
microscope,

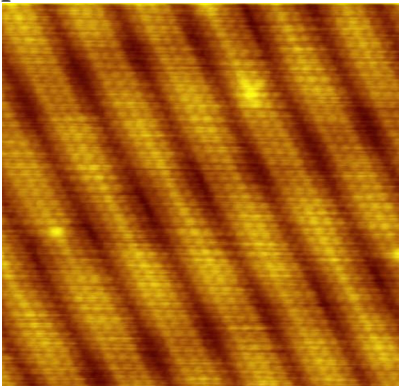
exhibits Brownian motion. Brownian motion is due to fluctuations in the number of atoms and molecules colliding with a small mass, causing it to move about in complex paths. This is nearly direct evidence for the existence of atoms, providing a satisfactory alternative explanation cannot be found.

It was Albert Einstein who, starting in his epochal year of 1905, published several papers that explained precisely how Brownian motion could be used to measure the size of atoms and molecules. (In 1905 Einstein created special relativity, proposed photons as quanta of EM radiation, and produced a theory of Brownian motion that allowed the size of atoms to be determined. All of this was done in his spare time, since he worked days as a patent examiner. Any one of these very basic works could have been the crowning achievement of an entire career—yet Einstein did even more in later years.) Their sizes were only approximately known to be 10^{-10} m, based on a comparison of latent heat of vaporization and surface tension made in about 1805 by Thomas Young of double-slit fame and the famous astronomer and mathematician Simon Laplace.

Using Einstein's ideas, the French physicist Jean-Baptiste Perrin (1870–1942) carefully observed Brownian motion; not only did he confirm Einstein's theory, he also produced accurate sizes for atoms and molecules.

Since molecular weights and densities of materials were well established, knowing atomic and molecular sizes allowed a precise value for Avogadro's number to be obtained. (If we know how big an atom is, we know how many fit into a certain volume.) Perrin also used these ideas to explain atomic and molecular agitation effects in sedimentation, and he received the 1926 Nobel Prize for his achievements. Most scientists were already convinced of the existence of atoms, but the accurate observation and analysis of Brownian motion was conclusive—it was the first truly direct evidence.

A huge array of direct and indirect evidence for the existence of atoms now exists. For example, it has become possible to accelerate ions (much as electrons are accelerated in cathode-ray tubes) and to detect them individually as well as measure their masses (see [More Applications of Magnetism](#) for a discussion of mass spectrometers). Other devices that observe individual atoms, such as the scanning tunneling electron microscope, will be discussed elsewhere. (See [\[link\]](#).) All of our understanding of the properties of matter is based on and consistent with the atom. The atom's substructures, such as electron shells and the nucleus, are both interesting and important. The nucleus in turn has a substructure, as do the particles of which it is composed. These topics, and the question of whether there is a smallest basic structure to matter, will be explored in later parts of the text.



Individual atoms
can be detected
with devices such
as the scanning
tunneling electron

microscope that
produced this
image of individual
gold atoms on a
graphite substrate.
(credit: Erwin
Rossen, Eindhoven
University of
Technology, via
Wikimedia
Commons)

Section Summary

- Atoms are the smallest unit of elements; atoms combine to form molecules, the smallest unit of compounds.
- The first direct observation of atoms was in Brownian motion.
- Analysis of Brownian motion gave accurate sizes for atoms (10^{-10} m on average) and a precise value for Avogadro's number.

Conceptual Questions

Exercise:

Problem:

Name three different types of evidence for the existence of atoms.

Exercise:

Problem:

Explain why patterns observed in the periodic table of the elements are evidence for the existence of atoms, and why Brownian motion is a more direct type of evidence for their existence.

Exercise:

Problem: If atoms exist, why can't we see them with visible light?

Problems & Exercises

Exercise:

Problem:

Using the given charge-to-mass ratios for electrons and protons, and knowing the magnitudes of their charges are equal, what is the ratio of the proton's mass to the electron's? (Note that since the charge-to-mass ratios are given to only three-digit accuracy, your answer may differ from the accepted ratio in the fourth digit.)

Solution:

$$1.84 \times 10^3$$

Exercise:

Problem:

(a) Calculate the mass of a proton using the charge-to-mass ratio given for it in this chapter and its known charge. (b) How does your result compare with the proton mass given in this chapter?

Exercise:

Problem:

If someone wanted to build a scale model of the atom with a nucleus 1.00 m in diameter, how far away would the nearest electron need to be?

Solution:

50 km

Glossary

atom

basic unit of matter, which consists of a central, positively charged nucleus surrounded by negatively charged electrons

Brownian motion

the continuous random movement of particles of matter suspended in a liquid or gas

Discovery of the Parts of the Atom: Electrons and Nuclei

- Describe how electrons were discovered.
- Explain the Millikan oil drop experiment.
- Describe Rutherford's gold foil experiment.
- Describe Rutherford's planetary model of the atom.

Just as atoms are a substructure of matter, electrons and nuclei are substructures of the atom. The experiments that were used to discover electrons and nuclei reveal some of the basic properties of atoms and can be readily understood using ideas such as electrostatic and magnetic force, already covered in previous chapters.

Note:

Charges and Electromagnetic Forces

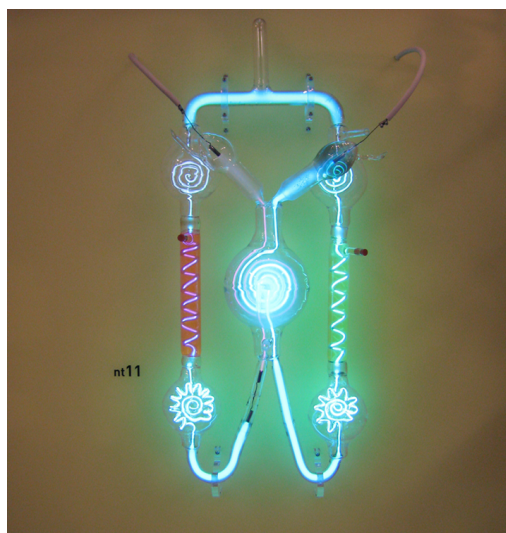
In previous discussions, we have noted that positive charge is associated with nuclei and negative charge with electrons. We have also covered many aspects of the electric and magnetic forces that affect charges. We will now explore the discovery of the electron and nucleus as substructures of the atom and examine their contributions to the properties of atoms.

The Electron

Gas discharge tubes, such as that shown in [\[link\]](#), consist of an evacuated glass tube containing two metal electrodes and a rarefied gas. When a high voltage is applied to the electrodes, the gas glows. These tubes were the precursors to today's neon lights. They were first studied seriously by Heinrich Geissler, a German inventor and glassblower, starting in the 1860s. The English scientist William Crookes, among others, continued to study what for some time were called Crookes tubes, wherein electrons are freed from atoms and molecules in the rarefied gas inside the tube and are accelerated from the cathode (negative) to the anode (positive) by the high potential. These "*cathode rays*" collide with the gas atoms and molecules and excite them, resulting in the emission of electromagnetic (EM)

radiation that makes the electrons' path visible as a ray that spreads and fades as it moves away from the cathode.

Gas discharge tubes today are most commonly called **cathode-ray tubes**, because the rays originate at the cathode. Crookes showed that the electrons carry momentum (they can make a small paddle wheel rotate). He also found that their normally straight path is bent by a magnet in the direction expected for a negative charge moving away from the cathode. These were the first direct indications of electrons and their charge.



A gas discharge tube
glows when a high
voltage is applied to it.
Electrons emitted from
the cathode are
accelerated toward the
anode; they excite atoms
and molecules in the gas,
which glow in response.
Once called Geissler
tubes and later Crookes
tubes, they are now
known as cathode-ray

tubes (CRTs) and are found in older TVs, computer screens, and x-ray machines. When a magnetic field is applied, the beam bends in the direction expected for negative charge. (credit: Paul Downey, Flickr)

The English physicist J. J. Thomson (1856–1940) improved and expanded the scope of experiments with gas discharge tubes. (See [\[link\]](#) and [\[link\]](#).) He verified the negative charge of the cathode rays with both magnetic and electric fields. Additionally, he collected the rays in a metal cup and found an excess of negative charge. Thomson was also able to measure the ratio of the charge of the electron to its mass, q_e/m_e —an important step to finding the actual values of both q_e and m_e . [\[link\]](#) shows a cathode-ray tube, which produces a narrow beam of electrons that passes through charging plates connected to a high-voltage power supply. An electric field $\mathbf{E}$ is produced between the charging plates, and the cathode-ray tube is placed between the poles of a magnet so that the electric field $\mathbf{E}$ is perpendicular to the magnetic field $\mathbf{B}$ of the magnet. These fields, being perpendicular to each other, produce opposing forces on the electrons. As discussed for mass spectrometers in [More Applications of Magnetism](#), if the net force due to the fields vanishes, then the velocity of the charged particle is $v = E/B$. In this manner, Thomson determined the velocity of the electrons and then moved the beam up and down by adjusting the electric field.



J. J. Thomson (credit:
www.firstworldwar.com
, via Wikimedia
Commons)

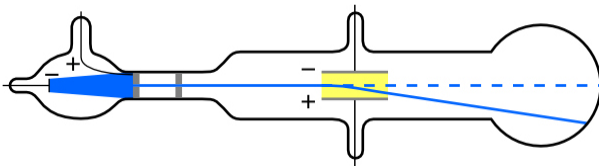
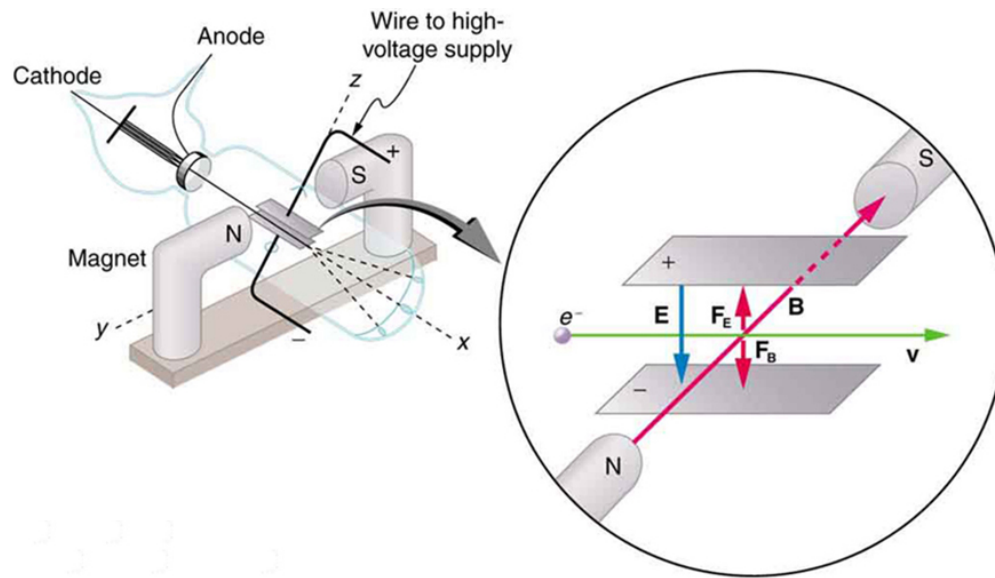


Diagram of Thomson's CRT.
(credit: Kurzon, Wikimedia
Commons)



This schematic shows the electron beam in a CRT passing through crossed electric and magnetic fields and causing phosphor to glow when striking the end of the tube.

To see how the amount of deflection is used to calculate q_e/m_e , note that the deflection is proportional to the electric force on the electron:

Equation:

$$F = q_e E.$$

But the vertical deflection is also related to the electron's mass, since the electron's acceleration is

Equation:

$$a = \frac{F}{m_e}.$$

The value of F is not known, since q_e was not yet known. Substituting the expression for electric force into the expression for acceleration yields

Equation:

$$a = \frac{F}{m_e} = \frac{q_e E}{m_e}.$$

Gathering terms, we have

Equation:

$$\frac{q_e}{m_e} = \frac{a}{E}.$$

The deflection is analyzed to get a , and E is determined from the applied voltage and distance between the plates; thus, $\frac{q_e}{m_e}$ can be determined. With the velocity known, another measurement of $\frac{q_e}{m_e}$ can be obtained by bending the beam of electrons with the magnetic field. Since $F_{\text{mag}} = q_e vB = m_e a$, we have $q_e/m_e = a/vB$. Consistent results are obtained using magnetic deflection.

What is so important about q_e/m_e , the ratio of the electron's charge to its mass? The value obtained is

Equation:

$$\frac{q_e}{m_e} = -1.76 \times 10^{11} \text{ C/kg (electron)}.$$

This is a huge number, as Thomson realized, and it implies that the electron has a very small mass. It was known from electroplating that about 10^8 C/kg is needed to plate a material, a factor of about 1000 less than the charge per kilogram of electrons. Thomson went on to do the same experiment for positively charged hydrogen ions (now known to be bare protons) and found a charge per kilogram about 1000 times smaller than that for the electron, implying that the proton is about 1000 times more massive than the electron. Today, we know more precisely that

Equation:

$$\frac{q_p}{m_p} = 9.58 \times 10^7 \text{ C/kg (proton),}$$

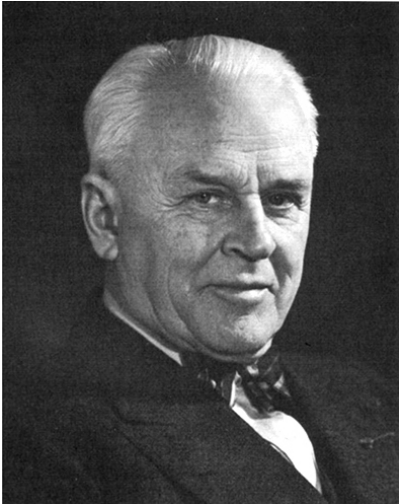
where q_p is the charge of the proton and m_p is its mass. This ratio (to four significant figures) is 1836 times less charge per kilogram than for the electron. Since the charges of electrons and protons are equal in magnitude, this implies $m_p = 1836m_e$.

Thomson performed a variety of experiments using differing gases in discharge tubes and employing other methods, such as the photoelectric effect, for freeing electrons from atoms. He always found the same properties for the electron, proving it to be an independent particle. For his work, the important pieces of which he began to publish in 1897, Thomson was awarded the 1906 Nobel Prize in Physics. In retrospect, it is difficult to appreciate how astonishing it was to find that the atom has a substructure. Thomson himself said, “It was only when I was convinced that the experiment left no escape from it that I published my belief in the existence of bodies smaller than atoms.”

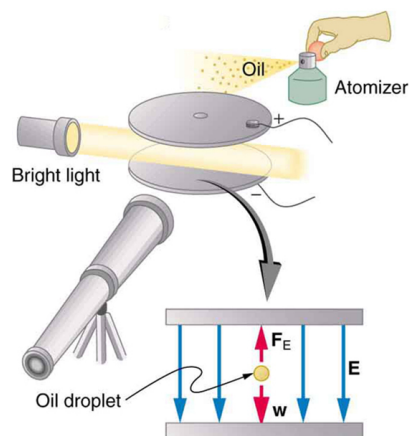
Thomson attempted to measure the charge of individual electrons, but his method could determine its charge only to the order of magnitude expected.

Since Faraday’s experiments with electroplating in the 1830s, it had been known that about 100,000 C per mole was needed to plate singly ionized ions. Dividing this by the number of ions per mole (that is, by Avogadro’s number), which was approximately known, the charge per ion was calculated to be about 1.6×10^{-19} C, close to the actual value.

An American physicist, Robert Millikan (1868–1953) (see [\[link\]](#)), decided to improve upon Thomson’s experiment for measuring q_e and was eventually forced to try another approach, which is now a classic experiment performed by students. The Millikan oil drop experiment is shown in [\[link\]](#).



Robert Millikan
(credit: Unknown
Author, via
Wikimedia
Commons)



The Millikan oil
drop experiment
produced the first
accurate direct
measurement of the

charge on
electrons, one of
the most
fundamental
constants in nature.

Fine drops of oil
become charged
when sprayed.

Their movement is
observed between
metal plates with a
potential applied to
oppose the
gravitational force.

The balance of
gravitational and
electric forces
allows the
calculation of the
charge on a drop.

The charge is found
to be quantized in
units of
 $-1.6 \times 10^{-19} \text{ C}$,
thus determining
directly the charge
of the excess and
missing electrons
on the oil drops.

In the Millikan oil drop experiment, fine drops of oil are sprayed from an atomizer. Some of these are charged by the process and can then be suspended between metal plates by a voltage between the plates. In this situation, the weight of the drop is balanced by the electric force:

Equation:

$$m_{\text{drop}}g = q_e E$$

The electric field is produced by the applied voltage, hence, $E = V/d$, and V is adjusted to just balance the drop's weight. The drops can be seen as points of reflected light using a microscope, but they are too small to directly measure their size and mass. The mass of the drop is determined by observing how fast it falls when the voltage is turned off. Since air resistance is very significant for these submicroscopic drops, the more massive drops fall faster than the less massive, and sophisticated sedimentation calculations can reveal their mass. Oil is used rather than water, because it does not readily evaporate, and so mass is nearly constant. Once the mass of the drop is known, the charge of the electron is given by rearranging the previous equation:

Equation:

$$q = \frac{m_{\text{drop}}g}{E} = \frac{m_{\text{drop}}gd}{V},$$

where d is the separation of the plates and V is the voltage that holds the drop motionless. (The same drop can be observed for several hours to see that it really is motionless.) By 1913 Millikan had measured the charge of the electron q_e to an accuracy of 1%, and he improved this by a factor of 10 within a few years to a value of -1.60×10^{-19} C. He also observed that all charges were multiples of the basic electron charge and that sudden changes could occur in which electrons were added or removed from the drops. For this very fundamental direct measurement of q_e and for his studies of the photoelectric effect, Millikan was awarded the 1923 Nobel Prize in Physics.

With the charge of the electron known and the charge-to-mass ratio known, the electron's mass can be calculated. It is

Equation:

$$m = \frac{q_e}{\left(\frac{q_e}{m_e}\right)}.$$

Substituting known values yields

Equation:

$$m_e = \frac{-1.60 \times 10^{-19} \text{ C}}{-1.76 \times 10^{11} \text{ C/kg}}$$

or

Equation:

$$m_e = 9.11 \times 10^{-31} \text{ kg (electron's mass),}$$

where the round-off errors have been corrected. The mass of the electron has been verified in many subsequent experiments and is now known to an accuracy of better than one part in one million. It is an incredibly small mass and remains the smallest known mass of any particle that has mass. (Some particles, such as photons, are massless and cannot be brought to rest, but travel at the speed of light.) A similar calculation gives the masses of other particles, including the proton. To three digits, the mass of the proton is now known to be

Equation:

$$m_p = 1.67 \times 10^{-27} \text{ kg (proton's mass),}$$

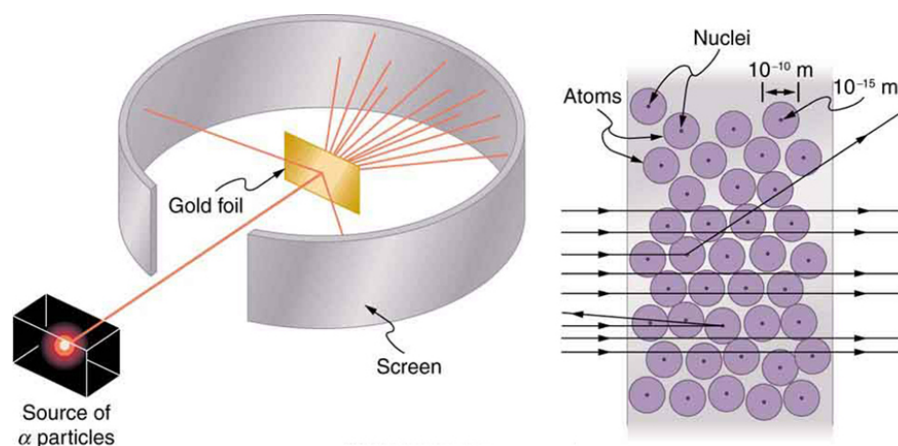
which is nearly identical to the mass of a hydrogen atom. What Thomson and Millikan had done was to prove the existence of one substructure of atoms, the electron, and further to show that it had only a tiny fraction of the mass of an atom. The nucleus of an atom contains most of its mass, and the nature of the nucleus was completely unanticipated.

Another important characteristic of quantum mechanics was also beginning to emerge. All electrons are identical to one another. The charge and mass of electrons are not average values; rather, they are unique values that all electrons have. This is true of other fundamental entities at the submicroscopic level. All protons are identical to one another, and so on.

The Nucleus

Here, we examine the first direct evidence of the size and mass of the nucleus. In later chapters, we will examine many other aspects of nuclear physics, but the basic information on nuclear size and mass is so important to understanding the atom that we consider it here.

Nuclear radioactivity was discovered in 1896, and it was soon the subject of intense study by a number of the best scientists in the world. Among them was New Zealander Lord Ernest Rutherford, who made numerous fundamental discoveries and earned the title of “father of nuclear physics.” Born in Nelson, Rutherford did his postgraduate studies at the Cavendish Laboratories in England before taking up a position at McGill University in Canada where he did the work that earned him a Nobel Prize in Chemistry in 1908. In the area of atomic and nuclear physics, there is much overlap between chemistry and physics, with physics providing the fundamental enabling theories. He returned to England in later years and had six future Nobel Prize winners as students. Rutherford used nuclear radiation to directly examine the size and mass of the atomic nucleus. The experiment he devised is shown in [\[link\]](#). A radioactive source that emits alpha radiation was placed in a lead container with a hole in one side to produce a beam of alpha particles, which are a type of ionizing radiation ejected by the nuclei of a radioactive source. A thin gold foil was placed in the beam, and the scattering of the alpha particles was observed by the glow they caused when they struck a phosphor screen.



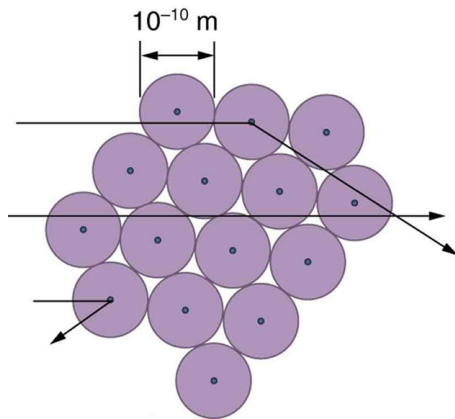
Rutherford's experiment gave direct evidence for the size and mass of the nucleus by scattering alpha particles from a thin gold foil. Alpha particles with energies of about 5 MeV are emitted from a radioactive source (which is a small metal container in which a specific amount of a radioactive material is sealed), are collimated into a beam, and fall upon the foil. The number of particles that penetrate the foil or scatter to various angles indicates that gold nuclei are very small and contain nearly all of the gold atom's mass. This is particularly indicated by the alpha particles that scatter to very large angles, much like a soccer ball bouncing off a goalie's head.

Alpha particles were known to be the doubly charged positive nuclei of helium atoms that had kinetic energies on the order of 5 MeV when emitted in nuclear decay, which is the disintegration of the nucleus of an unstable nuclide by the spontaneous emission of charged particles. These particles interact with matter mostly via the Coulomb force, and the manner in which they scatter from nuclei can reveal nuclear size and mass. This is analogous to observing how a bowling ball is scattered by an object you cannot see directly. Because the alpha particle's energy is so large compared with the typical energies associated with atoms (MeV versus eV), you would expect the alpha particles to simply crash through a thin foil much like a supersonic bowling ball would crash through a few dozen rows of bowling pins. Thomson had envisioned the atom to be a small sphere in which equal amounts of positive and negative charge were distributed evenly. The incident massive alpha particles would suffer only small deflections in such a model. Instead, Rutherford and his collaborators found that alpha particles occasionally were scattered to large angles, some even back in the direction from which they came! Detailed analysis using conservation of momentum and energy—particularly of the small number that came straight back—implied that gold nuclei are very small compared with the size of a gold atom, contain almost all of the atom's mass, and are tightly bound. Since

the gold nucleus is several times more massive than the alpha particle, a head-on collision would scatter the alpha particle straight back toward the source. In addition, the smaller the nucleus, the fewer alpha particles that would hit one head on.

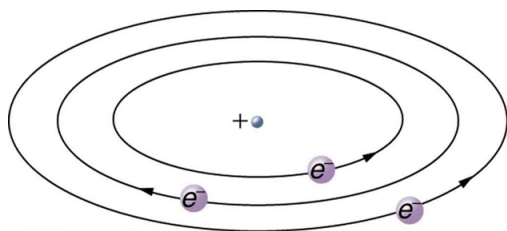
Although the results of the experiment were published by his colleagues in 1909, it took Rutherford two years to convince himself of their meaning. Like Thomson before him, Rutherford was reluctant to accept such radical results. Nature on a small scale is so unlike our classical world that even those at the forefront of discovery are sometimes surprised. Rutherford later wrote: “It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration, I realized that this scattering backwards ... [meant] ... the greatest part of the mass of the atom was concentrated in a tiny nucleus.” In 1911, Rutherford published his analysis together with a proposed model of the atom. The size of the nucleus was determined to be about 10^{-15} m, or 100,000 times smaller than the atom. This implies a huge density, on the order of 10^{15} g/cm³, vastly unlike any macroscopic matter. Also implied is the existence of previously unknown nuclear forces to counteract the huge repulsive Coulomb forces among the positive charges in the nucleus. Huge forces would also be consistent with the large energies emitted in nuclear radiation.

The small size of the nucleus also implies that the atom is mostly empty inside. In fact, in Rutherford’s experiment, most alphas went straight through the gold foil with very little scattering, since electrons have such small masses and since the atom was mostly empty with nothing for the alpha to hit. There were already hints of this at the time Rutherford performed his experiments, since energetic electrons had been observed to penetrate thin foils more easily than expected. [\[link\]](#) shows a schematic of the atoms in a thin foil with circles representing the size of the atoms (about 10^{-10} m) and dots representing the nuclei. (The dots are not to scale—if they were, you would need a microscope to see them.) Most alpha particles miss the small nuclei and are only slightly scattered by electrons. Occasionally, (about once in 8000 times in Rutherford’s experiment), an alpha hits a nucleus head-on and is scattered straight backward.



An expanded view of the atoms in the gold foil in Rutherford's experiment. Circles represent the atoms (about 10^{-10} m in diameter), while the dots represent the nuclei (about 10^{-15} m in diameter). To be visible, the dots are much larger than scale. Most alpha particles crash through but are relatively unaffected because of their high energy and the electron's small mass. Some, however, head straight toward a nucleus and are scattered straight back. A detailed analysis gives the size and mass of the nucleus.

Based on the size and mass of the nucleus revealed by his experiment, as well as the mass of electrons, Rutherford proposed the **planetary model of the atom**. The planetary model of the atom pictures low-mass electrons orbiting a large-mass nucleus. The sizes of the electron orbits are large compared with the size of the nucleus, with mostly vacuum inside the atom. This picture is analogous to how low-mass planets in our solar system orbit the large-mass Sun at distances large compared with the size of the sun. In the atom, the attractive Coulomb force is analogous to gravitation in the planetary system. (See [\[link\]](#).) Note that a model or mental picture is needed to explain experimental results, since the atom is too small to be directly observed with visible light.



Rutherford's planetary model of the atom incorporates the characteristics of the nucleus, electrons, and the size of the atom. This model was the first to recognize the structure of atoms, in which low-mass electrons orbit a very small, massive nucleus in orbits much larger than the nucleus. The atom is mostly empty and is analogous to our planetary system.

Rutherford's planetary model of the atom was crucial to understanding the characteristics of atoms, and their interactions and energies, as we shall see in the next few sections. Also, it was an indication of how different nature is from the familiar classical world on the small, quantum mechanical scale. The discovery of a substructure to all matter in the form of atoms and molecules was now being taken a step further to reveal a substructure of atoms that was simpler than the 92 elements then known. We have continued to search for deeper substructures, such as those inside the nucleus, with some success. In later chapters, we will follow this quest in the discussion of quarks and other elementary particles, and we will look at the direction the search seems now to be heading.

Note:

PhET Explorations: Rutherford Scattering

How did Rutherford figure out the structure of the atom without being able to see it? Simulate the famous experiment in which he disproved the Plum Pudding model of the atom by observing alpha particles bouncing off atoms and determining that they must have a small core.

https://phet.colorado.edu/sims/html/rutherford-scattering/latest/rutherford-scattering_en.html

Section Summary

- Atoms are composed of negatively charged electrons, first proved to exist in cathode-ray-tube experiments, and a positively charged nucleus.
- All electrons are identical and have a charge-to-mass ratio of

Equation:

$$\frac{q_e}{m_e} = -1.76 \times 10^{11} \text{ C/kg.}$$

- The positive charge in the nuclei is carried by particles called protons, which have a charge-to-mass ratio of

Equation:

$$\frac{q_p}{m_p} = 9.57 \times 10^7 \text{ C/kg.}$$

- Mass of electron,

Equation:

$$m_e = 9.11 \times 10^{-31} \text{ kg.}$$

- Mass of proton,

Equation:

$$m_p = 1.67 \times 10^{-27} \text{ kg.}$$

- The planetary model of the atom pictures electrons orbiting the nucleus in the same way that planets orbit the sun.

Conceptual Questions

Exercise:

Problem:

What two pieces of evidence allowed the first calculation of m_e , the mass of the electron?

- The ratios q_e/m_e and q_p/m_p .
- The values of q_e and E_B .
- The ratio q_e/m_e and q_e .

Justify your response.

Exercise:

Problem:

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

Problem Exercises**Exercise:****Problem:**

Rutherford found the size of the nucleus to be about 10^{-15} m. This implied a huge density. What would this density be for gold?

Solution:

$$6 \times 10^{20} \text{ kg/m}^3$$

Exercise:**Problem:**

In Millikan's oil-drop experiment, one looks at a small oil drop held motionless between two plates. Take the voltage between the plates to be 2033 V, and the plate separation to be 2.00 cm. The oil drop (of density 0.81 g/cm^3) has a diameter of 4.0×10^{-6} m. Find the charge on the drop, in terms of electron units.

Exercise:**Problem:**

(a) An aspiring physicist wants to build a scale model of a hydrogen atom for her science fair project. If the atom is 1.00 m in diameter, how big should she try to make the nucleus?

(b) How easy will this be to do?

Solution:

(a) $10.0\ \mu\text{m}$

(b) It isn't hard to make one of approximately this size. It would be harder to make it exactly $10.0\ \mu\text{m}$.

Glossary

cathode-ray tube

a vacuum tube containing a source of electrons and a screen to view images

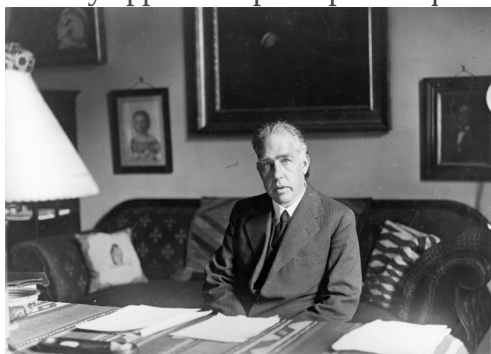
planetary model of the atom

the most familiar model or illustration of the structure of the atom

Bohr's Theory of the Hydrogen Atom

- Describe the mysteries of atomic spectra.
- Explain Bohr's theory of the hydrogen atom.
- Explain Bohr's planetary model of the atom.
- Illustrate energy state using the energy-level diagram.
- Describe the triumphs and limits of Bohr's theory.

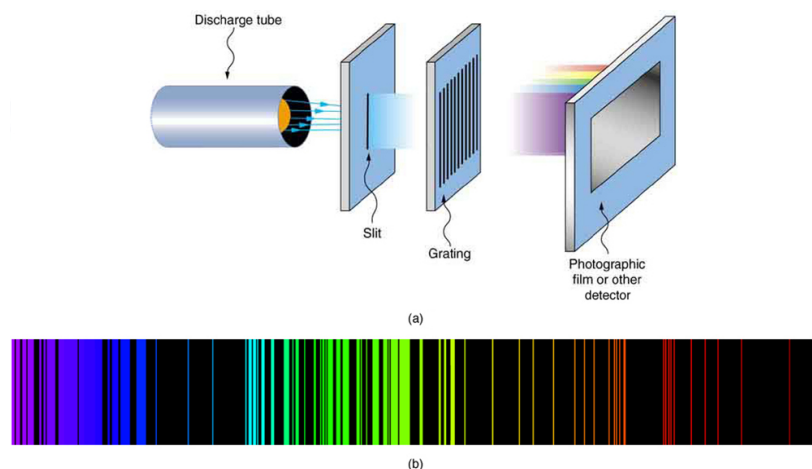
The great Danish physicist Niels Bohr (1885–1962) made immediate use of Rutherford's planetary model of the atom. ([link](#)). Bohr became convinced of its validity and spent part of 1912 at Rutherford's laboratory. In 1913, after returning to Copenhagen, he began publishing his theory of the simplest atom, hydrogen, based on the planetary model of the atom. For decades, many questions had been asked about atomic characteristics. From their sizes to their spectra, much was known about atoms, but little had been explained in terms of the laws of physics. Bohr's theory explained the atomic spectrum of hydrogen and established new and broadly applicable principles in quantum mechanics.



Niels Bohr, Danish physicist, used the planetary model of the atom to explain the atomic spectrum and size of the hydrogen atom. His many contributions to the development of atomic physics and quantum mechanics, his personal influence on many students and colleagues, and his personal integrity, especially in the face of Nazi oppression, earned him a prominent place in history. (credit: Unknown Author, via Wikimedia Commons)

Mysteries of Atomic Spectra

As noted in [Quantization of Energy](#), the energies of some small systems are quantized. Atomic and molecular emission and absorption spectra have been known for over a century to be discrete (or quantized). (See [\[link\]](#).) Maxwell and others had realized that there must be a connection between the spectrum of an atom and its structure, something like the resonant frequencies of musical instruments. But, in spite of years of efforts by many great minds, no one had a workable theory. (It was a running joke that any theory of atomic and molecular spectra could be destroyed by throwing a book of data at it, so complex were the spectra.) Following Einstein's proposal of photons with quantized energies directly proportional to their wavelengths, it became even more evident that electrons in atoms can exist only in discrete orbits.



Part (a) shows, from left to right, a discharge tube, slit, and diffraction grating producing a line spectrum. Part (b) shows the emission line spectrum for iron. The discrete lines imply quantized energy states for the atoms that produce them. The line spectrum for each element is unique, providing a powerful and much used analytical tool, and many line spectra were well known for many years before they could be explained with physics.
(credit for (b): Yttrium91, Wikimedia Commons)

In some cases, it had been possible to devise formulas that described the emission spectra. As you might expect, the simplest atom—hydrogen, with its single electron—has a relatively simple spectrum. The hydrogen spectrum had been observed in the infrared (IR), visible, and ultraviolet (UV), and several series of spectral lines had been observed. (See [\[link\]](#).) These series are named after early researchers who studied them in particular depth.

The observed **hydrogen-spectrum wavelengths** can be calculated using the following formula:

Equation:

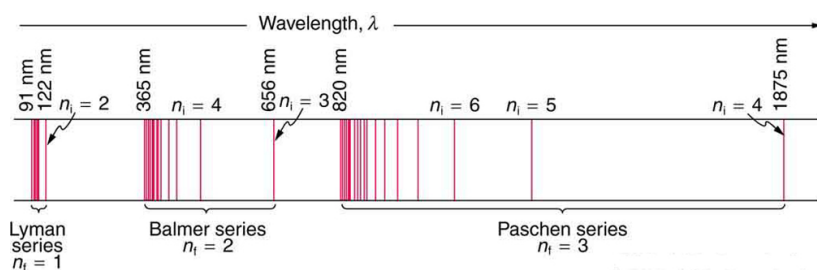
$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right),$$

where λ is the wavelength of the emitted EM radiation and R is the **Rydberg constant**, determined by the experiment to be

Equation:

$$R = 1.097 \times 10^7 / \text{m (or m}^{-1}\text{)}.$$

The constant n_f is a positive integer associated with a specific series. For the Lyman series, $n_f = 1$; for the Balmer series, $n_f = 2$; for the Paschen series, $n_f = 3$; and so on. The Lyman series is entirely in the UV, while part of the Balmer series is visible with the remainder UV. The Paschen series and all the rest are entirely IR. There are apparently an unlimited number of series, although they lie progressively farther into the infrared and become difficult to observe as n_f increases. The constant n_i is a positive integer, but it must be greater than n_f . Thus, for the Balmer series, $n_f = 2$ and $n_i = 3, 4, 5, 6, \dots$. Note that n_i can approach infinity. While the formula in the wavelengths equation was just a recipe designed to fit data and was not based on physical principles, it did imply a deeper meaning. Balmer first devised the formula for his series alone, and it was later found to describe all the other series by using different values of n_f . Bohr was the first to comprehend the deeper meaning. Again, we see the interplay between experiment and theory in physics. Experimentally, the spectra were well established, an equation was found to fit the experimental data, but the theoretical foundation was missing.



A schematic of the hydrogen spectrum shows several series named for those who contributed most to their determination. Part of the Balmer series is in the visible spectrum, while the Lyman series is entirely in the UV, and the Paschen series and others are in the IR. Values of n_f and n_i are shown for some of the lines.

Example:**Calculating Wave Interference of a Hydrogen Line**

What is the distance between the slits of a grating that produces a first-order maximum for the second Balmer line at an angle of 15° ?

Strategy and Concept

For an Integrated Concept problem, we must first identify the physical principles involved. In this example, we need to know (a) the wavelength of light as well as (b) conditions for an interference maximum for the pattern from a double slit. Part (a) deals with a topic of the present chapter, while part (b) considers the wave interference material of [Wave Optics](#).

Solution for (a)

Hydrogen spectrum wavelength. The Balmer series requires that $n_f = 2$. The first line in the series is taken to be for $n_i = 3$, and so the second would have $n_i = 4$.

The calculation is a straightforward application of the wavelength equation. Entering the determined values for n_f and n_i yields

Equation:

$$\begin{aligned}\frac{1}{\lambda} &= R\left(\frac{1}{n_f^2} - \frac{1}{n_i^2}\right) \\ &= (1.097 \times 10^7 \text{ m}^{-1})\left(\frac{1}{2^2} - \frac{1}{4^2}\right) \\ &= 2.057 \times 10^6 \text{ m}^{-1}.\end{aligned}$$

Inverting to find λ gives

Equation:

$$\begin{aligned}\lambda &= \frac{1}{2.057 \times 10^6 \text{ m}^{-1}} = 486 \times 10^{-9} \text{ m} \\ &= 486 \text{ nm}.\end{aligned}$$

Discussion for (a)

This is indeed the experimentally observed wavelength, corresponding to the second (blue-green) line in the Balmer series. More impressive is the fact that the same simple recipe predicts *all* of the hydrogen spectrum lines, including new ones observed in subsequent experiments. What is nature telling us?

Solution for (b)

Double-slit interference ([Wave Optics](#)). To obtain constructive interference for a double slit, the path length difference from two slits must be an integral multiple of the wavelength. This condition was expressed by the equation

Equation:

$$d \sin \theta = m\lambda,$$

where d is the distance between slits and θ is the angle from the original direction of the beam. The number m is the order of the interference; $m = 1$ in this example. Solving for d and entering known values yields

Equation:

$$d = \frac{(1)(486 \text{ nm})}{\sin 15^\circ} = 1.88 \times 10^{-6} \text{ m}.$$

Discussion for (b)

This number is similar to those used in the interference examples of [Introduction to Quantum Physics](#) (and is close to the spacing between slits in commonly used diffraction glasses).

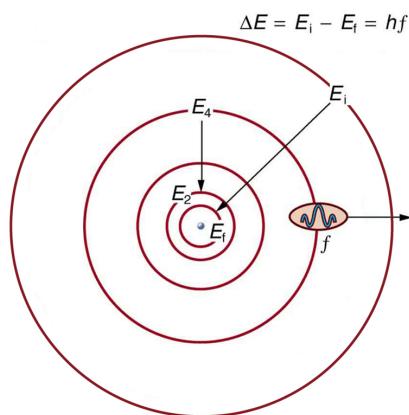
Bohr's Solution for Hydrogen

Bohr was able to derive the formula for the hydrogen spectrum using basic physics, the planetary model of the atom, and some very important new proposals. His first proposal is that only certain orbits are allowed: we say that *the orbits of electrons in atoms are quantized*. Each orbit has a different energy, and electrons can move to a higher orbit by absorbing energy and drop to a lower orbit by emitting energy. If the orbits are quantized, the amount of energy absorbed or emitted is also quantized, producing discrete spectra. Photon absorption and emission are among the primary methods of transferring energy into and out of atoms. The energies of the photons are quantized, and their energy is explained as being equal to the change in energy of the electron when it moves from one orbit to another. In equation form, this is

Equation:

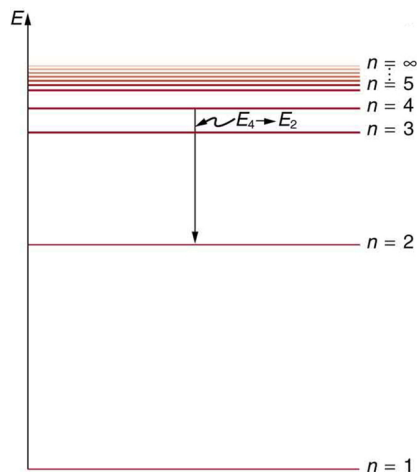
$$\Delta E = hf = E_i - E_f.$$

Here, ΔE is the change in energy between the initial and final orbits, and hf is the energy of the absorbed or emitted photon. It is quite logical (that is, expected from our everyday experience) that energy is involved in changing orbits. A blast of energy is required for the space shuttle, for example, to climb to a higher orbit. What is not expected is that atomic orbits should be quantized. This is not observed for satellites or planets, which can have any orbit given the proper energy. (See [\[link\]](#).)



The planetary model of the atom, as modified by Bohr, has the orbits of the electrons quantized. Only certain orbits are allowed, explaining why atomic spectra are discrete (quantized). The energy carried away from an atom by a photon comes from the electron dropping from one allowed orbit to another and is thus quantized. This is likewise true for atomic absorption of photons.

[\[link\]](#) shows an **energy-level diagram**, a convenient way to display energy states. In the present discussion, we take these to be the allowed energy levels of the electron. Energy is plotted vertically with the lowest or ground state at the bottom and with excited states above. Given the energies of the lines in an atomic spectrum, it is possible (although sometimes very difficult) to determine the energy levels of an atom. Energy-level diagrams are used for many systems, including molecules and nuclei. A theory of the atom or any other system must predict its energies based on the physics of the system.



An energy-level diagram plots energy vertically and is useful in visualizing the energy states of a system and the transitions between them.

This diagram is for the hydrogen-atom electrons, showing a transition between two orbits having energies E_4 and E_2 .

Bohr was clever enough to find a way to calculate the electron orbital energies in hydrogen. This was an important first step that has been improved upon, but it is well worth repeating here, because it does correctly describe many characteristics of hydrogen. Assuming circular orbits, Bohr proposed that the **angular momentum L of an electron in its orbit is quantized**, that is, it has only specific, discrete values. The value for L is given by the formula

Equation:

$$L = m_e v r_n = n \frac{h}{2\pi} (n = 1, 2, 3, \dots),$$

where L is the angular momentum, m_e is the electron's mass, r_n is the radius of the n th orbit, and h is Planck's constant. Note that angular momentum is $L = I\omega$. For a small object at a radius r , $I = mr^2$ and $\omega = v/r$, so that $L = (mr^2)(v/r) = mvr$. Quantization says that this value of mvr can only be equal to $h/2$, $2h/2$, $3h/2$, etc. At the time, Bohr himself did not know why angular momentum should be quantized, but using this assumption he was

able to calculate the energies in the hydrogen spectrum, something no one else had done at the time.

From Bohr's assumptions, we will now derive a number of important properties of the hydrogen atom from the classical physics we have covered in the text. We start by noting the centripetal force causing the electron to follow a circular path is supplied by the Coulomb force. To be more general, we note that this analysis is valid for any single-electron atom. So, if a nucleus has Z protons ($Z = 1$ for hydrogen, 2 for helium, etc.) and only one electron, that atom is called a **hydrogen-like atom**. The spectra of hydrogen-like ions are similar to hydrogen, but shifted to higher energy by the greater attractive force between the electron and nucleus. The magnitude of the centripetal force is $m_e v^2 / r_n$, while the Coulomb force is $k(Zq_e)(q_e)/r_n^2$. The tacit assumption here is that the nucleus is more massive than the stationary electron, and the electron orbits about it. This is consistent with the planetary model of the atom. Equating these,

Equation:

$$k \frac{Zq_e^2}{r_n^2} = \frac{m_e v^2}{r_n} \text{ (Coulomb = centripetal).}$$

Angular momentum quantization is stated in an earlier equation. We solve that equation for v , substitute it into the above, and rearrange the expression to obtain the radius of the orbit. This yields:

Equation:

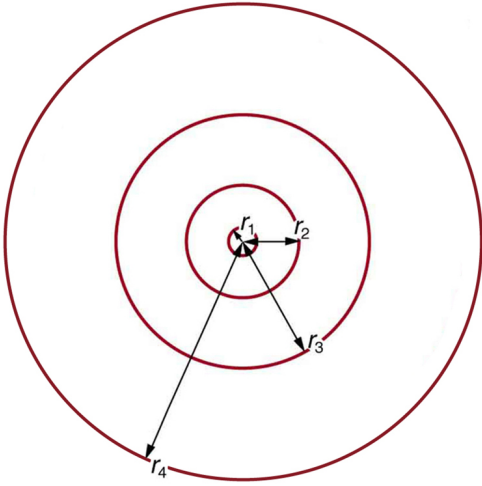
$$r_n = \frac{n^2}{Z} a_B, \text{ for allowed orbits } (n = 1, 2, 3, \dots),$$

where a_B is defined to be the **Bohr radius**, since for the lowest orbit ($n = 1$) and for hydrogen ($Z = 1$), $r_1 = a_B$. It is left for this chapter's Problems and Exercises to show that the Bohr radius is

Equation:

$$a_B = \frac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 \times 10^{-10} \text{ m.}$$

These last two equations can be used to calculate the **radii of the allowed (quantized) electron orbits in any hydrogen-like atom**. It is impressive that the formula gives the correct size of hydrogen, which is measured experimentally to be very close to the Bohr radius. The earlier equation also tells us that the orbital radius is proportional to n^2 , as illustrated in [\[link\]](#).



The allowed electron orbits in hydrogen have the radii shown. These radii were first calculated by Bohr and are given by the equation $r_n = \frac{n^2}{Z} a_B$. The lowest orbit has the experimentally verified diameter of a hydrogen atom.

To get the electron orbital energies, we start by noting that the electron energy is the sum of its kinetic and potential energy:

Equation:

$$E_n = \text{KE} + \text{PE}.$$

Kinetic energy is the familiar $\text{KE} = (1/2)m_e v^2$, assuming the electron is not moving at relativistic speeds. Potential energy for the electron is electrical, or $\text{PE} = q_e V$, where V is the potential due to the nucleus, which looks like a point charge. The nucleus has a positive charge Zq_e ; thus, $V = kZq_e/r_n$, recalling an earlier equation for the potential due to a point charge. Since the electron's charge is negative, we see that $\text{PE} = -kZq_e/r_n$. Entering the expressions for KE and PE, we find

Equation:

$$E_n = \frac{1}{2}m_e v^2 - k \frac{Zq_e^2}{r_n}.$$

Now we substitute r_n and v from earlier equations into the above expression for energy. Algebraic manipulation yields

Equation:

$$E_n = -\frac{Z^2}{n^2} E_0 (n = 1, 2, 3, \dots)$$

for the orbital **energies of hydrogen-like atoms**. Here, E_0 is the **ground-state energy** ($n = 1$) for hydrogen ($Z = 1$) and is given by

Equation:

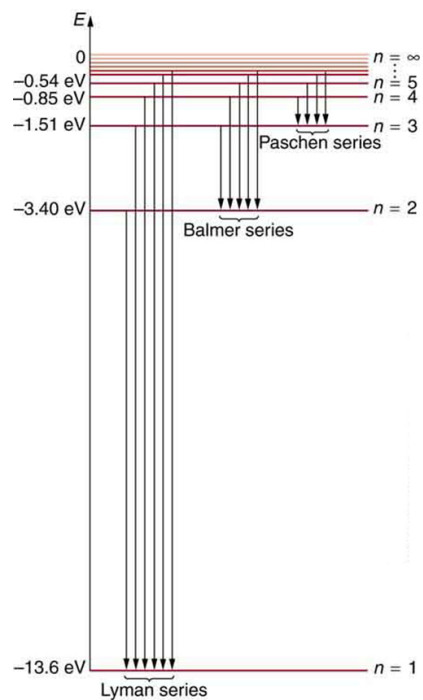
$$E_0 = \frac{2\pi^2 q_e^4 m_e k^2}{h^2} = 13.6 \text{ eV}.$$

Thus, for hydrogen,

Equation:

$$E_n = -\frac{13.6 \text{ eV}}{n^2} (n = 1, 2, 3, \dots).$$

[\[link\]](#) shows an energy-level diagram for hydrogen that also illustrates how the various spectral series for hydrogen are related to transitions between energy levels.



Energy-level diagram for

hydrogen showing the Lyman, Balmer, and Paschen series of transitions. The orbital energies are calculated using the above equation, first derived by Bohr.

Electron total energies are negative, since the electron is bound to the nucleus, analogous to being in a hole without enough kinetic energy to escape. As n approaches infinity, the total energy becomes zero. This corresponds to a free electron with no kinetic energy, since r_n gets very large for large n , and the electric potential energy thus becomes zero. Thus, 13.6 eV is needed to ionize hydrogen (to go from -13.6 eV to 0, or unbound), an experimentally verified number. Given more energy, the electron becomes unbound with some kinetic energy. For example, giving 15.0 eV to an electron in the ground state of hydrogen strips it from the atom and leaves it with 1.4 eV of kinetic energy.

Finally, let us consider the energy of a photon emitted in a downward transition, given by the equation to be

Equation:

$$\Delta E = hf = E_i - E_f.$$

Substituting $E_n = (-13.6 \text{ eV}/n^2)$, we see that

Equation:

$$hf = (13.6 \text{ eV}) \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

Dividing both sides of this equation by hc gives an expression for $1/\lambda$:

Equation:

$$\frac{hf}{hc} = \frac{f}{c} = \frac{1}{\lambda} = \frac{(13.6 \text{ eV})}{hc} \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right).$$

It can be shown that

Equation:

$$\left(\frac{13.6 \text{ eV}}{hc} \right) = \frac{(13.6 \text{ eV})(1.602 \times 10^{-19} \text{ J/eV})}{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})(2.998 \times 10^8 \text{ m/s})} = 1.097 \times 10^7 \text{ m}^{-1} = R$$

is the **Rydberg constant**. Thus, we have used Bohr's assumptions to derive the formula first proposed by Balmer years earlier as a recipe to fit experimental data.

Equation:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

We see that Bohr's theory of the hydrogen atom answers the question as to why this previously known formula describes the hydrogen spectrum. It is because the energy levels are proportional to $1/n^2$, where n is a non-negative integer. A downward transition releases energy, and so n_i must be greater than n_f . The various series are those where the transitions end on a certain level. For the Lyman series, $n_f = 1$ — that is, all the transitions end in the ground state (see also [link](#)). For the Balmer series, $n_f = 2$, or all the transitions end in the first excited state; and so on. What was once a recipe is now based in physics, and something new is emerging—angular momentum is quantized.

Triumphs and Limits of the Bohr Theory

Bohr did what no one had been able to do before. Not only did he explain the spectrum of hydrogen, he correctly calculated the size of the atom from basic physics. Some of his ideas are broadly applicable. Electron orbital energies are quantized in all atoms and molecules. Angular momentum is quantized. The electrons do not spiral into the nucleus, as expected classically (accelerated charges radiate, so that the electron orbits classically would decay quickly, and the electrons would sit on the nucleus—matter would collapse). These are major triumphs.

But there are limits to Bohr's theory. It cannot be applied to multielectron atoms, even one as simple as a two-electron helium atom. Bohr's model is what we call *semiclassical*. The orbits are quantized (nonclassical) but are assumed to be simple circular paths (classical). As quantum mechanics was developed, it became clear that there are no well-defined orbits; rather, there are clouds of probability. Bohr's theory also did not explain that some spectral lines are doublets (split into two) when examined closely. We shall examine many of these aspects of quantum mechanics in more detail, but it should be kept in mind that Bohr did not fail. Rather, he made very important steps along the path to greater knowledge and laid the foundation for all of atomic physics that has since evolved.

Note:

PhET Explorations: Models of the Hydrogen Atom

How did scientists figure out the structure of atoms without looking at them? Try out different models by shooting light at the atom. Check how the prediction of the model

matches the experimental results.

<https://archive.cnx.org/specials/d77cc1d0-33e4-11e6-b016-6726afecd2be/hydrogen-atom/#sim-hydrogen-atom>

Section Summary

- The planetary model of the atom pictures electrons orbiting the nucleus in the way that planets orbit the sun. Bohr used the planetary model to develop the first reasonable theory of hydrogen, the simplest atom. Atomic and molecular spectra are quantized, with hydrogen spectrum wavelengths given by the formula

Equation:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right),$$

where λ is the wavelength of the emitted EM radiation and R is the Rydberg constant, which has the value

Equation:

$$R = 1.097 \times 10^7 \text{ m}^{-1}.$$

- The constants n_i and n_f are positive integers, and n_i must be greater than n_f .
- Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantized, with energy for transitions between orbits given by

Equation:

$$\Delta E = hf = E_i - E_f,$$

where ΔE is the change in energy between the initial and final orbits and hf is the energy of an absorbed or emitted photon. It is useful to plot orbital energies on a vertical graph called an energy-level diagram.

- Bohr proposed that the allowed orbits are circular and must have quantized orbital angular momentum given by

Equation:

$$L = m_e v r_n = n \frac{h}{2\pi} (n = 1, 2, 3 \dots),$$

where L is the angular momentum, r_n is the radius of the n th orbit, and h is Planck's constant. For all one-electron (hydrogen-like) atoms, the radius of an orbit is given by

Equation:

$$r_n = \frac{n^2}{Z} a_B (\text{allowed orbits } n = 1, 2, 3, \dots),$$

Z is the atomic number of an element (the number of electrons it has when neutral) and a_B is defined to be the Bohr radius, which is

Equation:

$$a_B = \frac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 \times 10^{-10} \text{ m}.$$

- Furthermore, the energies of hydrogen-like atoms are given by
Equation:

$$E_n = -\frac{Z^2}{n^2} E_0 (n = 1, 2, 3 \dots),$$

where E_0 is the ground-state energy and is given by

Equation:

$$E_0 = \frac{2\pi^2 q_e^4 m_e k^2}{h^2} = 13.6 \text{ eV}.$$

Thus, for hydrogen,

Equation:

$$E_n = -\frac{13.6 \text{ eV}}{n^2} (n = 1, 2, 3 \dots).$$

- The Bohr Theory gives accurate values for the energy levels in hydrogen-like atoms, but it has been improved upon in several respects.

Conceptual Questions

Exercise:

Problem:

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

Exercise:

Problem:

Explain how Bohr's rule for the quantization of electron orbital angular momentum differs from the actual rule.

Exercise:

Problem:

What is a hydrogen-like atom, and how are the energies and radii of its electron orbits related to those in hydrogen?

Problems & Exercises**Exercise:****Problem:**

By calculating its wavelength, show that the first line in the Lyman series is UV radiation.

Solution:

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right) \Rightarrow \lambda = \frac{1}{R} \left[\frac{(n_i \cdot n_f)^2}{n_i^2 - n_f^2} \right]; n_i = 2, n_f = 1, \text{ so that}$$

$$\lambda = \left(\frac{\text{m}}{1.097 \times 10^7} \right) \left[\frac{(2 \times 1)^2}{2^2 - 1^2} \right] = 1.22 \times 10^{-7} \text{ m} = 122 \text{ nm}, \text{ which is UV radiation.}$$

Exercise:**Problem:**

Find the wavelength of the third line in the Lyman series, and identify the type of EM radiation.

Exercise:**Problem:**

Look up the values of the quantities in $a_B = \frac{h^2}{4\pi^2 m_e k q_e^2}$, and verify that the Bohr radius a_B is $0.529 \times 10^{-10} \text{ m}$.

Solution:

$$a_B = \frac{h^2}{4\pi^2 m_e k Z q_e^2} = \frac{(6.626 \times 10^{-34} \text{ J}\cdot\text{s})^2}{4\pi^2 (9.109 \times 10^{-31} \text{ kg})(8.988 \times 10^9 \text{ N}\cdot\text{m}^2/\text{C}^2)(1)(1.602 \times 10^{-19} \text{ C})^2} = 0.529 \times 10^{-10} \text{ m}$$

Exercise:

Problem: Verify that the ground state energy E_0 is 13.6 eV by using $E_0 = \frac{2\pi^2 q_e^4 m_e k^2}{h^2}$.

Exercise:

Problem:

If a hydrogen atom has its electron in the $n = 4$ state, how much energy in eV is needed to ionize it?

Solution:

0.850 eV

Exercise:**Problem:**

A hydrogen atom in an excited state can be ionized with less energy than when it is in its ground state. What is n for a hydrogen atom if 0.850 eV of energy can ionize it?

Exercise:**Problem:**

Find the radius of a hydrogen atom in the $n = 2$ state according to Bohr's theory.

Solution:

$2.12 \times 10^{-10} \text{ m}$

Exercise:**Problem:**

Show that $(13.6 \text{ eV})/hc = 1.097 \times 10^7 \text{ m} = R$ (Rydberg's constant), as discussed in the text.

Exercise:**Problem:**

What is the smallest-wavelength line in the Balmer series? Is it in the visible part of the spectrum?

Solution:

365 nm

It is in the ultraviolet.

Exercise:**Problem:**

Show that the entire Paschen series is in the infrared part of the spectrum. To do this, you only need to calculate the shortest wavelength in the series.

Exercise:

Problem:

Do the Balmer and Lyman series overlap? To answer this, calculate the shortest-wavelength Balmer line and the longest-wavelength Lyman line.

Solution:

No overlap

365 nm

122 nm

Exercise:

Problem:

(a) Which line in the Balmer series is the first one in the UV part of the spectrum?

(b) How many Balmer series lines are in the visible part of the spectrum?

(c) How many are in the UV?

Exercise:

Problem:

A wavelength of $4.653\ \mu\text{m}$ is observed in a hydrogen spectrum for a transition that ends in the $n_f = 5$ level. What was n_i for the initial level of the electron?

Solution:

7

Exercise:

Problem:

A singly ionized helium ion has only one electron and is denoted He^+ . What is the ion's radius in the ground state compared to the Bohr radius of hydrogen atom?

Exercise:

Problem:

A beryllium ion with a single electron (denoted Be^{3+}) is in an excited state with radius the same as that of the ground state of hydrogen.

(a) What is n for the Be^{3+} ion?

(b) How much energy in eV is needed to ionize the ion from this excited state?

Solution:

(a) 2

(b) 54.4 eV

Exercise:

Problem:

Atoms can be ionized by thermal collisions, such as at the high temperatures found in the solar corona. One such ion is C^{+5} , a carbon atom with only a single electron.

(a) By what factor are the energies of its hydrogen-like levels greater than those of hydrogen?

(b) What is the wavelength of the first line in this ion's Paschen series?

(c) What type of EM radiation is this?

Exercise:

Problem:

Verify Equations $r_n = \frac{n^2}{Z} a_B$ and $a_B = \frac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 \times 10^{-10} \text{ m}$ using the approach stated in the text. That is, equate the Coulomb and centripetal forces and then insert an expression for velocity from the condition for angular momentum quantization.

Solution:

$\frac{kZq_e^2}{r_n^2} = \frac{m_e V^2}{r_n}$, so that $r_n = \frac{kZq_e^2}{m_e V^2} = \frac{kZq_e^2}{m_e} \frac{1}{V^2}$. From the equation $m_e v r_n = n \frac{h}{2\pi}$, we can substitute for the velocity, giving: $r_n = \frac{kZq_e^2}{m_e} \cdot \frac{4\pi^2 m_e^2 r_n^2}{n^2 h^2}$ so that $r_n = \frac{n^2}{Z} \frac{h^2}{4\pi^2 m_e k q_e^2} = \frac{n^2}{Z} a_B$, where $a_B = \frac{h^2}{4\pi^2 m_e k q_e^2}$.

Exercise:

Problem:

The wavelength of the four Balmer series lines for hydrogen are found to be 410.3, 434.2, 486.3, and 656.5 nm. What average percentage difference is found between these wavelength numbers and those predicted by $\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$? It is amazing how well a simple formula (disconnected originally from theory) could duplicate this phenomenon.

Glossary

hydrogen spectrum wavelengths

the wavelengths of visible light from hydrogen; can be calculated by

$$\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$$

Rydberg constant

a physical constant related to the atomic spectra with an established value of

$$1.097 \times 10^7 \text{ m}^{-1}$$

double-slit interference

an experiment in which waves or particles from a single source impinge upon two slits so that the resulting interference pattern may be observed

energy-level diagram

a diagram used to analyze the energy level of electrons in the orbits of an atom

Bohr radius

the mean radius of the orbit of an electron around the nucleus of a hydrogen atom in its ground state

hydrogen-like atom

any atom with only a single electron

energies of hydrogen-like atoms

Bohr formula for energies of electron states in hydrogen-like atoms:

$$E_n = -\frac{Z^2}{n^2} E_0 (n = 1, 2, 3, \dots)$$

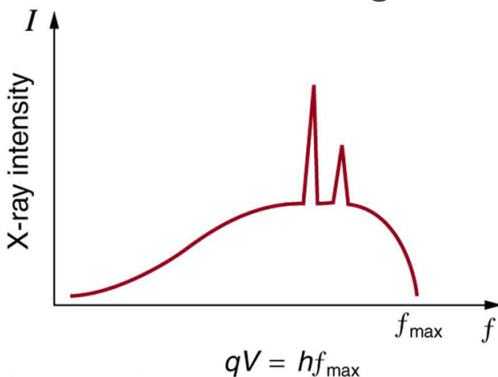
X Rays: Atomic Origins and Applications

- Define x-ray tube and its spectrum.
- Show the x-ray characteristic energy.
- Specify the use of x rays in medical observations.
- Explain the use of x rays in CT scanners in diagnostics.

Each type of atom (or element) has its own characteristic electromagnetic spectrum. **X rays** lie at the high-frequency end of an atom's spectrum and are characteristic of the atom as well. In this section, we explore characteristic x rays and some of their important applications.

We have previously discussed x rays as a part of the electromagnetic spectrum in [Photon Energies and the Electromagnetic Spectrum](#). That module illustrated how an x-ray tube (a specialized CRT) produces x rays. Electrons emitted from a hot filament are accelerated with a high voltage, gaining significant kinetic energy and striking the anode.

There are two processes by which x rays are produced in the anode of an x-ray tube. In one process, the deceleration of electrons produces x rays, and these x rays are called *bremsstrahlung*, or braking radiation. The second process is atomic in nature and produces *characteristic x rays*, so called because they are characteristic of the anode material. The x-ray spectrum in [\[link\]](#) is typical of what is produced by an x-ray tube, showing a broad curve of bremsstrahlung radiation with characteristic x-ray peaks on it.



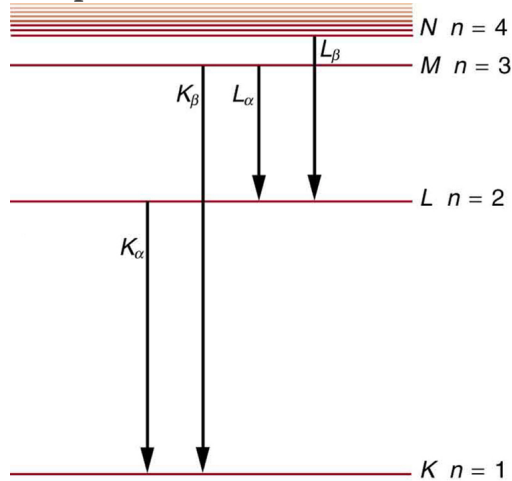
X-ray spectrum obtained when energetic electrons strike a material, such as

in the anode of a CRT.
The smooth part of the
spectrum is
bremsstrahlung radiation,
while the peaks are
characteristic of the
anode material. A
different anode material
would have characteristic
x-ray peaks at different
frequencies.

The spectrum in [\[link\]](#) is collected over a period of time in which many electrons strike the anode, with a variety of possible outcomes for each hit. The broad range of x-ray energies in the bremsstrahlung radiation indicates that an incident electron's energy is not usually converted entirely into photon energy. The highest-energy x ray produced is one for which all of the electron's energy was converted to photon energy. Thus the accelerating voltage and the maximum x-ray energy are related by conservation of energy. Electric potential energy is converted to kinetic energy and then to photon energy, so that $E_{\text{max}} = hf_{\text{max}} = q_e V$. Units of electron volts are convenient. For example, a 100-kV accelerating voltage produces x-ray photons with a maximum energy of 100 keV.

Some electrons excite atoms in the anode. Part of the energy that they deposit by collision with an atom results in one or more of the atom's inner electrons being knocked into a higher orbit or the atom being ionized. When the anode's atoms de-excite, they emit characteristic electromagnetic radiation. The most energetic of these are produced when an inner-shell vacancy is filled—that is, when an $n = 1$ or $n = 2$ shell electron has been excited to a higher level, and another electron falls into the vacant spot. A *characteristic x ray* (see [Photon Energies and the Electromagnetic Spectrum](#)) is electromagnetic (EM) radiation emitted by an atom when an inner-shell vacancy is filled. [\[link\]](#) shows a representative energy-level diagram that illustrates the labeling of characteristic x rays. X rays created

when an electron falls into an $n = 1$ shell vacancy are called K_α when they come from the next higher level; that is, an $n = 2$ to $n = 1$ transition. The labels $K, L, M, \dots$ come from the older alphabetical labeling of shells starting with K rather than using the principal quantum numbers 1, 2, 3, ... A more energetic K_β x ray is produced when an electron falls into an $n = 1$ shell vacancy from the $n = 3$ shell; that is, an $n = 3$ to $n = 1$ transition. Similarly, when an electron falls into the $n = 2$ shell from the $n = 3$ shell, an L_α x ray is created. The energies of these x rays depend on the energies of electron states in the particular atom and, thus, are characteristic of that element: every element has its own set of x-ray energies. This property can be used to identify elements, for example, to find trace (small) amounts of an element in an environmental or biological sample.



A characteristic x ray is emitted when an electron fills an inner-shell vacancy, as shown for several transitions in this approximate energy level diagram for a multiple-electron atom.

Characteristic x rays are labeled according to the shell that had the vacancy and the shell from which

the electron came. A K_α x ray, for example, is produced when an electron coming from the $n = 2$ shell fills the $n = 1$ shell vacancy.

Example:**Characteristic X-Ray Energy**

Calculate the approximate energy of a K_α x ray from a tungsten anode in an x-ray tube.

Strategy

How do we calculate energies in a multiple-electron atom? In the case of characteristic x rays, the following approximate calculation is reasonable. Characteristic x rays are produced when an inner-shell vacancy is filled. Inner-shell electrons are nearer the nucleus than others in an atom and thus feel little net effect from the others. This is similar to what happens inside a charged conductor, where its excess charge is distributed over the surface so that it produces no electric field inside. It is reasonable to assume the inner-shell electrons have hydrogen-like energies, as given by

$E_n = -\frac{Z^2}{n^2} E_0$ ($n = 1, 2, 3, \dots$). As noted, a K_α x ray is produced by an $n = 2$ to $n = 1$ transition. Since there are two electrons in a filled K shell, a vacancy would leave one electron, so that the effective charge would be $Z - 1$ rather than Z . For tungsten, $Z = 74$, so that the effective charge is 73.

Solution

$E_n = -\frac{Z^2}{n^2} E_0$ ($n = 1, 2, 3, \dots$) gives the orbital energies for hydrogen-like atoms to be $E_n = -(Z^2/n^2)E_0$, where $E_0 = 13.6$ eV. As noted, the effective Z is 73. Now the K_α x-ray energy is given by

Equation:

$$E_{K_\alpha} = \Delta E = E_i - E_f = E_2 - E_1,$$

where

Equation:

$$E_1 = -\frac{Z^2}{1^2} E_0 = -\frac{73^2}{1} (13.6 \text{ eV}) = -72.5 \text{ keV}$$

and

Equation:

$$E_2 = -\frac{Z^2}{2^2} E_0 = -\frac{73^2}{4} (13.6 \text{ eV}) = -18.1 \text{ keV}.$$

Thus,

Equation:

$$E_{K_\alpha} = -18.1 \text{ keV} - (-72.5 \text{ keV}) = 54.4 \text{ keV}.$$

Discussion

This large photon energy is typical of characteristic x rays from heavy elements. It is large compared with other atomic emissions because it is produced when an inner-shell vacancy is filled, and inner-shell electrons are tightly bound. Characteristic x ray energies become progressively larger for heavier elements because their energy increases approximately as Z^2 . Significant accelerating voltage is needed to create these inner-shell vacancies. In the case of tungsten, at least 72.5 kV is needed, because other shells are filled and you cannot simply bump one electron to a higher filled shell. Tungsten is a common anode material in x-ray tubes; so much of the energy of the impinging electrons is absorbed, raising its temperature, that a high-melting-point material like tungsten is required.

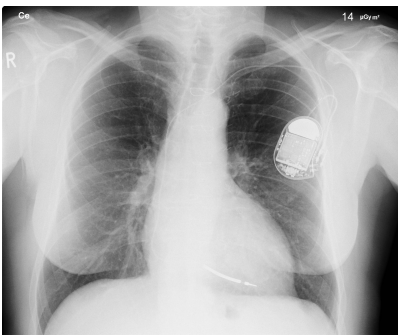
Medical and Other Diagnostic Uses of X-rays

All of us can identify diagnostic uses of x-ray photons. Among these are the universal dental and medical x rays that have become an essential part of medical diagnostics. (See [\[link\]](#) and [\[link\]](#).) X rays are also used to inspect

our luggage at airports, as shown in [\[link\]](#), and for early detection of cracks in crucial aircraft components. An x ray is not only a noun meaning high-energy photon, it also is an image produced by x rays, and it has been made into a familiar verb—to be x-rayed.



An x-ray image reveals
fillings in a person's
teeth. (credit: Dmitry G,
Wikimedia Commons)



This x-ray image of
a person's chest
shows many
details, including
an artificial
pacemaker. (credit:
Sunzi99,

Wikimedia
Commons)



This x-ray image
shows the contents of
a piece of luggage.

The denser the
material, the darker
the shadow. (credit:
IDuke, Wikimedia
Commons)

The most common x-ray images are simple shadows. Since x-ray photons have high energies, they penetrate materials that are opaque to visible light. The more energy an x-ray photon has, the more material it will penetrate. So an x-ray tube may be operated at 50.0 kV for a chest x ray, whereas it may need to be operated at 100 kV to examine a broken leg in a cast. The depth of penetration is related to the density of the material as well as to the energy of the photon. The denser the material, the fewer x-ray photons get through and the darker the shadow. Thus x rays excel at detecting breaks in bones and in imaging other physiological structures, such as some tumors, that differ in density from surrounding material. Because of their high photon energy, x rays produce significant ionization in materials and

damage cells in biological organisms. Modern uses minimize exposure to the patient and eliminate exposure to others. Biological effects of x rays will be explored in the next chapter along with other types of ionizing radiation such as those produced by nuclei.

As the x-ray energy increases, the Compton effect (see [Photon Momentum](#)) becomes more important in the attenuation of the x rays. Here, the x ray scatters from an outer electron shell of the atom, giving the ejected electron some kinetic energy while losing energy itself. The probability for attenuation of the x rays depends upon the number of electrons present (the material's density) as well as the thickness of the material. Chemical composition of the medium, as characterized by its atomic number Z , is not important here. Low-energy x rays provide better contrast (sharper images). However, due to greater attenuation and less scattering, they are more absorbed by thicker materials. Greater contrast can be achieved by injecting a substance with a large atomic number, such as barium or iodine. The structure of the part of the body that contains the substance (e.g., the gastrointestinal tract or the abdomen) can easily be seen this way.

Breast cancer is the second-leading cause of death among women worldwide. Early detection can be very effective, hence the importance of x-ray diagnostics. A mammogram cannot diagnose a malignant tumor, only give evidence of a lump or region of increased density within the breast. X-ray absorption by different types of soft tissue is very similar, so contrast is difficult; this is especially true for younger women, who typically have denser breasts. For older women who are at greater risk of developing breast cancer, the presence of more fat in the breast gives the lump or tumor more contrast. MRI (Magnetic resonance imaging) has recently been used as a supplement to conventional x rays to improve detection and eliminate false positives. The subject's radiation dose from x rays will be treated in a later chapter.

A standard x ray gives only a two-dimensional view of the object. Dense bones might hide images of soft tissue or organs. If you took another x ray from the side of the person (the first one being from the front), you would gain additional information. While shadow images are sufficient in many applications, far more sophisticated images can be produced with modern

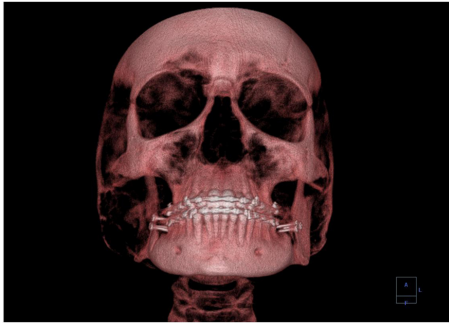
technology. [\[link\]](#) shows the use of a computed tomography (CT) scanner, also called computed axial tomography (CAT) scanner. X rays are passed through a narrow section (called a slice) of the patient's body (or body part) over a range of directions. An array of many detectors on the other side of the patient registers the x rays. The system is then rotated around the patient and another image is taken, and so on. The x-ray tube and detector array are mechanically attached and so rotate together. Complex computer image processing of the relative absorption of the x rays along different directions produces a highly-detailed image. Different slices are taken as the patient moves through the scanner on a table. Multiple images of different slices can also be computer analyzed to produce three-dimensional information, sometimes enhancing specific types of tissue, as shown in [\[link\]](#). G. Hounsfield (UK) and A. Cormack (US) won the Nobel Prize in Medicine in 1979 for their development of computed tomography.



A patient being positioned in a CT scanner aboard the hospital ship USNS Mercy. The CT scanner passes x rays through slices of the patient's body (or body part) over a range of directions.

The relative absorption of the x rays along different

directions is computer analyzed to produce highly detailed images. Three-dimensional information can be obtained from multiple slices. (credit: Rebecca Moat, U.S. Navy)



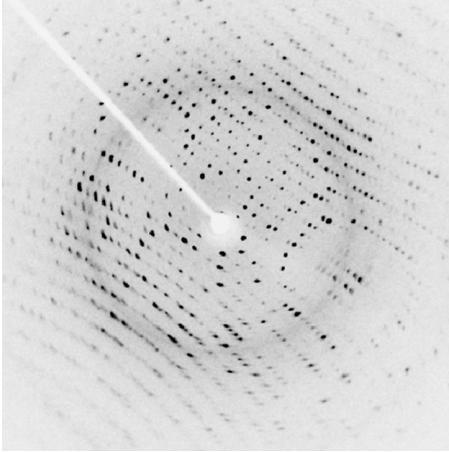
This three-dimensional image of a skull was produced by computed tomography, involving analysis of several x-ray slices of the head. (credit: Emailshankar, Wikimedia Commons)

X-Ray Diffraction and Crystallography

Since x-ray photons are very energetic, they have relatively short wavelengths. For example, the 54.4-keV K_{α} x ray of [\[link\]](#) has a

wavelength $\lambda = hc/E = 0.0228 \text{ nm}$. Thus, typical x-ray photons act like rays when they encounter macroscopic objects, like teeth, and produce sharp shadows; however, since atoms are on the order of 0.1 nm in size, x rays can be used to detect the location, shape, and size of atoms and molecules. The process is called **x-ray diffraction**, because it involves the diffraction and interference of x rays to produce patterns that can be analyzed for information about the structures that scattered the x rays. Perhaps the most famous example of x-ray diffraction is the discovery of the double-helix structure of DNA in 1953 by an international team of scientists working at the Cavendish Laboratory—American James Watson, Englishman Francis Crick, and New Zealand-born Maurice Wilkins. Using x-ray diffraction data produced by Rosalind Franklin, they were the first to discern the structure of DNA that is so crucial to life. For this, Watson, Crick, and Wilkins were awarded the 1962 Nobel Prize in Physiology or Medicine. There is much debate and controversy over the issue that Rosalind Franklin was not included in the prize.

[\[link\]](#) shows a diffraction pattern produced by the scattering of x rays from a crystal. This process is known as x-ray crystallography because of the information it can yield about crystal structure, and it was the type of data Rosalind Franklin supplied to Watson and Crick for DNA. Not only do x rays confirm the size and shape of atoms, they give information on the atomic arrangements in materials. For example, current research in high-temperature superconductors involves complex materials whose lattice arrangements are crucial to obtaining a superconducting material. These can be studied using x-ray crystallography.



X-ray diffraction from
the crystal of a protein,
hen egg lysozyme,
produced this
interference pattern.
Analysis of the pattern
yields information
about the structure of
the protein. (credit:
Del45, Wikimedia
Commons)

Historically, the scattering of x rays from crystals was used to prove that x rays are energetic EM waves. This was suspected from the time of the discovery of x rays in 1895, but it was not until 1912 that the German Max von Laue (1879–1960) convinced two of his colleagues to scatter x rays from crystals. If a diffraction pattern is obtained, he reasoned, then the x rays must be waves, and their wavelength could be determined. (The spacing of atoms in various crystals was reasonably well known at the time, based on good values for Avogadro's number.) The experiments were convincing, and the 1914 Nobel Prize in Physics was given to von Laue for his suggestion leading to the proof that x rays are EM waves. In 1915, the unique father-and-son team of Sir William Henry Bragg and his son Sir William Lawrence Bragg were awarded a joint Nobel Prize for inventing

the x-ray spectrometer and the then-new science of x-ray analysis. The elder Bragg had migrated to Australia from England just after graduating in mathematics. He learned physics and chemistry during his career at the University of Adelaide. The younger Bragg was born in Adelaide but went back to the Cavendish Laboratories in England to a career in x-ray and neutron crystallography; he provided support for Watson, Crick, and Wilkins for their work on unraveling the mysteries of DNA and to Max Perutz for his 1962 Nobel Prize-winning work on the structure of hemoglobin. Here again, we witness the enabling nature of physics—establishing instruments and designing experiments as well as solving mysteries in the biomedical sciences.

Certain other uses for x rays will be studied in later chapters. X rays are useful in the treatment of cancer because of the inhibiting effect they have on cell reproduction. X rays observed coming from outer space are useful in determining the nature of their sources, such as neutron stars and possibly black holes. Created in nuclear bomb explosions, x rays can also be used to detect clandestine atmospheric tests of these weapons. X rays can cause excitations of atoms, which then fluoresce (emitting characteristic EM radiation), making x-ray-induced fluorescence a valuable analytical tool in a range of fields from art to archaeology.

Section Summary

- X rays are relatively high-frequency EM radiation. They are produced by transitions between inner-shell electron levels, which produce x rays characteristic of the atomic element, or by decelerating electrons.
- X rays have many uses, including medical diagnostics and x-ray diffraction.

Conceptual Questions

Exercise:

Problem:

Explain why characteristic x rays are the most energetic in the EM emission spectrum of a given element.

Exercise:**Problem:**

Why does the energy of characteristic x rays become increasingly greater for heavier atoms?

Exercise:**Problem:**

Observers at a safe distance from an atmospheric test of a nuclear bomb feel its heat but receive none of its copious x rays. Why is air opaque to x rays but transparent to infrared?

Exercise:**Problem:**

Lasers are used to burn and read CDs. Explain why a laser that emits blue light would be capable of burning and reading more information than one that emits infrared.

Exercise:**Problem:**

Crystal lattices can be examined with x rays but not UV. Why?

Exercise:**Problem:**

CT scanners do not detect details smaller than about 0.5 mm. Is this limitation due to the wavelength of x rays? Explain.

Problem Exercises

Exercise:**Problem:**

(a) What is the shortest-wavelength x-ray radiation that can be generated in an x-ray tube with an applied voltage of 50.0 kV? (b) Calculate the photon energy in eV. (c) Explain the relationship of the photon energy to the applied voltage.

Solution:

(a) $0.248 \times 10^{-10} \text{ m}$

(b) 50.0 keV

(c) The photon energy is simply the applied voltage times the electron charge, so the value of the voltage in volts is the same as the value of the energy in electron volts.

Exercise:**Problem:**

A color television tube also generates some x rays when its electron beam strikes the screen. What is the shortest wavelength of these x rays, if a 30.0-kV potential is used to accelerate the electrons? (Note that TVs have shielding to prevent these x rays from exposing viewers.)

Exercise:**Problem:**

An x ray tube has an applied voltage of 100 kV. (a) What is the most energetic x-ray photon it can produce? Express your answer in electron volts and joules. (b) Find the wavelength of such an X-ray.

Solution:

(a) $100 \times 10^3 \text{ eV}$, $1.60 \times 10^{-14} \text{ J}$

(b) $0.124 \times 10^{-10} \text{ m}$

Exercise:

Problem:

The maximum characteristic x-ray photon energy comes from the capture of a free electron into a K shell vacancy. What is this photon energy in keV for tungsten, assuming the free electron has no initial kinetic energy?

Exercise:

Problem:

What are the approximate energies of the K_α and K_β x rays for copper?

Solution:

(a) 8.00 keV

(b) 9.48 keV

Glossary

x rays

a form of electromagnetic radiation

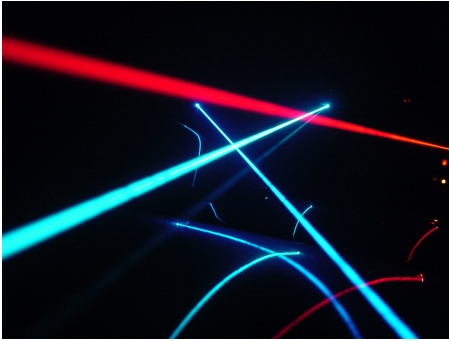
x-ray diffraction

a technique that provides the detailed information about crystallographic structure of natural and manufactured materials

Applications of Atomic Excitations and De-Excitations

- Define and discuss fluorescence.
- Define metastable.
- Describe how laser emission is produced.
- Explain population inversion.
- Define and discuss holography.

Many properties of matter and phenomena in nature are directly related to atomic energy levels and their associated excitations and de-excitations. The color of a rose, the output of a laser, and the transparency of air are but a few examples. (See [\[link\]](#).) While it may not appear that glow-in-the-dark pajamas and lasers have much in common, they are in fact different applications of similar atomic de-excitations.



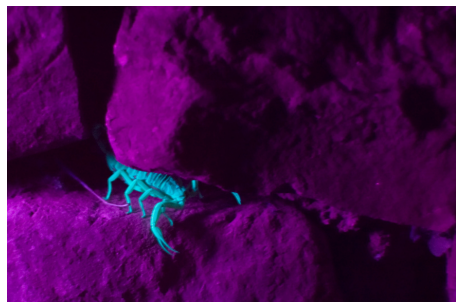
Light from a laser is based on a particular type of atomic de-excitation. (credit: Jeff Keyzer)

The color of a material is due to the ability of its atoms to absorb certain wavelengths while reflecting or reemitting others. A simple red material, for example a tomato, absorbs all visible wavelengths except red. This is because the atoms of its hydrocarbon pigment (lycopene) have levels separated by a variety of energies corresponding to all visible photon energies except red. Air is another interesting example. It is transparent to visible light, because there are few energy levels that visible photons can

excite in air molecules and atoms. Visible light, thus, cannot be absorbed. Furthermore, visible light is only weakly scattered by air, because visible wavelengths are so much greater than the sizes of the air molecules and atoms. Light must pass through kilometers of air to scatter enough to cause red sunsets and blue skies.

Fluorescence and Phosphorescence

The ability of a material to emit various wavelengths of light is similarly related to its atomic energy levels. [\[link\]](#) shows a scorpion illuminated by a UV lamp, sometimes called a black light. Some rocks also glow in black light, the particular colors being a function of the rock's mineral composition. Black lights are also used to make certain posters glow.



Objects glow in the visible spectrum when illuminated by an ultraviolet (black) light. Emissions are characteristic of the mineral involved, since they are related to its energy levels. In the case of scorpions, proteins near the surface of their skin give off the characteristic blue

glow. This is a colorful example of fluorescence in which excitation is induced by UV radiation while de-excitation occurs in the form of visible light. (credit: Ken Bosma, Flickr)

In the fluorescence process, an atom is excited to a level several steps above its ground state by the absorption of a relatively high-energy UV photon. This is called **atomic excitation**. Once it is excited, the atom can de-excite in several ways, one of which is to re-emit a photon of the same energy as excited it, a single step back to the ground state. This is called **atomic de-excitation**. All other paths of de-excitation involve smaller steps, in which lower-energy (longer wavelength) photons are emitted. Some of these may be in the visible range, such as for the scorpion in [\[link\]](#). **Fluorescence** is defined to be any process in which an atom or molecule, excited by a photon of a given energy, and de-excites by emission of a lower-energy photon.

Fluorescence can be induced by many types of energy input. Fluorescent paint, dyes, and even soap residues in clothes make colors seem brighter in sunlight by converting some UV into visible light. X rays can induce fluorescence, as is done in x-ray fluoroscopy to make brighter visible images. Electric discharges can induce fluorescence, as in so-called neon lights and in gas-discharge tubes that produce atomic and molecular spectra. Common fluorescent lights use an electric discharge in mercury vapor to cause atomic emissions from mercury atoms. The inside of a fluorescent light is coated with a fluorescent material that emits visible light over a broad spectrum of wavelengths. By choosing an appropriate coating, fluorescent lights can be made more like sunlight or like the reddish glow of candlelight, depending on needs. Fluorescent lights are more efficient in converting electrical energy into visible light than incandescent filaments

(about four times as efficient), the blackbody radiation of which is primarily in the infrared due to temperature limitations.

This atom is excited to one of its higher levels by absorbing a UV photon. It can de-excite in a single step, re-emitting a photon of the same energy, or in several steps. The process is called fluorescence if the atom de-excites in smaller steps, emitting energy different from that which excited it. Fluorescence can be induced by a variety of energy inputs, such as UV, x-rays, and electrical discharge.

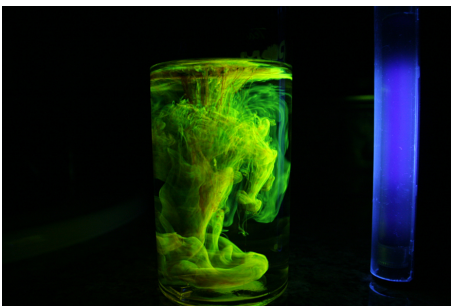
The spectacular Waitomo caves on North Island in New Zealand provide a natural habitat for glow-worms. The glow-worms hang up to 70 silk threads of about 30 or 40 cm each to trap prey that fly towards them in the dark. The fluorescence process is very efficient, with nearly 100% of the energy input turning into light. (In comparison, fluorescent lights are about 20% efficient.)

Fluorescence has many uses in biology and medicine. It is commonly used to label and follow a molecule within a cell. Such tagging allows one to study the structure of DNA and proteins. Fluorescent dyes and antibodies are usually used to tag the molecules, which are then illuminated with UV light and their emission of visible light is observed. Since the fluorescence of each element is characteristic, identification of elements within a sample can be done this way.

[\[link\]](#) shows a commonly used fluorescent dye called fluorescein. Below that, [\[link\]](#) reveals the diffusion of a fluorescent dye in water by observing it under UV light.



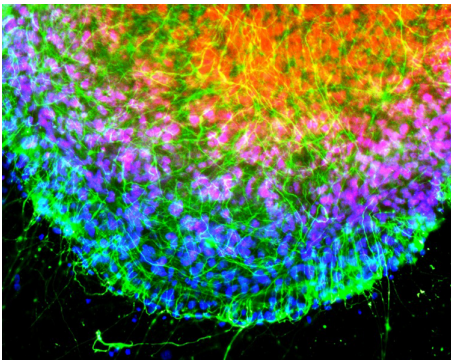
Fluorescein, shown here in powder form, is used to dye laboratory samples.
(credit: Benjah-bmm27, Wikimedia Commons)



Here, fluorescent powder is added to a beaker of water. The mixture gives off a bright glow under ultraviolet light.
(credit: Bricksnite, Wikimedia Commons)

Note:**Nano-Crystals**

Recently, a new class of fluorescent materials has appeared—“nano-crystals.” These are single-crystal molecules less than 100 nm in size. The smallest of these are called “quantum dots.” These semiconductor indicators are very small (2–6 nm) and provide improved brightness. They also have the advantage that all colors can be excited with the same incident wavelength. They are brighter and more stable than organic dyes and have a longer lifetime than conventional phosphors. They have become an excellent tool for long-term studies of cells, including migration and morphology. ([link](#).)



Microscopic image of chicken cells using nano-crystals of a fluorescent dye. Cell nuclei exhibit blue fluorescence while neurofilaments exhibit green. (credit: Weerapong Prasongchean, Wikimedia Commons)

Once excited, an atom or molecule will usually spontaneously de-excite quickly. (The electrons raised to higher levels are attracted to lower ones by the positive charge of the nucleus.) Spontaneous de-excitation has a very short mean lifetime of typically about 10^{-8} s. However, some levels have significantly longer lifetimes, ranging up to milliseconds to minutes or even hours. These energy levels are inhibited and are slow in de-exciting because their quantum numbers differ greatly from those of available lower levels. Although these level lifetimes are short in human terms, they are many orders of magnitude longer than is typical and, thus, are said to be **metastable**, meaning relatively stable. **Phosphorescence** is the de-excitation of a metastable state. Glow-in-the-dark materials, such as luminous dials on some watches and clocks and on children's toys and pajamas, are made of phosphorescent substances. Visible light excites the atoms or molecules to metastable states that decay slowly, releasing the stored excitation energy partially as visible light. In some ceramics, atomic excitation energy can be frozen in after the ceramic has cooled from its firing. It is very slowly released, but the ceramic can be induced to phosphoresce by heating—a process called “thermoluminescence.” Since the release is slow, thermoluminescence can be used to date antiquities. The less light emitted, the older the ceramic. (See [\[link\]](#).)



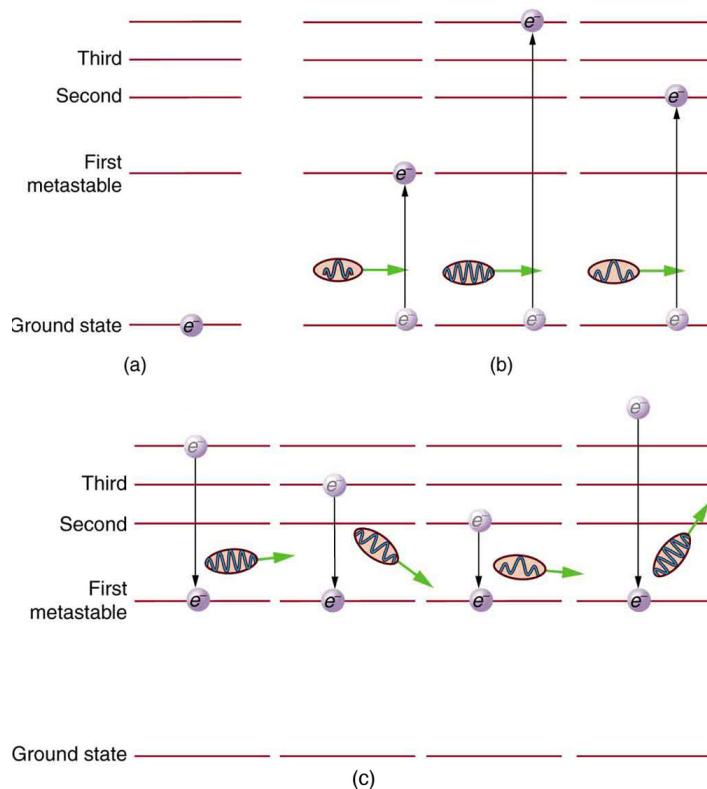
Atoms frozen in an excited state when this Chinese ceramic figure was fired can be stimulated to de-excite and emit EM radiation by heating a sample of the ceramic—a process called thermoluminescence. Since the states slowly de-excite over centuries, the amount of thermoluminescence decreases with age, making it possible to use this effect to date and authenticate antiquities. This figure dates from the 11th century. (credit: Vassil, Wikimedia Commons)

Lasers

Lasers today are commonplace. Lasers are used to read bar codes at stores and in libraries, laser shows are staged for entertainment, laser printers produce high-quality images at relatively low cost, and lasers send prodigious numbers of telephone messages through optical fibers. Among other things, lasers are also employed in surveying, weapons guidance,

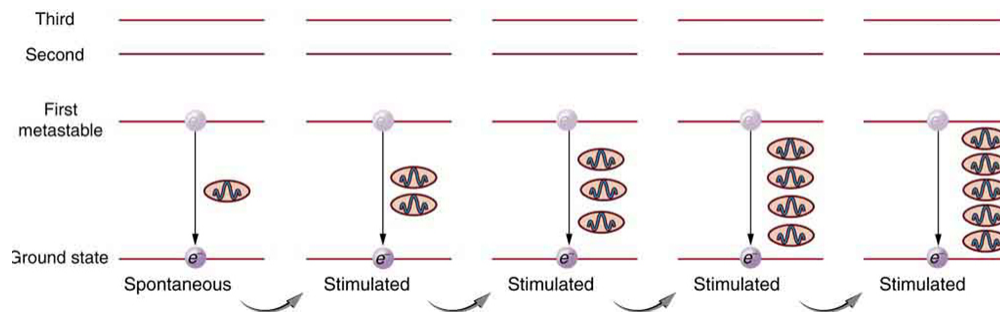
tumor eradication, retinal welding, and for reading music CDs and computer CD-ROMs.

Why do lasers have so many varied applications? The answer is that lasers produce single-wavelength EM radiation that is also very coherent—that is, the emitted photons are in phase. Laser output can, thus, be more precisely manipulated than incoherent mixed-wavelength EM radiation from other sources. The reason laser output is so pure and coherent is based on how it is produced, which in turn depends on a metastable state in the lasing material. Suppose a material had the energy levels shown in [\[link\]](#). When energy is put into a large collection of these atoms, electrons are raised to all possible levels. Most return to the ground state in less than about 10^{-8} s, but those in the metastable state linger. This includes those electrons originally excited to the metastable state and those that fell into it from above. It is possible to get a majority of the atoms into the metastable state, a condition called a **population inversion**.



(a) Energy-level diagram for an atom showing the first few states, one of which is metastable. (b) Massive energy input excites atoms to a variety of states. (c) Most states decay quickly, leaving electrons only in the metastable and ground state. If a majority of electrons are in the metastable state, a population inversion has been achieved.

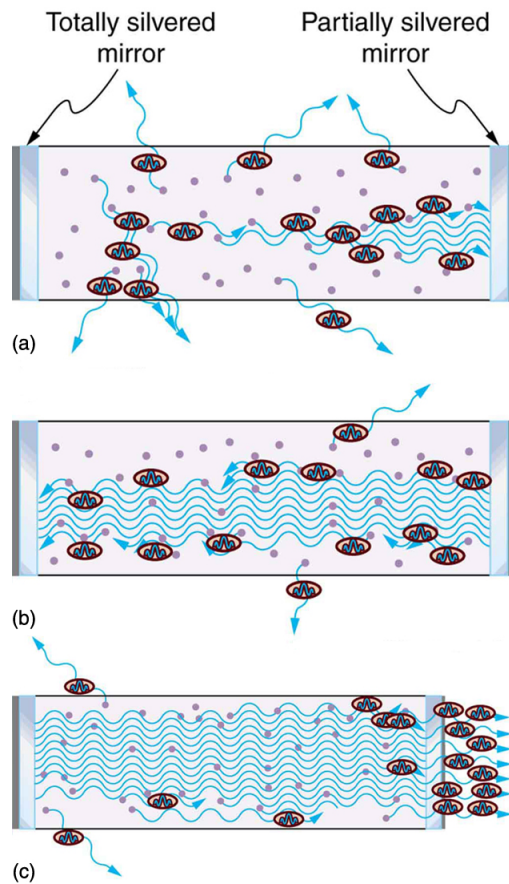
Once a population inversion is achieved, a very interesting thing can happen, as shown in [\[link\]](#). An electron spontaneously falls from the metastable state, emitting a photon. This photon finds another atom in the metastable state and stimulates it to decay, emitting a second photon of *the same wavelength and in phase* with the first, and so on. **Stimulated emission** is the emission of electromagnetic radiation in the form of photons of a given frequency, triggered by photons of the same frequency. For example, an excited atom, with an electron in an energy orbit higher than normal, releases a photon of a specific frequency when the electron drops back to a lower energy orbit. If this photon then strikes another electron in the same high-energy orbit in another atom, another photon of the same frequency is released. The emitted photons and the triggering photons are always in phase, have the same polarization, and travel in the same direction. The probability of absorption of a photon is the same as the probability of stimulated emission, and so a majority of atoms must be in the metastable state to produce energy. Einstein (again Einstein, and back in 1917!) was one of the important contributors to the understanding of stimulated emission of radiation. Among other things, Einstein was the first to realize that stimulated emission and absorption are equally probable. The laser acts as a temporary energy storage device that subsequently produces a massive energy output of single-wavelength, in-phase photons.



One atom in the metastable state spontaneously decays to a lower level, producing a photon that goes on to stimulate another atom to de-excite. The second photon has exactly the same energy and wavelength as the first and is in phase with it. Both go on to stimulate the emission of other photons. A population inversion is necessary for there to be a net production rather than a net absorption of the photons.

The name **laser** is an acronym for light amplification by stimulated emission of radiation, the process just described. The process was proposed and developed following the advances in quantum physics. A joint Nobel Prize was awarded in 1964 to American Charles Townes (1915–), and Nikolay Basov (1922–2001) and Aleksandr Prokhorov (1916–2002), from the Soviet Union, for the development of lasers. The Nobel Prize in 1981 went to Arthur Schawlow (1921-1999) for pioneering laser applications. The original devices were called masers, because they produced microwaves. The first working laser was created in 1960 at Hughes Research labs (CA) by T. Maiman. It used a pulsed high-powered flash lamp and a ruby rod to produce red light. Today the name laser is used for all such devices developed to produce a variety of wavelengths, including microwave, infrared, visible, and ultraviolet radiation. [\[link\]](#) shows how a laser can be constructed to enhance the stimulated emission of radiation. Energy input can be from a flash tube, electrical discharge, or other sources, in a process sometimes called optical pumping. A large percentage of the original pumping energy is dissipated in other forms, but a population inversion must be achieved. Mirrors can be used to enhance stimulated

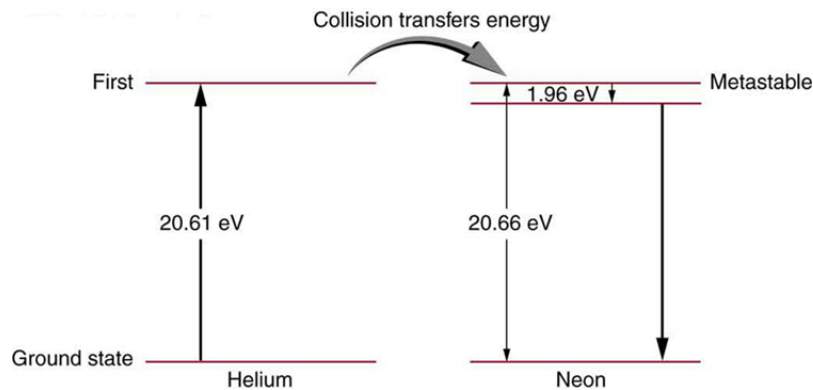
emission by multiple passes of the radiation back and forth through the lasing material. One of the mirrors is semitransparent to allow some of the light to pass through. The laser output from a laser is a mere 1% of the light passing back and forth in a laser.



Typical laser construction has a method of pumping energy into the lasing material to produce a population inversion. (a) Spontaneous emission begins with some photons escaping and others stimulating further emissions. (b) and (c)

Mirrors are used to enhance the probability of stimulated emission by passing photons through the material several times.

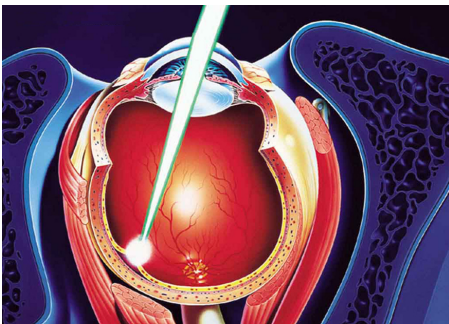
Lasers are constructed from many types of lasing materials, including gases, liquids, solids, and semiconductors. But all lasers are based on the existence of a metastable state or a phosphorescent material. Some lasers produce continuous output; others are pulsed in bursts as brief as 10^{-14} s. Some laser outputs are fantastically powerful—some greater than 10^{12} W—but the more common, everyday lasers produce something on the order of 10^{-3} W. The helium-neon laser that produces a familiar red light is very common. [\[link\]](#) shows the energy levels of helium and neon, a pair of noble gases that work well together. An electrical discharge is passed through a helium-neon gas mixture in which the number of atoms of helium is ten times that of neon. The first excited state of helium is metastable and, thus, stores energy. This energy is easily transferred by collision to neon atoms, because they have an excited state at nearly the same energy as that in helium. That state in neon is also metastable, and this is the one that produces the laser output. (The most likely transition is to the nearby state, producing 1.96 eV photons, which have a wavelength of 633 nm and appear red.) A population inversion can be produced in neon, because there are so many more helium atoms and these put energy into the neon. Helium-neon lasers often have continuous output, because the population inversion can be maintained even while lasing occurs. Probably the most common lasers in use today, including the common laser pointer, are semiconductor or diode lasers, made of silicon. Here, energy is pumped into the material by passing a current in the device to excite the electrons. Special coatings on the ends and fine cleavings of the semiconductor material allow light to bounce back and forth and a tiny fraction to emerge as laser light. Diode lasers can usually run continually and produce outputs in the milliwatt range.



Energy levels in helium and neon. In the common helium-neon laser, an electrical discharge pumps energy into the metastable states of both atoms. The gas mixture has about ten times more helium atoms than neon atoms. Excited helium atoms easily de-excite by transferring energy to neon in a collision. A population inversion in neon is achieved, allowing lasing by the neon to occur.

There are many medical applications of lasers. Lasers have the advantage that they can be focused to a small spot. They also have a well-defined wavelength. Many types of lasers are available today that provide wavelengths from the ultraviolet to the infrared. This is important, as one needs to be able to select a wavelength that will be preferentially absorbed by the material of interest. Objects appear a certain color because they absorb all other visible colors incident upon them. What wavelengths are absorbed depends upon the energy spacing between electron orbitals in that molecule. Unlike the hydrogen atom, biological molecules are complex and have a variety of absorption wavelengths or lines. But these can be determined and used in the selection of a laser with the appropriate wavelength. Water is transparent to the visible spectrum but will absorb light in the UV and IR regions. Blood (hemoglobin) strongly reflects red but absorbs most strongly in the UV.

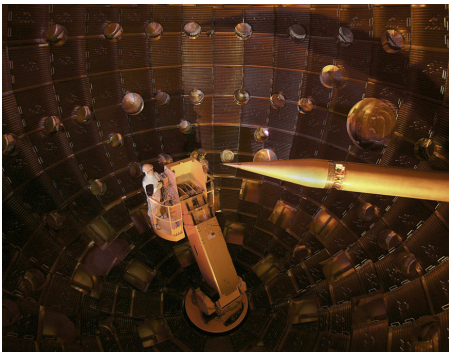
Laser surgery uses a wavelength that is strongly absorbed by the tissue it is focused upon. One example of a medical application of lasers is shown in [\[link\]](#). A detached retina can result in total loss of vision. Burns made by a laser focused to a small spot on the retina form scar tissue that can hold the retina in place, salvaging the patient's vision. Other light sources cannot be focused as precisely as a laser due to refractive dispersion of different wavelengths. Similarly, laser surgery in the form of cutting or burning away tissue is made more accurate because laser output can be very precisely focused and is preferentially absorbed because of its single wavelength. Depending upon what part or layer of the retina needs repairing, the appropriate type of laser can be selected. For the repair of tears in the retina, a green argon laser is generally used. This light is absorbed well by tissues containing blood, so coagulation or "welding" of the tear can be done.



A detached retina is burned by a laser designed to focus on a small spot on the retina, the resulting scar tissue holding it in place. The lens of the eye is used to focus the light, as is the device bringing the laser output to the eye.

In dentistry, the use of lasers is rising. Lasers are most commonly used for surgery on the soft tissue of the mouth. They can be used to remove ulcers, stop bleeding, and reshape gum tissue. Their use in cutting into bones and teeth is not quite so common; here the erbium YAG (yttrium aluminum garnet) laser is used.

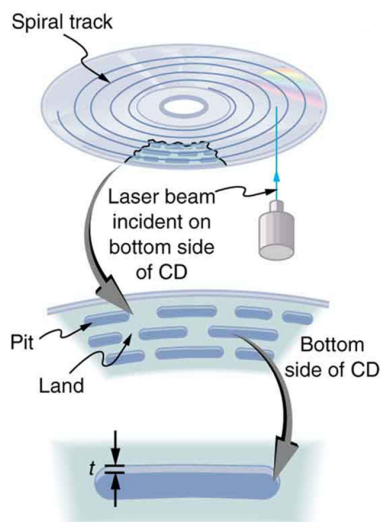
The massive combination of lasers shown in [\[link\]](#) can be used to induce nuclear fusion, the energy source of the sun and hydrogen bombs. Since lasers can produce very high power in very brief pulses, they can be used to focus an enormous amount of energy on a small glass sphere containing fusion fuel. Not only does the incident energy increase the fuel temperature significantly so that fusion can occur, it also compresses the fuel to great density, enhancing the probability of fusion. The compression or implosion is caused by the momentum of the impinging laser photons.



This system of lasers
at Lawrence
Livermore Laboratory
is used to ignite
nuclear fusion. A
tremendous burst of
energy is focused on a
small fuel pellet,
which is imploded to
the high density and
temperature needed to
make the fusion

reaction proceed.
(credit: Lawrence
Livermore National
Laboratory, Lawrence
Livermore National
Security, LLC, and the
Department of
Energy)

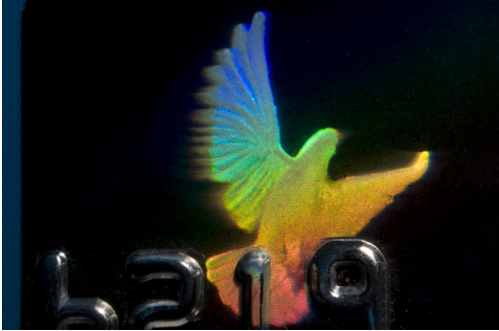
Music CDs are now so common that vinyl records are quaint antiquities. CDs (and DVDs) store information digitally and have a much larger information-storage capacity than vinyl records. An entire encyclopedia can be stored on a single CD. [\[link\]](#) illustrates how the information is stored and read from the CD. Pits made in the CD by a laser can be tiny and very accurately spaced to record digital information. These are read by having an inexpensive solid-state infrared laser beam scatter from pits as the CD spins, revealing their digital pattern and the information encoded upon them.



A CD has digital
information

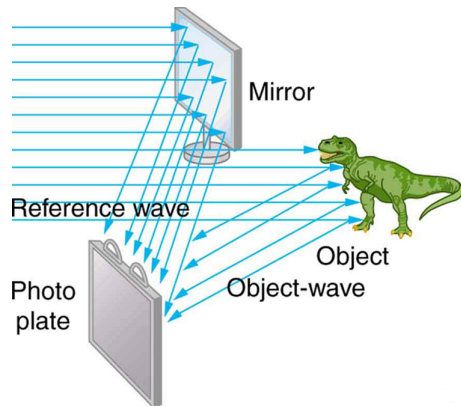
stored in the form
of laser-created
pits on its
surface. These in
turn can be read
by detecting the
laser light
scattered from the
pit. Large
information
capacity is
possible because
of the precision
of the laser.
Shorter-
wavelength lasers
enable greater
storage capacity.

Holograms, such as those in [\[link\]](#), are true three-dimensional images recorded on film by lasers. Holograms are used for amusement, decoration on novelty items and magazine covers, security on credit cards and driver's licenses (a laser and other equipment is needed to reproduce them), and for serious three-dimensional information storage. You can see that a hologram is a true three-dimensional image, because objects change relative position in the image when viewed from different angles.



Credit cards commonly have holograms for logos, making them difficult to reproduce (credit: Dominic Alves, Flickr)

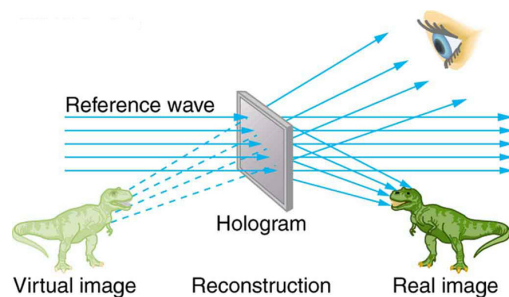
The name **hologram** means “entire picture” (from the Greek *holo*, as in holistic), because the image is three-dimensional. **Holography** is the process of producing holograms and, although they are recorded on photographic film, the process is quite different from normal photography. Holography uses light interference or wave optics, whereas normal photography uses geometric optics. [\[link\]](#) shows one method of producing a hologram. Coherent light from a laser is split by a mirror, with part of the light illuminating the object. The remainder, called the reference beam, shines directly on a piece of film. Light scattered from the object interferes with the reference beam, producing constructive and destructive interference. As a result, the exposed film looks foggy, but close examination reveals a complicated interference pattern stored on it. Where the interference was constructive, the film (a negative actually) is darkened. Holography is sometimes called lensless photography, because it uses the wave characteristics of light as contrasted to normal photography, which uses geometric optics and so requires lenses.



Production of a hologram. Single-wavelength coherent light from a laser produces a well-defined interference pattern on a piece of film. The laser beam is split by a partially silvered mirror, with part of the light illuminating the object and the remainder shining directly on the film.

Light falling on a hologram can form a three-dimensional image. The process is complicated in detail, but the basics can be understood as shown in [\[link\]](#), in which a laser of the same type that exposed the film is now used to illuminate it. The myriad tiny exposed regions of the film are dark and block the light, while less exposed regions allow light to pass. The film thus acts much like a collection of diffraction gratings with various spacings. Light passing through the hologram is diffracted in various directions, producing both real and virtual images of the object used to expose the film. The interference pattern is the same as that produced by the object. Moving

your eye to various places in the interference pattern gives you different perspectives, just as looking directly at the object would. The image thus looks like the object and is three-dimensional like the object.



A transmission hologram is one that produces real and virtual images when a laser of the same type as that which exposed the hologram is passed through it. Diffraction from various parts of the film produces the same interference pattern as the object that was used to expose it.

The hologram illustrated in [\[link\]](#) is a transmission hologram. Holograms that are viewed with reflected light, such as the white light holograms on credit cards, are reflection holograms and are more common. White light holograms often appear a little blurry with rainbow edges, because the diffraction patterns of various colors of light are at slightly different locations due to their different wavelengths. Further uses of holography include all types of 3-D information storage, such as of statues in museums and engineering studies of structures and 3-D images of human organs. Invented in the late 1940s by Dennis Gabor (1900–1970), who won the

1971 Nobel Prize in Physics for his work, holography became far more practical with the development of the laser. Since lasers produce coherent single-wavelength light, their interference patterns are more pronounced. The precision is so great that it is even possible to record numerous holograms on a single piece of film by just changing the angle of the film for each successive image. This is how the holograms that move as you walk by them are produced—a kind of lensless movie.

In a similar way, in the medical field, holograms have allowed complete 3-D holographic displays of objects from a stack of images. Storing these images for future use is relatively easy. With the use of an endoscope, high-resolution 3-D holographic images of internal organs and tissues can be made.

Section Summary

- An important atomic process is fluorescence, defined to be any process in which an atom or molecule is excited by absorbing a photon of a given energy and de-excited by emitting a photon of a lower energy.
- Some states live much longer than others and are termed metastable.
- Phosphorescence is the de-excitation of a metastable state.
- Lasers produce coherent single-wavelength EM radiation by stimulated emission, in which a metastable state is stimulated to decay.
- Lasing requires a population inversion, in which a majority of the atoms or molecules are in their metastable state.

Conceptual Questions

Exercise:

Problem:

How do the allowed orbits for electrons in atoms differ from the allowed orbits for planets around the sun? Explain how the correspondence principle applies here.

Exercise:

Problem:

Atomic and molecular spectra are discrete. What does discrete mean, and how are discrete spectra related to the quantization of energy and electron orbits in atoms and molecules?

Exercise:**Problem:**

Hydrogen gas can only absorb EM radiation that has an energy corresponding to a transition in the atom, just as it can only emit these discrete energies. When a spectrum is taken of the solar corona, in which a broad range of EM wavelengths are passed through very hot hydrogen gas, the absorption spectrum shows all the features of the emission spectrum. But when such EM radiation passes through room-temperature hydrogen gas, only the Lyman series is absorbed. Explain the difference.

Exercise:**Problem:**

Lasers are used to burn and read CDs. Explain why a laser that emits blue light would be capable of burning and reading more information than one that emits infrared.

Exercise:**Problem:**

The coating on the inside of fluorescent light tubes absorbs ultraviolet light and subsequently emits visible light. An inventor claims that he is able to do the reverse process. Is the inventor's claim possible?

Exercise:**Problem:**

What is the difference between fluorescence and phosphorescence?

Exercise:

Problem:

How can you tell that a hologram is a true three-dimensional image and that those in 3-D movies are not?

Problem Exercises**Exercise:****Problem:**

[\[link\]](#) shows the energy-level diagram for neon. (a) Verify that the energy of the photon emitted when neon goes from its metastable state to the one immediately below is equal to 1.96 eV. (b) Show that the wavelength of this radiation is 633 nm. (c) What wavelength is emitted when the neon makes a direct transition to its ground state?

Solution:

(a) 1.96 eV

(b) $(1240 \text{ eV}\cdot\text{nm})/(1.96 \text{ eV}) = 633 \text{ nm}$

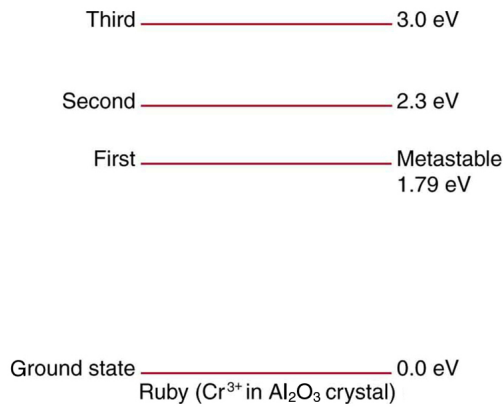
(c) 60.0 nm

Exercise:**Problem:**

A helium-neon laser is pumped by electric discharge. What wavelength electromagnetic radiation would be needed to pump it? See [\[link\]](#) for energy-level information.

Exercise:**Problem:**

Ruby lasers have chromium atoms doped in an aluminum oxide crystal. The energy level diagram for chromium in a ruby is shown in [\[link\]](#). What wavelength is emitted by a ruby laser?



Chromium atoms in an aluminum oxide crystal have these energy levels, one of which is metastable. This is the basis of a ruby laser. Visible light can pump the atom into an excited state above the metastable state to achieve a population inversion.

Solution:

693 nm

Exercise:

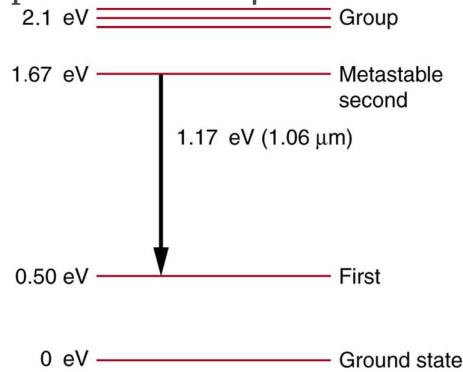
Problem:

(a) What energy photons can pump chromium atoms in a ruby laser from the ground state to its second and third excited states? (b) What are the wavelengths of these photons? Verify that they are in the visible part of the spectrum.

Exercise:

Problem:

Some of the most powerful lasers are based on the energy levels of neodymium in solids, such as glass, as shown in [\[link\]](#). (a) What average wavelength light can pump the neodymium into the levels above its metastable state? (b) Verify that the 1.17 eV transition produces 1.06 μm radiation.



Neodymium atoms in glass have these energy levels, one of which is metastable.

The group of levels above the metastable state is convenient for achieving a population inversion, since photons of many different energies can be absorbed by atoms in the ground state.

Solution:

(a) 590 nm

(b) $(1240 \text{ eV}\cdot\text{nm})/(1.17 \text{ eV}) = 1.06 \mu\text{m}$

Glossary

metastable

a state whose lifetime is an order of magnitude longer than the most short-lived states

atomic excitation

a state in which an atom or ion acquires the necessary energy to promote one or more of its electrons to electronic states higher in energy than their ground state

atomic de-excitation

process by which an atom transfers from an excited electronic state back to the ground state electronic configuration; often occurs by emission of a photon

laser

acronym for light amplification by stimulated emission of radiation

phosphorescence

the de-excitation of a metastable state

population inversion

the condition in which the majority of atoms in a sample are in a metastable state

stimulated emission

emission by atom or molecule in which an excited state is stimulated to decay, most readily caused by a photon of the same energy that is necessary to excite the state

hologram

means *entire picture* (from the Greek word *holo*, as in holistic), because the image produced is three dimensional

holography

the process of producing holograms

fluorescence

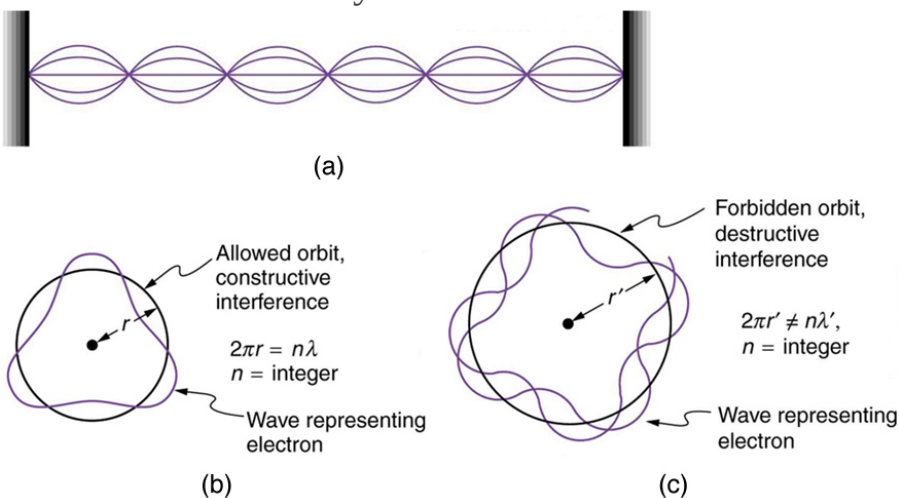
any process in which an atom or molecule, excited by a photon of a given energy, de-excites by emission of a lower-energy photon

The Wave Nature of Matter Causes Quantization

- Explain Bohr's model of atom.
- Define and describe quantization of angular momentum.
- Calculate the angular momentum for an orbit of atom.
- Define and describe the wave-like properties of matter.

After visiting some of the applications of different aspects of atomic physics, we now return to the basic theory that was built upon Bohr's atom. Einstein once said it was important to keep asking the questions we eventually teach children not to ask. Why is angular momentum quantized? You already know the answer. Electrons have wave-like properties, as de Broglie later proposed. They can exist only where they interfere constructively, and only certain orbits meet proper conditions, as we shall see in the next module.

Following Bohr's initial work on the hydrogen atom, a decade was to pass before de Broglie proposed that matter has wave properties. The wave-like properties of matter were subsequently confirmed by observations of electron interference when scattered from crystals. Electrons can exist only in locations where they interfere constructively. How does this affect electrons in atomic orbits? When an electron is bound to an atom, its wavelength must fit into a small space, something like a standing wave on a string. (See [\[link\]](#).) Allowed orbits are those orbits in which an electron constructively interferes with itself. Not all orbits produce constructive interference. Thus only certain orbits are allowed—the orbits are quantized.



(a) Waves on a string have a wavelength related to the length of the string, allowing them to interfere constructively. (b) If we imagine the string bent into a closed circle, we get a rough idea of how electrons in circular orbits can interfere constructively. (c) If the wavelength does not fit into the circumference, the electron interferes destructively; it cannot exist in such an orbit.

For a circular orbit, constructive interference occurs when the electron's wavelength fits neatly into the circumference, so that wave crests always align with crests and wave troughs align with troughs, as shown in [\[link\]](#) (b). More precisely, when an integral multiple of the electron's wavelength equals the circumference of the orbit, constructive interference is obtained. In equation form, the *condition for constructive interference and an allowed electron orbit* is

Equation:

$$n\lambda_n = 2\pi r_n (n = 1, 2, 3 \dots),$$

where λ_n is the electron's wavelength and r_n is the radius of that circular orbit. The de Broglie wavelength is $\lambda = h/p = h/mv$, and so here $\lambda = h/m_e v$. Substituting this into the previous condition for constructive interference produces an interesting result:

Equation:

$$\frac{nh}{m_e v} = 2\pi r_n.$$

Rearranging terms, and noting that $L = mvr$ for a circular orbit, we obtain the quantization of angular momentum as the condition for allowed orbits:

Equation:

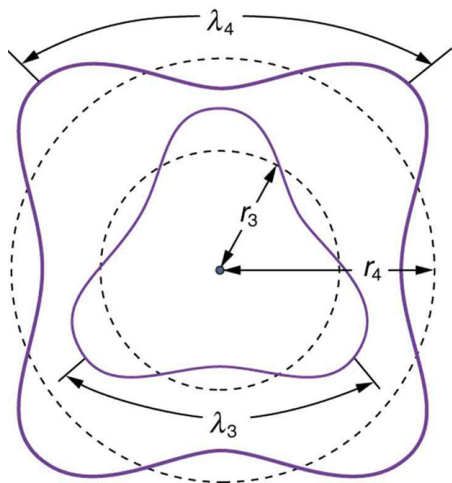
$$L = m_e v r_n = n \frac{h}{2\pi} (n = 1, 2, 3 \dots).$$

This is what Bohr was forced to hypothesize as the rule for allowed orbits, as stated earlier. We now realize that it is the condition for constructive interference of an electron in a circular orbit. [\[link\]](#) illustrates this for $n = 3$ and $n = 4$.

Note:

Waves and Quantization

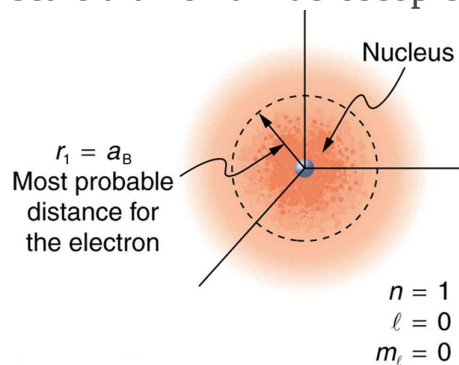
The wave nature of matter is responsible for the quantization of energy levels in bound systems. Only those states where matter interferes constructively exist, or are “allowed.” Since there is a lowest orbit where this is possible in an atom, the electron cannot spiral into the nucleus. It cannot exist closer to or inside the nucleus. The wave nature of matter is what prevents matter from collapsing and gives atoms their sizes.



The third and fourth
allowed circular orbits
have three and four
wavelengths,

respectively, in their
circumferences.

Because of the wave character of matter, the idea of well-defined orbits gives way to a model in which there is a cloud of probability, consistent with Heisenberg's uncertainty principle. [\[link\]](#) shows how this applies to the ground state of hydrogen. If you try to follow the electron in some well-defined orbit using a probe that has a small enough wavelength to get some details, you will instead knock the electron out of its orbit. Each measurement of the electron's position will find it to be in a definite location somewhere near the nucleus. Repeated measurements reveal a cloud of probability like that in the figure, with each speck the location determined by a single measurement. There is not a well-defined, circular-orbit type of distribution. Nature again proves to be different on a small scale than on a macroscopic scale.



The ground state of a
hydrogen atom has a
probability cloud
describing the position
of its electron. The
probability of finding
the electron is
proportional to the
darkness of the cloud.
The electron can be
closer or farther than

the Bohr radius, but it is very unlikely to be a great distance from the nucleus.

There are many examples in which the wave nature of matter causes quantization in bound systems such as the atom. Whenever a particle is confined or bound to a small space, its allowed wavelengths are those which fit into that space. For example, the particle in a box model describes a particle free to move in a small space surrounded by impenetrable barriers. This is true in blackbody radiators (atoms and molecules) as well as in atomic and molecular spectra. Various atoms and molecules will have different sets of electron orbits, depending on the size and complexity of the system. When a system is large, such as a grain of sand, the tiny particle waves in it can fit in so many ways that it becomes impossible to see that the allowed states are discrete. Thus the correspondence principle is satisfied. As systems become large, they gradually look less grainy, and quantization becomes less evident. Unbound systems (small or not), such as an electron freed from an atom, do not have quantized energies, since their wavelengths are not constrained to fit in a certain volume.

Note:

PhET Explorations: Quantum Wave Interference

When do photons, electrons, and atoms behave like particles and when do they behave like waves? Watch waves spread out and interfere as they pass through a double slit, then get detected on a screen as tiny dots. Use quantum detectors to explore how measurements change the waves and the patterns they produce on the screen.

[Quantum](#)
[Wave](#)

Section Summary

- Quantization of orbital energy is caused by the wave nature of matter. Allowed orbits in atoms occur for constructive interference of electrons in the orbit, requiring an integral number of wavelengths to fit in an orbit's circumference; that is,

Equation:

$$n\lambda_n = 2\pi r_n (n = 1, 2, 3 \dots),$$

where λ_n is the electron's de Broglie wavelength.

- Owing to the wave nature of electrons and the Heisenberg uncertainty principle, there are no well-defined orbits; rather, there are clouds of probability.
- Bohr correctly proposed that the energy and radii of the orbits of electrons in atoms are quantized, with energy for transitions between orbits given by

Equation:

$$\Delta E = hf = E_i - E_f,$$

where ΔE is the change in energy between the initial and final orbits and hf is the energy of an absorbed or emitted photon.

- It is useful to plot orbit energies on a vertical graph called an energy-level diagram.
- The allowed orbits are circular, Bohr proposed, and must have quantized orbital angular momentum given by

Equation:

$$L = m_e v r_n = n \frac{h}{2\pi} (n = 1, 2, 3 \dots),$$

where L is the angular momentum, r_n is the radius of orbit n , and h is Planck's constant.

Conceptual Questions

Exercise:

Problem:

How is the de Broglie wavelength of electrons related to the quantization of their orbits in atoms and molecules?

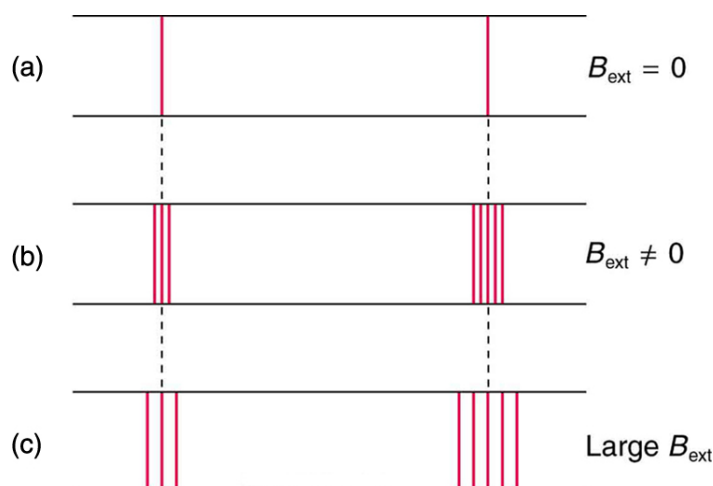
Patterns in Spectra Reveal More Quantization

- State and discuss the Zeeman effect.
- Define orbital magnetic field.
- Define orbital angular momentum.
- Define space quantization.

High-resolution measurements of atomic and molecular spectra show that the spectral lines are even more complex than they first appear. In this section, we will see that this complexity has yielded important new information about electrons and their orbits in atoms.

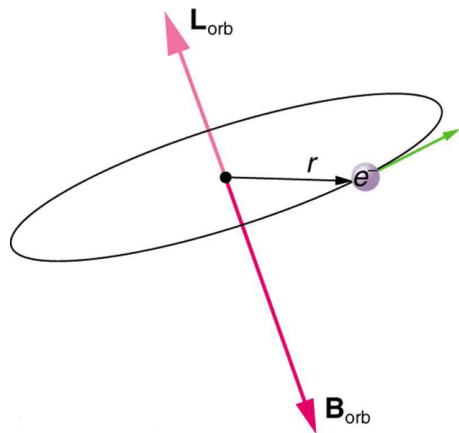
In order to explore the substructure of atoms (and knowing that magnetic fields affect moving charges), the Dutch physicist Hendrik Lorentz (1853–1930) suggested that his student Pieter Zeeman (1865–1943) study how spectra might be affected by magnetic fields. What they found became known as the **Zeeman effect**, which involved spectral lines being split into two or more separate emission lines by an external magnetic field, as shown in [\[link\]](#). For their discoveries, Zeeman and Lorentz shared the 1902 Nobel Prize in Physics.

Zeeman splitting is complex. Some lines split into three lines, some into five, and so on. But one general feature is that the amount the split lines are separated is proportional to the applied field strength, indicating an interaction with a moving charge. The splitting means that the quantized energy of an orbit is affected by an external magnetic field, causing the orbit to have several discrete energies instead of one. Even without an external magnetic field, very precise measurements showed that spectral lines are doublets (split into two), apparently by magnetic fields within the atom itself.

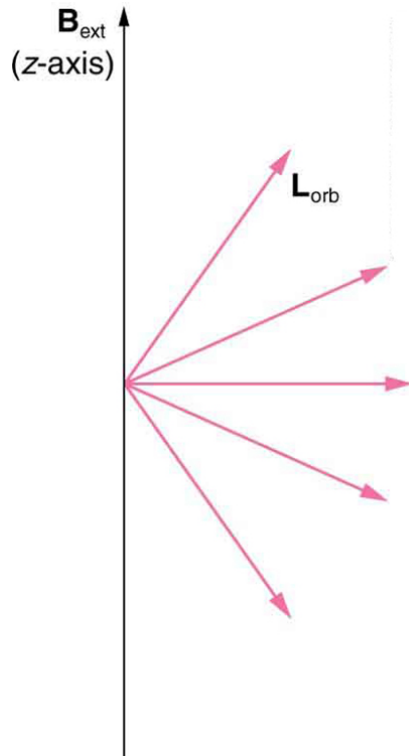


The Zeeman effect is the splitting of spectral lines when a magnetic field is applied. The number of lines formed varies, but the spread is proportional to the strength of the applied field. (a) Two spectral lines with no external magnetic field. (b) The lines split when the field is applied. (c) The splitting is greater when a stronger field is applied.

Bohr's theory of circular orbits is useful for visualizing how an electron's orbit is affected by a magnetic field. The circular orbit forms a current loop, which creates a magnetic field of its own, $\mathbf{B}_{\text{orb}}$ as seen in [\[link\]](#). Note that the **orbital magnetic field** $\mathbf{B}_{\text{orb}}$ and the **orbital angular momentum** $\mathbf{L}_{\text{orb}}$ are along the same line. The external magnetic field and the orbital magnetic field interact; a torque is exerted to align them. A torque rotating a system through some angle does work so that there is energy associated with this interaction. Thus, orbits at different angles to the external magnetic field have different energies. What is remarkable is that the energies are quantized—the magnetic field splits the spectral lines into several discrete lines that have different energies. This means that only certain angles are allowed between the orbital angular momentum and the external field, as seen in [\[link\]](#).



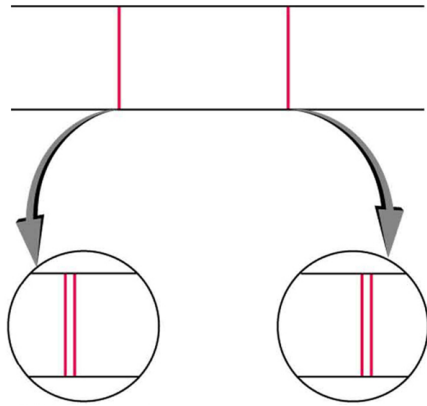
The approximate picture of an electron in a circular orbit illustrates how the current loop produces its own magnetic field, called $\mathbf{B}_{\text{orb}}$. It also shows how $\mathbf{B}_{\text{orb}}$ is along the same line as the orbital angular momentum $\mathbf{L}_{\text{orb}}$.



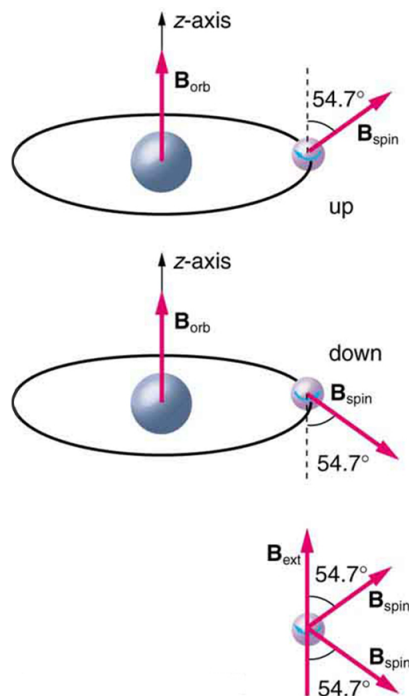
Only certain angles are allowed between the orbital angular momentum and an external magnetic field. This is implied by the fact that the Zeeman effect splits spectral lines into several discrete lines. Each line is associated with an angle between the external magnetic field and magnetic fields due to electrons and their orbits.

We already know that the magnitude of angular momentum is quantized for electron orbits in atoms. The new insight is that the *direction of the orbital angular momentum is also quantized*. The fact that the orbital angular momentum can have only certain directions is called **space quantization**. Like many aspects of quantum mechanics, this quantization of direction is totally unexpected. On the macroscopic scale, orbital angular momentum, such as that of the moon around the earth, can have any magnitude and be in any direction.

Detailed treatment of space quantization began to explain some complexities of atomic spectra, but certain patterns seemed to be caused by something else. As mentioned, spectral lines are actually closely spaced doublets, a characteristic called **fine structure**, as shown in [\[link\]](#). The doublet changes when a magnetic field is applied, implying that whatever causes the doublet interacts with a magnetic field. In 1925, Sem Goudsmit and George Uhlenbeck, two Dutch physicists, successfully argued that electrons have properties analogous to a macroscopic charge spinning on its axis. Electrons, in fact, have an internal or intrinsic angular momentum called **intrinsic spin $\mathbf{S}$** . Since electrons are charged, their intrinsic spin creates an **intrinsic magnetic field $\mathbf{B}_{\text{int}}$** , which interacts with their orbital magnetic field $\mathbf{B}_{\text{orb}}$. Furthermore, *electron intrinsic spin is quantized in magnitude and direction*, analogous to the situation for orbital angular momentum. The spin of the electron can have only one magnitude, and its direction can be at only one of two angles relative to a magnetic field, as seen in [\[link\]](#). We refer to this as spin up or spin down for the electron. Each spin direction has a different energy; hence, spectroscopic lines are split into two. Spectral doublets are now understood as being due to electron spin.



Fine structure. Upon close examination, spectral lines are doublets, even in the absence of an external magnetic field. The electron has an intrinsic magnetic field that interacts with its orbital magnetic field.



The intrinsic magnetic field B_{int} of an electron is attributed to its spin, S , roughly pictured to be due to its charge spinning on its axis. This is only a crude model, since electrons seem to have no size. The spin and intrinsic magnetic field of the electron can make only one of two angles with another magnetic field, such as that created by the electron's orbital

motion. Space is quantized for spin as well as for orbital angular momentum.

These two new insights—that the direction of angular momentum, whether orbital or spin, is quantized, and that electrons have intrinsic spin—help to explain many of the complexities of atomic and molecular spectra. In magnetic resonance imaging, it is the way that the intrinsic magnetic field of hydrogen and biological atoms interact with an external field that underlies the diagnostic fundamentals.

Section Summary

- The Zeeman effect—the splitting of lines when a magnetic field is applied—is caused by other quantized entities in atoms.
- Both the magnitude and direction of orbital angular momentum are quantized.
- The same is true for the magnitude and direction of the intrinsic spin of electrons.

Conceptual Questions

Exercise:

Problem:

What is the Zeeman effect, and what type of quantization was discovered because of this effect?

Glossary

Zeeman effect

the effect of external magnetic fields on spectral lines

intrinsic spin

the internal or intrinsic angular momentum of electrons

orbital angular momentum

an angular momentum that corresponds to the quantum analog of classical angular momentum

fine structure

the splitting of spectral lines of the hydrogen spectrum when the spectral lines are examined at very high resolution

space quantization

the fact that the orbital angular momentum can have only certain directions

intrinsic magnetic field

the magnetic field generated due to the intrinsic spin of electrons

orbital magnetic field

the magnetic field generated due to the orbital motion of electrons

Quantum Numbers and Rules

- Define quantum number.
- Calculate angle of angular momentum vector with an axis.
- Define spin quantum number.

Physical characteristics that are quantized—such as energy, charge, and angular momentum—are of such importance that names and symbols are given to them. The values of quantized entities are expressed in terms of **quantum numbers**, and the rules governing them are of the utmost importance in determining what nature is and does. This section covers some of the more important quantum numbers and rules—all of which apply in chemistry, material science, and far beyond the realm of atomic physics, where they were first discovered. Once again, we see how physics makes discoveries which enable other fields to grow.

The *energy states of bound systems are quantized*, because the particle wavelength can fit into the bounds of the system in only certain ways. This was elaborated for the hydrogen atom, for which the allowed energies are expressed as $E_n \propto 1/n^2$, where $n = 1, 2, 3, \dots$. We define n to be the principal quantum number that labels the basic states of a system. The lowest-energy state has $n = 1$, the first excited state has $n = 2$, and so on. Thus the allowed values for the principal quantum number are

Equation:

$$n = 1, 2, 3, \dots$$

This is more than just a numbering scheme, since the energy of the system, such as the hydrogen atom, can be expressed as some function of n , as can other characteristics (such as the orbital radii of the hydrogen atom).

The fact that the *magnitude of angular momentum is quantized* was first recognized by Bohr in relation to the hydrogen atom; it is now known to be true in general. With the development of quantum mechanics, it was found that the magnitude of angular momentum L can have only the values

Equation:

$$L = \sqrt{l(l+1)} \frac{h}{2\pi} \quad (l = 0, 1, 2, \dots, n-1),$$

where l is defined to be the **angular momentum quantum number**. The rule for l in atoms is given in the parentheses. Given n , the value of l can be any integer from zero up to $n - 1$. For example, if $n = 4$, then l can be 0, 1, 2, or 3.

Note that for $n = 1$, l can only be zero. This means that the ground-state angular momentum for hydrogen is actually zero, not $h/2\pi$ as Bohr proposed. The picture of circular orbits is not valid, because there would be angular momentum for any circular orbit. A more valid picture is the cloud of probability shown for the ground state of hydrogen in [\[link\]](#). The electron actually spends time in and near the nucleus. The reason the electron does not remain in the nucleus is related to Heisenberg's uncertainty principle—the electron's energy would have to be much too large to be confined to the small space of the nucleus. Now the first excited state of hydrogen has $n = 2$, so that l can be either 0 or 1, according to the rule in $L = \sqrt{l(l+1)} \frac{h}{2\pi}$. Similarly, for $n = 3$, l can be 0, 1, or 2. It is often most convenient to state the value of l , a simple integer, rather than calculating the value of L from $L = \sqrt{l(l+1)} \frac{h}{2\pi}$. For example, for $l = 2$, we see that

Equation:

$$L = \sqrt{2(2+1)} \frac{h}{2\pi} = \sqrt{6} \frac{h}{2\pi} = 0.390h = 2.58 \times 10^{-34} \text{ J} \cdot \text{s}.$$

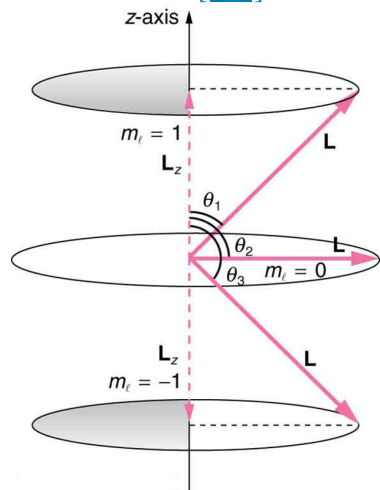
It is much simpler to state $l = 2$.

As recognized in the Zeeman effect, the *direction of angular momentum is quantized*. We now know this is true in all circumstances. It is found that the component of angular momentum along one direction in space, usually called the z -axis, can have only certain values of L_z . The direction in space must be related to something physical, such as the direction of the magnetic field at that location. This is an aspect of relativity. Direction has no meaning if there is nothing that varies with direction, as does magnetic force. The allowed values of L_z are

Equation:

$$L_z = m_l \frac{h}{2\pi} \quad (m_l = -l, -l+1, \dots, -1, 0, 1, \dots, l-1, l),$$

where L_z is the z -**component of the angular momentum** and m_l is the angular momentum projection quantum number. The rule in parentheses for the values of m_l is that it can range from $-l$ to l in steps of one. For example, if $l = 2$, then m_l can have the five values $-2, -1, 0, 1$, and 2 . Each m_l corresponds to a different energy in the presence of a magnetic field, so that they are related to the splitting of spectral lines into discrete parts, as discussed in the preceding section. If the z -component of angular momentum can have only certain values, then the angular momentum can have only certain directions, as illustrated in [\[link\]](#).



The component of a given angular momentum along the z -axis (defined by the direction of a magnetic field) can have only certain values; these are shown here for $l = 1$, for which $m_l = -1, 0$, and $+1$.

The direction of L is quantized in the sense that it can have only certain angles relative to the z -axis.

Example:

What Are the Allowed Directions?

Calculate the angles that the angular momentum vector $\mathbf{L}$ can make with the z -axis for $l = 1$, as illustrated in [\[link\]](#).

Strategy

[\[link\]](#) represents the vectors $\mathbf{L}$ and $\mathbf{L}_z$ as usual, with arrows proportional to their magnitudes and pointing in the correct directions. $\mathbf{L}$ and $\mathbf{L}_z$ form a right triangle, with $\mathbf{L}$ being the hypotenuse and $\mathbf{L}_z$ the adjacent side. This means that the ratio of $\mathbf{L}_z$ to $\mathbf{L}$ is the cosine of the angle of interest. We can find $\mathbf{L}$ and $\mathbf{L}_z$ using $L = \sqrt{l(l+1)}\frac{h}{2\pi}$ and $L_z = m_l\frac{h}{2\pi}$.

Solution

We are given $l = 1$, so that m_l can be $+1$, 0 , or -1 . Thus L has the value given by $L = \sqrt{l(l+1)}\frac{h}{2\pi}$.

Equation:

$$L = \frac{\sqrt{l(l+1)}h}{2\pi} = \frac{\sqrt{2}h}{2\pi}$$

L_z can have three values, given by $L_z = m_l\frac{h}{2\pi}$.

Equation:

$$L_z = m_l\frac{h}{2\pi} = \begin{matrix} \frac{h}{2\pi}, & m_l = +1 \\ 0, & m_l = 0 \\ -\frac{h}{2\pi}, & m_l = -1 \end{matrix}$$

As can be seen in [\[link\]](#), $\cos \theta = L_z/L$, and so for $m_l = +1$, we have

Equation:

$$\cos \theta_1 = \frac{L_z}{L} = \frac{\frac{h}{2\pi}}{\frac{\sqrt{2}h}{2\pi}} = \frac{1}{\sqrt{2}} = 0.707.$$

Thus,

Equation:

$$\theta_1 = \cos^{-1}0.707 = 45.0^\circ.$$

Similarly, for $m_l = 0$, we find $\cos \theta_2 = 0$; thus,

Equation:

$$\theta_2 = \cos^{-1}0 = 90.0^\circ.$$

And for $m_l = -1$,

Equation:

$$\cos \theta_3 = \frac{L_z}{L} = \frac{-\frac{h}{2\pi}}{\frac{\sqrt{2}h}{2\pi}} = -\frac{1}{\sqrt{2}} = -0.707,$$

so that

Equation:

$$\theta_3 = \cos^{-1}(-0.707) = 135.0^\circ.$$

Discussion

The angles are consistent with the figure. Only the angle relative to the z -axis is quantized. L can point in any direction as long as it makes the proper angle with the z -axis. Thus the angular momentum vectors lie on cones as illustrated. This behavior is not observed on the large scale. To see how the correspondence principle holds here, consider that the smallest angle (θ_1 in the example) is for the maximum value of $m_l = 0$, namely $m_l = l$. For that smallest angle,

Equation:

$$\cos \theta = \frac{L_z}{L} = \frac{l}{\sqrt{l(l+1)}},$$

which approaches 1 as l becomes very large. If $\cos \theta = 1$, then $\theta = 0^\circ$. Furthermore, for large l , there are many values of m_l , so that all angles become possible as l gets very large.

Intrinsic Spin Angular Momentum Is Quantized in Magnitude and Direction

There are two more quantum numbers of immediate concern. Both were first discovered for electrons in conjunction with fine structure in atomic spectra. It is now well established that electrons and other fundamental particles have *intrinsic spin*, roughly analogous to a planet spinning on its axis. This spin is a fundamental characteristic of particles, and only one magnitude of intrinsic spin is allowed for a given type of particle. Intrinsic angular momentum is quantized independently of orbital angular momentum. Additionally, the direction of the spin is also quantized. It has been found that the **magnitude of the intrinsic (internal) spin angular momentum**, S , of an electron is given by

Equation:

$$S = \sqrt{s(s+1)} \frac{h}{2\pi} \quad (s = 1/2 \text{ for electrons}),$$

where s is defined to be the **spin quantum number**. This is very similar to the quantization of L given in $L = \sqrt{l(l+1)} \frac{h}{2\pi}$, except that the only value allowed for s for electrons is $1/2$.

The *direction of intrinsic spin is quantized*, just as is the direction of orbital angular momentum. The direction of spin angular momentum along one direction in space, again called the z -axis, can have only the values

Equation:

$$S_z = m_s \frac{h}{2\pi} \left(m_s = -\frac{1}{2}, +\frac{1}{2} \right)$$

for electrons. S_z is the **z -component of spin angular momentum** and m_s is the **spin projection quantum number**. For electrons, s can only be $1/2$, and m_s can be either $+1/2$ or $-1/2$. Spin projection $m_s = +1/2$ is referred to as *spin up*, whereas $m_s = -1/2$ is called *spin down*. These are illustrated in [\[link\]](#).

Note:

Intrinsic Spin

In later chapters, we will see that intrinsic spin is a characteristic of all subatomic particles. For some particles s is half-integral, whereas for others s is integral—there are crucial differences between half-integral spin particles and integral spin particles. Protons and neutrons, like electrons, have $s = 1/2$, whereas photons have $s = 1$, and other particles called pions have $s = 0$, and so on.

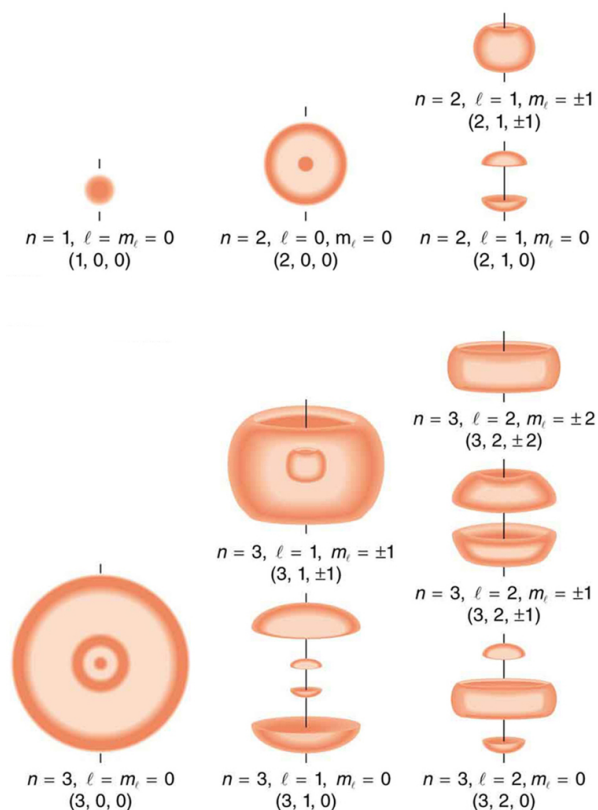
To summarize, the state of a system, such as the precise nature of an electron in an atom, is determined by its particular quantum numbers. These are expressed in the form (n, l, m_l, m_s) —see [\[link\]](#) For *electrons in atoms*, the principal quantum number can have the values $n = 1, 2, 3, \dots$. Once n is known, the values of the angular momentum quantum number are limited to $l = 1, 2, 3, \dots, n - 1$. For a given value of l , the angular momentum projection quantum number can have only the values $m_l = -l, -l + 1, \dots, -1, 0, 1, \dots, l - 1, l$. Electron spin is independent of n, l , and m_l , always having $s = 1/2$. The spin projection quantum number can have two values, $m_s = 1/2$ or $-1/2$.

Name	Symbol	Allowed values
Principal quantum number	n	1, 2, 3, ...
Angular momentum	l	0, 1, 2, ... $n - 1$
Angular momentum projection	m_l	$-l, -l + 1, \dots, -1, 0, 1, \dots, l - 1, l$ (or 0, $\pm 1, \pm 2, \dots, \pm l$)

Name	Symbol	Allowed values
Spin ^{footnote} The spin quantum number s is usually not stated, since it is always $1/2$ for electrons	s	$1/2$ (electrons)
Spin projection	m_s	$-1/2, +1/2$

Atomic Quantum Numbers

[\[link\]](#) shows several hydrogen states corresponding to different sets of quantum numbers. Note that these clouds of probability are the locations of electrons as determined by making repeated measurements—each measurement finds the electron in a definite location, with a greater chance of finding the electron in some places rather than others. With repeated measurements, the pattern of probability shown in the figure emerges. The clouds of probability do not look like nor do they correspond to classical orbits. The uncertainty principle actually prevents us and nature from knowing how the electron gets from one place to another, and so an orbit really does not exist as such. Nature on a small scale is again much different from that on the large scale.



Probability clouds for the electron in the ground state and several excited states of hydrogen. The nature of these states is determined by their sets of quantum numbers, here given as (n, l, m_l) . The ground state is $(0, 0, 0)$; one of the possibilities for the second excited state is $(3, 2, 1)$. The probability of finding the electron is indicated by the shade of color; the darker the coloring the greater the chance of finding the electron.

We will see that the quantum numbers discussed in this section are valid for a broad range of particles and other systems, such as nuclei. Some quantum numbers, such as intrinsic spin, are related to fundamental classifications of subatomic particles, and they obey laws that will give us further insight into the substructure of matter and its interactions.

Note:

PhET Explorations: Stern-Gerlach Experiment

The classic Stern-Gerlach Experiment shows that atoms have a property called spin. Spin is a kind of intrinsic angular momentum, which has no classical counterpart. When the z-component of the spin is

measured, one always gets one of two values: spin up or spin down.

https://phet.colorado.edu/sims/stern-gerlach/stern-gerlach_en.html

Section Summary

- Quantum numbers are used to express the allowed values of quantized entities. The principal quantum number n labels the basic states of a system and is given by

Equation:

$$n = 1, 2, 3, \dots$$

- The magnitude of angular momentum is given by

Equation:

$$L = \sqrt{l(l+1)} \frac{h}{2\pi} \quad (l = 0, 1, 2, \dots, n-1),$$

where l is the angular momentum quantum number. The direction of angular momentum is quantized, in that its component along an axis defined by a magnetic field, called the z -axis is given by

Equation:

$$L_z = m_l \frac{h}{2\pi} \quad (m_l = -l, -l+1, \dots, -1, 0, 1, \dots, l-1, l),$$

where L_z is the z -component of the angular momentum and m_l is the angular momentum projection quantum number. Similarly, the electron's intrinsic spin angular momentum S is given by

Equation:

$$S = \sqrt{s(s+1)} \frac{h}{2\pi} \quad (s = 1/2 \text{ for electrons}),$$

s is defined to be the spin quantum number. Finally, the direction of the electron's spin along the z -axis is given by

Equation:

$$S_z = m_s \frac{h}{2\pi} \quad \left(m_s = -\frac{1}{2}, +\frac{1}{2} \right),$$

where S_z is the z -component of spin angular momentum and m_s is the spin projection quantum number. Spin projection $m_s = +1/2$ is referred to as spin up, whereas $m_s = -1/2$ is called spin down. [\[link\]](#) summarizes the atomic quantum numbers and their allowed values.

Conceptual Questions

Exercise:

Problem: Define the quantum numbers n , l , m_l , s , and m_s .

Exercise:

Problem: For a given value of n , what are the allowed values of l ?

Exercise:

Problem:

For a given value of l , what are the allowed values of m_l ? What are the allowed values of m_l for a given value of n ? Give an example in each case.

Exercise:

Problem:

List all the possible values of s and m_s for an electron. Are there particles for which these values are different? The same?

Problem Exercises

Exercise:

Problem:

If an atom has an electron in the $n = 5$ state with $m_l = 3$, what are the possible values of l ?

Solution:

$l = 4, 3$ are possible since $l < n$ and $|m_l| \leq l$.

Exercise:

Problem: An atom has an electron with $m_l = 2$. What is the smallest value of n for this electron?

Exercise:

Problem: What are the possible values of m_l for an electron in the $n = 4$ state?

Solution:

$n = 4 \Rightarrow l = 3, 2, 1, 0 \Rightarrow m_l = \pm 3, \pm 2, \pm 1, 0$ are possible.

Exercise:

Problem:

What, if any, constraints does a value of $m_l = 1$ place on the other quantum numbers for an electron in an atom?

Exercise:

Problem:

(a) Calculate the magnitude of the angular momentum for an $l = 1$ electron. (b) Compare your answer to the value Bohr proposed for the $n = 1$ state.

Solution:

(a) $1.49 \times 10^{-34} \text{ J} \cdot \text{s}$

(b) $1.06 \times 10^{-34} \text{ J} \cdot \text{s}$

Exercise:**Problem:**

(a) What is the magnitude of the angular momentum for an $l = 1$ electron? (b) Calculate the magnitude of the electron's spin angular momentum. (c) What is the ratio of these angular momenta?

Exercise:

Problem: Repeat [\[link\]](#) for $l = 3$.

Solution:

(a) $3.66 \times 10^{-34} \text{ J} \cdot \text{s}$

(b) $s = 9.13 \times 10^{-35} \text{ J} \cdot \text{s}$

(c) $\frac{L}{s} = \frac{\sqrt{12}}{\sqrt{3/4}} = 4$

Exercise:**Problem:**

(a) How many angles can L make with the z -axis for an $l = 2$ electron? (b) Calculate the value of the smallest angle.

Exercise:

Problem: What angles can the spin S of an electron make with the z -axis?

Solution:

$$\theta = 54.7^\circ, 125.3^\circ$$

Glossary

quantum numbers

the values of quantized entities, such as energy and angular momentum

angular momentum quantum number

a quantum number associated with the angular momentum of electrons

spin quantum number

the quantum number that parameterizes the intrinsic angular momentum (or spin angular momentum, or simply spin) of a given particle

spin projection quantum number

quantum number that can be used to calculate the intrinsic electron angular momentum along the z -axis

z -component of spin angular momentum

component of intrinsic electron spin along the z -axis

magnitude of the intrinsic (internal) spin angular momentum

given by $S = \sqrt{s(s+1)} \frac{h}{2\pi}$

z -component of the angular momentum

component of orbital angular momentum of electron along the z -axis

The Pauli Exclusion Principle

- Define the composition of an atom along with its electrons, neutrons, and protons.
- Explain the Pauli exclusion principle and its application to the atom.
- Specify the shell and subshell symbols and their positions.
- Define the position of electrons in different shells of an atom.
- State the position of each element in the periodic table according to shell filling.

Multiple-Electron Atoms

All atoms except hydrogen are multiple-electron atoms. The physical and chemical properties of elements are directly related to the number of electrons a neutral atom has. The periodic table of the elements groups elements with similar properties into columns. This systematic organization is related to the number of electrons in a neutral atom, called the **atomic number**, Z . We shall see in this section that the exclusion principle is key to the underlying explanations, and that it applies far beyond the realm of atomic physics.

In 1925, the Austrian physicist Wolfgang Pauli (see [link](#)) proposed the following rule: No two electrons can have the same set of quantum numbers. That is, no two electrons can be in the same state. This statement is known as the **Pauli exclusion principle**, because it excludes electrons from being in the same state. The Pauli exclusion principle is extremely powerful and very broadly applicable. It applies to any identical particles with half-integral intrinsic spin—that is, having $s = 1/2, 3/2, \dots$. Thus no two electrons can have the same set of quantum numbers.

Note:

Pauli Exclusion Principle

No two electrons can have the same set of quantum numbers. That is, no two electrons can be in the same state.

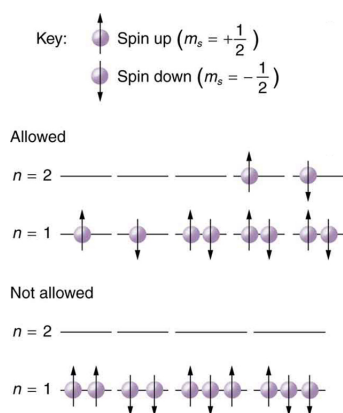


The Austrian physicist Wolfgang Pauli (1900–1958) played a major role in the development of quantum mechanics. He proposed the exclusion principle; hypothesized the existence of an important particle,

called the neutrino,
before it was directly
observed; made
fundamental
contributions to
several areas of
theoretical physics;
and influenced many
students who went
on to do important
work of their own.
(credit: Nobel
Foundation, via
Wikimedia
Commons)

Let us examine how the exclusion principle applies to electrons in atoms. The quantum numbers involved were defined in [Quantum Numbers and Rules](#) as n , l , m_l , s , and m_s . Since s is always $1/2$ for electrons, it is redundant to list s , and so we omit it and specify the state of an electron by a set of four numbers (n, l, m_l, m_s) . For example, the quantum numbers $(2, 1, 0, -1/2)$ completely specify the state of an electron in an atom.

Since no two electrons can have the same set of quantum numbers, there are limits to how many of them can be in the same energy state. Note that n determines the energy state in the absence of a magnetic field. So we first choose n , and then we see how many electrons can be in this energy state or energy level. Consider the $n = 1$ level, for example. The only value l can have is 0 (see [link](#) for a list of possible values once n is known), and thus m_l can only be 0. The spin projection m_s can be either $+1/2$ or $-1/2$, and so there can be two electrons in the $n = 1$ state. One has quantum numbers $(1, 0, 0, +1/2)$, and the other has $(1, 0, 0, -1/2)$. [link](#) illustrates that there can be one or two electrons having $n = 1$, but not three.



The Pauli exclusion
principle explains why
some configurations of
electrons are allowed
while others are not.
Since electrons cannot
have the same set of
quantum numbers, a
maximum of two can be

in the $n = 1$ level, and a third electron must reside in the higher-energy $n = 2$ level. If there are two electrons in the $n = 1$ level, their spins must be in opposite directions. (More precisely, their spin projections must differ.)

Shells and Subshells

Because of the Pauli exclusion principle, only hydrogen and helium can have all of their electrons in the $n = 1$ state. Lithium (see the periodic table) has three electrons, and so one must be in the $n = 2$ level. This leads to the concept of shells and shell filling. As we progress up in the number of electrons, we go from hydrogen to helium, lithium, beryllium, boron, and so on, and we see that there are limits to the number of electrons for each value of n . Higher values of the shell n correspond to higher energies, and they can allow more electrons because of the various combinations of l , m_l , and m_s that are possible. Each value of the principal quantum number n thus corresponds to an atomic **shell** into which a limited number of electrons can go. Shells and the number of electrons in them determine the physical and chemical properties of atoms, since it is the outermost electrons that interact most with anything outside the atom.

The probability clouds of electrons with the lowest value of l are closest to the nucleus and, thus, more tightly bound. Thus when shells fill, they start with $l = 0$, progress to $l = 1$, and so on. Each value of l thus corresponds to a **subshell**.

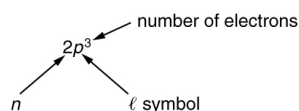
The table given below lists symbols traditionally used to denote shells and subshells.

Shell	Subshell	
n	l	Symbol
1	0	s
2	1	p
3	2	d
4	3	f

Shell	Subshell	
5	4	<i>g</i>
	5	<i>h</i>
	6 ^[footnote] It is unusual to deal with subshells having <i>l</i> greater than 6, but when encountered, they continue to be labeled in alphabetical order.	<i>i</i>

Shell and Subshell Symbols

To denote shells and subshells, we write nl with a number for n and a letter for l . For example, an electron in the $n = 1$ state must have $l = 0$, and it is denoted as a $1s$ electron. Two electrons in the $n = 1$ state is denoted as $1s^2$. Another example is an electron in the $n = 2$ state with $l = 1$, written as $2p$. The case of three electrons with these quantum numbers is written $2p^3$. This notation, called spectroscopic notation, is generalized as shown in [\[link\]](#).



Counting the number of possible combinations of quantum numbers allowed by the exclusion principle, we can determine how many electrons it takes to fill each subshell and shell.

Example:

How Many Electrons Can Be in This Shell?

List all the possible sets of quantum numbers for the $n = 2$ shell, and determine the number of electrons that can be in the shell and each of its subshells.

Strategy

Given $n = 2$ for the shell, the rules for quantum numbers limit l to be 0 or 1. The shell therefore has two subshells, labeled $2s$ and $2p$. Since the lowest l subshell fills first, we start with the $2s$ subshell possibilities and then proceed with the $2p$ subshell.

Solution

It is convenient to list the possible quantum numbers in a table, as shown below.

n	ℓ	m_ℓ	m_s	Subshell	Total in subshell	Total in shell
2	0	0	+1/2	2s	2	8
2	0	0	-1/2			
2	1	1	+1/2	2p	6	
2	1	1	-1/2			
2	1	0	+1/2			
2	1	0	-1/2			
2	1	-1	+1/2			
2	1	-1	-1/2			

Discussion

It is laborious to make a table like this every time we want to know how many electrons can be in a shell or subshell. There exist general rules that are easy to apply, as we shall now see.

The number of electrons that can be in a subshell depends entirely on the value of l . Once l is known, there are a fixed number of values of m_l , each of which can have two values for m_s . First, since m_l goes from $-l$ to l in steps of 1, there are $2l + 1$ possibilities. This number is multiplied by 2, since each electron can be spin up or spin down. Thus the *maximum number of electrons that can be in a subshell* is $2(2l + 1)$.

For example, the $2s$ subshell in [\[link\]](#) has a maximum of 2 electrons in it, since $2(2l + 1) = 2(0 + 1) = 2$ for this subshell. Similarly, the $2p$ subshell has a maximum of 6 electrons, since $2(2l + 1) = 2(2 + 1) = 6$. For a shell, the maximum number is the sum of what can fit in the subshells. Some algebra shows that the *maximum number of electrons that can be in a shell* is $2n^2$.

For example, for the first shell $n = 1$, and so $2n^2 = 2$. We have already seen that only two electrons can be in the $n = 1$ shell. Similarly, for the second shell, $n = 2$, and so $2n^2 = 8$. As found in [\[link\]](#), the total number of electrons in the $n = 2$ shell is 8.

Example:

Subshells and Totals for $n = 3$

How many subshells are in the $n = 3$ shell? Identify each subshell, calculate the maximum number of electrons that will fit into each, and verify that the total is $2n^2$.

Strategy

Subshells are determined by the value of l ; thus, we first determine which values of l are allowed, and then we apply the equation “maximum number of electrons that can be in a subshell = $2(2l + 1)$ ” to find the number of electrons in each subshell.

Solution

Since $n = 3$, we know that l can be 0, 1, or 2; thus, there are three possible subshells. In standard notation, they are labeled the $3s$, $3p$, and $3d$ subshells. We have already seen that 2 electrons can be in an s state, and 6 in a p state, but let us use the equation “maximum number of electrons that can be in a subshell = $2(2l + 1)$ ” to calculate the maximum number in each:

Equation:

$$\begin{aligned} 3s \text{ has } l = 0; \text{ thus, } 2(2l + 1) &= 2(0 + 1) = 2 \\ 3p \text{ has } l = 1; \text{ thus, } 2(2l + 1) &= 2(2 + 1) = 6 \\ 3d \text{ has } l = 2; \text{ thus, } 2(2l + 1) &= 2(4 + 1) = 10 \\ \text{Total} &= 18 \\ &(\text{in the } n = 3 \text{ shell}) \end{aligned}$$

The equation “maximum number of electrons that can be in a shell = $2n^2$ ” gives the maximum number in the $n = 3$ shell to be

Equation:

$$\text{Maximum number of electrons} = 2n^2 = 2(3)^2 = 2(9) = 18.$$

Discussion

The total number of electrons in the three possible subshells is thus the same as the formula $2n^2$. In standard (spectroscopic) notation, a filled $n = 3$ shell is denoted as $3s^23p^63d^{10}$. Shells do not fill in a simple manner. Before the $n = 3$ shell is completely filled, for example, we begin to find electrons in the $n = 4$ shell.

Shell Filling and the Periodic Table

[\[link\]](#) shows electron configurations for the first 20 elements in the periodic table, starting with hydrogen and its single electron and ending with calcium. The Pauli exclusion principle determines the maximum number of electrons allowed in each shell and subshell. But the order in which the shells and subshells are filled is complicated because of the large numbers of interactions between electrons.

Element	Number of electrons (Z)	Ground state configuration					
H	1	$1s^1$					
He	2	$1s^2$					
Li	3	$1s^2$	$2s^1$				
Be	4	"	$2s^2$				
B	5	"	$2s^2$	$2p^1$			
C	6	"	$2s^2$	$2p^2$			
N	7	"	$2s^2$	$2p^3$			
O	8	"	$2s^2$	$2p^4$			
F	9	"	$2s^2$	$2p^5$			
Ne	10	"	$2s^2$	$2p^6$			
Na	11	"	$2s^2$	$2p^6$	$3s^1$		
Mg	12	"	"	"	$3s^2$		
Al	13	"	"	"	$3s^2$	$3p^1$	
Si	14	"	"	"	$3s^2$	$3p^2$	

Element	Number of electrons (Z)	Ground state configuration					
P	15	"	"	"	$3s^2$	$3p^3$	
S	16	"	"	"	$3s^2$	$3p^4$	
Cl	17	"	"	"	$3s^2$	$3p^5$	
Ar	18	"	"	"	$3s^2$	$3p^6$	
K	19	"	"	"	$3s^2$	$3p^6$	$4s^1$
Ca	20	"	"	"	"	"	$4s^2$

Electron Configurations of Elements Hydrogen Through Calcium

Examining the above table, you can see that as the number of electrons in an atom increases from 1 in hydrogen to 2 in helium and so on, the lowest-energy shell gets filled first—that is, the $n = 1$ shell fills first, and then the $n = 2$ shell begins to fill. Within a shell, the subshells fill starting with the lowest l , or with the s subshell, then the p , and so on, usually until all subshells are filled. The first exception to this occurs for potassium, where the $4s$ subshell begins to fill before any electrons go into the $3d$ subshell. The next exception is not shown in [\[link\]](#); it occurs for rubidium, where the $5s$ subshell starts to fill before the $4d$ subshell. The reason for these exceptions is that $l = 0$ electrons have probability clouds that penetrate closer to the nucleus and, thus, are more tightly bound (lower in energy).

[\[link\]](#) shows the periodic table of the elements, through element 118. Of special interest are elements in the main groups, namely, those in the columns numbered 1, 2, 13, 14, 15, 16, 17, and 18.

The image shows a comprehensive periodic table from NIST (National Institute of Standards and Technology). It includes the following features:

- Group and Period Labels:** Groups are labeled at the top (1A to 8A, plus transition metals), and periods are labeled on the left (1 to 7).
- Element Symbols and Names:** Each element is represented by its symbol and full name (e.g., H, Helium; Li, Lithium).
- Atomic Properties:** Key data points are provided for each element, including atomic weight, atomic number, and standard atomic weight.
- Physical and Chemical Data:** Additional data such as melting point, boiling point, and density are included for many elements.
- Color Coding:** Elements are color-coded by groups: 1A (pink), 2A (light blue), 3A (light green), 4A (light yellow), 5A (light orange), 6A (light purple), 7A (light blue), 8A (light pink), 9A (light green), 10A (light yellow), 11A (light orange), 12A (light purple), 13A (light blue), 14A (light green), 15A (light orange), 16A (light purple), 17A (light blue), 18A (light pink).
- Legend:** A legend at the top right identifies the color coding for different groups.
- Footnote:** A note at the bottom states: "Based upon: 1. 1 indicates the mass number of the longest-lived isotope. For a description of the data, visit physics.nist.gov/ptable. NIST SP 900 (September 2016)".

Periodic table of the elements (credit:

The number of electrons in the outermost subshell determines the atom's chemical properties, since it is these electrons that are farthest from the nucleus and thus interact most with other atoms. If the outermost subshell can accept or give up an electron easily, then the atom will be highly reactive chemically. Each group in the periodic table is characterized by its outermost electron configuration. Perhaps the most familiar is Group 18 (Group VIII), the noble gases (helium, neon, argon, etc.). These gases are all characterized by a filled outer subshell that is particularly stable. This means that they have large ionization energies and do not readily give up an electron. Furthermore, if they were to accept an extra electron, it would be in a significantly higher level and thus loosely bound. Chemical reactions often involve sharing electrons. Noble gases can be forced into unstable chemical compounds only under high pressure and temperature.

Group 17 (Group VII) contains the halogens, such as fluorine, chlorine, iodine and bromine, each of which has one less electron than a neighboring noble gas. Each halogen has 5 p electrons (a p^5 configuration), while the p subshell can hold 6 electrons. This means the halogens have one vacancy in their outermost subshell. They thus readily accept an extra electron (it becomes tightly bound, closing the shell as in noble gases) and are highly reactive chemically. The halogens are also likely to form singly negative ions, such as Cl^- , fitting an extra electron into the vacancy in the outer subshell. In contrast, alkali metals, such as sodium and potassium, all have a single s electron in their outermost subshell (an s^1 configuration) and are members of Group 1 (Group I). These elements easily give up their extra electron and are thus highly reactive chemically. As you might expect, they also tend to form singly positive ions, such as Na^+ , by losing their loosely bound outermost electron. They are metals (conductors), because the loosely bound outer electron can move freely.

Of course, other groups are also of interest. Carbon, silicon, and germanium, for example, have similar chemistries and are in Group 4 (Group IV). Carbon, in particular, is extraordinary in its ability to form many types of bonds and to be part of long chains, such as inorganic molecules. The large group of what are called transitional elements is characterized by the filling of the d subshells and crossing of energy levels. Heavier groups, such as the lanthanide series, are more complex—their shells do not fill in simple order. But the groups recognized by chemists such as Mendeleev have an explanation in the substructure of atoms.

Note:

PhET Explorations: Stern-Gerlach Experiment

Build an atom out of protons, neutrons, and electrons, and see how the element, charge, and mass change. Then play a game to test your ideas!

https://phet.colorado.edu/sims/html/build-an-atom/latest/build-an-atom_en.html

Section Summary

- The state of a system is completely described by a complete set of quantum numbers. This set is written as (n, l, m_l, m_s) .
- The Pauli exclusion principle says that no two electrons can have the same set of quantum numbers; that is, no two electrons can be in the same state.
- This exclusion limits the number of electrons in atomic shells and subshells. Each value of n corresponds to a shell, and each value of l corresponds to a subshell.
- The maximum number of electrons that can be in a subshell is $2(2l + 1)$.
- The maximum number of electrons that can be in a shell is $2n^2$.

Conceptual Questions

Exercise:**Problem:**

Identify the shell, subshell, and number of electrons for the following: (a) $2p^3$. (b) $4d^9$. (c) $3s^1$. (d) $5g^{16}$.

Exercise:**Problem:**

Which of the following are not allowed? State which rule is violated for any that are not allowed. (a) $1p^3$ (b) $2p^8$ (c) $3g^{11}$ (d) $4f^2$

Problem Exercises**Exercise:**

Problem: (a) How many electrons can be in the $n = 4$ shell?

(b) What are its subshells, and how many electrons can be in each?

Solution:

(a) 32. (b) 2 in s , 6 in p , 10 in d , and 14 in f , for a total of 32.

Exercise:

Problem: (a) What is the minimum value of l for a subshell that has 11 electrons in it?

(b) If this subshell is in the $n = 5$ shell, what is the spectroscopic notation for this atom?

Exercise:**Problem:**

(a) If one subshell of an atom has 9 electrons in it, what is the minimum value of l ? (b) What is the spectroscopic notation for this atom, if this subshell is part of the $n = 3$ shell?

Solution:

(a) 2

(b) $3d^9$

Exercise:**Problem:**

(a) List all possible sets of quantum numbers (n, l, m_l, m_s) for the $n = 3$ shell, and determine the number of electrons that can be in the shell and each of its subshells.

(b) Show that the number of electrons in the shell equals $2n^2$ and that the number in each subshell is $2(2l + 1)$.

Exercise:**Problem:**

Which of the following spectroscopic notations are not allowed? (a) $5s^1$ (b) $1d^1$ (c) $4s^3$ (d) $3p^7$ (e) $5g^{15}$. State which rule is violated for each that is not allowed.

Solution:

(b) $n \geq l$ is violated,

(c) cannot have 3 electrons in s subshell since $3 > (2l + 1) = 2$

(d) cannot have 7 electrons in p subshell since $7 > (2l + 1) = 2(2 + 1) = 6$

Exercise:**Problem:**

Which of the following spectroscopic notations are allowed (that is, which violate none of the rules regarding values of quantum numbers)? (a) $1s^1$ (b) $1d^3$ (c) $4s^2$ (d) $3p^7$ (e) $6h^{20}$

Exercise:**Problem:**

(a) Using the Pauli exclusion principle and the rules relating the allowed values of the quantum numbers (n, l, m_l, m_s), prove that the maximum number of electrons in a subshell is $2n^2$.

(b) In a similar manner, prove that the maximum number of electrons in a shell is $2n^2$.

Solution:

(a) The number of different values of m_l is $\pm l, \pm (l - 1), \dots, 0$ for each $l > 0$ and one for $l = 0 \Rightarrow (2l + 1)$. Also an overall factor of 2 since each m_l can have m_s equal to either $+1/2$ or $-1/2 \Rightarrow 2(2l + 1)$.

(b) for each value of l , you get $2(2l + 1)$

$$= 0, 1, 2, \dots, (n-1) \Rightarrow 2\{[(2)(0) + 1] + [(2)(1) + 1] + \dots + [(2)(n-1) + 1]\} = 2[1 + 3 + \dots + (2n-3) +$$

n terms

to see that the expression in the box is $= n^2$, imagine taking $(n - 1)$ from the last term and adding it to first term $= 2[1 + (n-1) + 3 + \dots + (2n-3) + (2n-1) - (n-1)] = 2[n + 3 + \dots + (2n-3) + n]$. Now take $(n - 3)$ from penultimate term and add to the second term $2[n + n + \dots + n + n] = 2n^2$.

n terms

Exercise:**Problem: Integrated Concepts**

Estimate the density of a nucleus by calculating the density of a proton, taking it to be a sphere 1.2 fm in diameter. Compare your result with the value estimated in this chapter.

Exercise:**Problem: Integrated Concepts**

The electric and magnetic forces on an electron in the CRT in [\[link\]](#) are supposed to be in opposite directions. Verify this by determining the direction of each force for the situation shown. Explain how you obtain the directions (that is, identify the rules used).

Solution:

The electric force on the electron is up (toward the positively charged plate). The magnetic force is down (by the RHR).

Exercise:

Problem:

- (a) What is the distance between the slits of a diffraction grating that produces a first-order maximum for the first Balmer line at an angle of 20.0° ?
- (b) At what angle will the fourth line of the Balmer series appear in first order?
- (c) At what angle will the second-order maximum be for the first line?

Exercise:**Problem: Integrated Concepts**

A galaxy moving away from the earth has a speed of $0.0100c$. What wavelength do we observe for an $n_i = 7$ to $n_f = 2$ transition for hydrogen in that galaxy?

Solution:

401 nm

Exercise:**Problem: Integrated Concepts**

Calculate the velocity of a star moving relative to the earth if you observe a wavelength of 91.0 nm for ionized hydrogen capturing an electron directly into the lowest orbital (that is, a $n_i = \infty$ to $n_f = 1$, or a Lyman series transition).

Exercise:**Problem: Integrated Concepts**

In a Millikan oil-drop experiment using a setup like that in [\[link\]](#), a 500-V potential difference is applied to plates separated by 2.50 cm. (a) What is the mass of an oil drop having two extra electrons that is suspended motionless by the field between the plates? (b) What is the diameter of the drop, assuming it is a sphere with the density of olive oil?

Solution:

(a) 6.54×10^{-16} kg

(b) 5.54×10^{-7} m

Exercise:**Problem: Integrated Concepts**

What double-slit separation would produce a first-order maximum at 3.00° for 25.0-keV x rays? The small answer indicates that the wave character of x rays is best determined by having them interact with very small objects such as atoms and molecules.

Exercise:**Problem: Integrated Concepts**

In a laboratory experiment designed to duplicate Thomson's determination of q_e/m_e , a beam of electrons having a velocity of 6.00×10^7 m/s enters a 5.00×10^{-3} T magnetic field. The beam moves perpendicular

to the field in a path having a 6.80-cm radius of curvature. Determine q_e/m_e from these observations, and compare the result with the known value.

Solution:

1.76×10^{11} C/kg, which agrees with the known value of 1.759×10^{11} C/kg to within the precision of the measurement

Exercise:

Problem: Integrated Concepts

Find the value of l , the orbital angular momentum quantum number, for the moon around the earth. The extremely large value obtained implies that it is impossible to tell the difference between adjacent quantized orbits for macroscopic objects.

Exercise:

Problem: Integrated Concepts

Particles called muons exist in cosmic rays and can be created in particle accelerators. Muons are very similar to electrons, having the same charge and spin, but they have a mass 207 times greater. When muons are captured by an atom, they orbit just like an electron but with a smaller radius, since the mass in

$$a_B = \frac{h^2}{4\pi^2 m_e k q_e^2} = 0.529 \times 10^{-10} \text{ m is } 207 m_e.$$

(a) Calculate the radius of the $n = 1$ orbit for a muon in a uranium ion ($Z = 92$).

(b) Compare this with the 7.5-fm radius of a uranium nucleus. Note that since the muon orbits inside the electron, it falls into a hydrogen-like orbit. Since your answer is less than the radius of the nucleus, you can see that the photons emitted as the muon falls into its lowest orbit can give information about the nucleus.

Solution:

(a) 2.78 fm

(b) 0.37 of the nuclear radius.

Exercise:

Problem: Integrated Concepts

Calculate the minimum amount of energy in joules needed to create a population inversion in a helium-neon laser containing 1.00×10^{-4} moles of neon.

Exercise:

Problem: Integrated Concepts

A carbon dioxide laser used in surgery emits infrared radiation with a wavelength of 10.6 μm . In 1.00 ms, this laser raised the temperature of 1.00 cm^3 of flesh to 100°C and evaporated it.

(a) How many photons were required? You may assume flesh has the same heat of vaporization as water. (b) What was the minimum power output during the flash?

Solution:

(a) 1.34×10^{23}

(b) 2.52 MW

Exercise:

Problem: Integrated Concepts

Suppose an MRI scanner uses 100-MHz radio waves.

- (a) Calculate the photon energy.
- (b) How does this compare to typical molecular binding energies?

Exercise:

Problem: Integrated Concepts

- (a) An excimer laser used for vision correction emits 193-nm UV. Calculate the photon energy in eV.
- (b) These photons are used to evaporate corneal tissue, which is very similar to water in its properties. Calculate the amount of energy needed per molecule of water to make the phase change from liquid to gas. That is, divide the heat of vaporization in kJ/kg by the number of water molecules in a kilogram.
- (c) Convert this to eV and compare to the photon energy. Discuss the implications.

Solution:

- (a) 6.42 eV
- (b) 7.27×10^{-20} J/molecule
- (c) 0.454 eV, 14.1 times less than a single UV photon. Therefore, each photon will evaporate approximately 14 molecules of tissue. This gives the surgeon a rather precise method of removing corneal tissue from the surface of the eye.

Exercise:

Problem: Integrated Concepts

A neighboring galaxy rotates on its axis so that stars on one side move toward us as fast as 200 km/s, while those on the other side move away as fast as 200 km/s. This causes the EM radiation we receive to be Doppler shifted by velocities over the entire range of ± 200 km/s. What range of wavelengths will we observe for the 656.0-nm line in the Balmer series of hydrogen emitted by stars in this galaxy. (This is called line broadening.)

Exercise:

Problem: Integrated Concepts

A pulsar is a rapidly spinning remnant of a supernova. It rotates on its axis, sweeping hydrogen along with it so that hydrogen on one side moves toward us as fast as 50.0 km/s, while that on the other side moves away as fast as 50.0 km/s. This means that the EM radiation we receive will be Doppler shifted over a range of ± 50.0 km/s. What range of wavelengths will we observe for the 91.20-nm line in the Lyman series of hydrogen? (Such line broadening is observed and actually provides part of the evidence for rapid rotation.)

Solution:

91.18 nm to 91.22 nm

Exercise:

Problem: Integrated Concepts

Prove that the velocity of charged particles moving along a straight path through perpendicular electric and magnetic fields is $v = E/B$. Thus crossed electric and magnetic fields can be used as a velocity selector independent of the charge and mass of the particle involved.

Exercise:**Problem: Unreasonable Results**

(a) What voltage must be applied to an X-ray tube to obtain 0.0100-nm-wavelength X-rays for use in exploring the details of nuclei? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $1.24 \times 10^{11} \text{ V}$

(b) The voltage is extremely large compared with any practical value.

(c) The assumption of such a short wavelength by this method is unreasonable.

Exercise:**Problem: Unreasonable Results**

A student in a physics laboratory observes a hydrogen spectrum with a diffraction grating for the purpose of measuring the wavelengths of the emitted radiation. In the spectrum, she observes a yellow line and finds its wavelength to be 589 nm. (a) Assuming this is part of the Balmer series, determine n_i , the principal quantum number of the initial state. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Exercise:**Problem: Construct Your Own Problem**

The solar corona is so hot that most atoms in it are ionized. Consider a hydrogen-like atom in the corona that has only a single electron. Construct a problem in which you calculate selected spectral energies and wavelengths of the Lyman, Balmer, or other series of this atom that could be used to identify its presence in a very hot gas. You will need to choose the atomic number of the atom, identify the element, and choose which spectral lines to consider.

Exercise:**Problem: Construct Your Own Problem**

Consider the Doppler-shifted hydrogen spectrum received from a rapidly receding galaxy. Construct a problem in which you calculate the energies of selected spectral lines in the Balmer series and examine whether they can be described with a formula like that in the equation $\frac{1}{\lambda} = R \left(\frac{1}{n_f^2} - \frac{1}{n_i^2} \right)$, but with a different constant R .

Glossary

atomic number

the number of protons in the nucleus of an atom

Pauli exclusion principle

a principle that states that no two electrons can have the same set of quantum numbers; that is, no two electrons can be in the same state

shell

a probability cloud for electrons that has a single principal quantum number

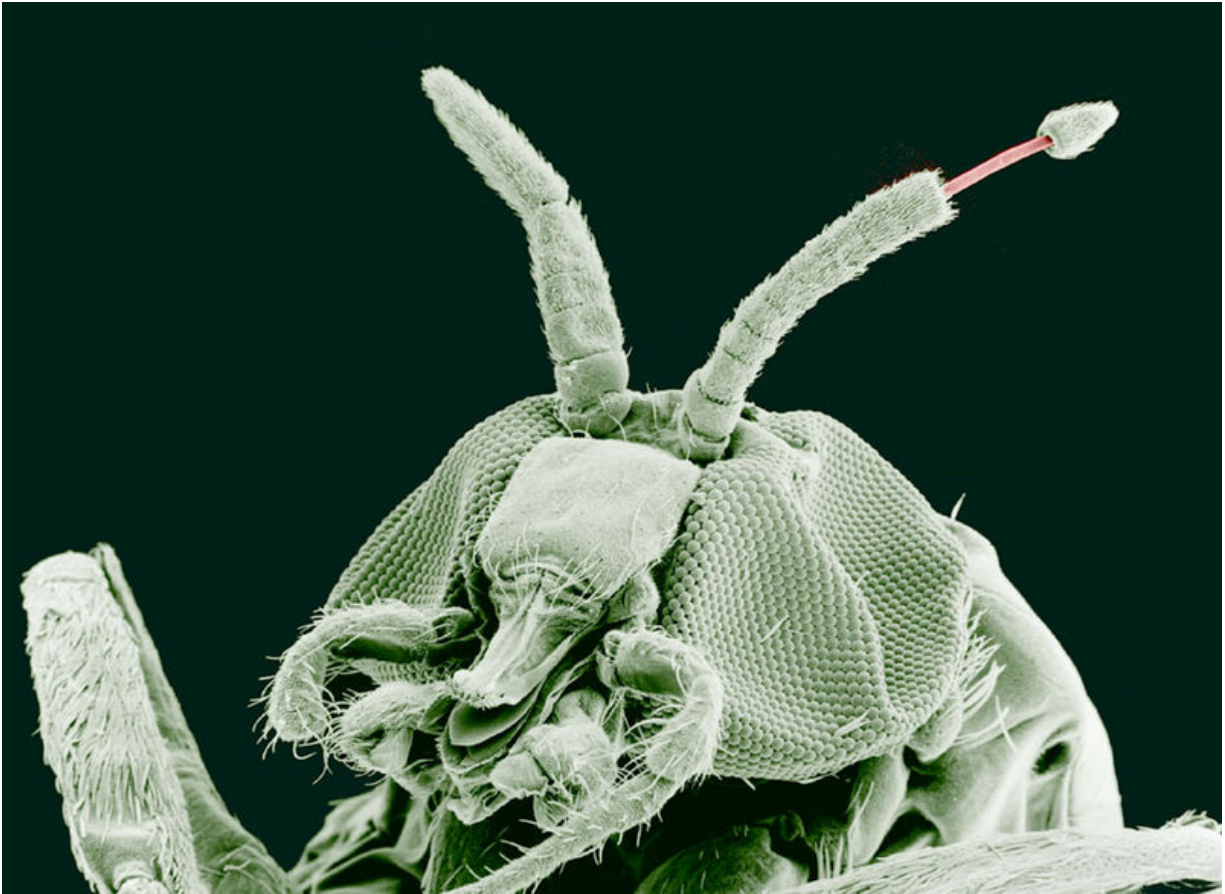
subshell

the probability cloud for electrons that has a single angular momentum quantum number l

Introduction to Quantum Physics

class="introduction"

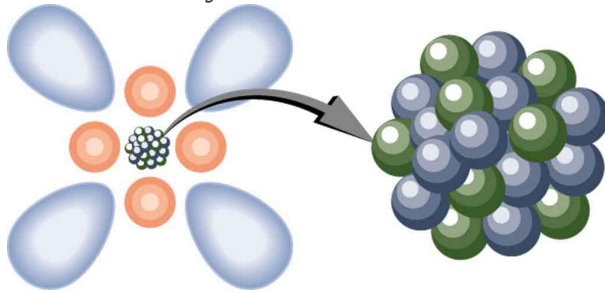
A black fly
imaged by
an electron
microscope
is as
monstrous
as any
science-
fiction
creature.
(credit:
U.S.
Departmen
t of
Agriculture
via
Wikimedia
Commons)



Quantum mechanics is the branch of physics needed to deal with submicroscopic objects. Because these objects are smaller than we can observe directly with our senses and generally must be observed with the aid of instruments, parts of quantum mechanics seem as foreign and bizarre as parts of relativity. But, like relativity, quantum mechanics has been shown to be valid—truth is often stranger than fiction.

Certain aspects of quantum mechanics are familiar to us. We accept as fact that matter is composed of atoms, the smallest unit of an element, and that these atoms combine to form molecules, the smallest unit of a compound. (See [\[link\]](#).) While we cannot see the individual water molecules in a stream, for example, we are aware that this is because molecules are so small and so numerous in that stream. When introducing atoms, we commonly say that electrons orbit atoms in discrete shells around a tiny nucleus, itself composed of smaller particles called protons and neutrons. We are also aware that electric charge comes in tiny units carried almost entirely by electrons and protons. As with water molecules in a stream, we

do not notice individual charges in the current through a lightbulb, because the charges are so small and so numerous in the macroscopic situations we sense directly.



Atoms and their substructure are familiar examples of objects that require quantum mechanics to be fully explained. Certain of their characteristics, such as the discrete electron shells, are classical physics explanations.

In quantum mechanics we conceptualize discrete “electron clouds” around the nucleus.

Note:

Making Connections: Realms of Physics

Classical physics is a good approximation of modern physics under conditions first discussed in the [The Nature of Science and Physics](#).

Quantum mechanics is valid in general, and it must be used rather than classical physics to describe small objects, such as atoms.

Atoms, molecules, and fundamental electron and proton charges are all examples of physical entities that are **quantized**—that is, they appear only in certain discrete values and do not have every conceivable value.

Quantized is the opposite of continuous. We cannot have a fraction of an atom, or part of an electron's charge, or 14-1/3 cents, for example. Rather, everything is built of integral multiples of these substructures. Quantum physics is the branch of physics that deals with small objects and the quantization of various entities, including energy and angular momentum. Just as with classical physics, quantum physics has several subfields, such as mechanics and the study of electromagnetic forces. The **correspondence principle** states that in the classical limit (large, slow-moving objects), **quantum mechanics** becomes the same as classical physics. In this chapter, we begin the development of quantum mechanics and its description of the strange submicroscopic world. In later chapters, we will examine many areas, such as atomic and nuclear physics, in which quantum mechanics is crucial.

Glossary

quantized

the fact that certain physical entities exist only with particular discrete values and not every conceivable value

correspondence principle

in the classical limit (large, slow-moving objects), quantum mechanics becomes the same as classical physics

quantum mechanics

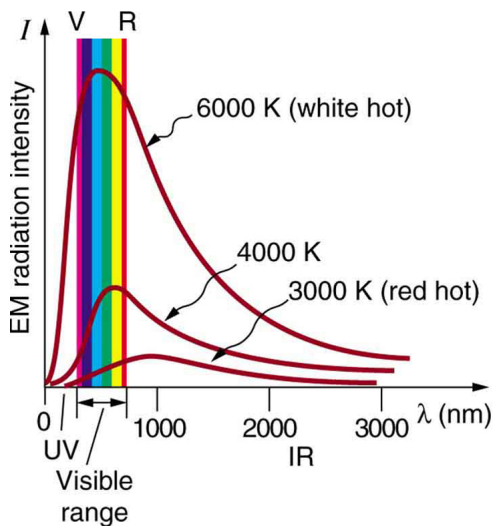
the branch of physics that deals with small objects and with the quantization of various entities, especially energy

Quantization of Energy

- Explain Max Planck's contribution to the development of quantum mechanics.
- Explain why atomic spectra indicate quantization.

Planck's Contribution

Energy is quantized in some systems, meaning that the system can have only certain energies and not a continuum of energies, unlike the classical case. This would be like having only certain speeds at which a car can travel because its kinetic energy can have only certain values. We also find that some forms of energy transfer take place with discrete lumps of energy. While most of us are familiar with the quantization of matter into lumps called atoms, molecules, and the like, we are less aware that energy, too, can be quantized. Some of the earliest clues about the necessity of quantum mechanics over classical physics came from the quantization of energy.



Graphs of blackbody radiation (from an ideal radiator) at three different radiator temperatures. The intensity or rate of

radiation emission
increases dramatically
with temperature, and the
peak of the spectrum
shifts toward the visible
and ultraviolet parts of
the spectrum. The shape
of the spectrum cannot be
described with classical
physics.

Where is the quantization of energy observed? Let us begin by considering the emission and absorption of electromagnetic (EM) radiation. The EM spectrum radiated by a hot solid is linked directly to the solid's temperature. (See [\[link\]](#).) An ideal radiator is one that has an emissivity of 1 at all wavelengths and, thus, is jet black. Ideal radiators are therefore called **blackbodies**, and their EM radiation is called **blackbody radiation**. It was discussed that the total intensity of the radiation varies as T^4 , the fourth power of the absolute temperature of the body, and that the peak of the spectrum shifts to shorter wavelengths at higher temperatures. All of this seems quite continuous, but it was the curve of the spectrum of intensity versus wavelength that gave a clue that the energies of the atoms in the solid are quantized. In fact, providing a theoretical explanation for the experimentally measured shape of the spectrum was a mystery at the turn of the century. When this “ultraviolet catastrophe” was eventually solved, the answers led to new technologies such as computers and the sophisticated imaging techniques described in earlier chapters. Once again, physics as an enabling science changed the way we live.

The German physicist Max Planck (1858–1947) used the idea that atoms and molecules in a body act like oscillators to absorb and emit radiation. The energies of the oscillating atoms and molecules had to be quantized to correctly describe the shape of the blackbody spectrum. Planck deduced that the energy of an oscillator having a frequency f is given by

Equation:

$$E = \left(n + \frac{1}{2} \right) hf.$$

Here n is any nonnegative integer (0, 1, 2, 3, ...). The symbol h stands for **Planck's constant**, given by

Equation:

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}.$$

The equation $E = \left(n + \frac{1}{2} \right) hf$ means that an oscillator having a frequency f (emitting and absorbing EM radiation of frequency f) can have its energy increase or decrease only in *discrete* steps of size

Equation:

$$\Delta E = hf.$$

It might be helpful to mention some macroscopic analogies of this quantization of energy phenomena. This is like a pendulum that has a characteristic oscillation frequency but can swing with only certain amplitudes. Quantization of energy also resembles a standing wave on a string that allows only particular harmonics described by integers. It is also similar to going up and down a hill using discrete stair steps rather than being able to move up and down a continuous slope. Your potential energy takes on discrete values as you move from step to step.

Using the quantization of oscillators, Planck was able to correctly describe the experimentally known shape of the blackbody spectrum. This was the first indication that energy is sometimes quantized on a small scale and earned him the Nobel Prize in Physics in 1918. Although Planck's theory comes from observations of a macroscopic object, its analysis is based on atoms and molecules. It was such a revolutionary departure from classical physics that Planck himself was reluctant to accept his own idea that energy states are not continuous. The general acceptance of Planck's energy quantization was greatly enhanced by Einstein's explanation of the photoelectric effect (discussed in the next section), which took energy

quantization a step further. Planck was fully involved in the development of both early quantum mechanics and relativity. He quickly embraced Einstein's special relativity, published in 1905, and in 1906 Planck was the first to suggest the correct formula for relativistic momentum, $p = \gamma mu$.



The German physicist Max Planck had a major influence on the early development of quantum mechanics, being the first to recognize that energy is sometimes quantized. Planck also made important contributions to special relativity and classical physics.
(credit: Library of Congress, Prints and Photographs Division via Wikimedia Commons)

Note that Planck's constant h is a very small number. So for an infrared frequency of 10^{14} Hz being emitted by a blackbody, for example, the difference between energy levels is only $\Delta E = hf = (6.63 \times 10^{-34} \text{ J}\cdot\text{s})(10^{14} \text{ Hz}) = 6.63 \times 10^{-20} \text{ J}$, or about 0.4 eV. This 0.4 eV of energy is significant compared with typical atomic

energies, which are on the order of an electron volt, or thermal energies, which are typically fractions of an electron volt. But on a macroscopic or classical scale, energies are typically on the order of joules. Even if macroscopic energies are quantized, the quantum steps are too small to be noticed. This is an example of the correspondence principle. For a large object, quantum mechanics produces results indistinguishable from those of classical physics.

Atomic Spectra

Now let us turn our attention to the *emission and absorption of EM radiation by gases*. The Sun is the most common example of a body containing gases emitting an EM spectrum that includes visible light. We also see examples in neon signs and candle flames. Studies of emissions of hot gases began more than two centuries ago, and it was soon recognized that these emission spectra contained huge amounts of information. The type of gas and its temperature, for example, could be determined. We now know that these EM emissions come from electrons transitioning between energy levels in individual atoms and molecules; thus, they are called **atomic spectra**. Atomic spectra remain an important analytical tool today. [\[link\]](#) shows an example of an emission spectrum obtained by passing an electric discharge through a material. One of the most important characteristics of these spectra is that they are discrete. By this we mean that only certain wavelengths, and hence frequencies, are emitted. This is called a line spectrum. If frequency and energy are associated as $\Delta E = hf$, the energies of the electrons in the emitting atoms and molecules are quantized. This is discussed in more detail later in this chapter.



Emission spectrum of oxygen. When an electrical discharge is passed through a substance, its atoms and molecules absorb energy, which is reemitted as EM radiation. The discrete nature of these emissions implies that the energy states of the atoms

and molecules are quantized. Such atomic spectra were used as analytical tools for many decades before it was understood why they are quantized. (credit: Teravolt, Wikimedia Commons)

It was a major puzzle that atomic spectra are quantized. Some of the best minds of 19th-century science failed to explain why this might be. Not until the second decade of the 20th century did an answer based on quantum mechanics begin to emerge. Again a macroscopic or classical body of gas was involved in the studies, but the effect, as we shall see, is due to individual atoms and molecules.

Note:

PhET Explorations: Models of the Hydrogen Atom

How did scientists figure out the structure of atoms without looking at them? Try out different models by shooting light at the atom. Check how the prediction of the model matches the experimental results.

<https://archive.cnx.org/specials/d77cc1d0-33e4-11e6-b016-6726afecd2be/hydrogen-atom/#sim-hydrogen-atom>

Section Summary

- The first indication that energy is sometimes quantized came from blackbody radiation, which is the emission of EM radiation by an object with an emissivity of 1.
- Planck recognized that the energy levels of the emitting atoms and molecules were quantized, with only the allowed values of $E = (n + \frac{1}{2})hf$, where n is any non-negative integer (0, 1, 2, 3, ...).
- h is Planck's constant, whose value is $h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$.
- Thus, the oscillatory absorption and emission energies of atoms and molecules in a blackbody could increase or decrease only in steps of

size $\Delta E = hf$ where f is the frequency of the oscillatory nature of the absorption and emission of EM radiation.

- Another indication of energy levels being quantized in atoms and molecules comes from the lines in atomic spectra, which are the EM emissions of individual atoms and molecules.

Conceptual Questions

Exercise:

Problem:

Give an example of a physical entity that is quantized. State specifically what the entity is and what the limits are on its values.

Exercise:

Problem:

Give an example of a physical entity that is not quantized, in that it is continuous and may have a continuous range of values.

Exercise:

Problem:

What aspect of the blackbody spectrum forced Planck to propose quantization of energy levels in its atoms and molecules?

Exercise:

Problem:

If Planck's constant were large, say 10^{34} times greater than it is, we would observe macroscopic entities to be quantized. Describe the motions of a child's swing under such circumstances.

Exercise:

Problem: Why don't we notice quantization in everyday events?

Problems & Exercises

Exercise:

Problem:

A LiBr molecule oscillates with a frequency of 1.7×10^{13} Hz. (a) What is the difference in energy in eV between allowed oscillator states? (b) What is the approximate value of n for a state having an energy of 1.0 eV?

Solution:

(a) 0.070 eV

(b) 14

Exercise:

Problem:

The difference in energy between allowed oscillator states in HBr molecules is 0.330 eV. What is the oscillation frequency of this molecule?

Exercise:

Problem:

A physicist is watching a 15-kg orangutan at a zoo swing lazily in a tire at the end of a rope. He (the physicist) notices that each oscillation takes 3.00 s and hypothesizes that the energy is quantized. (a) What is the difference in energy in joules between allowed oscillator states? (b) What is the value of n for a state where the energy is 5.00 J? (c) Can the quantization be observed?

Solution:

(a) 2.21×10^{-34} J

(b) 2.26×10^{34}

(c) No

Glossary

blackbody

an ideal radiator, which can radiate equally well at all wavelengths

blackbody radiation

the electromagnetic radiation from a blackbody

Planck's constant

$$h = 6.626 \times 10^{-34} \text{ J} \cdot \text{s}$$

atomic spectra

the electromagnetic emission from atoms and molecules

The Photoelectric Effect

- Describe a typical photoelectric-effect experiment.
- Determine the maximum kinetic energy of photoelectrons ejected by photons of one energy or wavelength, when given the maximum kinetic energy of photoelectrons for a different photon energy or wavelength.

When light strikes materials, it can eject electrons from them. This is called the **photoelectric effect**, meaning that light (*photo*) produces electricity. One common use of the photoelectric effect is in light meters, such as those that adjust the automatic iris on various types of cameras. In a similar way, another use is in solar cells, as you probably have in your calculator or have seen on a roof top or a roadside sign. These make use of the photoelectric effect to convert light into electricity for running different devices.



The
photoelectric
effect can be
observed by
allowing
light to fall
on the metal
plate in this
evacuated
tube.

Electrons
ejected by
the light are
collected on
the collector
wire and

measured as
a current. A
retarding
voltage
between the
collector
wire and
plate can
then be
adjusted so
as to
determine the
energy of the
ejected
electrons. For
example, if it
is sufficiently
negative, no
electrons will
reach the
wire. (credit:
P.P. Urone)

This effect has been known for more than a century and can be studied using a device such as that shown in [\[link\]](#). This figure shows an evacuated tube with a metal plate and a collector wire that are connected by a variable voltage source, with the collector more negative than the plate. When light (or other EM radiation) strikes the plate in the evacuated tube, it may eject electrons. If the electrons have energy in electron volts (eV) greater than the potential difference between the plate and the wire in volts, some electrons will be collected on the wire. Since the electron energy in eV is qV , where q is the electron charge and V is the potential difference, the electron energy can be measured by adjusting the retarding voltage between the wire and the plate. The voltage that stops the electrons from reaching the wire equals the energy in eV. For example, if -3.00 V barely stops the electrons,

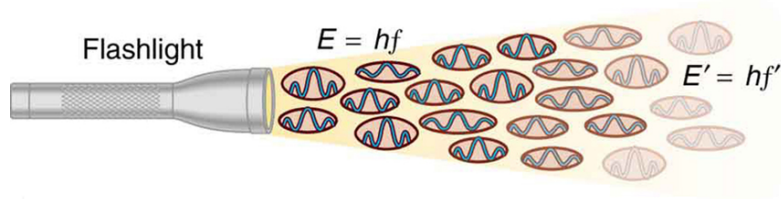
their energy is 3.00 eV. The number of electrons ejected can be determined by measuring the current between the wire and plate. The more light, the more electrons; a little circuitry allows this device to be used as a light meter.

What is really important about the photoelectric effect is what Albert Einstein deduced from it. Einstein realized that there were several characteristics of the photoelectric effect that could be explained only if *EM radiation is itself quantized*: the apparently continuous stream of energy in an EM wave is actually composed of energy quanta called photons. In his explanation of the photoelectric effect, Einstein defined a quantized unit or quantum of EM energy, which we now call a **photon**, with an energy proportional to the frequency of EM radiation. In equation form, the **photon energy** is

Equation:

$$E = hf,$$

where E is the energy of a photon of frequency f and h is Planck's constant. This revolutionary idea looks similar to Planck's quantization of energy states in blackbody oscillators, but it is quite different. It is the quantization of EM radiation itself. EM waves are composed of photons and are not continuous smooth waves as described in previous chapters on optics. Their energy is absorbed and emitted in lumps, not continuously. This is exactly consistent with Planck's quantization of energy levels in blackbody oscillators, since these oscillators increase and decrease their energy in steps of hf by absorbing and emitting photons having $E = hf$. We do not observe this with our eyes, because there are so many photons in common light sources that individual photons go unnoticed. (See [\[link\]](#).) The next section of the text ([Photon Energies and the Electromagnetic Spectrum](#)) is devoted to a discussion of photons and some of their characteristics and implications. For now, we will use the photon concept to explain the photoelectric effect, much as Einstein did.



An EM wave of frequency f is composed of photons, or individual quanta of EM radiation. The energy of each photon is $E = hf$, where h is Planck's constant and f is the frequency of the EM radiation. Higher intensity means more photons per unit area.

The flashlight emits large numbers of photons of many different frequencies, hence others have energy $E' = hf'$, and so on.

The photoelectric effect has the properties discussed below. All these properties are consistent with the idea that individual photons of EM radiation are absorbed by individual electrons in a material, with the electron gaining the photon's energy. Some of these properties are inconsistent with the idea that EM radiation is a simple wave. For simplicity, let us consider what happens with monochromatic EM radiation in which all photons have the same energy hf .

1. If we vary the frequency of the EM radiation falling on a material, we find the following: For a given material, there is a threshold frequency f_0 for the EM radiation below which no electrons are ejected, regardless of intensity. Individual photons interact with individual electrons. Thus if the photon energy is too small to break an electron away, no electrons will be ejected. If EM radiation was a simple wave, sufficient energy could be obtained by increasing the intensity.
2. *Once EM radiation falls on a material, electrons are ejected without delay.* As soon as an individual photon of a sufficiently high frequency is absorbed by an individual electron, the electron is ejected. If the EM

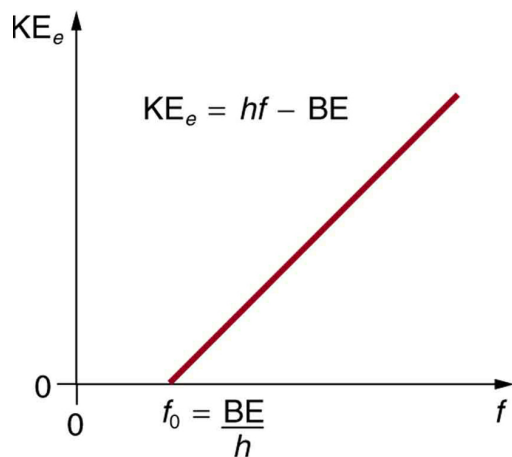
radiation were a simple wave, several minutes would be required for sufficient energy to be deposited to the metal surface to eject an electron.

3. The number of electrons ejected per unit time is proportional to the intensity of the EM radiation and to no other characteristic. High-intensity EM radiation consists of large numbers of photons per unit area, with all photons having the same characteristic energy hf .
4. If we vary the intensity of the EM radiation and measure the energy of ejected electrons, we find the following: *The maximum kinetic energy of ejected electrons is independent of the intensity of the EM radiation.* Since there are so many electrons in a material, it is extremely unlikely that two photons will interact with the same electron at the same time, thereby increasing the energy given it. Instead (as noted in 3 above), increased intensity results in more electrons of the same energy being ejected. If EM radiation were a simple wave, a higher intensity could give more energy, and higher-energy electrons would be ejected.
5. The kinetic energy of an ejected electron equals the photon energy minus the binding energy of the electron in the specific material. An individual photon can give all of its energy to an electron. The photon's energy is partly used to break the electron away from the material. The remainder goes into the ejected electron's kinetic energy. In equation form, this is given by

Equation:

$$KE_e = hf - BE,$$

where KE_e is the maximum kinetic energy of the ejected electron, hf is the photon's energy, and BE is the **binding energy** of the electron to the particular material. (BE is sometimes called the *work function* of the material.) This equation, due to Einstein in 1905, explains the properties of the photoelectric effect quantitatively. An individual photon of EM radiation (it does not come any other way) interacts with an individual electron, supplying enough energy, BE , to break it away, with the remainder going to kinetic energy. The binding energy is $BE = hf_0$, where f_0 is the threshold frequency for the particular material. [\[link\]](#) shows a graph of maximum KE_e versus the frequency of incident EM radiation falling on a particular material.



Photoelectric effect. A graph of the kinetic energy of an ejected electron, KE_e , versus the frequency of EM radiation impinging on a certain material. There is a threshold frequency below which no electrons are ejected, because the individual photon interacting with an individual electron has insufficient energy to break it away. Above the threshold energy, KE_e increases linearly with f , consistent with $KE_e = hf - BE$. The slope of this line is h — the data can be used to determine Planck's constant experimentally. Einstein gave the first successful explanation of such data by proposing

the idea of photons—
quanta of EM radiation.

Einstein's idea that EM radiation is quantized was crucial to the beginnings of quantum mechanics. It is a far more general concept than its explanation of the photoelectric effect might imply. All EM radiation can also be modeled in the form of photons, and the characteristics of EM radiation are entirely consistent with this fact. (As we will see in the next section, many aspects of EM radiation, such as the hazards of ultraviolet (UV) radiation, can be explained *only* by photon properties.) More famous for modern relativity, Einstein planted an important seed for quantum mechanics in 1905, the same year he published his first paper on special relativity. His explanation of the photoelectric effect was the basis for the Nobel Prize awarded to him in 1921. Although his other contributions to theoretical physics were also noted in that award, special and general relativity were not fully recognized in spite of having been partially verified by experiment by 1921. Although hero-worshipped, this great man never received Nobel recognition for his most famous work—relativity.

Example:

Calculating Photon Energy and the Photoelectric Effect: A Violet Light

(a) What is the energy in joules and electron volts of a photon of 420-nm violet light? (b) What is the maximum kinetic energy of electrons ejected from calcium by 420-nm violet light, given that the binding energy (or work function) of electrons for calcium metal is 2.71 eV?

Strategy

To solve part (a), note that the energy of a photon is given by $E = hf$. For part (b), once the energy of the photon is calculated, it is a straightforward application of $KE_e = hf - BE$ to find the ejected electron's maximum kinetic energy, since BE is given.

Solution for (a)

Photon energy is given by

Equation:

$$E = hf$$

Since we are given the wavelength rather than the frequency, we solve the familiar relationship $c = f\lambda$ for the frequency, yielding

Equation:

$$f = \frac{c}{\lambda}.$$

Combining these two equations gives the useful relationship

Equation:

$$E = \frac{hc}{\lambda}.$$

Now substituting known values yields

Equation:

$$E = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{420 \times 10^{-9} \text{ m}} = 4.74 \times 10^{-19} \text{ J}.$$

Converting to eV, the energy of the photon is

Equation:

$$E = (4.74 \times 10^{-19} \text{ J}) \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} = 2.96 \text{ eV}.$$

Solution for (b)

Finding the kinetic energy of the ejected electron is now a simple application of the equation $\text{KE}_e = hf - \text{BE}$. Substituting the photon energy and binding energy yields

Equation:

$$\text{KE}_e = hf - \text{BE} = 2.96 \text{ eV} - 2.71 \text{ eV} = 0.246 \text{ eV}.$$

Discussion

The energy of this 420-nm photon of violet light is a tiny fraction of a joule, and so it is no wonder that a single photon would be difficult for us to sense directly—humans are more attuned to energies on the order of joules. But looking at the energy in electron volts, we can see that this photon has enough energy to affect atoms and molecules. A DNA molecule can be broken with about 1 eV of energy, for example, and typical atomic and molecular energies are on the order of eV, so that the UV photon in this example could have biological effects. The ejected electron (called a *photoelectron*) has a rather low energy, and it would not travel far, except in a vacuum. The electron would be stopped by a retarding potential of but 0.26 eV. In fact, if the photon wavelength were longer and its energy less than 2.71 eV, then the formula would give a negative kinetic energy, an impossibility. This simply means that the 420-nm photons with their 2.96-eV energy are not much above the frequency threshold. You can show for yourself that the threshold wavelength is 459 nm (blue light). This means that if calcium metal is used in a light meter, the meter will be insensitive to wavelengths longer than those of blue light. Such a light meter would be completely insensitive to red light, for example.

Note:

PhET Explorations: Photoelectric Effect

See how light knocks electrons off a metal target, and recreate the experiment that spawned the field of quantum mechanics.

<https://archive.cnx.org/specials/cf1152da-eae8-11e5-b874-f779884a9994/photoelectric-effect/#sim-photoelectric-effect>

Section Summary

- The photoelectric effect is the process in which EM radiation ejects electrons from a material.
- Einstein proposed photons to be quanta of EM radiation having energy $E = hf$, where f is the frequency of the radiation.

- All EM radiation is composed of photons. As Einstein explained, all characteristics of the photoelectric effect are due to the interaction of individual photons with individual electrons.
- The maximum kinetic energy KE_e of ejected electrons (photoelectrons) is given by $KE_e = hf - BE$, where hf is the photon energy and BE is the binding energy (or work function) of the electron to the particular material.

Conceptual Questions

Exercise:

Problem:

Is visible light the only type of EM radiation that can cause the photoelectric effect?

Exercise:

Problem:

Which aspects of the photoelectric effect cannot be explained without photons? Which can be explained without photons? Are the latter inconsistent with the existence of photons?

Exercise:

Problem:

Is the photoelectric effect a direct consequence of the wave character of EM radiation or of the particle character of EM radiation? Explain briefly.

Exercise:

Problem:

Insulators (nonmetals) have a higher BE than metals, and it is more difficult for photons to eject electrons from insulators. Discuss how this relates to the free charges in metals that make them good conductors.

Exercise:**Problem:**

If you pick up and shake a piece of metal that has electrons in it free to move as a current, no electrons fall out. Yet if you heat the metal, electrons can be boiled off. Explain both of these facts as they relate to the amount and distribution of energy involved with shaking the object as compared with heating it.

Problems & Exercises**Exercise:****Problem:**

What is the longest-wavelength EM radiation that can eject a photoelectron from silver, given that the binding energy is 4.73 eV? Is this in the visible range?

Solution:

263 nm

Exercise:**Problem:**

Find the longest-wavelength photon that can eject an electron from potassium, given that the binding energy is 2.24 eV. Is this visible EM radiation?

Exercise:**Problem:**

What is the binding energy in eV of electrons in magnesium, if the longest-wavelength photon that can eject electrons is 337 nm?

Solution:

3.69 eV

Exercise:

Problem:

Calculate the binding energy in eV of electrons in aluminum, if the longest-wavelength photon that can eject them is 304 nm.

Exercise:

Problem:

What is the maximum kinetic energy in eV of electrons ejected from sodium metal by 450-nm EM radiation, given that the binding energy is 2.28 eV?

Solution:

0.483 eV

Exercise:

Problem:

UV radiation having a wavelength of 120 nm falls on gold metal, to which electrons are bound by 4.82 eV. What is the maximum kinetic energy of the ejected photoelectrons?

Exercise:

Problem:

Violet light of wavelength 400 nm ejects electrons with a maximum kinetic energy of 0.860 eV from sodium metal. What is the binding energy of electrons to sodium metal?

Solution:

2.25 eV

Exercise:

Problem:

UV radiation having a 300-nm wavelength falls on uranium metal, ejecting 0.500-eV electrons. What is the binding energy of electrons to uranium metal?

Exercise:**Problem:**

What is the wavelength of EM radiation that ejects 2.00-eV electrons from calcium metal, given that the binding energy is 2.71 eV? What type of EM radiation is this?

Solution:

- (a) 264 nm
- (b) Ultraviolet

Exercise:**Problem:**

Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is 2.24 eV. Are these photons visible?

Exercise:**Problem:**

What is the maximum velocity of electrons ejected from a material by 80-nm photons, if they are bound to the material by 4.73 eV?

Solution:

$$1.95 \times 10^6 \text{ m/s}$$

Exercise:

Problem:

Photoelectrons from a material with a binding energy of 2.71 eV are ejected by 420-nm photons. Once ejected, how long does it take these electrons to travel 2.50 cm to a detection device?

Exercise:**Problem:**

A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV?

Solution:

(a) $4.02 \times 10^{15} \text{ /s}$

(b) 0.256 mW

Exercise:**Problem:**

(a) Calculate the number of photoelectrons per second ejected from a 1.00-mm^2 area of sodium metal by 500-nm EM radiation having an intensity of 1.30 kW/m^2 (the intensity of sunlight above the Earth's atmosphere). (b) Given that the binding energy is 2.28 eV, what power is carried away by the electrons? (c) The electrons carry away less power than brought in by the photons. Where does the other power go? How can it be recovered?

Exercise:**Problem: Unreasonable Results**

Red light having a wavelength of 700 nm is projected onto magnesium metal to which electrons are bound by 3.68 eV. (a) Use $\text{KE}_e = hf - \text{BE}$ to calculate the kinetic energy of the ejected electrons.

(b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) -1.90 eV

(b) Negative kinetic energy

(c) That the electrons would be knocked free.

Exercise:

Problem: Unreasonable Results

(a) What is the binding energy of electrons to a material from which 4.00-eV electrons are ejected by 400-nm EM radiation? (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Glossary

photoelectric effect

the phenomenon whereby some materials eject electrons when light is shined on them

photon

a quantum, or particle, of electromagnetic radiation

photon energy

the amount of energy a photon has; $E = hf$

binding energy

also called the *work function*; the amount of energy necessary to eject an electron from a material

Photon Energies and the Electromagnetic Spectrum

- Explain the relationship between the energy of a photon in joules or electron volts and its wavelength or frequency.
- Calculate the number of photons per second emitted by a monochromatic source of specific wavelength and power.

Ionizing Radiation

A photon is a quantum of EM radiation. Its energy is given by $E = hf$ and is related to the frequency f and wavelength λ of the radiation by

Equation:

$$E = hf = \frac{hc}{\lambda} (\text{energy of a photon}),$$

where E is the energy of a single photon and c is the speed of light. When working with small systems, energy in eV is often useful. Note that Planck's constant in these units is

Equation:

$$h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}.$$

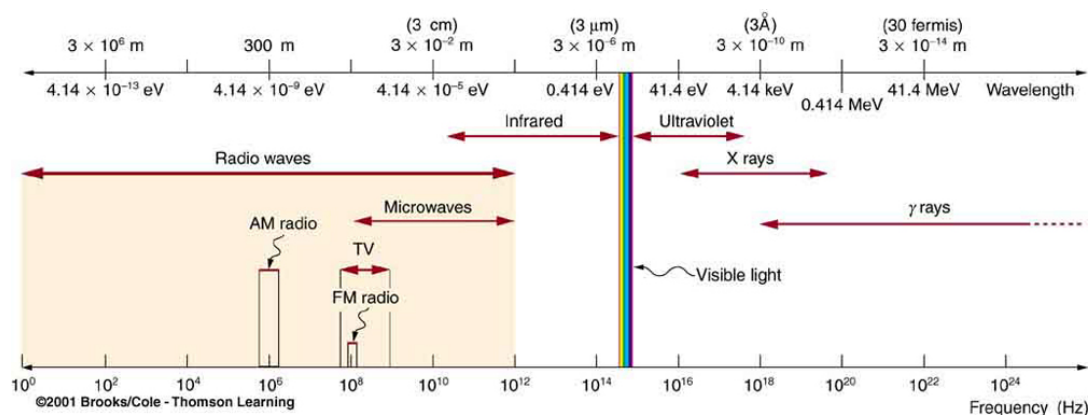
Since many wavelengths are stated in nanometers (nm), it is also useful to know that

Equation:

$$hc = 1240 \text{ eV} \cdot \text{nm}.$$

These will make many calculations a little easier.

All EM radiation is composed of photons. [\[link\]](#) shows various divisions of the EM spectrum plotted against wavelength, frequency, and photon energy. Previously in this book, photon characteristics were alluded to in the discussion of some of the characteristics of UV, x rays, and γ rays, the first of which start with frequencies just above violet in the visible spectrum. It was noted that these types of EM radiation have characteristics much different than visible light. We can now see that such properties arise because photon energy is larger at high frequencies.



The EM spectrum, showing major categories as a function of photon energy in eV, as well as wavelength and frequency. Certain characteristics of EM radiation are directly attributable to photon energy alone.

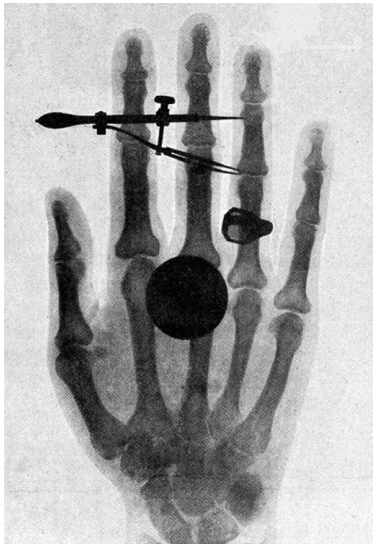
Rotational energies of molecules	10^{-5} eV
Vibrational energies of molecules	0.1 eV
Energy between outer electron shells in atoms	1 eV
Binding energy of a weakly bound molecule	1 eV
Energy of red light	2 eV
Binding energy of a tightly bound molecule	10 eV
Energy to ionize atom or molecule	10 to 1000 eV

Representative Energies for Submicroscopic Effects (Order of Magnitude Only)

Photons act as individual quanta and interact with individual electrons, atoms, molecules, and so on. The energy a photon carries is, thus, crucial to the effects it has. [\[link\]](#) lists representative submicroscopic energies in eV. When we compare photon energies from the EM spectrum in [\[link\]](#) with energies in the table, we can see how effects vary with the type of EM radiation.

Gamma rays, a form of nuclear and cosmic EM radiation, can have the highest frequencies and, hence, the highest photon energies in the EM spectrum. For example, a γ -ray photon with $f = 10^{21}$ Hz has an energy $E = hf = 6.63 \times 10^{-13} \text{ J} = 4.14 \text{ MeV}$. This is sufficient energy to ionize thousands of atoms and molecules, since only 10 to 1000 eV are needed per ionization. In fact, γ rays are one type of **ionizing radiation**, as are x rays and UV, because they produce ionization in materials that absorb

them. Because so much ionization can be produced, a single γ -ray photon can cause significant damage to biological tissue, killing cells or damaging their ability to properly reproduce. When cell reproduction is disrupted, the result can be cancer, one of the known effects of exposure to ionizing radiation. Since cancer cells are rapidly reproducing, they are exceptionally sensitive to the disruption produced by ionizing radiation. This means that ionizing radiation has positive uses in cancer treatment as well as risks in producing cancer.



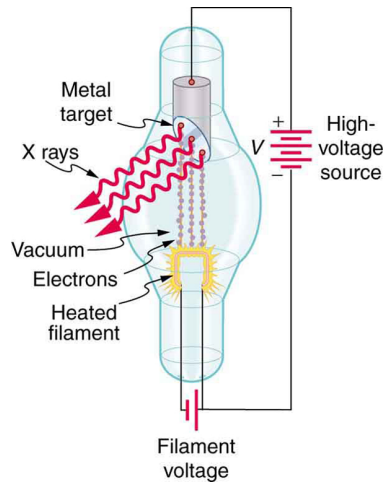
One of the first x-ray images, taken by Röntgen himself. The hand belongs to Bertha Röntgen, his wife.
(credit: Wilhelm Conrad Röntgen, via Wikimedia Commons)

High photon energy also enables γ rays to penetrate materials, since a collision with a single atom or molecule is unlikely to absorb all the γ ray's energy. This can make γ rays useful as a probe, and they are sometimes used in medical imaging. **x rays**, as you can see in [\[link\]](#), overlap with the low-frequency end of the γ ray range. Since x rays have energies of keV and up, individual x-ray photons also can produce large amounts of ionization. At lower photon energies, x rays are not as penetrating as γ rays and are slightly less hazardous. X rays are ideal for medical imaging, their most common use, and a fact that was recognized immediately upon their discovery in 1895 by the German physicist W. C. Roentgen (1845–1923). (See [\[link\]](#).) Within one year of their discovery, x rays (for a time called Roentgen rays) were used for medical diagnostics. Roentgen received the 1901 Nobel Prize for the discovery of x rays.

Note:

Connections: Conservation of Energy

Once again, we find that conservation of energy allows us to consider the initial and final forms that energy takes, without having to make detailed calculations of the intermediate steps. [\[link\]](#) is solved by considering only the initial and final forms of energy.



X rays are produced when energetic electrons strike the copper anode of this cathode ray tube (CRT). Electrons (shown here as separate particles) interact individually with the material they strike, sometimes producing photons of EM radiation.

While γ rays originate in nuclear decay, x rays are produced by the process shown in [\[link\]](#). Electrons ejected by thermal agitation from a hot filament in a vacuum tube are accelerated through a high voltage, gaining kinetic energy from the electrical potential energy. When they strike the anode, the electrons convert their kinetic energy to a variety of forms, including thermal energy. But since an accelerated charge radiates EM waves, and since the electrons act individually, photons are also produced. Some of these x-ray photons obtain the kinetic energy of the electron. The accelerated electrons originate at the cathode, so such a tube is called a cathode ray tube (CRT), and various versions of them are found in older TV and computer screens as well as in x-ray machines.

Example:

X-ray Photon Energy and X-ray Tube Voltage

Find the maximum energy in eV of an x-ray photon produced by electrons accelerated through a potential difference of 50.0 kV in a CRT like the one in [\[link\]](#).

Strategy

Electrons can give all of their kinetic energy to a single photon when they strike the anode of a CRT. (This is something like the photoelectric effect in reverse.) The kinetic energy of the electron comes from electrical potential energy. Thus we can simply equate the maximum photon energy to the electrical potential energy—that is, $hf = qV$. (We do not have to calculate each step from beginning to end if we know that all of the starting energy qV is converted to the final form hf .)

Solution

The maximum photon energy is $hf = qV$, where q is the charge of the electron and V is the accelerating voltage. Thus,

Equation:

$$hf = (1.60 \times 10^{-19} \text{ C})(50.0 \times 10^3 \text{ V}).$$

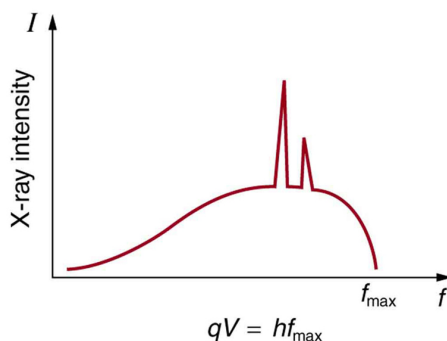
From the definition of the electron volt, we know $1 \text{ eV} = 1.60 \times 10^{-19} \text{ J}$, where $1 \text{ J} = 1 \text{ C} \cdot \text{V}$. Gathering factors and converting energy to eV yields

Equation:

$$hf = (50.0 \times 10^3)(1.60 \times 10^{-19} \text{ C} \cdot \text{V}) \left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ C} \cdot \text{V}} \right) = (50.0 \times 10^3)(1 \text{ eV}) = 50.0 \text{ keV}.$$

Discussion

This example produces a result that can be applied to many similar situations. If you accelerate a single elementary charge, like that of an electron, through a potential given in volts, then its energy in eV has the same numerical value. Thus a 50.0-kV potential generates 50.0 keV electrons, which in turn can produce photons with a maximum energy of 50 keV. Similarly, a 100-kV potential in an x-ray tube can generate up to 100-keV x-ray photons. Many x-ray tubes have adjustable voltages so that various energy x rays with differing energies, and therefore differing abilities to penetrate, can be generated.



X-ray spectrum obtained when energetic electrons strike a material. The smooth part of the spectrum is bremsstrahlung, while the peaks are characteristic of the anode material. Both are atomic processes that produce energetic

photons known as x-ray
photons.

[\[link\]](#) shows the spectrum of x rays obtained from an x-ray tube. There are two distinct features to the spectrum. First, the smooth distribution results from electrons being decelerated in the anode material. A curve like this is obtained by detecting many photons, and it is apparent that the maximum energy is unlikely. This decelerating process produces radiation that is called **bremsstrahlung** (German for *braking radiation*). The second feature is the existence of sharp peaks in the spectrum; these are called **characteristic x rays**, since they are characteristic of the anode material. Characteristic x rays come from atomic excitations unique to a given type of anode material. They are akin to lines in atomic spectra, implying the energy levels of atoms are quantized. Phenomena such as discrete atomic spectra and characteristic x rays are explored further in [Atomic Physics](#).

Ultraviolet radiation (approximately 4 eV to 300 eV) overlaps with the low end of the energy range of x rays, but UV is typically lower in energy. UV comes from the de-excitation of atoms that may be part of a hot solid or gas. These atoms can be given energy that they later release as UV by numerous processes, including electric discharge, nuclear explosion, thermal agitation, and exposure to x rays. A UV photon has sufficient energy to ionize atoms and molecules, which makes its effects different from those of visible light. UV thus has some of the same biological effects as γ rays and x rays. For example, it can cause skin cancer and is used as a sterilizer. The major difference is that several UV photons are required to disrupt cell reproduction or kill a bacterium, whereas single γ -ray and X-ray photons can do the same damage. But since UV does have the energy to alter molecules, it can do what visible light cannot. One of the beneficial aspects of UV is that it triggers the production of vitamin D in the skin, whereas visible light has insufficient energy per photon to alter the molecules that trigger this production. Infantile jaundice is treated by exposing the baby to UV (with eye protection), called phototherapy, the beneficial effects of which are thought to be related to its ability to help prevent the buildup of potentially toxic bilirubin in the blood.

Example:

Photon Energy and Effects for UV

Short-wavelength UV is sometimes called vacuum UV, because it is strongly absorbed by air and must be studied in a vacuum. Calculate the photon energy in eV for 100-nm vacuum UV, and estimate the number of molecules it could ionize or break apart.

Strategy

Using the equation $E = hf$ and appropriate constants, we can find the photon energy and compare it with energy information in [\[link\]](#).

Solution

The energy of a photon is given by

Equation:

$$E = hf = \frac{hc}{\lambda}.$$

Using $hc = 1240 \text{ eV} \cdot \text{nm}$, we find that

Equation:

$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{100 \text{ nm}} = 12.4 \text{ eV}.$$

Discussion

According to [\[link\]](#), this photon energy might be able to ionize an atom or molecule, and it is about what is needed to break up a tightly bound molecule, since they are bound by approximately 10 eV. This photon energy could destroy about a dozen weakly bound molecules. Because of its high photon energy, UV disrupts atoms and molecules it interacts with. One good consequence is that all but the longest-wavelength UV is strongly absorbed and is easily blocked by sunglasses. In fact, most of the Sun's UV is absorbed by a thin layer of ozone in the upper atmosphere, protecting sensitive organisms on Earth. Damage to our ozone layer by the addition of such chemicals as CFC's has reduced this protection for us.

Visible Light

The range of photon energies for **visible light** from red to violet is 1.63 to 3.26 eV, respectively (left for this chapter's Problems and Exercises to verify). These energies are on the order of those between outer electron shells in atoms and molecules. This means that these photons can be absorbed by atoms and molecules. A *single* photon can actually stimulate the retina, for example, by altering a receptor molecule that then triggers a nerve impulse. Photons can be absorbed or emitted only by atoms and molecules that have precisely the correct quantized energy step to do so. For example, if a red photon of frequency f encounters a molecule that has an energy step, ΔE , equal to hf , then the photon can be absorbed. Violet flowers absorb red and reflect violet; this implies there is no energy step between levels in the receptor molecule equal to the violet photon's energy, but there is an energy step for the red.

There are some noticeable differences in the characteristics of light between the two ends of the visible spectrum that are due to photon energies. Red light has insufficient photon energy to expose most black-and-white film, and it is thus used to illuminate darkrooms where such film is developed. Since violet light has a higher photon energy, dyes that absorb violet tend to fade more quickly than those that do not. (See [\[link\]](#).) Take a look at some faded color posters in a storefront some time, and you will notice that the blues and violets are the last to fade. This is because other dyes, such as red and green dyes, absorb blue and violet photons, the higher energies of which break up their weakly bound molecules. (Complex molecules such as those in dyes and DNA tend to be weakly bound.) Blue and violet dyes reflect those colors and, therefore, do not absorb these more energetic photons, thus suffering less molecular damage.



Why do the reds, yellows,
and greens fade before
the blues and violets
when exposed to the Sun,
as with this poster? The
answer is related to
photon energy. (credit:
Deb Collins, Flickr)

Transparent materials, such as some glasses, do not absorb any visible light, because there is no energy step in the atoms or molecules that could absorb the light. Since individual photons interact with individual atoms, it is nearly impossible to have two photons absorbed simultaneously to reach a large energy step. Because of its lower photon energy, visible light can sometimes pass through many kilometers of a substance, while higher frequencies like UV, x ray, and γ rays are absorbed, because they have sufficient photon energy to ionize the material.

Example:

How Many Photons per Second Does a Typical Light Bulb Produce?

Assuming that 10.0% of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, calculate the number of visible photons emitted per second.

Strategy

Power is energy per unit time, and so if we can find the energy per photon, we can determine the number of photons per second. This will best be done in joules, since power is given in watts, which are joules per second.

Solution

The power in visible light production is 10.0% of 100 W, or 10.0 J/s. The energy of the average visible photon is found by substituting the given average wavelength into the formula

Equation:

$$E = \frac{hc}{\lambda}.$$

This produces

Equation:

$$E = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m/s})}{580 \times 10^{-9} \text{ m}} = 3.43 \times 10^{-19} \text{ J}.$$

The number of visible photons per second is thus

Equation:

$$\text{photon/s} = \frac{10.0 \text{ J/s}}{3.43 \times 10^{-19} \text{ J/photon}} = 2.92 \times 10^{19} \text{ photon/s}.$$

Discussion

This incredible number of photons per second is verification that individual photons are insignificant in ordinary human experience. It is also a verification of the correspondence principle—on the macroscopic scale, quantization becomes essentially continuous or classical. Finally, there are so many photons emitted by a 100-W lightbulb that it can be seen by the unaided eye many kilometers away.

Lower-Energy Photons

Infrared radiation (IR) has even lower photon energies than visible light and cannot significantly alter atoms and molecules. IR can be absorbed and emitted by atoms and molecules, particularly between closely spaced states. IR is extremely strongly absorbed by water, for example, because water molecules have many states separated by energies on the order of 10^{-5} eV to 10^{-2} eV, well within the IR and microwave energy ranges. This is why in the IR range, skin is almost jet black, with an emissivity near 1—there are many states in water molecules in the skin that can absorb a large range of IR photon energies. Not all molecules have this property. Air, for example, is nearly transparent to many IR frequencies.

Microwaves are the highest frequencies that can be produced by electronic circuits, although they are also produced naturally. Thus microwaves are similar to IR but do not extend to as high frequencies. There are states in water and other molecules that have the same frequency and energy as microwaves, typically about 10^{-5} eV. This is one reason why food absorbs microwaves more strongly than many other materials, making microwave ovens an efficient way of putting energy directly into food.

Photon energies for both IR and microwaves are so low that huge numbers of photons are involved in any significant energy transfer by IR or microwaves (such as warming yourself with a heat lamp or cooking pizza in the microwave). Visible light, IR, microwaves, and all lower frequencies cannot produce ionization with single photons and do not ordinarily have the hazards of higher frequencies. When visible, IR, or microwave radiation is hazardous, such as the inducement of cataracts by microwaves, the hazard is due to huge numbers of photons acting together (not to an accumulation of photons, such as sterilization by weak UV). The negative effects of visible, IR, or microwave radiation can be thermal effects, which could be produced by any heat source. But one difference is that at very high intensity, strong electric and magnetic fields can be produced by photons acting together. Such electromagnetic fields (EMF) can actually ionize materials.

Note:**Misconception Alert: High-Voltage Power Lines**

Although some people think that living near high-voltage power lines is hazardous to one's health, ongoing studies of the transient field effects produced by these lines show their strengths to be insufficient to cause damage. Demographic studies also fail to show significant correlation of ill effects with high-voltage power lines. The American Physical Society issued a report over 10 years ago on power-line fields, which concluded that the scientific literature and reviews of panels show no consistent, significant link between cancer and power-line fields. They also felt that the "diversion of resources to eliminate a threat which has no persuasive scientific basis is disturbing."

It is virtually impossible to detect individual photons having frequencies below microwave frequencies, because of their low photon energy. But the photons are there. A continuous EM wave can be modeled as photons. At low frequencies, EM waves are generally treated as time- and position-varying electric and magnetic fields with no discernible quantization. This is another example of the correspondence principle in situations involving huge numbers of photons.

Note:**PhET Explorations: Color Vision**

Make a whole rainbow by mixing red, green, and blue light. Change the wavelength of a monochromatic beam or filter white light. View the light as a solid beam, or see the individual photons.

https://phet.colorado.edu/sims/html/color-vision/latest/color-vision_en.html

Section Summary

- Photon energy is responsible for many characteristics of EM radiation, being particularly noticeable at high frequencies.
- Photons have both wave and particle characteristics.

Conceptual Questions**Exercise:**

Problem: Why are UV, x rays, and γ rays called ionizing radiation?

Exercise:**Problem:**

How can treating food with ionizing radiation help keep it from spoiling? UV is not very penetrating. What else could be used?

Exercise:

Problem:

Some television tubes are CRTs. They use an approximately 30-kV accelerating potential to send electrons to the screen, where the electrons stimulate phosphors to emit the light that forms the pictures we watch. Would you expect x rays also to be created?

Exercise:**Problem:**

Tanning salons use “safe” UV with a longer wavelength than some of the UV in sunlight. This “safe” UV has enough photon energy to trigger the tanning mechanism. Is it likely to be able to cause cell damage and induce cancer with prolonged exposure?

Exercise:**Problem:**

Your pupils dilate when visible light intensity is reduced. Does wearing sunglasses that lack UV blockers increase or decrease the UV hazard to your eyes? Explain.

Exercise:**Problem:**

One could feel heat transfer in the form of infrared radiation from a large nuclear bomb detonated in the atmosphere 75 km from you. However, none of the profusely emitted x rays or γ rays reaches you. Explain.

Exercise:

Problem: Can a single microwave photon cause cell damage? Explain.

Exercise:**Problem:**

In an x-ray tube, the maximum photon energy is given by $hf = qV$. Would it be technically more correct to say $hf = qV + BE$, where BE is the binding energy of electrons in the target anode? Why isn't the energy stated the latter way?

Problems & Exercises**Exercise:****Problem:**

What is the energy in joules and eV of a photon in a radio wave from an AM station that has a 1530-kHz broadcast frequency?

Solution:

$$6.34 \times 10^{-9} \text{ eV}, 1.01 \times 10^{-27} \text{ J}$$

Exercise:

Problem:

(a) Find the energy in joules and eV of photons in radio waves from an FM station that has a 90.0-MHz broadcast frequency. (b) What does this imply about the number of photons per second that the radio station must broadcast?

Exercise:

Problem: Calculate the frequency in hertz of a 1.00-MeV γ -ray photon.

Solution:

$$2.42 \times 10^{20} \text{ Hz}$$

Exercise:**Problem:**

(a) What is the wavelength of a 1.00-eV photon? (b) Find its frequency in hertz. (c) Identify the type of EM radiation.

Exercise:**Problem:**

Do the unit conversions necessary to show that $hc = 1240 \text{ eV} \cdot \text{nm}$, as stated in the text.

Solution:**Equation:**

$$\begin{aligned} hc &= (6.62607 \times 10^{-34} \text{ J} \cdot \text{s}) (2.99792 \times 10^8 \text{ m/s}) \left(\frac{10^9 \text{ nm}}{1 \text{ m}} \right) \left(\frac{1.00000 \text{ eV}}{1.60218 \times 10^{-19} \text{ J}} \right) \\ &= 1239.84 \text{ eV} \cdot \text{nm} \\ &\approx 1240 \text{ eV} \cdot \text{nm} \end{aligned}$$

Exercise:**Problem:**

Confirm the statement in the text that the range of photon energies for visible light is 1.63 to 3.26 eV, given that the range of visible wavelengths is 380 to 760 nm.

Exercise:**Problem:**

(a) Calculate the energy in eV of an IR photon of frequency $2.00 \times 10^{13} \text{ Hz}$. (b) How many of these photons would need to be absorbed simultaneously by a tightly bound molecule to break it apart? (c) What is the energy in eV of a γ ray of frequency $3.00 \times 10^{20} \text{ Hz}$? (d) How many tightly bound molecules could a single such γ ray break apart?

Solution:

(a) 0.0829 eV

- (b) 121
- (c) 1.24 MeV
- (d) 1.24×10^5

Exercise:

Problem: Prove that, to three-digit accuracy, $h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$, as stated in the text.

Exercise:

Problem:

(a) What is the maximum energy in eV of photons produced in a CRT using a 25.0-kV accelerating potential, such as a color TV? (b) What is their frequency?

Solution:

- (a) $25.0 \times 10^3 \text{ eV}$
- (b) $6.04 \times 10^{18} \text{ Hz}$

Exercise:

Problem:

What is the accelerating voltage of an x-ray tube that produces x rays with a shortest wavelength of 0.0103 nm?

Exercise:

Problem:

(a) What is the ratio of power outputs by two microwave ovens having frequencies of 950 and 2560 MHz, if they emit the same number of photons per second? (b) What is the ratio of photons per second if they have the same power output?

Solution:

- (a) 2.69
- (b) 0.371

Exercise:

Problem:

How many photons per second are emitted by the antenna of a microwave oven, if its power output is 1.00 kW at a frequency of 2560 MHz?

Exercise:

Problem:

Some satellites use nuclear power. (a) If such a satellite emits a 1.00-W flux of γ rays having an average energy of 0.500 MeV, how many are emitted per second? (b) These γ rays affect other satellites. How far away must another satellite be to only receive one γ ray per second per square meter?

Solution:

(a) 1.25×10^{13} photons/s

(b) 997 km

Exercise:**Problem:**

(a) If the power output of a 650-kHz radio station is 50.0 kW, how many photons per second are produced? (b) If the radio waves are broadcast uniformly in all directions, find the number of photons per second per square meter at a distance of 100 km. Assume no reflection from the ground or absorption by the air.

Exercise:**Problem:**

How many x-ray photons per second are created by an x-ray tube that produces a flux of x rays having a power of 1.00 W? Assume the average energy per photon is 75.0 keV.

Solution:

8.33×10^{13} photons/s

Exercise:**Problem:**

(a) How far away must you be from a 650-kHz radio station with power 50.0 kW for there to be only one photon per second per square meter? Assume no reflections or absorption, as if you were in deep outer space. (b) Discuss the implications for detecting intelligent life in other solar systems by detecting their radio broadcasts.

Exercise:**Problem:**

Assuming that 10.0% of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, and that the photons spread out uniformly and are not absorbed by the atmosphere, how far away would you be if 500 photons per second enter the 3.00-mm diameter pupil of your eye? (This number easily stimulates the retina.)

Solution:

181 km

Exercise:

Problem:Construct Your Own Problem

Consider a laser pen. Construct a problem in which you calculate the number of photons per second emitted by the pen. Among the things to be considered are the laser pen's wavelength and power output. Your instructor may also wish for you to determine the minimum diffraction spreading in the beam and the number of photons per square centimeter the pen can project at some large distance. In this latter case, you will also need to consider the output size of the laser beam, the distance to the object being illuminated, and any absorption or scattering along the way.

Glossary

gamma ray

also γ -ray; highest-energy photon in the EM spectrum

ionizing radiation

radiation that ionizes materials that absorb it

x ray

EM photon between γ -ray and UV in energy

bremsstrahlung

German for *braking radiation*; produced when electrons are decelerated

characteristic x rays

x rays whose energy depends on the material they were produced in

ultraviolet radiation

UV; ionizing photons slightly more energetic than violet light

visible light

the range of photon energies the human eye can detect

infrared radiation

photons with energies slightly less than red light

microwaves

photons with wavelengths on the order of a micron (μm)

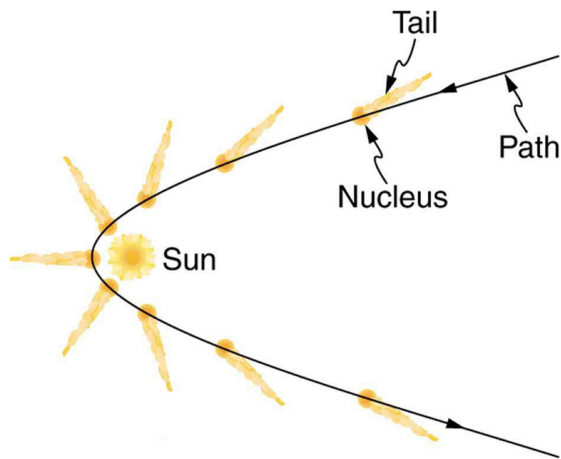
Photon Momentum

- Relate the linear momentum of a photon to its energy or wavelength, and apply linear momentum conservation to simple processes involving the emission, absorption, or reflection of photons.
- Account qualitatively for the increase of photon wavelength that is observed, and explain the significance of the Compton wavelength.

Measuring Photon Momentum

The quantum of EM radiation we call a **photon** has properties analogous to those of particles we can see, such as grains of sand. A photon interacts as a unit in collisions or when absorbed, rather than as an extensive wave.

Massive quanta, like electrons, also act like macroscopic particles—something we expect, because they are the smallest units of matter. Particles carry momentum as well as energy. Despite photons having no mass, there has long been evidence that EM radiation carries momentum. (Maxwell and others who studied EM waves predicted that they would carry momentum.) It is now a well-established fact that photons *do* have momentum. In fact, photon momentum is suggested by the photoelectric effect, where photons knock electrons out of a substance. [\[link\]](#) shows macroscopic evidence of photon momentum.



The tails of the Hale-Bopp comet point away from the Sun, evidence that light has momentum. Dust emanating from the body of the comet forms this tail. Particles of dust are pushed away from the Sun by light reflecting from them. The blue ionized gas tail is also produced by photons interacting with atoms in the comet material. (credit: Geoff Chester, U.S. Navy, via Wikimedia Commons)

[\[link\]](#) shows a comet with two prominent tails. What most people do not know about the tails is that they always point *away* from the Sun rather than trailing behind the comet (like the tail of Bo Peep's sheep). Comet tails are composed of gases and dust evaporated from the body of the comet and ionized gas. The dust particles recoil away from the Sun when photons scatter from them. Evidently, photons carry momentum in the direction of their motion (away from the Sun), and some of this momentum is transferred to dust particles in collisions. Gas atoms and molecules in the blue tail are most affected by other particles of radiation, such as protons and electrons emanating from the Sun, rather than by the momentum of photons.

Note:

Connections: Conservation of Momentum

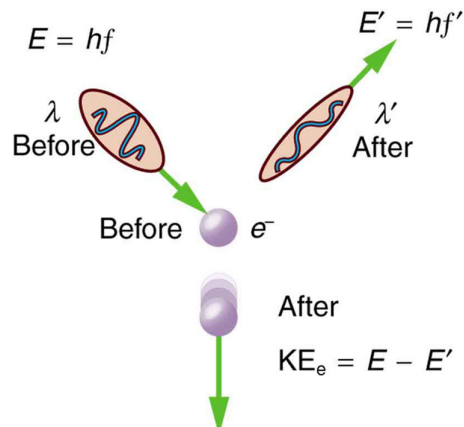
Not only is momentum conserved in all realms of physics, but all types of particles are found to have momentum. We expect particles with mass to have momentum, but now we see that massless particles including photons also carry momentum.

Momentum is conserved in quantum mechanics just as it is in relativity and classical physics. Some of the earliest direct experimental evidence of this came from scattering of x-ray photons by electrons in substances, named Compton scattering after the American physicist Arthur H. Compton (1892–1962). Around 1923, Compton observed that x rays scattered from materials had a decreased energy and correctly analyzed this as being due to the scattering of photons from electrons. This phenomenon could be handled as a collision between two particles—a photon and an electron at rest in the material. Energy and momentum are conserved in the collision. (See [\[link\]](#)) He won a Nobel Prize in 1929 for the discovery of this scattering, now called the **Compton effect**, because it helped prove that **photon momentum** is given by

Equation:

$$p = \frac{h}{\lambda},$$

where h is Planck's constant and λ is the photon wavelength. (Note that relativistic momentum given as $p = \gamma mu$ is valid only for particles having mass.)



The Compton effect is the name given to the scattering of a photon by an electron. Energy and momentum are conserved, resulting in a reduction of both for the scattered photon. Studying this effect, Compton verified that photons have momentum.

We can see that photon momentum is small, since $p = h/\lambda$ and h is very small. It is for this reason that we do not ordinarily observe photon

momentum. Our mirrors do not recoil when light reflects from them (except perhaps in cartoons). Compton saw the effects of photon momentum because he was observing x rays, which have a small wavelength and a relatively large momentum, interacting with the lightest of particles, the electron.

Example:

Electron and Photon Momentum Compared

(a) Calculate the momentum of a visible photon that has a wavelength of 500 nm. (b) Find the velocity of an electron having the same momentum. (c) What is the energy of the electron, and how does it compare with the energy of the photon?

Strategy

Finding the photon momentum is a straightforward application of its definition: $p = \frac{h}{\lambda}$. If we find the photon momentum is small, then we can assume that an electron with the same momentum will be nonrelativistic, making it easy to find its velocity and kinetic energy from the classical formulas.

Solution for (a)

Photon momentum is given by the equation:

Equation:

$$p = \frac{h}{\lambda}.$$

Entering the given photon wavelength yields

Equation:

$$p = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{500 \times 10^{-9} \text{ m}} = 1.33 \times 10^{-27} \text{ kg} \cdot \text{m/s}.$$

Solution for (b)

Since this momentum is indeed small, we will use the classical expression $p = mv$ to find the velocity of an electron with this momentum. Solving for v and using the known value for the mass of an electron gives

Equation:

$$v = \frac{p}{m} = \frac{1.33 \times 10^{-27} \text{ kg} \cdot \text{m/s}}{9.11 \times 10^{-31} \text{ kg}} = 1460 \text{ m/s} \approx 1460 \text{ m/s}.$$

Solution for (c)

The electron has kinetic energy, which is classically given by

Equation:

$$\text{KE}_e = \frac{1}{2}mv^2.$$

Thus,

Equation:

$$\text{KE}_e = \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(1455 \text{ m/s})^2 = 9.64 \times 10^{-25} \text{ J}.$$

Converting this to eV by multiplying by $(1 \text{ eV})/(1.602 \times 10^{-19} \text{ J})$ yields

Equation:

$$\text{KE}_e = 6.02 \times 10^{-6} \text{ eV}.$$

The photon energy E is

Equation:

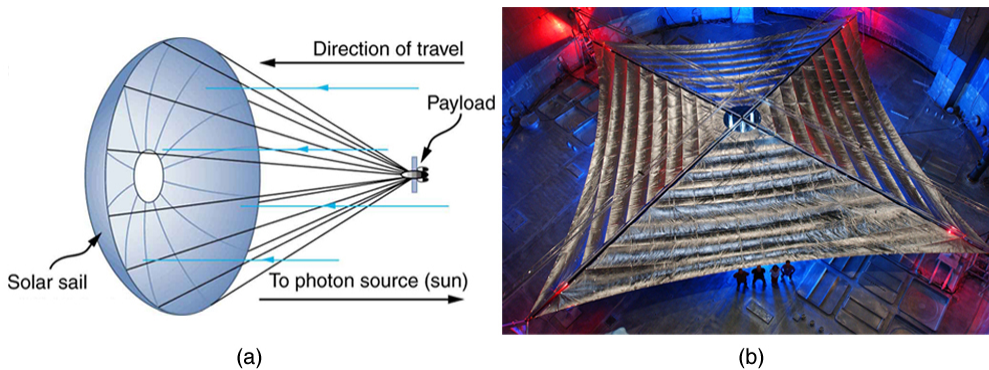
$$E = \frac{hc}{\lambda} = \frac{1240 \text{ eV} \cdot \text{nm}}{500 \text{ nm}} = 2.48 \text{ eV},$$

which is about five orders of magnitude greater.

Discussion

Photon momentum is indeed small. Even if we have huge numbers of them, the total momentum they carry is small. An electron with the same momentum has a 1460 m/s velocity, which is clearly nonrelativistic. A more massive particle with the same momentum would have an even smaller velocity. This is borne out by the fact that it takes far less energy to give an electron the same momentum as a photon. But on a quantum-mechanical scale, especially for high-energy photons interacting with small

masses, photon momentum is significant. Even on a large scale, photon momentum can have an effect if there are enough of them and if there is nothing to prevent the slow recoil of matter. Comet tails are one example, but there are also proposals to build space sails that use huge low-mass mirrors (made of aluminized Mylar) to reflect sunlight. In the vacuum of space, the mirrors would gradually recoil and could actually take spacecraft from place to place in the solar system. (See [\[link\]](#).)



(a) Space sails have been proposed that use the momentum of sunlight reflecting from gigantic low-mass sails to propel spacecraft about the solar system. A Russian test model of this (the Cosmos 1) was launched in 2005, but did not make it into orbit due to a rocket failure. (b) A U.S. version of this, labeled LightSail-1, is scheduled for trial launches in the first part of this decade. It will have a 40-m² sail. (credit: Kim Newton/NASA)

Relativistic Photon Momentum

There is a relationship between photon momentum p and photon energy E that is consistent with the relation given previously for the relativistic total energy of a particle as $E^2 = (pc)^2 + (mc)^2$. We know m is zero for a photon, but p is not, so that $E^2 = (pc)^2 + (mc)^2$ becomes

Equation:

$$E = pc,$$

or

Equation:

$$p = \frac{E}{c} \text{ (photons).}$$

To check the validity of this relation, note that $E = hc/\lambda$ for a photon. Substituting this into $p = E/c$ yields

Equation:

$$p = (hc/\lambda)/c = \frac{h}{\lambda},$$

as determined experimentally and discussed above. Thus, $p = E/c$ is equivalent to Compton's result $p = h/\lambda$. For a further verification of the relationship between photon energy and momentum, see [\[link\]](#).

Note:

Photon Detectors

Almost all detection systems talked about thus far—eyes, photographic plates, photomultiplier tubes in microscopes, and CCD cameras—rely on particle-like properties of photons interacting with a sensitive area. A change is caused and either the change is cascaded or zillions of points are recorded to form an image we detect. These detectors are used in biomedical imaging systems, and there is ongoing research into improving the efficiency of receiving photons, particularly by cooling detection systems and reducing thermal effects.

Example:**Photon Energy and Momentum**

Show that $p = E/c$ for the photon considered in the [\[link\]](#).

Strategy

We will take the energy E found in [\[link\]](#), divide it by the speed of light, and see if the same momentum is obtained as before.

Solution

Given that the energy of the photon is 2.48 eV and converting this to joules, we get

Equation:

$$p = \frac{E}{c} = \frac{(2.48 \text{ eV})(1.60 \times 10^{-19} \text{ J/eV})}{3.00 \times 10^8 \text{ m/s}} = 1.33 \times 10^{-27} \text{ kg} \cdot \text{m/s}.$$

Discussion

This value for momentum is the same as found before (note that unrounded values are used in all calculations to avoid even small rounding errors), an expected verification of the relationship $p = E/c$. This also means the relationship between energy, momentum, and mass given by $E^2 = (pc)^2 + (mc)^2$ applies to both matter and photons. Once again, note that p is not zero, even when m is.

Note:**Problem-Solving Suggestion**

Note that the forms of the constants $h = 4.14 \times 10^{-15} \text{ eV} \cdot \text{s}$ and $hc = 1240 \text{ eV} \cdot \text{nm}$ may be particularly useful for this section's Problems and Exercises.

Section Summary

- Photons have momentum, given by $p = \frac{h}{\lambda}$, where λ is the photon wavelength.

- Photon energy and momentum are related by $p = \frac{E}{c}$, where $E = hf = hc/\lambda$ for a photon.

Conceptual Questions

Exercise:

Problem:

Which formula may be used for the momentum of all particles, with or without mass?

Exercise:

Problem:

Is there any measurable difference between the momentum of a photon and the momentum of matter?

Exercise:

Problem:

Why don't we feel the momentum of sunlight when we are on the beach?

Problems & Exercises

Exercise:

Problem:

- (a) Find the momentum of a 4.00-cm-wavelength microwave photon.
- (b) Discuss why you expect the answer to (a) to be very small.

Solution:

- (a) $1.66 \times 10^{-32} \text{ kg} \cdot \text{m/s}$

(b) The wavelength of microwave photons is large, so the momentum they carry is very small.

Exercise:

Problem:

(a) What is the momentum of a 0.0100-nm-wavelength photon that could detect details of an atom? (b) What is its energy in MeV?

Exercise:

Problem:

(a) What is the wavelength of a photon that has a momentum of $5.00 \times 10^{-29} \text{ kg} \cdot \text{m/s}$? (b) Find its energy in eV.

Solution:

(a) $13.3 \text{ } \mu\text{m}$

(b) $9.38 \times 10^{-2} \text{ eV}$

Exercise:

Problem:

(a) A γ -ray photon has a momentum of $8.00 \times 10^{-21} \text{ kg} \cdot \text{m/s}$. What is its wavelength? (b) Calculate its energy in MeV.

Exercise:

Problem:

(a) Calculate the momentum of a photon having a wavelength of $2.50 \text{ } \mu\text{m}$. (b) Find the velocity of an electron having the same momentum. (c) What is the kinetic energy of the electron, and how does it compare with that of the photon?

Solution:

(a) $2.65 \times 10^{-28} \text{ kg} \cdot \text{m/s}$

(b) 291 m/s

(c) electron 3.86×10^{-26} J, photon 7.96×10^{-20} J, ratio 2.06×10^6

Exercise:

Problem:

Repeat the previous problem for a 10.0-nm-wavelength photon.

Exercise:

Problem:

(a) Calculate the wavelength of a photon that has the same momentum as a proton moving at 1.00% of the speed of light. (b) What is the energy of the photon in MeV? (c) What is the kinetic energy of the proton in MeV?

Solution:

(a) 1.32×10^{-13} m

(b) 9.39 MeV

(c) 4.70×10^{-2} MeV

Exercise:

Problem:

(a) Find the momentum of a 100-keV x-ray photon. (b) Find the equivalent velocity of a neutron with the same momentum. (c) What is the neutron's kinetic energy in keV?

Exercise:

Problem:

Take the ratio of relativistic rest energy, $E = \gamma mc^2$, to relativistic momentum, $p = \gamma mu$, and show that in the limit that mass approaches zero, you find $E/p = c$.

Solution:

$E = \gamma mc^2$ and $P = \gamma mu$, so

Equation:

$$\frac{E}{P} = \frac{\gamma mc^2}{\gamma mu} = \frac{c^2}{u}.$$

As the mass of particle approaches zero, its velocity u will approach c , so that the ratio of energy to momentum in this limit is

Equation:

$$\lim_{m \rightarrow 0} \frac{E}{P} = \frac{c^2}{c} = c$$

which is consistent with the equation for photon energy.

Exercise:**Problem: Construct Your Own Problem**

Consider a space sail such as mentioned in [\[link\]](#). Construct a problem in which you calculate the light pressure on the sail in N/m^2 produced by reflecting sunlight. Also calculate the force that could be produced and how much effect that would have on a spacecraft. Among the things to be considered are the intensity of sunlight, its average wavelength, the number of photons per square meter this implies, the area of the space sail, and the mass of the system being accelerated.

Exercise:**Problem: Unreasonable Results**

A car feels a small force due to the light it sends out from its headlights, equal to the momentum of the light divided by the time in which it is emitted. (a) Calculate the power of each headlight, if they

exert a total force of 2.00×10^{-2} N backward on the car. (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) 3.00×10^6 W

(b) Headlights are way too bright.

(c) Force is too large.

Glossary

photon momentum

the amount of momentum a photon has, calculated by $p = \frac{h}{\lambda} = \frac{E}{c}$

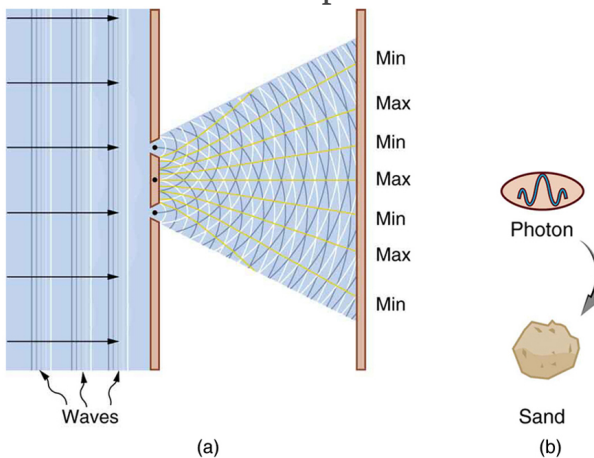
Compton effect

the phenomenon whereby x rays scattered from materials have decreased energy

The Particle-Wave Duality

- Explain what the term particle-wave duality means, and why it is applied to EM radiation.

We have long known that EM radiation is a wave, capable of interference and diffraction. We now see that light can be modeled as photons, which are massless particles. This may seem contradictory, since we ordinarily deal with large objects that never act like both wave and particle. An ocean wave, for example, looks nothing like a rock. To understand small-scale phenomena, we make analogies with the large-scale phenomena we observe directly. When we say something behaves like a wave, we mean it shows interference effects analogous to those seen in overlapping water waves. (See [\[link\]](#).) Two examples of waves are sound and EM radiation. When we say something behaves like a particle, we mean that it interacts as a discrete unit with no interference effects. Examples of particles include electrons, atoms, and photons of EM radiation. How do we talk about a phenomenon that acts like both a particle and a wave?



(a) The interference pattern for light through a double slit is a wave property understood by analogy to water waves. (b) The properties of photons having quantized energy and momentum and acting as a concentrated unit are

understood by analogy to
macroscopic particles.

There is no doubt that EM radiation interferes and has the properties of wavelength and frequency. There is also no doubt that it behaves as particles—photons with discrete energy. We call this twofold nature the **particle-wave duality**, meaning that EM radiation has both particle and wave properties. This so-called duality is simply a term for properties of the photon analogous to phenomena we can observe directly, on a macroscopic scale. If this term seems strange, it is because we do not ordinarily observe details on the quantum level directly, and our observations yield either particle *or* wavelike properties, but never both simultaneously.

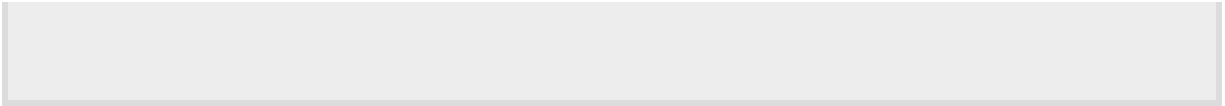
Since we have a particle-wave duality for photons, and since we have seen connections between photons and matter in that both have momentum, it is reasonable to ask whether there is a particle-wave duality for matter as well. If the EM radiation we once thought to be a pure wave has particle properties, is it possible that matter has wave properties? The answer is yes. The consequences are tremendous, as we will begin to see in the next section.

Note:

PhET Explorations: Quantum Wave Interference

When do photons, electrons, and atoms behave like particles and when do they behave like waves? Watch waves spread out and interfere as they pass through a double slit, then get detected on a screen as tiny dots. Use quantum detectors to explore how measurements change the waves and the patterns they produce on the screen.

[Quantum](#)
[Wave](#)
[Interferenc](#)
[e](#)



Section Summary

- EM radiation can behave like either a particle or a wave.
- This is termed particle-wave duality.

Glossary

particle-wave duality

the property of behaving like either a particle or a wave; the term for the phenomenon that all particles have wave characteristics

The Wave Nature of Matter

- Describe the Davisson-Germer experiment, and explain how it provides evidence for the wave nature of electrons.

De Broglie Wavelength

In 1923 a French physics graduate student named Prince Louis-Victor de Broglie (1892–1987) made a radical proposal based on the hope that nature is symmetric. If EM radiation has both particle and wave properties, then nature would be symmetric if matter also had both particle and wave properties. If what we once thought of as an unequivocal wave (EM radiation) is also a particle, then what we think of as an unequivocal particle (matter) may also be a wave. De Broglie's suggestion, made as part of his doctoral thesis, was so radical that it was greeted with some skepticism. A copy of his thesis was sent to Einstein, who said it was not only probably correct, but that it might be of fundamental importance. With the support of Einstein and a few other prominent physicists, de Broglie was awarded his doctorate.

De Broglie took both relativity and quantum mechanics into account to develop the proposal that *all particles have a wavelength*, given by

Equation:

$$\lambda = \frac{h}{p} \text{ (matter and photons),}$$

where h is Planck's constant and p is momentum. This is defined to be the **de Broglie wavelength**. (Note that we already have this for photons, from the equation $p = h/\lambda$.) The hallmark of a wave is interference. If matter is a wave, then it must exhibit constructive and destructive interference. Why isn't this ordinarily observed? The answer is that in order to see significant interference effects, a wave must interact with an object about the same size as its wavelength. Since h is very small, λ is also small, especially for macroscopic objects. A 3-kg bowling ball moving at 10 m/s, for example, has

Equation:

$$\lambda = h/p = (6.63 \times 10^{-34} \text{ J}\cdot\text{s})/[(3 \text{ kg})(10 \text{ m/s})] = 2 \times 10^{-35} \text{ m}.$$

This means that to see its wave characteristics, the bowling ball would have to interact with something about 10^{-35} m in size—far smaller than anything known. When waves interact with objects much larger than their wavelength, they show negligible interference effects and move in straight lines (such as light rays in geometric optics). To get easily observed interference effects from particles of matter, the longest wavelength and hence smallest mass possible would be useful. Therefore, this effect was first observed with electrons.

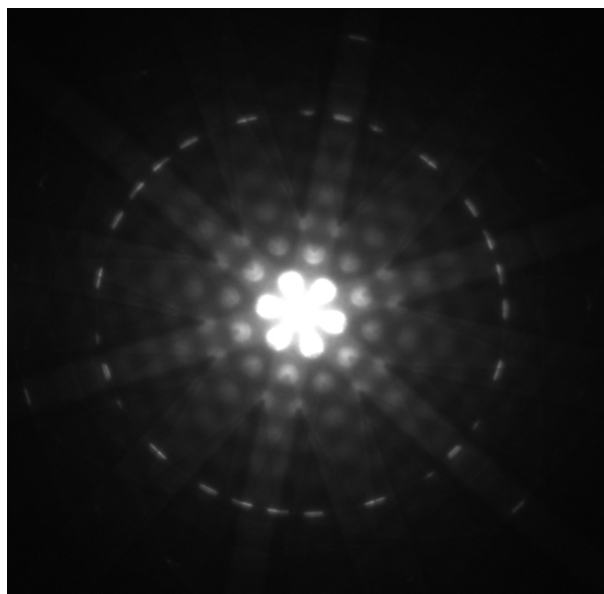
American physicists Clinton J. Davisson and Lester H. Germer in 1925 and, independently, British physicist G. P. Thomson (son of J. J. Thomson, discoverer of the electron) in 1926 scattered electrons from crystals and found diffraction patterns. These patterns are exactly consistent with interference of electrons having the de Broglie wavelength and are somewhat analogous to light interacting with a diffraction grating. (See [\[link\]](#).)

Note:**Connections: Waves**

All microscopic particles, whether massless, like photons, or having mass, like electrons, have wave properties. The relationship between momentum and wavelength is fundamental for all particles.

De Broglie's proposal of a wave nature for all particles initiated a remarkably productive era in which the foundations for quantum mechanics were laid. In 1926, the Austrian physicist Erwin Schrödinger (1887–1961) published four papers in which the wave nature of particles was treated explicitly with wave equations. At the same time, many others began important work. Among them was German physicist Werner Heisenberg

(1901–1976) who, among many other contributions to quantum mechanics, formulated a mathematical treatment of the wave nature of matter that used matrices rather than wave equations. We will deal with some specifics in later sections, but it is worth noting that de Broglie's work was a watershed for the development of quantum mechanics. De Broglie was awarded the Nobel Prize in 1929 for his vision, as were Davisson and G. P. Thomson in 1937 for their experimental verification of de Broglie's hypothesis.



This diffraction pattern was obtained for electrons diffracted by crystalline silicon. Bright regions are those of constructive interference, while dark regions are those of destructive interference. (credit: Ndtthe, Wikimedia Commons)

Example:

Electron Wavelength versus Velocity and Energy

For an electron having a de Broglie wavelength of 0.167 nm (appropriate for interacting with crystal lattice structures that are about this size): (a)

Calculate the electron's velocity, assuming it is nonrelativistic. (b)

Calculate the electron's kinetic energy in eV.

Strategy

For part (a), since the de Broglie wavelength is given, the electron's velocity can be obtained from $\lambda = h/p$ by using the nonrelativistic formula for momentum, $p = mv$. For part (b), once v is obtained (and it has been verified that v is nonrelativistic), the classical kinetic energy is simply $(1/2)mv^2$.

Solution for (a)

Substituting the nonrelativistic formula for momentum ($p = mv$) into the de Broglie wavelength gives

Equation:

$$\lambda = \frac{h}{p} = \frac{h}{mv}.$$

Solving for v gives

Equation:

$$v = \frac{h}{m\lambda}.$$

Substituting known values yields

Equation:

$$v = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(0.167 \times 10^{-9} \text{ m})} = 4.36 \times 10^6 \text{ m/s}.$$

Solution for (b)

While fast compared with a car, this electron's speed is not highly relativistic, and so we can comfortably use the classical formula to find the electron's kinetic energy and convert it to eV as requested.

Equation:

$$\begin{aligned}
 \text{KE} &= \frac{1}{2}mv^2 \\
 &= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(4.36 \times 10^6 \text{ m/s})^2 \\
 &= (86.4 \times 10^{-18} \text{ J}) \left(\frac{1 \text{ eV}}{1.602 \times 10^{-19} \text{ J}} \right) \\
 &= 54.0 \text{ eV}
 \end{aligned}$$

Discussion

This low energy means that these 0.167-nm electrons could be obtained by accelerating them through a 54.0-V electrostatic potential, an easy task. The results also confirm the assumption that the electrons are nonrelativistic, since their velocity is just over 1% of the speed of light and the kinetic energy is about 0.01% of the rest energy of an electron (0.511 MeV). If the electrons had turned out to be relativistic, we would have had to use more involved calculations employing relativistic formulas.

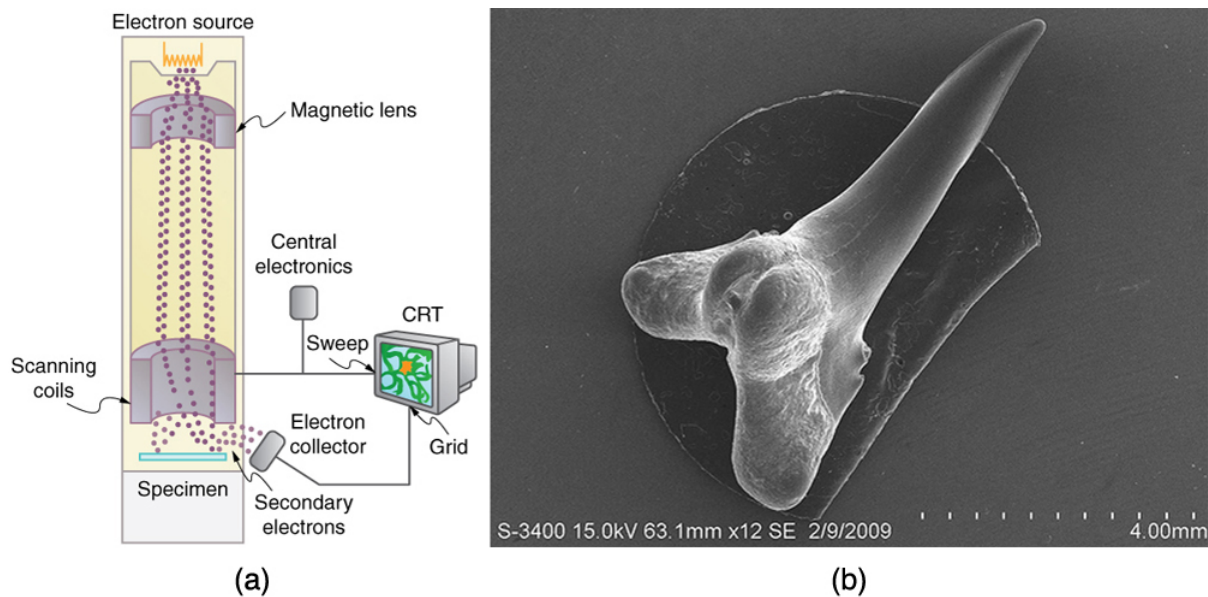
Electron Microscopes

One consequence or use of the wave nature of matter is found in the electron microscope. As we have discussed, there is a limit to the detail observed with any probe having a wavelength. Resolution, or observable detail, is limited to about one wavelength. Since a potential of only 54 V can produce electrons with sub-nanometer wavelengths, it is easy to get electrons with much smaller wavelengths than those of visible light (hundreds of nanometers). Electron microscopes can, thus, be constructed to detect much smaller details than optical microscopes. (See [\[link\]](#).)

There are basically two types of electron microscopes. The transmission electron microscope (TEM) accelerates electrons that are emitted from a hot filament (the cathode). The beam is broadened and then passes through the sample. A magnetic lens focuses the beam image onto a fluorescent screen, a photographic plate, or (most probably) a CCD (light sensitive camera), from which it is transferred to a computer. The TEM is similar to the optical microscope, but it requires a thin sample examined in a vacuum. However it can resolve details as small as 0.1 nm (10^{-10} m), providing magnifications

of 100 million times the size of the original object. The TEM has allowed us to see individual atoms and structure of cell nuclei.

The scanning electron microscope (SEM) provides images by using secondary electrons produced by the primary beam interacting with the surface of the sample (see [\[link\]](#)). The SEM also uses magnetic lenses to focus the beam onto the sample. However, it moves the beam around electrically to “scan” the sample in the x and y directions. A CCD detector is used to process the data for each electron position, producing images like the one at the beginning of this chapter. The SEM has the advantage of not requiring a thin sample and of providing a 3-D view. However, its resolution is about ten times less than a TEM.



Schematic of a scanning electron microscope (SEM) (a) used to observe small details, such as those seen in this image of a tooth of a *Himipristis*, a type of shark (b). (credit: Dallas Krentzel, Flickr)

Electrons were the first particles with mass to be directly confirmed to have the wavelength proposed by de Broglie. Subsequently, protons, helium nuclei, neutrons, and many others have been observed to exhibit

interference when they interact with objects having sizes similar to their de Broglie wavelength. The de Broglie wavelength for massless particles was well established in the 1920s for photons, and it has since been observed that all massless particles have a de Broglie wavelength $\lambda = h/p$. The wave nature of all particles is a universal characteristic of nature. We shall see in following sections that implications of the de Broglie wavelength include the quantization of energy in atoms and molecules, and an alteration of our basic view of nature on the microscopic scale. The next section, for example, shows that there are limits to the precision with which we may make predictions, regardless of how hard we try. There are even limits to the precision with which we may measure an object's location or energy.

Note:

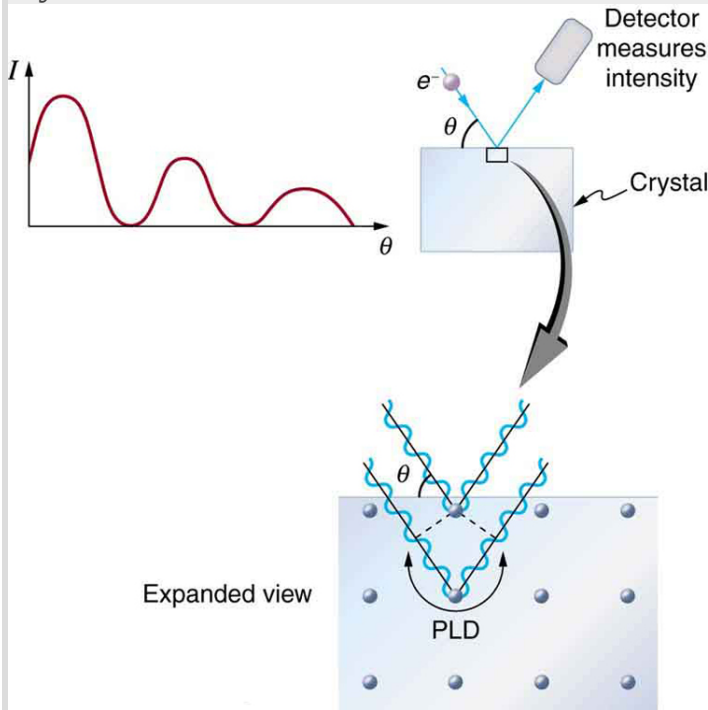
Making Connections: A Submicroscopic Diffraction Grating

The wave nature of matter allows it to exhibit all the characteristics of other, more familiar, waves. Diffraction gratings, for example, produce diffraction patterns for light that depend on grating spacing and the wavelength of the light. This effect, as with most wave phenomena, is most pronounced when the wave interacts with objects having a size similar to its wavelength. For gratings, this is the spacing between multiple slits.)

When electrons interact with a system having a spacing similar to the electron wavelength, they show the same types of interference patterns as light does for diffraction gratings, as shown at top left in [\[link\]](#).

Atoms are spaced at regular intervals in a crystal as parallel planes, as shown in the bottom part of [\[link\]](#). The spacings between these planes act like the openings in a diffraction grating. At certain incident angles, the paths of electrons scattering from successive planes differ by one wavelength and, thus, interfere constructively. At other angles, the path length differences are not an integral wavelength, and there is partial to total destructive interference. This type of scattering from a large crystal with well-defined lattice planes can produce dramatic interference patterns. It is called *Bragg reflection*, for the father-and-son team who first explored and analyzed it in some detail. The expanded view also shows the path-length differences and indicates how these depend on incident angle θ in a

manner similar to the diffraction patterns for x rays reflecting from a crystal.



The diffraction pattern at top left is produced by scattering electrons from a crystal and is graphed as a function of incident angle relative to the regular array of atoms in a crystal, as shown at bottom. Electrons scattering from the second layer of atoms travel farther than those scattered from the top layer. If the path length difference (PLD) is an integral wavelength, there is constructive interference.

Let us take the spacing between parallel planes of atoms in the crystal to be d . As mentioned, if the path length difference (PLD) for the electrons is a whole number of wavelengths, there will be constructive interference—that is, $\text{PLD} = n\lambda (n = 1, 2, 3, \dots)$. Because $AB = BC = d \sin \theta$, we have constructive interference when $n\lambda = 2d \sin \theta$. This relationship is

called the *Bragg equation* and applies not only to electrons but also to x rays.

The wavelength of matter is a submicroscopic characteristic that explains a macroscopic phenomenon such as Bragg reflection. Similarly, the wavelength of light is a submicroscopic characteristic that explains the macroscopic phenomenon of diffraction patterns.

Section Summary

- Particles of matter also have a wavelength, called the de Broglie wavelength, given by $\lambda = \frac{h}{p}$, where p is momentum.
- Matter is found to have the same *interference characteristics* as any other wave.

Conceptual Questions

Exercise:

Problem:

How does the interference of water waves differ from the interference of electrons? How are they analogous?

Exercise:

Problem: Describe one type of evidence for the wave nature of matter.

Exercise:

Problem:

Describe one type of evidence for the particle nature of EM radiation.

Problems & Exercises

Exercise:

Problem:

At what velocity will an electron have a wavelength of 1.00 m?

Solution:

$$7.28 \times 10^{-4} \text{ m}$$

Exercise:**Problem:**

What is the wavelength of an electron moving at 3.00% of the speed of light?

Exercise:**Problem:**

At what velocity does a proton have a 6.00-fm wavelength (about the size of a nucleus)? Assume the proton is nonrelativistic. (1 femtometer = 10^{-15} m.)

Solution:

$$6.62 \times 10^7 \text{ m/s}$$

Exercise:**Problem:**

What is the velocity of a 0.400-kg billiard ball if its wavelength is 7.50 cm (large enough for it to interfere with other billiard balls)?

Exercise:**Problem:**

Find the wavelength of a proton moving at 1.00% of the speed of light.

Solution:

$$1.32 \times 10^{-13} \text{ m}$$

Exercise:**Problem:**

Experiments are performed with ultracold neutrons having velocities as small as 1.00 m/s. (a) What is the wavelength of such a neutron? (b) What is its kinetic energy in eV?

Exercise:**Problem:**

(a) Find the velocity of a neutron that has a 6.00-fm wavelength (about the size of a nucleus). Assume the neutron is nonrelativistic. (b) What is the neutron's kinetic energy in MeV?

Solution:

(a) $6.62 \times 10^7 \text{ m/s}$

(b) 22.9 MeV

Exercise:**Problem:**

What is the wavelength of an electron accelerated through a 30.0-kV potential, as in a TV tube?

Exercise:**Problem:**

What is the kinetic energy of an electron in a TEM having a 0.0100-nm wavelength?

Solution:

Equation: 15.1 keV

Exercise:

Problem:

(a) Calculate the velocity of an electron that has a wavelength of $1.00\text{ }\mu\text{m}$. (b) Through what voltage must the electron be accelerated to have this velocity?

Exercise:**Problem:**

The velocity of a proton emerging from a Van de Graaff accelerator is 25.0% of the speed of light. (a) What is the proton's wavelength? (b) What is its kinetic energy, assuming it is nonrelativistic? (c) What was the equivalent voltage through which it was accelerated?

Solution:

(a) 5.29 fm

(b) $4.70 \times 10^{-12}\text{ J}$

(c) 29.4 MV

Exercise:**Problem:**

The kinetic energy of an electron accelerated in an x-ray tube is 100 keV . Assuming it is nonrelativistic, what is its wavelength?

Exercise:**Problem: Unreasonable Results**

(a) Assuming it is nonrelativistic, calculate the velocity of an electron with a 0.100-fm wavelength (small enough to detect details of a nucleus). (b) What is unreasonable about this result? (c) Which assumptions are unreasonable or inconsistent?

Solution:

(a) $7.28 \times 10^{12} \text{ m/s}$

(b) This is thousands of times the speed of light (an impossibility).

(c) The assumption that the electron is non-relativistic is unreasonable at this wavelength.

Glossary

de Broglie wavelength

the wavelength possessed by a particle of matter, calculated by

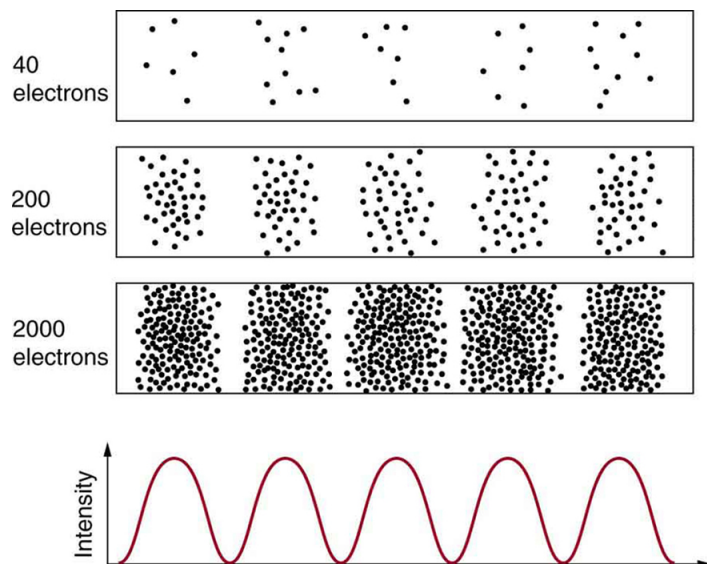
$$\lambda = h/p$$

Probability: The Heisenberg Uncertainty Principle

- Use both versions of Heisenberg's uncertainty principle in calculations.
- Explain the implications of Heisenberg's uncertainty principle for measurements.

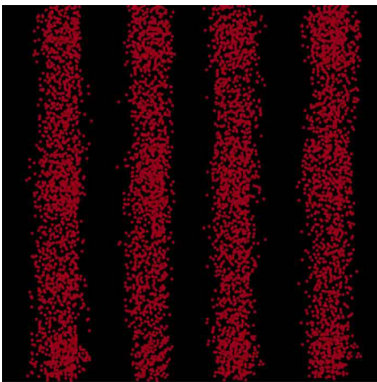
Probability Distribution

Matter and photons are waves, implying they are spread out over some distance. What is the position of a particle, such as an electron? Is it at the center of the wave? The answer lies in how you measure the position of an electron. Experiments show that you will find the electron at some definite location, unlike a wave. But if you set up exactly the same situation and measure it again, you will find the electron in a different location, often far outside any experimental uncertainty in your measurement. Repeated measurements will display a statistical distribution of locations that appears wavelike. (See [\[link\]](#).)

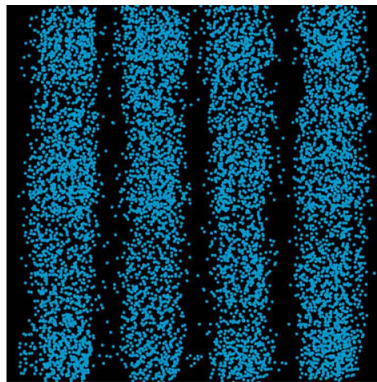


The building up of the diffraction pattern of electrons scattered from a crystal surface. Each electron arrives

at a definite location, which cannot be precisely predicted. The overall distribution shown at the bottom can be predicted as the diffraction of waves having the de Broglie wavelength of the electrons.



(a) Electrons



(b) Protons

Double-slit interference for electrons (a) and protons (b) is identical for equal wavelengths and equal slit separations. Both patterns are probability distributions in the sense that they are built up by individual particles traversing the apparatus, the paths of which are not individually predictable.

After de Broglie proposed the wave nature of matter, many physicists, including Schrödinger and Heisenberg, explored the consequences. The idea quickly emerged that, *because of its wave character, a particle's trajectory and destination cannot be precisely predicted for each particle individually*. However, each particle goes to a definite place (as illustrated in [\[link\]](#)). After compiling enough data, you get a distribution related to the

particle's wavelength and diffraction pattern. There is a certain *probability* of finding the particle at a given location, and the overall pattern is called a **probability distribution**. Those who developed quantum mechanics devised equations that predicted the probability distribution in various circumstances.

It is somewhat disquieting to think that you cannot predict exactly where an individual particle will go, or even follow it to its destination. Let us explore what happens if we try to follow a particle. Consider the double-slit patterns obtained for electrons and photons in [\[link\]](#). First, we note that these patterns are identical, following $d \sin \theta = m\lambda$, the equation for double-slit constructive interference developed in [Photon Energies and the Electromagnetic Spectrum](#), where d is the slit separation and λ is the electron or photon wavelength.

Both patterns build up statistically as individual particles fall on the detector. This can be observed for photons or electrons—for now, let us concentrate on electrons. You might imagine that the electrons are interfering with one another as any waves do. To test this, you can lower the intensity until there is never more than one electron between the slits and the screen. The same interference pattern builds up! This implies that a particle's probability distribution spans both slits, and the particles actually interfere with themselves. Does this also mean that the electron goes through both slits? An electron is a basic unit of matter that is not divisible. But it is a fair question, and so we should look to see if the electron traverses one slit or the other, or both. One possibility is to have coils around the slits that detect charges moving through them. What is observed is that an electron always goes through one slit or the other; it does not split to go through both. But there is a catch. If you determine that the electron went through one of the slits, you no longer get a double slit pattern—instead, you get single slit interference. There is no escape by using another method of determining which slit the electron went through. Knowing the particle went through one slit forces a single-slit pattern. If you do not observe which slit the electron goes through, you obtain a double-slit pattern.

Heisenberg Uncertainty

How does knowing which slit the electron passed through change the pattern? The answer is fundamentally important—*measurement affects the system being observed*. Information can be lost, and in some cases it is impossible to measure two physical quantities simultaneously to exact precision. For example, you can measure the position of a moving electron by scattering light or other electrons from it. Those probes have momentum themselves, and by scattering from the electron, they change its momentum *in a manner that loses information*. There is a limit to absolute knowledge, even in principle.



Werner Heisenberg was one of the best of those physicists who developed early quantum mechanics. Not only did his work enable a description of nature on the very small scale, it also changed our

view of the
availability of
knowledge.
Although he is
universally
recognized for his
brilliance and the
importance of his
work (he received
the Nobel Prize in
1932, for example),
Heisenberg
remained in
Germany during
World War II and
headed the German
effort to build a
nuclear bomb,
permanently
alienating himself
from most of the
scientific
community. (credit:
Author Unknown,
via Wikimedia
Commons)

It was Werner Heisenberg who first stated this limit to knowledge in 1929 as a result of his work on quantum mechanics and the wave characteristics of all particles. (See [\[link\]](#)). Specifically, consider simultaneously measuring the position and momentum of an electron (it could be any particle). There is an **uncertainty in position** Δx that is approximately equal to the wavelength of the particle. That is,
Equation:

$$\Delta x \approx \lambda.$$

As discussed above, a wave is not located at one point in space. If the electron's position is measured repeatedly, a spread in locations will be observed, implying an uncertainty in position Δx . To detect the position of the particle, we must interact with it, such as having it collide with a detector. In the collision, the particle will lose momentum. This change in momentum could be anywhere from close to zero to the total momentum of the particle, $p = h/\lambda$. It is not possible to tell how much momentum will be transferred to a detector, and so there is an **uncertainty in momentum** Δp , too. In fact, the uncertainty in momentum may be as large as the momentum itself, which in equation form means that

Equation:

$$\Delta p \approx \frac{h}{\lambda}.$$

The uncertainty in position can be reduced by using a shorter-wavelength electron, since $\Delta x \approx \lambda$. But shortening the wavelength increases the uncertainty in momentum, since $\Delta p \approx h/\lambda$. Conversely, the uncertainty in momentum can be reduced by using a longer-wavelength electron, but this increases the uncertainty in position. Mathematically, you can express this trade-off by multiplying the uncertainties. The wavelength cancels, leaving

Equation:

$$\Delta x \Delta p \approx h.$$

So if one uncertainty is reduced, the other must increase so that their product is $\approx h$.

With the use of advanced mathematics, Heisenberg showed that the best that can be done in a *simultaneous measurement of position and momentum* is

Equation:

$$\Delta x \Delta p \geq \frac{h}{4\pi}.$$

This is known as the **Heisenberg uncertainty principle**. It is impossible to measure position x and momentum p simultaneously with uncertainties Δx and Δp that multiply to be less than $h/4\pi$. Neither uncertainty can be zero. Neither uncertainty can become small without the other becoming large. A small wavelength allows accurate position measurement, but it increases the momentum of the probe to the point that it further disturbs the momentum of a system being measured. For example, if an electron is scattered from an atom and has a wavelength small enough to detect the position of electrons in the atom, its momentum can knock the electrons from their orbits in a manner that loses information about their original motion. It is therefore impossible to follow an electron in its orbit around an atom. If you measure the electron's position, you will find it in a definite location, but the atom will be disrupted. Repeated measurements on identical atoms will produce interesting probability distributions for electrons around the atom, but they will not produce motion information. The probability distributions are referred to as electron clouds or orbitals. The shapes of these orbitals are often shown in general chemistry texts and are discussed in [The Wave Nature of Matter Causes Quantization](#).

Example:

Heisenberg Uncertainty Principle in Position and Momentum for an Atom

(a) If the position of an electron in an atom is measured to an accuracy of 0.0100 nm, what is the electron's uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV?

Strategy

The uncertainty in position is the accuracy of the measurement, or $\Delta x = 0.0100$ nm. Thus the smallest uncertainty in momentum Δp can be calculated using $\Delta x \Delta p \geq h/4\pi$. Once the uncertainty in momentum Δp is found, the uncertainty in velocity can be found from $\Delta p = m\Delta v$.

Solution for (a)

Using the equals sign in the uncertainty principle to express the minimum uncertainty, we have

Equation:

$$\Delta x \Delta p = \frac{h}{4\pi}.$$

Solving for Δp and substituting known values gives

Equation:

$$\Delta p = \frac{h}{4\pi\Delta x} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi(1.00 \times 10^{-11} \text{ m})} = 5.28 \times 10^{-24} \text{ kg} \cdot \text{m/s}.$$

Thus,

Equation:

$$\Delta p = 5.28 \times 10^{-24} \text{ kg} \cdot \text{m/s} = m\Delta v.$$

Solving for Δv and substituting the mass of an electron gives

Equation:

$$\Delta v = \frac{\Delta p}{m} = \frac{5.28 \times 10^{-24} \text{ kg} \cdot \text{m/s}}{9.11 \times 10^{-31} \text{ kg}} = 5.79 \times 10^6 \text{ m/s}.$$

Solution for (b)

Although large, this velocity is not highly relativistic, and so the electron's kinetic energy is

Equation:

$$\begin{aligned} \text{KE}_e &= \frac{1}{2}mv^2 \\ &= \frac{1}{2}(9.11 \times 10^{-31} \text{ kg})(5.79 \times 10^6 \text{ m/s})^2 \\ &= (1.53 \times 10^{-17} \text{ J})\left(\frac{1 \text{ eV}}{1.60 \times 10^{-19} \text{ J}}\right) = 95.5 \text{ eV}. \end{aligned}$$

Discussion

Since atoms are roughly 0.1 nm in size, knowing the position of an electron to 0.0100 nm localizes it reasonably well inside the atom. This

would be like being able to see details one-tenth the size of the atom. But the consequent uncertainty in velocity is large. You certainly could not follow it very well if its velocity is so uncertain. To get a further idea of how large the uncertainty in velocity is, we assumed the velocity of the electron was equal to its uncertainty and found this gave a kinetic energy of 95.5 eV. This is significantly greater than the typical energy difference between levels in atoms (see [\[link\]](#)), so that it is impossible to get a meaningful energy for the electron if we know its position even moderately well.

Why don't we notice Heisenberg's uncertainty principle in everyday life? The answer is that Planck's constant is very small. Thus the lower limit in the uncertainty of measuring the position and momentum of large objects is negligible. We can detect sunlight reflected from Jupiter and follow the planet in its orbit around the Sun. The reflected sunlight alters the momentum of Jupiter and creates an uncertainty in its momentum, but this is totally negligible compared with Jupiter's huge momentum. The correspondence principle tells us that the predictions of quantum mechanics become indistinguishable from classical physics for large objects, which is the case here.

Heisenberg Uncertainty for Energy and Time

There is another form of **Heisenberg's uncertainty principle** for *simultaneous measurements of energy and time*. In equation form,

Equation:

$$\Delta E \Delta t \geq \frac{h}{4\pi},$$

where ΔE is the **uncertainty in energy** and Δt is the **uncertainty in time**. This means that within a time interval Δt , it is not possible to measure energy precisely—there will be an uncertainty ΔE in the measurement. In order to measure energy more precisely (to make ΔE smaller), we must

increase Δt . This time interval may be the amount of time we take to make the measurement, or it could be the amount of time a particular state exists, as in the next [\[link\]](#).

Example:**Heisenberg Uncertainty Principle for Energy and Time for an Atom**

An atom in an excited state temporarily stores energy. If the lifetime of this excited state is measured to be 1.0×10^{-10} s, what is the minimum uncertainty in the energy of the state in eV?

Strategy

The minimum uncertainty in energy ΔE is found by using the equals sign in $\Delta E \Delta t \geq h/4\pi$ and corresponds to a reasonable choice for the uncertainty in time. The largest the uncertainty in time can be is the full lifetime of the excited state, or $\Delta t = 1.0 \times 10^{-10}$ s.

Solution

Solving the uncertainty principle for ΔE and substituting known values gives

Equation:

$$\Delta E = \frac{h}{4\pi\Delta t} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{4\pi(1.0 \times 10^{-10} \text{ s})} = 5.3 \times 10^{-25} \text{ J}.$$

Now converting to eV yields

Equation:

$$\Delta E = (5.3 \times 10^{-25} \text{ J}) \left(\frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) = 3.3 \times 10^{-6} \text{ eV}.$$

Discussion

The lifetime of 10^{-10} s is typical of excited states in atoms—on human time scales, they quickly emit their stored energy. An uncertainty in energy of only a few millionths of an eV results. This uncertainty is small compared with typical excitation energies in atoms, which are on the order of 1 eV. So here the uncertainty principle limits the accuracy with which

we can measure the lifetime and energy of such states, but not very significantly.

The uncertainty principle for energy and time can be of great significance if the lifetime of a system is very short. Then Δt is very small, and ΔE is consequently very large. Some nuclei and exotic particles have extremely short lifetimes (as small as 10^{-25} s), causing uncertainties in energy as great as many GeV (10^9 eV). Stored energy appears as increased rest mass, and so this means that there is significant uncertainty in the rest mass of short-lived particles. When measured repeatedly, a spread of masses or decay energies are obtained. The spread is ΔE . You might ask whether this uncertainty in energy could be avoided by not measuring the lifetime. The answer is no. Nature knows the lifetime, and so its brevity affects the energy of the particle. This is so well established experimentally that the uncertainty in decay energy is used to calculate the lifetime of short-lived states. Some nuclei and particles are so short-lived that it is difficult to measure their lifetime. But if their decay energy can be measured, its spread is ΔE , and this is used in the uncertainty principle ($\Delta E \Delta t \geq h/4\pi$) to calculate the lifetime Δt .

There is another consequence of the uncertainty principle for energy and time. If energy is uncertain by ΔE , then conservation of energy can be violated by ΔE for a time Δt . Neither the physicist nor nature can tell that conservation of energy has been violated, if the violation is temporary and smaller than the uncertainty in energy. While this sounds innocuous enough, we shall see in later chapters that it allows the temporary creation of matter from nothing and has implications for how nature transmits forces over very small distances.

Finally, note that in the discussion of particles and waves, we have stated that individual measurements produce precise or particle-like results. A definite position is determined each time we observe an electron, for example. But repeated measurements produce a spread in values consistent with wave characteristics. The great theoretical physicist Richard Feynman (1918–1988) commented, “What there are, are particles.” When you

observe enough of them, they distribute themselves as you would expect for a wave phenomenon. However, what there are as they travel we cannot tell because, when we do try to measure, we affect the traveling.

Section Summary

- Matter is found to have the same interference characteristics as any other wave.
- There is now a probability distribution for the location of a particle rather than a definite position.
- Another consequence of the wave character of all particles is the Heisenberg uncertainty principle, which limits the precision with which certain physical quantities can be known simultaneously. For position and momentum, the uncertainty principle is $\Delta x \Delta p \geq \frac{h}{4\pi}$, where Δx is the uncertainty in position and Δp is the uncertainty in momentum.
- For energy and time, the uncertainty principle is $\Delta E \Delta t \geq \frac{h}{4\pi}$ where ΔE is the uncertainty in energy and Δt is the uncertainty in time.
- These small limits are fundamentally important on the quantum-mechanical scale.

Conceptual Questions

Exercise:

Problem:

What is the Heisenberg uncertainty principle? Does it place limits on what can be known?

Problems & Exercises

Exercise:

Problem:

(a) If the position of an electron in a membrane is measured to an accuracy of $1.00\text{ }\mu\text{m}$, what is the electron's minimum uncertainty in velocity? (b) If the electron has this velocity, what is its kinetic energy in eV? (c) What are the implications of this energy, comparing it to typical molecular binding energies?

Solution:

(a) 57.9 m/s

(b) $9.55 \times 10^{-9}\text{ eV}$

(c) From [\[link\]](#), we see that typical molecular binding energies range from about 1 eV to 10 eV , therefore the result in part (b) is approximately 9 orders of magnitude smaller than typical molecular binding energies.

Exercise:**Problem:**

(a) If the position of a chlorine ion in a membrane is measured to an accuracy of $1.00\text{ }\mu\text{m}$, what is its minimum uncertainty in velocity, given its mass is $5.86 \times 10^{-26}\text{ kg}$? (b) If the ion has this velocity, what is its kinetic energy in eV, and how does this compare with typical molecular binding energies?

Exercise:**Problem:**

Suppose the velocity of an electron in an atom is known to an accuracy of $2.0 \times 10^3\text{ m/s}$ (reasonably accurate compared with orbital velocities). What is the electron's minimum uncertainty in position, and how does this compare with the approximate 0.1-nm size of the atom?

Solution:

29 nm,

290 times greater

Exercise:**Problem:**

The velocity of a proton in an accelerator is known to an accuracy of 0.250% of the speed of light. (This could be small compared with its velocity.) What is the smallest possible uncertainty in its position?

Exercise:**Problem:**

A relatively long-lived excited state of an atom has a lifetime of 3.00 ms. What is the minimum uncertainty in its energy?

Solution:

$$1.10 \times 10^{-13} \text{ eV}$$

Exercise:**Problem:**

(a) The lifetime of a highly unstable nucleus is 10^{-20} s. What is the smallest uncertainty in its decay energy? (b) Compare this with the rest energy of an electron.

Exercise:**Problem:**

The decay energy of a short-lived particle has an uncertainty of 1.0 MeV due to its short lifetime. What is the smallest lifetime it can have?

Solution:

$$3.3 \times 10^{-22} \text{ s}$$

Exercise:**Problem:**

The decay energy of a short-lived nuclear excited state has an uncertainty of 2.0 eV due to its short lifetime. What is the smallest lifetime it can have?

Exercise:**Problem:**

What is the approximate uncertainty in the mass of a muon, as determined from its decay lifetime?

Solution:

$$2.66 \times 10^{-46} \text{ kg}$$

Exercise:**Problem:**

Derive the approximate form of Heisenberg's uncertainty principle for energy and time, $\Delta E \Delta t \approx h$, using the following arguments: Since the position of a particle is uncertain by $\Delta x \approx \lambda$, where λ is the wavelength of the photon used to examine it, there is an uncertainty in the time the photon takes to traverse Δx . Furthermore, the photon has an energy related to its wavelength, and it can transfer some or all of this energy to the object being examined. Thus the uncertainty in the energy of the object is also related to λ . Find Δt and ΔE ; then multiply them to give the approximate uncertainty principle.

Glossary**Heisenberg's uncertainty principle**

a fundamental limit to the precision with which pairs of quantities (momentum and position, and energy and time) can be measured

uncertainty in energy

lack of precision or lack of knowledge of precise results in measurements of energy

uncertainty in time

lack of precision or lack of knowledge of precise results in measurements of time

uncertainty in momentum

lack of precision or lack of knowledge of precise results in measurements of momentum

uncertainty in position

lack of precision or lack of knowledge of precise results in measurements of position

probability distribution

the overall spatial distribution of probabilities to find a particle at a given location

The Particle-Wave Duality Reviewed

- Explain the concept of particle-wave duality, and its scope.

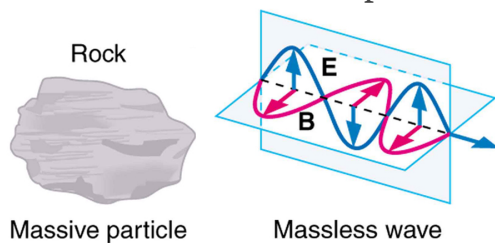
Particle-wave duality—the fact that all particles have wave properties—is one of the cornerstones of quantum mechanics. We first came across it in the treatment of photons, those particles of EM radiation that exhibit both particle and wave properties, but not at the same time. Later it was noted that particles of matter have wave properties as well. The dual properties of particles and waves are found for all particles, whether massless like photons, or having a mass like electrons. (See [\[link\]](#).)



On a quantum-mechanical scale (i.e., very small), particles with and without mass have wave properties. For example, both electrons and photons have wavelengths but also behave as particles.

There are many submicroscopic particles in nature. Most have mass and are expected to act as particles, or the smallest units of matter. All these masses have wave properties, with wavelengths given by the de Broglie relationship $\lambda = h/p$. So, too, do combinations of these particles, such as nuclei, atoms, and molecules. As a combination of masses becomes large, particularly if it is large enough to be called macroscopic, its wave nature becomes difficult to observe. This is consistent with our common experience with matter.

Some particles in nature are massless. We have only treated the photon so far, but all massless entities travel at the speed of light, have a wavelength, and exhibit particle and wave behaviors. They have momentum given by a rearrangement of the de Broglie relationship, $p = h/\lambda$. In large combinations of these massless particles (such large combinations are common only for photons or EM waves), there is mostly wave behavior upon detection, and the particle nature becomes difficult to observe. This is also consistent with experience. (See [\[link\]](#).)



On a classical scale (macroscopic), particles with mass behave as particles and not as waves. Particles without mass act as waves and not as particles.

The particle-wave duality is a universal attribute. It is another connection between matter and energy. Not only has modern physics been able to

describe nature for high speeds and small sizes, it has also discovered new connections and symmetries. There is greater unity and symmetry in nature than was known in the classical era—but they were dreamt of. A beautiful poem written by the English poet William Blake some two centuries ago contains the following four lines:

To see the World in a Grain of Sand

And a Heaven in a Wild Flower

Hold Infinity in the palm of your hand

And Eternity in an hour

Integrated Concepts

The problem set for this section involves concepts from this chapter and several others. Physics is most interesting when applied to general situations involving more than a narrow set of physical principles. For example, photons have momentum, hence the relevance of [Linear Momentum and Collisions](#). The following topics are involved in some or all of the problems in this section:

- [Dynamics: Newton's Laws of Motion](#)
- [Work, Energy, and Energy Resources](#)
- [Linear Momentum and Collisions](#)
- [Heat and Heat Transfer Methods](#)
- [Electric Potential and Electric Field](#)
- [Electric Current, Resistance, and Ohm's Law](#)
- [Wave Optics](#)
- [Special Relativity](#)

Note:

Problem-Solving Strategy

1. Identify which physical principles are involved.

2. Solve the problem using strategies outlined in the text.

[\[link\]](#) illustrates how these strategies are applied to an integrated-concept problem.

Example:

Recoil of a Dust Particle after Absorbing a Photon

The following topics are involved in this integrated concepts worked example:

Photons (quantum mechanics)

Linear Momentum

Topics

A 550-nm photon (visible light) is absorbed by a 1.00- μg particle of dust in outer space. (a) Find the momentum of such a photon. (b) What is the recoil velocity of the particle of dust, assuming it is initially at rest?

Strategy Step 1

To solve an *integrated-concept problem*, such as those following this example, we must first identify the physical principles involved and identify the chapters in which they are found. Part (a) of this example asks for the *momentum of a photon*, a topic of the present chapter. Part (b) considers *recoil following a collision*, a topic of [Linear Momentum and Collisions](#).

Strategy Step 2

The following solutions to each part of the example illustrate how specific problem-solving strategies are applied. These involve identifying knowns and unknowns, checking to see if the answer is reasonable, and so on.

Solution for (a)

The momentum of a photon is related to its wavelength by the equation:

Equation:

$$p = \frac{h}{\lambda}.$$

Entering the known value for Planck's constant h and given the wavelength λ , we obtain

Equation:

$$\begin{aligned} p &= \frac{6.63 \times 10^{-34} \text{ J}\cdot\text{s}}{550 \times 10^{-9} \text{ m}} \\ &= 1.21 \times 10^{-27} \text{ kg} \cdot \text{m/s}. \end{aligned}$$

Discussion for (a)

This momentum is small, as expected from discussions in the text and the fact that photons of visible light carry small amounts of energy and momentum compared with those carried by macroscopic objects.

Solution for (b)

Conservation of momentum in the absorption of this photon by a grain of dust can be analyzed using the equation:

Equation:

$$p_1 + p_2 = p'_1 + p'_2 (F_{\text{net}} = 0).$$

The net external force is zero, since the dust is in outer space. Let 1 represent the photon and 2 the dust particle. Before the collision, the dust is at rest (relative to some observer); after the collision, there is no photon (it is absorbed). So conservation of momentum can be written

Equation:

$$p_1 = p'_2 = mv,$$

where p_1 is the photon momentum before the collision and p'_2 is the dust momentum after the collision. The mass and recoil velocity of the dust are

m and v , respectively. Solving this for v , the requested quantity, yields

Equation:

$$v = \frac{p}{m},$$

where p is the photon momentum found in part (a). Entering known values (noting that a microgram is 10^{-9} kg) gives

Equation:

$$\begin{aligned} v &= \frac{1.21 \times 10^{-27} \text{ kg}\cdot\text{m/s}}{1.00 \times 10^{-9} \text{ kg}} \\ &= 1.21 \times 10^{-18} \text{ m/s.} \end{aligned}$$

Discussion

The recoil velocity of the particle of dust is extremely small. As we have noted, however, there are immense numbers of photons in sunlight and other macroscopic sources. In time, collisions and absorption of many photons could cause a significant recoil of the dust, as observed in comet tails.

Section Summary

- The particle-wave duality refers to the fact that all particles—those with mass and those without mass—have wave characteristics.
- This is a further connection between mass and energy.

Conceptual Questions

Exercise:

Problem:

In what ways are matter and energy related that were not known before the development of relativity and quantum mechanics?

Problems & Exercises

Exercise:

Problem: Integrated Concepts

The 54.0-eV electron in [\[link\]](#) has a 0.167-nm wavelength. If such electrons are passed through a double slit and have their first maximum at an angle of 25.0° , what is the slit separation d ?

Solution:

0.395 nm

Exercise:

Problem: Integrated Concepts

An electron microscope produces electrons with a 2.00-pm wavelength. If these are passed through a 1.00-nm single slit, at what angle will the first diffraction minimum be found?

Exercise:

Problem: Integrated Concepts

A certain heat lamp emits 200 W of mostly IR radiation averaging 1500 nm in wavelength. (a) What is the average photon energy in joules? (b) How many of these photons are required to increase the temperature of a person's shoulder by 2.0°C , assuming the affected mass is 4.0 kg with a specific heat of $0.83 \text{ kcal/kg}\cdot^\circ\text{C}$. Also assume no other significant heat transfer. (c) How long does this take?

Solution:

(a) $1.3 \times 10^{-19} \text{ J}$

(b) 2.1×10^{23}

(c) $1.4 \times 10^2 \text{ s}$

Exercise:

Problem: Integrated Concepts

On its high power setting, a microwave oven produces 900 W of 2560 MHz microwaves. (a) How many photons per second is this? (b) How many photons are required to increase the temperature of a 0.500-kg mass of pasta by 45.0°C , assuming a specific heat of $0.900 \text{ kcal/kg} \cdot ^\circ\text{C}$? Neglect all other heat transfer. (c) How long must the microwave operator wait for their pasta to be ready?

Exercise:

Problem: Integrated Concepts

(a) Calculate the amount of microwave energy in joules needed to raise the temperature of 1.00 kg of soup from 20.0°C to 100°C . (b) What is the total momentum of all the microwave photons it takes to do this? (c) Calculate the velocity of a 1.00-kg mass with the same momentum. (d) What is the kinetic energy of this mass?

Solution:

(a) $3.35 \times 10^5 \text{ J}$

(b) $1.12 \times 10^{-3} \text{ kg} \cdot \text{m/s}$

(c) $1.12 \times 10^{-3} \text{ m/s}$

(d) $6.23 \times 10^{-7} \text{ J}$

Exercise:

Problem: Integrated Concepts

- (a) What is γ for an electron emerging from the Stanford Linear Accelerator with a total energy of 50.0 GeV? (b) Find its momentum. (c) What is the electron's wavelength?

Exercise:

Problem: Integrated Concepts

- (a) What is γ for a proton having an energy of 1.00 TeV, produced by the Fermilab accelerator? (b) Find its momentum. (c) What is the proton's wavelength?

Solution:

- (a) 1.06×10^3
(b) $5.33 \times 10^{-16} \text{ kg} \cdot \text{m/s}$
(c) $1.24 \times 10^{-18} \text{ m}$

Exercise:

Problem: Integrated Concepts

An electron microscope passes 1.00-pm-wavelength electrons through a circular aperture 2.00 μm in diameter. What is the angle between two just-resolvable point sources for this microscope?

Exercise:

Problem: Integrated Concepts

- (a) Calculate the velocity of electrons that form the same pattern as 450-nm light when passed through a double slit. (b) Calculate the kinetic energy of each and compare them. (c) Would either be easier to generate than the other? Explain.

Solution:

(a) $1.62 \times 10^3 \text{ m/s}$

(b) $4.42 \times 10^{-19} \text{ J}$ for photon, $1.19 \times 10^{-24} \text{ J}$ for electron, photon energy is 3.71×10^5 times greater

(c) The light is easier to make because 450-nm light is blue light and therefore easy to make. Creating electrons with $7.43 \text{ } \mu\text{eV}$ of energy would not be difficult, but would require a vacuum.

Exercise:

Problem: Integrated Concepts

(a) What is the separation between double slits that produces a second-order minimum at 45.0° for 650-nm light? (b) What slit separation is needed to produce the same pattern for 1.00-keV protons.

Solution:

(a) $2.30 \times 10^{-6} \text{ m}$

(b) $3.20 \times 10^{-12} \text{ m}$

Exercise:

Problem: Integrated Concepts

A laser with a power output of 2.00 mW at a wavelength of 400 nm is projected onto calcium metal. (a) How many electrons per second are ejected? (b) What power is carried away by the electrons, given that the binding energy is 2.71 eV? (c) Calculate the current of ejected electrons. (d) If the photoelectric material is electrically insulated and acts like a 2.00-pF capacitor, how long will current flow before the capacitor voltage stops it?

Exercise:

Problem: Integrated Concepts

One problem with x rays is that they are not sensed. Calculate the temperature increase of a researcher exposed in a few seconds to a nearly fatal accidental dose of x rays under the following conditions. The energy of the x-ray photons is 200 keV, and 4.00×10^{13} of them are absorbed per kilogram of tissue, the specific heat of which is $0.830 \text{ kcal/kg} \cdot ^\circ\text{C}$. (Note that medical diagnostic x-ray machines *cannot* produce an intensity this great.)

Solution:

$$3.69 \times 10^{-4} \text{ }^\circ\text{C}$$

Exercise:

Problem: Integrated Concepts

A 1.00-fm photon has a wavelength short enough to detect some information about nuclei. (a) What is the photon momentum? (b) What is its energy in joules and MeV? (c) What is the (relativistic) velocity of an electron with the same momentum? (d) Calculate the electron's kinetic energy.

Exercise:

Problem: Integrated Concepts

The momentum of light is exactly reversed when reflected straight back from a mirror, assuming negligible recoil of the mirror. Thus the change in momentum is twice the photon momentum. Suppose light of intensity 1.00 kW/m^2 reflects from a mirror of area 2.00 m^2 . (a) Calculate the energy reflected in 1.00 s. (b) What is the momentum imparted to the mirror? (c) Using the most general form of Newton's second law, what is the force on the mirror? (d) Does the assumption of no mirror recoil seem reasonable?

Solution:

(a) 2.00 kJ

(b) $1.33 \times 10^{-5} \text{ kg} \cdot \text{m/s}$

(c) $1.33 \times 10^{-5} \text{ N}$

(d) yes

Exercise:

Problem: Integrated Concepts

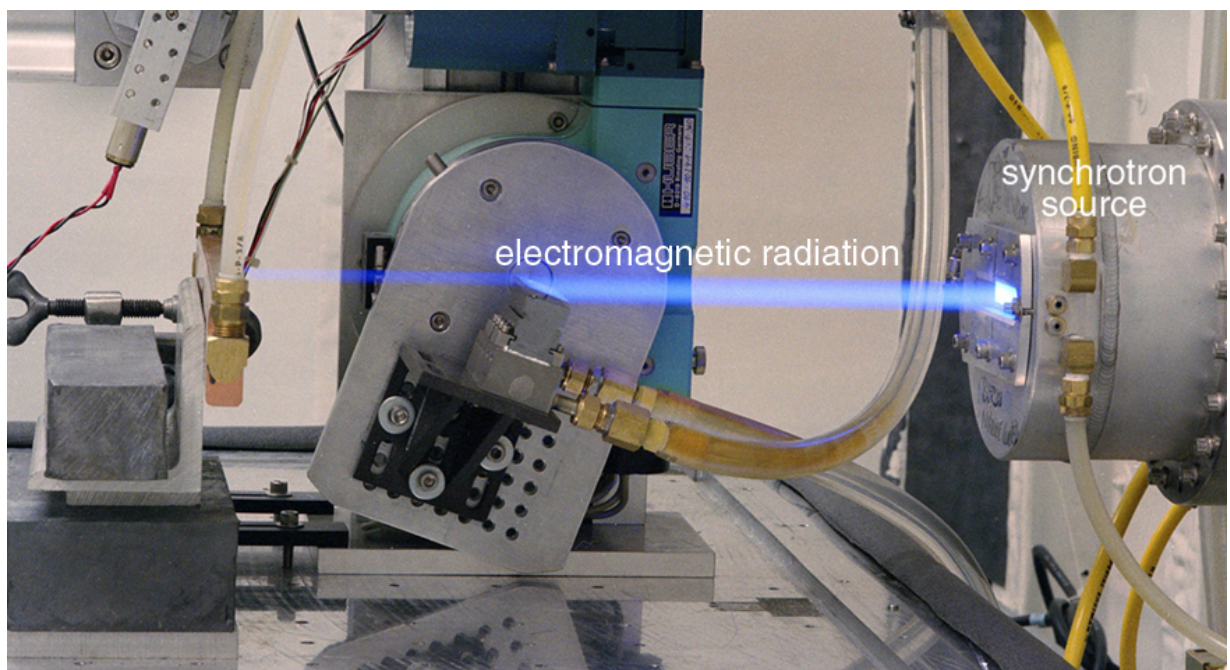
Sunlight above the Earth's atmosphere has an intensity of 1.30 kW/m^2 . If this is reflected straight back from a mirror that has only a small recoil, the light's momentum is exactly reversed, giving the mirror twice the incident momentum. (a) Calculate the force per square meter of mirror. (b) Very low mass mirrors can be constructed in the near weightlessness of space, and attached to a spaceship to sail it. Once done, the average mass per square meter of the spaceship is 0.100 kg . Find the acceleration of the spaceship if all other forces are balanced. (c) How fast is it moving 24 hours later?

Introduction to Radioactivity and Nuclear Physics

class="introduction"

- Define radioactivity.

The
synchrotron
source
produces
electromagnetic
radiation, as
evident from
the visible
glow. (credit:
United States
Department of
Energy, via
Wikimedia
Commons)

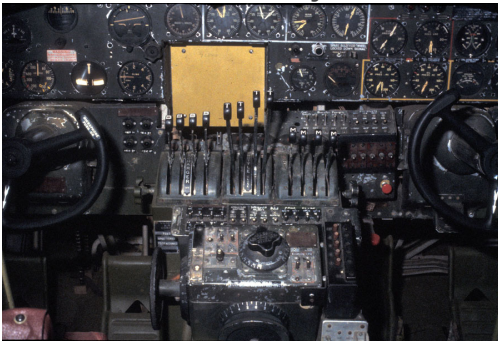


There is an ongoing quest to find substructures of matter. At one time, it was thought that atoms would be the ultimate substructure, but just when the first direct evidence of atoms was obtained, it became clear that they have a substructure and a tiny *nucleus*. The nucleus itself has spectacular characteristics. For example, certain nuclei are unstable, and their decay emits radiations with energies millions of times greater than atomic energies. Some of the mysteries of nature, such as why the core of the earth remains molten and how the sun produces its energy, are explained by nuclear phenomena. The exploration of *radioactivity* and the nucleus revealed fundamental and previously unknown particles, forces, and conservation laws. That exploration has evolved into a search for further underlying structures, such as quarks. In this chapter, the fundamentals of nuclear radioactivity and the nucleus are explored. The following two chapters explore the more important applications of nuclear physics in the field of medicine. We will also explore the basics of what we know about quarks and other substructures smaller than nuclei.

Nuclear Radioactivity

- Explain nuclear radiation.
- Explain the types of radiation—alpha emission, beta emission, and gamma emission.
- Explain the ionization of radiation in an atom.
- Define the range of radiation.

The discovery and study of nuclear radioactivity quickly revealed evidence of revolutionary new physics. In addition, uses for nuclear radiation also emerged quickly—for example, people such as Ernest Rutherford used it to determine the size of the nucleus and devices were painted with radon-doped paint to make them glow in the dark (see [\[link\]](#)). We therefore begin our study of nuclear physics with the discovery and basic features of nuclear radioactivity.



The dials of this World War II aircraft glow in the dark, because they are painted with radium-doped phosphorescent paint. It is a poignant reminder of the dual nature of radiation. Although radium paint dials are conveniently visible day and night, they emit radon, a radioactive gas that is hazardous and is not

directly sensed. (credit:
U.S. Air Force Photo)

Discovery of Nuclear Radioactivity

In 1896, the French physicist Antoine Henri Becquerel (1852–1908) accidentally found that a uranium-rich mineral called pitchblende emits invisible, penetrating rays that can darken a photographic plate enclosed in an opaque envelope. The rays therefore carry energy; but amazingly, the pitchblende emits them continuously without any energy input. This is an apparent violation of the law of conservation of energy, one that we now understand is due to the conversion of a small amount of mass into energy, as related in Einstein's famous equation $E = mc^2$. It was soon evident that Becquerel's rays originate in the nuclei of the atoms and have other unique characteristics. The emission of these rays is called **nuclear radioactivity** or simply **radioactivity**. The rays themselves are called **nuclear radiation**. A nucleus that spontaneously destroys part of its mass to emit radiation is said to **decay** (a term also used to describe the emission of radiation by atoms in excited states). A substance or object that emits nuclear radiation is said to be **radioactive**.

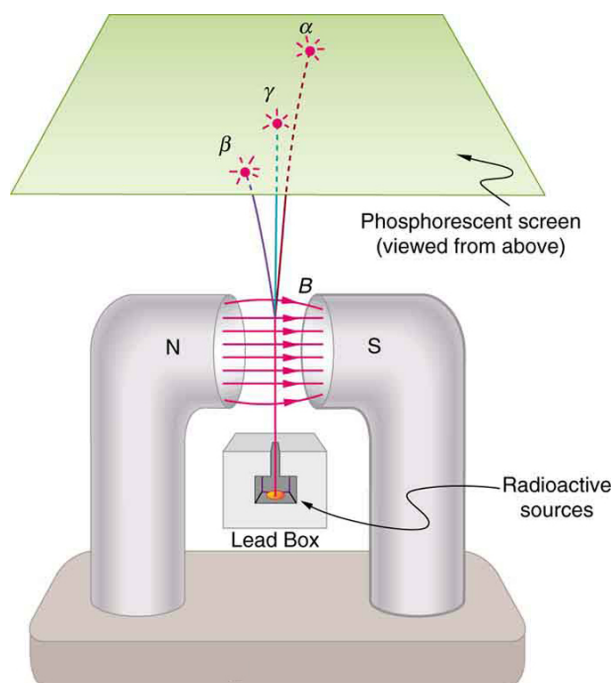
Two types of experimental evidence imply that Becquerel's rays originate deep in the heart (or nucleus) of an atom. First, the radiation is found to be associated with certain elements, such as uranium. Radiation does not vary with chemical state—that is, uranium is radioactive whether it is in the form of an element or compound. In addition, radiation does not vary with temperature, pressure, or ionization state of the uranium atom. Since all of these factors affect electrons in an atom, the radiation cannot come from electron transitions, as atomic spectra do. The huge energy emitted during each event is the second piece of evidence that the radiation cannot be atomic. Nuclear radiation has energies of the order of 10^6 eV per event, which is much greater than the typical atomic energies (a few eV), such as that observed in spectra and chemical reactions, and more than ten times as high as the most energetic characteristic x rays. Becquerel did not vigorously pursue his discovery for very long. In 1898, Marie Curie (1867–

1934), then a graduate student married the already well-known French physicist Pierre Curie (1859–1906), began her doctoral study of Becquerel's rays. She and her husband soon discovered two new radioactive elements, which she named *polonium* (after her native land) and *radium* (because it radiates). These two new elements filled holes in the periodic table and, further, displayed much higher levels of radioactivity per gram of material than uranium. Over a period of four years, working under poor conditions and spending their own funds, the Curies processed more than a ton of uranium ore to isolate a gram of radium salt. Radium became highly sought after, because it was about two million times as radioactive as uranium. Curie's radium salt glowed visibly from the radiation that took its toll on them and other unaware researchers. Shortly after completing her Ph.D., both Curies and Becquerel shared the 1903 Nobel Prize in physics for their work on radioactivity. Pierre was killed in a horse cart accident in 1906, but Marie continued her study of radioactivity for nearly 30 more years. Awarded the 1911 Nobel Prize in chemistry for her discovery of two new elements, she remains the only person to win Nobel Prizes in physics and chemistry. Marie's radioactive fingerprints on some pages of her notebooks can still expose film, and she suffered from radiation-induced lesions. She died of leukemia likely caused by radiation, but she was active in research almost until her death in 1934. The following year, her daughter and son-in-law, Irene and Frederic Joliot-Curie, were awarded the Nobel Prize in chemistry for their discovery of artificially induced radiation, adding to a remarkable family legacy.

Alpha, Beta, and Gamma

Research begun by people such as New Zealander Ernest Rutherford soon after the discovery of nuclear radiation indicated that different types of rays are emitted. Eventually, three types were distinguished and named **alpha** (α), **beta** (β), and **gamma** (γ), because, like x-rays, their identities were initially unknown. [\[link\]](#) shows what happens if the rays are passed through a magnetic field. The γ s are unaffected, while the α s and β s are deflected in opposite directions, indicating the α s are positive, the β s negative, and the γ s uncharged. Rutherford used both magnetic and electric fields to show that α s have a positive charge twice the magnitude of an electron, or $+2 |q_e|$. In the process, he found the α s charge to mass ratio to be several

thousand times smaller than the electron's. Later on, Rutherford collected α s from a radioactive source and passed an electric discharge through them, obtaining the spectrum of recently discovered helium gas. Among many important discoveries made by Rutherford and his collaborators was the proof that *α radiation is the emission of a helium nucleus*. Rutherford won the Nobel Prize in chemistry in 1908 for his early work. He continued to make important contributions until his death in 1934.



Alpha, beta, and gamma rays are passed through a magnetic field on the way to a phosphorescent screen. The α s and β s bend in opposite directions, while the γ s are unaffected, indicating a positive charge for α s, negative for β s, and neutral for γ s. Consistent results are obtained with electric fields. Collection of the radiation offers further

confirmation from the direct measurement of excess charge.

Other researchers had already proved that β s are negative and have the same mass and same charge-to-mass ratio as the recently discovered electron. By 1902, it was recognized that *β radiation is the emission of an electron*. Although β s are electrons, they do not exist in the nucleus before it decays and are not ejected atomic electrons—the electron is created in the nucleus at the instant of decay.

Since γ s remain unaffected by electric and magnetic fields, it is natural to think they might be photons. Evidence for this grew, but it was not until 1914 that this was proved by Rutherford and collaborators. By scattering γ radiation from a crystal and observing interference, they demonstrated that *γ radiation is the emission of a high-energy photon by a nucleus*. In fact, γ radiation comes from the de-excitation of a nucleus, just as an x ray comes from the de-excitation of an atom. The names " γ ray" and "x ray" identify the source of the radiation. At the same energy, γ rays and x rays are otherwise identical.

Type of Radiation	Range
α -Particles	A sheet of paper, a few cm of air, fractions of a mm of tissue

Type of Radiation	Range
β -Particles	A thin aluminum plate, or tens of cm of tissue
γ Rays	Several cm of lead or meters of concrete

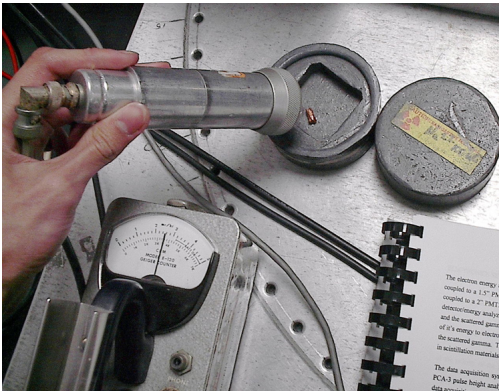
Properties of Nuclear Radiation

Ionization and Range

Two of the most important characteristics of α , β , and γ rays were recognized very early. All three types of nuclear radiation produce *ionization* in materials, but they penetrate different distances in materials—that is, they have different *ranges*. Let us examine why they have these characteristics and what are some of the consequences.

Like x rays, nuclear radiation in the form of α s, β s, and γ s has enough energy per event to ionize atoms and molecules in any material. The energy emitted in various nuclear decays ranges from a few keV to more than 10 MeV, while only a few eV are needed to produce ionization. The effects of x rays and nuclear radiation on biological tissues and other materials, such as solid state electronics, are directly related to the ionization they produce. All of them, for example, can damage electronics or kill cancer cells. In addition, methods for detecting x rays and nuclear radiation are based on ionization, directly or indirectly. All of them can ionize the air between the plates of a capacitor, for example, causing it to discharge. This is the basis of inexpensive personal radiation monitors, such as pictured in [\[link\]](#). Apart from α , β , and γ , there are other forms of nuclear radiation as well, and these also produce ionization with similar effects. We define **ionizing radiation** as any form of radiation that produces ionization

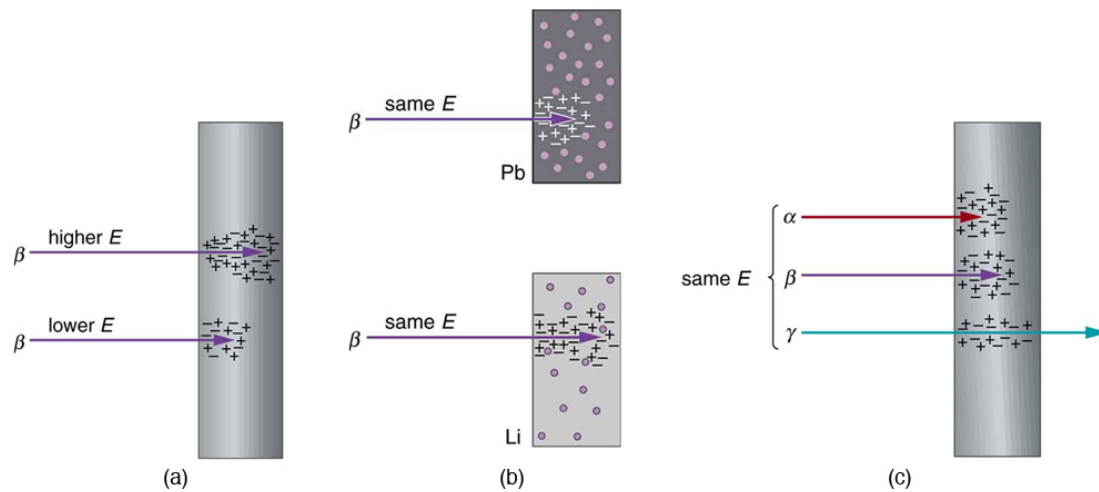
whether nuclear in origin or not, since the effects and detection of the radiation are related to ionization.



These dosimeters (literally, dose meters) are personal radiation monitors that detect the amount of radiation by the discharge of a rechargeable internal capacitor. The amount of discharge is related to the amount of ionizing radiation encountered, a measurement of dose. One dosimeter is shown in the charger. Its scale is read through an eyepiece on the top. (credit: L. Chang, Wikimedia Commons)

The **range of radiation** is defined to be the distance it can travel through a material. Range is related to several factors, including the energy of the

radiation, the material encountered, and the type of radiation (see [\[link\]](#)). The higher the *energy*, the greater the range, all other factors being the same. This makes good sense, since radiation loses its energy in materials primarily by producing ionization in them, and each ionization of an atom or a molecule requires energy that is removed from the radiation. The amount of ionization is, thus, directly proportional to the energy of the particle of radiation, as is its range.



The penetration or range of radiation depends on its energy, the material it encounters, and the type of radiation. (a) Greater energy means greater range. (b) Radiation has a smaller range in materials with high electron density. (c) Alphas have the smallest range, betas have a greater range, and gammas penetrate the farthest.

Radiation can be absorbed or shielded by materials, such as the lead aprons dentists drape on us when taking x rays. Lead is a particularly effective shield compared with other materials, such as plastic or air. How does the range of radiation depend on *material*? Ionizing radiation interacts best with charged particles in a material. Since electrons have small masses, they most readily absorb the energy of the radiation in collisions. The greater the

density of a material and, in particular, the greater the density of electrons within a material, the smaller the range of radiation.

Note:

Collisions

Conservation of energy and momentum often results in energy transfer to a less massive object in a collision. This was discussed in detail in [Work, Energy, and Energy Resources](#), for example.

Different *types* of radiation have different ranges when compared at the same energy and in the same material. Alphas have the shortest range, betas penetrate farther, and gammas have the greatest range. This is directly related to charge and speed of the particle or type of radiation. At a given energy, each α , β , or γ will produce the same number of ionizations in a material (each ionization requires a certain amount of energy on average). The more readily the particle produces ionization, the more quickly it will lose its energy. The effect of *charge* is as follows: The α has a charge of $+2q_e$, the β has a charge of $-q_e$, and the γ is uncharged. The electromagnetic force exerted by the α is thus twice as strong as that exerted by the β and it is more likely to produce ionization. Although chargeless, the γ does interact weakly because it is an electromagnetic wave, but it is less likely to produce ionization in any encounter. More quantitatively, the change in momentum Δp given to a particle in the material is $\Delta p = F\Delta t$, where F is the force the α , β , or γ exerts over a time Δt . The smaller the charge, the smaller is F and the smaller is the momentum (and energy) lost. Since the speed of alphas is about 5% to 10% of the speed of light, classical (non-relativistic) formulas apply.

The *speed* at which they travel is the other major factor affecting the range of α s, β s, and γ s. The faster they move, the less time they spend in the vicinity of an atom or a molecule, and the less likely they are to interact. Since α s and β s are particles with mass (helium nuclei and electrons, respectively), their energy is kinetic, given classically by $\frac{1}{2}mv^2$. The mass

of the β particle is thousands of times less than that of the α s, so that β s must travel much faster than α s to have the same energy. Since β s move faster (most at relativistic speeds), they have less time to interact than α s. Gamma rays are photons, which must travel at the speed of light. They are even less likely to interact than a β , since they spend even less time near a given atom (and they have no charge). The range of γ s is thus greater than the range of β s.

Alpha radiation from radioactive sources has a range much less than a millimeter of biological tissues, usually not enough to even penetrate the dead layers of our skin. On the other hand, the same α radiation can penetrate a few centimeters of air, so mere distance from a source prevents α radiation from reaching us. This makes α radiation relatively safe for our body compared to β and γ radiation. Typical β radiation can penetrate a few millimeters of tissue or about a meter of air. Beta radiation is thus hazardous even when not ingested. The range of β s in lead is about a millimeter, and so it is easy to store β sources in lead radiation-proof containers. Gamma rays have a much greater range than either α s or β s. In fact, if a given thickness of material, like a lead brick, absorbs 90% of the γ s, then a second lead brick will only absorb 90% of what got through the first. Thus, γ s do not have a well-defined range; we can only cut down the amount that gets through. Typically, γ s can penetrate many meters of air, go right through our bodies, and are effectively shielded (that is, reduced in intensity to acceptable levels) by many centimeters of lead. One benefit of γ s is that they can be used as radioactive tracers (see [\[link\]](#)).



This image of the concentration of a radioactive tracer in a patient's body reveals where the most active bone cells are, an indication of bone cancer. A short-lived radioactive substance that locates itself selectively is given to the patient, and the radiation is measured with an external detector. The emitted γ radiation has a sufficient range to leave the body—the range of α s and β s is too small for them to be observed outside the patient. (credit: Kieran Maher, Wikimedia Commons)

Note:**PhET Explorations: Beta Decay**

Build an atom out of protons, neutrons, and electrons, and see how the element, charge, and mass change. Then play a game to test your ideas!

<https://archive.cnx.org/specials/f0a27b96-f5c8-11e5-a22c-73f8c149bebf/beta-decay/#sim-multiple-atoms>

Section Summary

- Some nuclei are radioactive—they spontaneously decay destroying some part of their mass and emitting energetic rays, a process called nuclear radioactivity.
- Nuclear radiation, like x rays, is ionizing radiation, because energy sufficient to ionize matter is emitted in each decay.
- The range (or distance traveled in a material) of ionizing radiation is directly related to the charge of the emitted particle and its energy, with greater-charge and lower-energy particles having the shortest ranges.
- Radiation detectors are based directly or indirectly upon the ionization created by radiation, as are the effects of radiation on living and inert materials.

Conceptual Questions

Exercise:**Problem:**

Suppose the range for 5.0 MeV α ray is known to be 2.0 mm in a certain material. Does this mean that every 5.0 MeV α ray that strikes this material travels 2.0 mm, or does the range have an average value with some statistical fluctuations in the distances traveled? Explain.

Exercise:

Problem:

What is the difference between γ rays and characteristic x rays? Is either necessarily more energetic than the other? Which can be the most energetic?

Exercise:**Problem:**

Ionizing radiation interacts with matter by scattering from electrons and nuclei in the substance. Based on the law of conservation of momentum and energy, explain why electrons tend to absorb more energy than nuclei in these interactions.

Exercise:**Problem:**

What characteristics of radioactivity show it to be nuclear in origin and not atomic?

Exercise:**Problem:**

What is the source of the energy emitted in radioactive decay? Identify an earlier conservation law, and describe how it was modified to take such processes into account.

Exercise:**Problem:**

Consider [\[link\]](#). If an electric field is substituted for the magnetic field with positive charge instead of the north pole and negative charge instead of the south pole, in which directions will the α , β , and γ rays bend?

Exercise:

Problem:

Explain how an α particle can have a larger range in air than a β particle with the same energy in lead.

Exercise:**Problem:**

Arrange the following according to their ability to act as radiation shields, with the best first and worst last. Explain your ordering in terms of how radiation loses its energy in matter.

- (a) A solid material with low density composed of low-mass atoms.
- (b) A gas composed of high-mass atoms.
- (c) A gas composed of low-mass atoms.
- (d) A solid with high density composed of high-mass atoms.

Exercise:**Problem:**

Often, when people have to work around radioactive materials spills, we see them wearing white coveralls (usually a plastic material). What types of radiation (if any) do you think these suits protect the worker from, and how?

Glossary

alpha rays

one of the types of rays emitted from the nucleus of an atom

beta rays

one of the types of rays emitted from the nucleus of an atom

gamma rays

one of the types of rays emitted from the nucleus of an atom

ionizing radiation

radiation (whether nuclear in origin or not) that produces ionization
whether nuclear in origin or not

nuclear radiation

rays that originate in the nuclei of atoms, the first examples of which
were discovered by Becquerel

radioactivity

the emission of rays from the nuclei of atoms

radioactive

a substance or object that emits nuclear radiation

range of radiation

the distance that the radiation can travel through a material

Radiation Detection and Detectors

- Explain the working principle of a Geiger tube.
- Define and discuss radiation detectors.

It is well known that ionizing radiation affects us but does not trigger nerve impulses. Newspapers carry stories about unsuspecting victims of radiation poisoning who fall ill with radiation sickness, such as burns and blood count changes, but who never felt the radiation directly. This makes the detection of radiation by instruments more than an important research tool. This section is a brief overview of radiation detection and some of its applications.

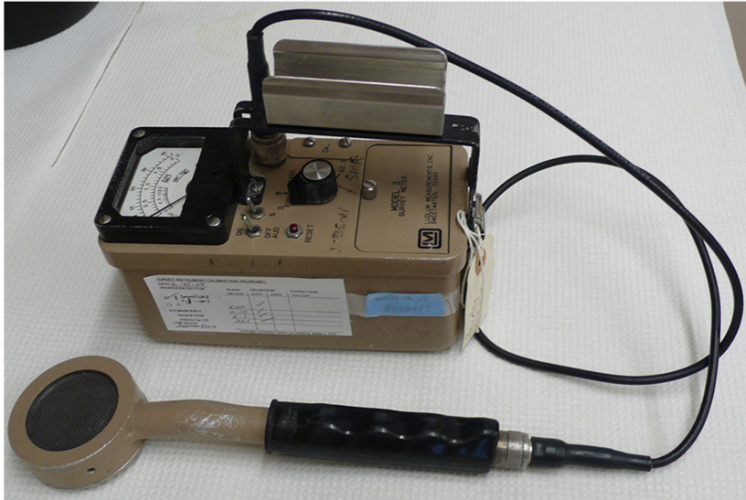
Human Application

The first direct detection of radiation was Becquerel's fogged photographic plate. Photographic film is still the most common detector of ionizing radiation, being used routinely in medical and dental x rays. Nuclear radiation is also captured on film, such as seen in [\[link\]](#). The mechanism for film exposure by ionizing radiation is similar to that by photons. A quantum of energy interacts with the emulsion and alters it chemically, thus exposing the film. The quantum come from an α -particle, β -particle, or photon, provided it has more than the few eV of energy needed to induce the chemical change (as does all ionizing radiation). The process is not 100% efficient, since not all incident radiation interacts and not all interactions produce the chemical change. The amount of film darkening is related to exposure, but the darkening also depends on the type of radiation, so that absorbers and other devices must be used to obtain energy, charge, and particle-identification information.

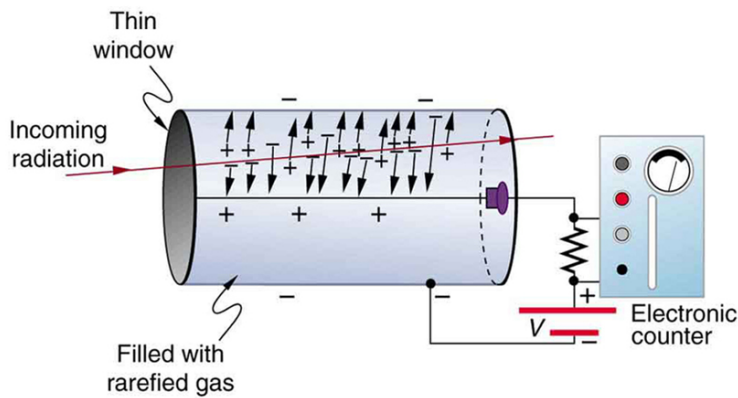


Film badges contain film similar to that used in this dental x-ray film and is sandwiched between various absorbers to determine the penetrating ability of the radiation as well as the amount.
(credit: Werneuchen, Wikimedia Commons)

Another very common **radiation detector** is the **Geiger tube**. The clicking and buzzing sound we hear in dramatizations and documentaries, as well as in our own physics labs, is usually an audio output of events detected by a Geiger counter. These relatively inexpensive radiation detectors are based on the simple and sturdy Geiger tube, shown schematically in [\[link\]](#)(b). A conducting cylinder with a wire along its axis is filled with an insulating gas so that a voltage applied between the cylinder and wire produces almost no current. Ionizing radiation passing through the tube produces free ion pairs that are attracted to the wire and cylinder, forming a current that is detected as a count. The word count implies that there is no information on energy, charge, or type of radiation with a simple Geiger counter. They do not detect every particle, since some radiation can pass through without producing enough ionization to be detected. However, Geiger counters are very useful in producing a prompt output that reveals the existence and relative intensity of ionizing radiation.



(a)

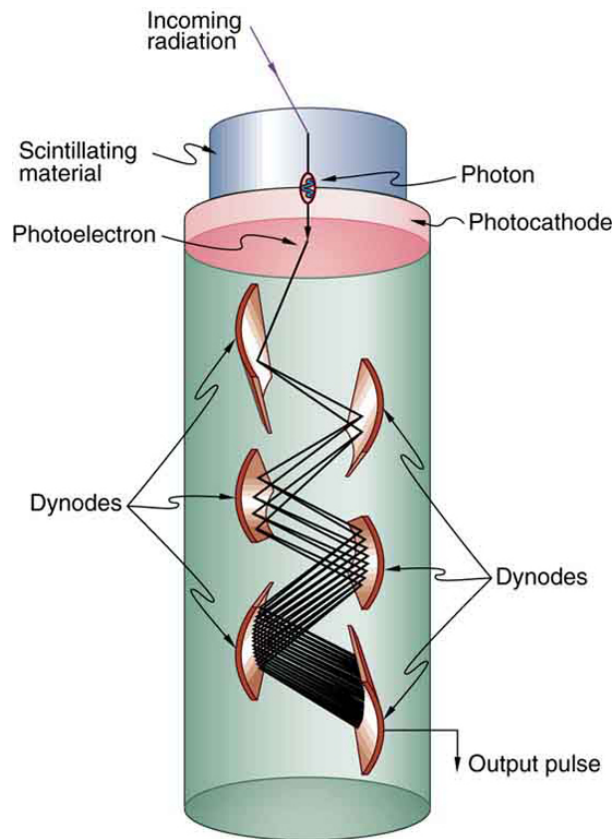


(b)

(a) Geiger counters such as this one are used for prompt monitoring of radiation levels, generally giving only relative intensity and not identifying the type or energy of the radiation. (credit: TimVickers, Wikimedia Commons) (b) Voltage applied between the cylinder and wire in a Geiger tube causes ions and electrons produced by radiation passing through the gas-filled cylinder to move towards them. The resulting current is detected and registered as a count.

Another radiation detection method records light produced when radiation interacts with materials. The energy of the radiation is sufficient to excite atoms in a material that may fluoresce, such as the phosphor used by Rutherford's group. Materials called **scintillators** use a more complex collaborative process to convert radiation energy into light. Scintillators may be liquid or solid, and they can be very efficient. Their light output can provide information about the energy, charge, and type of radiation. Scintillator light flashes are very brief in duration, enabling the detection of a huge number of particles in short periods of time. Scintillator detectors are used in a variety of research and diagnostic applications. Among these are the detection by satellite-mounted equipment of the radiation from distant galaxies, the analysis of radiation from a person indicating body burdens, and the detection of exotic particles in accelerator laboratories.

Light from a scintillator is converted into electrical signals by devices such as the **photomultiplier** tube shown schematically in [\[link\]](#). These tubes are based on the photoelectric effect, which is multiplied in stages into a cascade of electrons, hence the name photomultiplier. Light entering the photomultiplier strikes a metal plate, ejecting an electron that is attracted by a positive potential difference to the next plate, giving it enough energy to eject two or more electrons, and so on. The final output current can be made proportional to the energy of the light entering the tube, which is in turn proportional to the energy deposited in the scintillator. Very sophisticated information can be obtained with scintillators, including energy, charge, particle identification, direction of motion, and so on.



Photomultipliers use the photoelectric effect on the photocathode to convert the light output of a scintillator into an electrical signal. Each successive dynode has a more-positive potential than the last and attracts the ejected electrons, giving them more energy. The number of electrons is thus multiplied at each dynode, resulting in an easily detected output current.

Solid-state radiation detectors convert ionization produced in a semiconductor (like those found in computer chips) directly into an

electrical signal. Semiconductors can be constructed that do not conduct current in one particular direction. When a voltage is applied in that direction, current flows only when ionization is produced by radiation, similar to what happens in a Geiger tube. Further, the amount of current in a solid-state detector is closely related to the energy deposited and, since the detector is solid, it can have a high efficiency (since ionizing radiation is stopped in a shorter distance in solids fewer particles escape detection). As with scintillators, very sophisticated information can be obtained from solid-state detectors.

Note:

PhET Explorations: Radioactive Dating Game

Learn about different types of radiometric dating, such as carbon dating. Understand how decay and half life work to enable radiometric dating to work. Play a game that tests your ability to match the percentage of the dating element that remains to the age of the object.

<https://archive.cnx.org/specials/d709a8b0-068c-11e6-bcfe-f38266817c66/radioactive-dating-game/#sim-half-life>

Section Summary

- Radiation detectors are based directly or indirectly upon the ionization created by radiation, as are the effects of radiation on living and inert materials.

Conceptual Questions

Exercise:

Problem:

Is it possible for light emitted by a scintillator to be too low in frequency to be used in a photomultiplier tube? Explain.

Problems & Exercises

Exercise:

Problem:

The energy of 30.0 eV is required to ionize a molecule of the gas inside a Geiger tube, thereby producing an ion pair. Suppose a particle of ionizing radiation deposits 0.500 MeV of energy in this Geiger tube. What maximum number of ion pairs can it create?

Solution:

$$1.67 \times 10^4$$

Exercise:

Problem:

A particle of ionizing radiation creates 4000 ion pairs in the gas inside a Geiger tube as it passes through. What minimum energy was deposited, if 30.0 eV is required to create each ion pair?

Exercise:

Problem:

(a) Repeat [\[link\]](#), and convert the energy to joules or calories. (b) If all of this energy is converted to thermal energy in the gas, what is its temperature increase, assuming 50.0 cm³ of ideal gas at 0.250-atm pressure? (The small answer is consistent with the fact that the energy is large on a quantum mechanical scale but small on a macroscopic scale.)

Exercise:

Problem:

Suppose a particle of ionizing radiation deposits 1.0 MeV in the gas of a Geiger tube, all of which goes to creating ion pairs. Each ion pair requires 30.0 eV of energy. (a) The applied voltage sweeps the ions out of the gas in $1.00\ \mu\text{s}$. What is the current? (b) This current is smaller than the actual current since the applied voltage in the Geiger tube accelerates the separated ions, which then create other ion pairs in subsequent collisions. What is the current if this last effect multiplies the number of ion pairs by 900?

Glossary**Geiger tube**

a very common radiation detector that usually gives an audio output

photomultiplier

a device that converts light into electrical signals

radiation detector

a device that is used to detect and track the radiation from a radioactive reaction

scintillators

a radiation detection method that records light produced when radiation interacts with materials

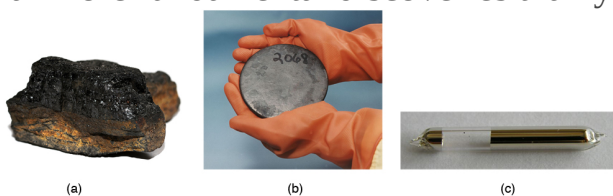
solid-state radiation detectors

semiconductors fabricated to directly convert incident radiation into electrical current

Substructure of the Nucleus

- Define and discuss the nucleus in an atom.
- Define atomic number.
- Define and discuss isotopes.
- Calculate the density of the nucleus.
- Explain nuclear force.

What is inside the nucleus? Why are some nuclei stable while others decay? (See [\[link\]](#).) Why are there different types of decay (α , β and γ)? Why are nuclear decay energies so large? Pursuing natural questions like these has led to far more fundamental discoveries than you might imagine.



Why is most of the carbon in this coal stable (a), while the uranium in the disk (b) slowly decays over billions of years? Why is cesium in this ampule (c) even less stable than the uranium, decaying in far less than 1/1,000,000 the time? What is the reason uranium and cesium undergo different types of decay (α and β , respectively)? (credits: (a) Bresson Thomas, Wikimedia Commons; (b) U.S. Department of Energy; (c) Tomihahndorf, Wikimedia Commons)

We have already identified **protons** as the particles that carry positive charge in the nuclei. However, there are actually *two* types of particles in the nuclei—the *proton* and the *neutron*, referred to collectively as **nucleons**, the constituents of nuclei. As its name implies, the **neutron** is a neutral particle ($q = 0$) that has

nearly the same mass and intrinsic spin as the proton. [\[link\]](#) compares the masses of protons, neutrons, and electrons. Note how close the proton and neutron masses are, but the neutron is slightly more massive once you look past the third digit. Both nucleons are much more massive than an electron. In fact, $m_p = 1836m_e$ (as noted in [Medical Applications of Nuclear Physics](#) and $m_n = 1839m_e$.

[\[link\]](#) also gives masses in terms of mass units that are more convenient than kilograms on the atomic and nuclear scale. The first of these is the *unified atomic mass unit* (u), defined as

Equation:

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg}.$$

This unit is defined so that a neutral carbon ^{12}C atom has a mass of exactly 12 u. Masses are also expressed in units of MeV/c^2 . These units are very convenient when considering the conversion of mass into energy (and vice versa), as is so prominent in nuclear processes. Using $E = mc^2$ and units of m in MeV/c^2 , we find that c^2 cancels and E comes out conveniently in MeV. For example, if the rest mass of a proton is converted entirely into energy, then

Equation:

$$E = mc^2 = (938.27 \text{ MeV}/c^2)c^2 = 938.27 \text{ MeV}.$$

It is useful to note that 1 u of mass converted to energy produces 931.5 MeV, or

Equation:

$$1 \text{ u} = 931.5 \text{ MeV}/c^2.$$

All properties of a nucleus are determined by the number of protons and neutrons it has. A specific combination of protons and neutrons is called a **nuclide** and is a unique nucleus. The following notation is used to represent a particular nuclide:

Equation:

$${}^A_Z\text{X}_N,$$

where the symbols A , X , Z , and N are defined as follows: The *number of protons in a nucleus* is the **atomic number** Z , as defined in [Medical Applications of Nuclear Physics](#). X is the *symbol for the element*, such as Ca for calcium. However, once Z is known, the element is known; hence, Z and X are redundant. For example, $Z = 20$ is always calcium, and calcium always has $Z = 20$. N is the *number of neutrons* in a nucleus. In the notation for a nuclide, the subscript N is usually omitted. The symbol A is defined as the number of nucleons or the *total number of protons and neutrons*,

Equation:

$$A = N + Z,$$

where A is also called the **mass number**. This name for A is logical; the mass of an atom is nearly equal to the mass of its nucleus, since electrons have so little mass. The mass of the nucleus turns out to be nearly equal to the sum of the masses of the protons and neutrons in it, which is proportional to A . In this context, it is particularly convenient to express masses in units of u. Both protons and neutrons have masses close to 1 u, and so the mass of an atom is close to A u. For example, in an oxygen nucleus with eight protons and eight neutrons, $A = 16$, and its mass is 16 u. As noticed, the unified atomic mass unit is defined so that a neutral carbon atom (actually a ^{12}C atom) has a mass of *exactly* 12 u. Carbon was chosen as the standard, partly because of its importance in organic chemistry (see [Appendix A](#)).

Particle	Symbol	kg	u	MeV c^2
Proton	p	1.67262×10^{-27}	1.007276	938.27
Neutron	n	1.67493×10^{-27}	1.008665	939.57

Particle	Symbol	kg	u	MeVc ²
Electron	e	9.1094×10^{-31}	0.00054858	0.511

Masses of the Proton, Neutron, and Electron

Let us look at a few examples of nuclides expressed in the ${}^A_Z\text{X}_N$ notation. The nucleus of the simplest atom, hydrogen, is a single proton, or ${}^1_1\text{H}$ (the zero for no neutrons is often omitted). To check this symbol, refer to the periodic table—you see that the atomic number Z of hydrogen is 1. Since you are given that there are no neutrons, the mass number A is also 1. Suppose you are told that the helium nucleus or α particle has two protons and two neutrons. You can then see that it is written ${}^4_2\text{He}_2$. There is a scarce form of hydrogen found in nature called deuterium; its nucleus has one proton and one neutron and, hence, twice the mass of common hydrogen. The symbol for deuterium is, thus, ${}^2_1\text{H}_1$ (sometimes D is used, as for deuterated water D_2O). An even rarer—and radioactive—form of hydrogen is called tritium, since it has a single proton and two neutrons, and it is written ${}^3_1\text{H}_2$. These three varieties of hydrogen have nearly identical chemistries, but the nuclei differ greatly in mass, stability, and other characteristics. Nuclei (such as those of hydrogen) having the same Z and different N s are defined to be **isotopes** of the same element.

There is some redundancy in the symbols A , X , Z , and N . If the element X is known, then Z can be found in a periodic table and is always the same for a given element. If both A and X are known, then N can also be determined (first find Z ; then, $N = A - Z$). Thus the simpler notation for nuclides is

Equation:

$${}^A\text{X},$$

which is sufficient and is most commonly used. For example, in this simpler notation, the three isotopes of hydrogen are ${}^1\text{H}$, ${}^2\text{H}$, and ${}^3\text{H}$, while the α particle is ${}^4\text{He}$. We read this backward, saying helium-4 for ${}^4\text{He}$, or uranium-238 for ${}^{238}\text{U}$. So for ${}^{238}\text{U}$, should we need to know, we can determine that $Z = 92$ for uranium from the periodic table, and, thus, $N = 238 - 92 = 146$.

A variety of experiments indicate that a nucleus behaves something like a tightly packed ball of nucleons, as illustrated in [\[link\]](#). These nucleons have large kinetic energies and, thus, move rapidly in very close contact. Nucleons can be separated by a large force, such as in a collision with another nucleus, but resist strongly being pushed closer together. The most compelling evidence that nucleons are closely packed in a nucleus is that the **radius of a nucleus**, r , is found to be given approximately by

Equation:

$$r = r_0 A^{1/3},$$

where $r_0 = 1.2$ fm and A is the mass number of the nucleus. Note that $r^3 \propto A$. Since many nuclei are spherical, and the volume of a sphere is $V = (4/3)\pi r^3$, we see that $V \propto A$ —that is, the volume of a nucleus is proportional to the number of nucleons in it. This is what would happen if you pack nucleons so closely that there is no empty space between them.



A model of the
nucleus.

Nucleons are held together by nuclear forces and resist both being pulled apart and pushed inside one another. The volume of the nucleus is the sum of the volumes of the nucleons in it, here shown in different colors to represent protons and neutrons.

Example:

How Small and Dense Is a Nucleus?

(a) Find the radius of an iron-56 nucleus. (b) Find its approximate density in kg/m^3 , approximating the mass of ^{56}Fe to be 56 u.

Strategy and Concept

(a) Finding the radius of ^{56}Fe is a straightforward application of $r = r_0 A^{1/3}$, given $A = 56$. (b) To find the approximate density, we assume the nucleus is spherical (this one actually is), calculate its volume using the radius found in part (a), and then find its density from $\rho = m / V$. Finally, we will need to convert density from units of u/fm^3 to kg/m^3 .

Solution

(a) The radius of a nucleus is given by

Equation:

$$r = r_0 A^{1/3}.$$

Substituting the values for r_0 and A yields

Equation:

$$\begin{aligned} r &= (1.2 \text{ fm})(56)^{1/3} = (1.2 \text{ fm})(3.83) \\ &= 4.6 \text{ fm}. \end{aligned}$$

(b) Density is defined to be $\rho = m / V$, which for a sphere of radius r is

Equation:

$$\rho = \frac{m}{V} = \frac{m}{(4/3)\pi r^3}.$$

Substituting known values gives

Equation:

$$\begin{aligned} \rho &= \frac{56 \text{ u}}{(1.33)(3.14)(4.6 \text{ fm})^3} \\ &= 0.138 \text{ u}/\text{fm}^3. \end{aligned}$$

Converting to units of kg/m^3 , we find

Equation:

$$\begin{aligned} \rho &= (0.138 \text{ u}/\text{fm}^3)(1.66 \times 10^{-27} \text{ kg}/\text{u})\left(\frac{1 \text{ fm}}{10^{-15} \text{ m}}\right) \\ &= 2.3 \times 10^{17} \text{ kg}/\text{m}^3. \end{aligned}$$

Discussion

(a) The radius of this medium-sized nucleus is found to be approximately 4.6 fm, and so its diameter is about 10 fm, or 10^{-14} m. In our discussion of Rutherford's discovery of the nucleus, we noticed that it is about 10^{-15} m in diameter (which is for lighter nuclei), consistent with this result to an order of magnitude. The nucleus is much smaller in diameter than the typical atom, which has a diameter of the order of 10^{-10} m.

(b) The density found here is so large as to cause disbelief. It is consistent with earlier discussions we have had about the nucleus being very small and containing nearly all of the mass of the atom. Nuclear densities, such as found here, are about 2×10^{14} times greater than that of water, which has a density of "only" 10^3 kg/m³. One cubic meter of nuclear matter, such as found in a neutron star, has the same mass as a cube of water 61 km on a side.

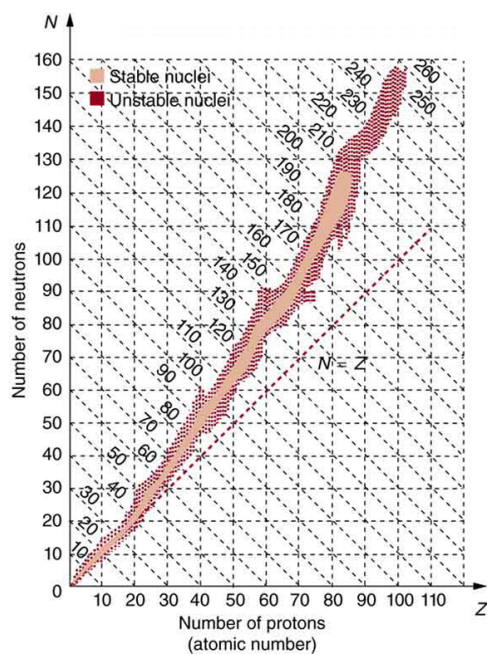
Nuclear Forces and Stability

What forces hold a nucleus together? The nucleus is very small and its protons, being positive, exert tremendous repulsive forces on one another. (The Coulomb force increases as charges get closer, since it is proportional to $1/r^2$, even at the tiny distances found in nuclei.) The answer is that two previously unknown forces hold the nucleus together and make it into a tightly packed ball of nucleons. These forces are called the *weak and strong nuclear forces*.

Nuclear forces are so short ranged that they fall to zero strength when nucleons are separated by only a few fm. However, like glue, they are strongly attracted when the nucleons get close to one another. The strong nuclear force is about 100 times more attractive than the repulsive EM force, easily holding the nucleons together. Nuclear forces become extremely repulsive if the nucleons get too close, making nucleons strongly resist being pushed inside one another, something like ball bearings.

The fact that nuclear forces are very strong is responsible for the very large energies emitted in nuclear decay. During decay, the forces do work, and since work is force times the distance ($W = Fd \cos \theta$), a large force can result in a large emitted energy. In fact, we know that there are *two* distinct nuclear forces because of the different types of nuclear decay—the strong nuclear force is responsible for α decay, while the weak nuclear force is responsible for β decay.

The many stable and unstable nuclei we have explored, and the hundreds we have not discussed, can be arranged in a table called the **chart of the nuclides**, a simplified version of which is shown in [\[link\]](#). Nuclides are located on a plot of N versus Z . Examination of a detailed chart of the nuclides reveals patterns in the characteristics of nuclei, such as stability, abundance, and types of decay, analogous to but more complex than the systematics in the periodic table of the elements.



Simplified chart of the nuclides, a graph of N versus Z for known nuclides. The patterns of stable and unstable nuclides reveal characteristics of the nuclear forces. The dashed line is for $N = Z$. Numbers along diagonals are mass numbers A .

In principle, a nucleus can have any combination of protons and neutrons, but [\[link\]](#) shows a definite pattern for those that are stable. For low-mass nuclei, there is a strong tendency for N and Z to be nearly equal. This means that the nuclear force is more attractive when $N = Z$. More detailed examination reveals greater stability when N and Z are even numbers—nuclear forces are more attractive when neutrons and protons are in pairs. For increasingly higher masses, there are progressively more neutrons than protons in stable nuclei. This is due to the ever-growing repulsion between protons. Since nuclear forces are short ranged, and the Coulomb force is long ranged, an excess of neutrons keeps the protons a little farther apart, reducing Coulomb repulsion. Decay modes of nuclides out of the region of stability consistently produce nuclides closer to the region of stability. There are more stable nuclei having certain numbers of protons and neutrons, called **magic numbers**. Magic numbers indicate a shell structure for the nucleus in which closed shells are more stable. Nuclear shell theory has been very successful in explaining nuclear energy levels, nuclear decay, and the greater stability of nuclei with closed shells. We have been producing ever-heavier transuranic elements since the early 1940s, and we have now produced the element with $Z = 118$. There are theoretical predictions of an island of relative stability for nuclei with such high Z s.



The German-born
American
physicist Maria
Goeppert Mayer
(1906–1972)

shared the 1963 Nobel Prize in physics with J. Jensen for the creation of the nuclear shell model. This successful nuclear model has nucleons filling shells analogous to electron shells in atoms. It was inspired by patterns observed in nuclear properties. (credit: Nobel Foundation via Wikimedia Commons)

Section Summary

- Two particles, both called nucleons, are found inside nuclei. The two types of nucleons are protons and neutrons; they are very similar, except that the proton is positively charged while the neutron is neutral. Some of their characteristics are given in [\[link\]](#) and compared with those of the electron. A mass unit convenient to atomic and nuclear processes is the unified atomic mass unit (u), defined to be

Equation:

$$1 \text{ u} = 1.6605 \times 10^{-27} \text{ kg} = 931.46 \text{ MeV}/c^2.$$

- A nuclide is a specific combination of protons and neutrons, denoted by
- Equation:**

$${}^A_Z\text{X}_N \text{ or simply } {}^A\text{X},$$

Z is the number of protons or atomic number, X is the symbol for the element, N is the number of neutrons, and A is the mass number or the total number of protons and neutrons,

Equation:

$$A = N + Z.$$

- Nuclides having the same Z but different N are isotopes of the same element.
- The radius of a nucleus, r , is approximately

Equation:

$$r = r_0 A^{1/3},$$

where $r_0 = 1.2$ fm. Nuclear volumes are proportional to A . There are two nuclear forces, the weak and the strong. Systematics in nuclear stability seen on the chart of the nuclides indicate that there are shell closures in nuclei for values of Z and N equal to the magic numbers, which correspond to highly stable nuclei.

Conceptual Questions

Exercise:

Problem:

The weak and strong nuclear forces are basic to the structure of matter. Why we do not experience them directly?

Exercise:

Problem:

Define and make clear distinctions between the terms neutron, nucleon, nucleus, nuclide, and neutrino.

Exercise:

Problem:

What are isotopes? Why do different isotopes of the same element have similar chemistries?

Problems & Exercises**Exercise:****Problem:**

Verify that a 2.3×10^{17} kg mass of water at normal density would make a cube 60 km on a side, as claimed in [\[link\]](#). (This mass at nuclear density would make a cube 1.0 m on a side.)

Solution:**Equation:**

$$\begin{aligned} m = \rho V = \rho d^3 &\Rightarrow a = \left(\frac{m}{\rho} \right)^{1/3} = \left(\frac{2.3 \times 10^{17} \text{ kg}}{1000 \text{ kg/m}^3} \right)^{\frac{1}{3}} \\ &= 61 \times 10^3 \text{ m} = 61 \text{ km} \end{aligned}$$

Exercise:**Problem:**

Find the length of a side of a cube having a mass of 1.0 kg and the density of nuclear matter, taking this to be $2.3 \times 10^{17} \text{ kg/m}^3$.

Exercise:

Problem: What is the radius of an α particle?

Solution:

1.9 fm

Exercise:

Problem:

Find the radius of a ^{238}Pu nucleus. ^{238}Pu is a manufactured nuclide that is used as a power source on some space probes.

Exercise:**Problem:**

- (a) Calculate the radius of ^{58}Ni , one of the most tightly bound stable nuclei.
- (b) What is the ratio of the radius of ^{58}Ni to that of ^{258}Ha , one of the largest nuclei ever made? Note that the radius of the largest nucleus is still much smaller than the size of an atom.
-

Solution:

- (a) 4.6 fm
- (b) 0.61 to 1

Exercise:**Problem:**

The unified atomic mass unit is defined to be $1\text{ u} = 1.6605 \times 10^{-27}\text{ kg}$. Verify that this amount of mass converted to energy yields 931.5 MeV. Note that you must use four-digit or better values for c and $|q_e|$.

Exercise:**Problem:**

What is the ratio of the velocity of a β particle to that of an α particle, if they have the same nonrelativistic kinetic energy?

Solution:

85.4 to 1

Exercise:

Problem:

If a 1.50-cm-thick piece of lead can absorb 90.0% of the γ rays from a radioactive source, how many centimeters of lead are needed to absorb all but 0.100% of the γ rays?

Exercise:**Problem:**

The detail observable using a probe is limited by its wavelength. Calculate the energy of a γ -ray photon that has a wavelength of 1×10^{-16} m, small enough to detect details about one-tenth the size of a nucleon. Note that a photon having this energy is difficult to produce and interacts poorly with the nucleus, limiting the practicability of this probe.

Solution:

12.4 GeV

Exercise:**Problem:**

(a) Show that if you assume the average nucleus is spherical with a radius $r = r_0 A^{1/3}$, and with a mass of A u, then its density is independent of A .

(b) Calculate that density in u/fm³ and kg/m³, and compare your results with those found in [\[link\]](#) for ⁵⁶Fe.

Exercise:**Problem:**

What is the ratio of the velocity of a 5.00-MeV β ray to that of an α particle with the same kinetic energy? This should confirm that β s travel much faster than α s even when relativity is taken into consideration. (See also [\[link\]](#).)

Solution:

19.3 to 1

Exercise:

Problem:

(a) What is the kinetic energy in MeV of a β ray that is traveling at $0.998c$? This gives some idea of how energetic a β ray must be to travel at nearly the same speed as a γ ray. (b) What is the velocity of the γ ray relative to the β ray?

Glossary

atomic mass

the total mass of the protons, neutrons, and electrons in a single atom

atomic number

number of protons in a nucleus

chart of the nuclides

a table comprising stable and unstable nuclei

isotopes

nuclei having the same Z and different N s

magic numbers

a number that indicates a shell structure for the nucleus in which closed shells are more stable

mass number

number of nucleons in a nucleus

neutron

a neutral particle that is found in a nucleus

nucleons

the particles found inside nuclei

nucleus

a region consisting of protons and neutrons at the center of an atom

nuclide

a type of atom whose nucleus has specific numbers of protons and neutrons

protons

the positively charged nucleons found in a nucleus

radius of a nucleus

the radius of a nucleus is $r = r_0 A^{1/3}$

Nuclear Decay and Conservation Laws

- Define and discuss nuclear decay.
- State the conservation laws.
- Explain parent and daughter nucleus.
- Calculate the energy emitted during nuclear decay.

Nuclear **decay** has provided an amazing window into the realm of the very small. Nuclear decay gave the first indication of the connection between mass and energy, and it revealed the existence of two of the four basic forces in nature. In this section, we explore the major modes of nuclear decay; and, like those who first explored them, we will discover evidence of previously unknown particles and conservation laws.

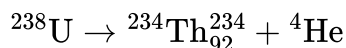
Some nuclides are stable, apparently living forever. Unstable nuclides decay (that is, they are radioactive), eventually producing a stable nuclide after many decays. We call the original nuclide the **parent** and its decay products the **daughters**. Some radioactive nuclides decay in a single step to a stable nucleus. For example, ^{60}Co is unstable and decays directly to ^{60}Ni , which is stable. Others, such as ^{238}U , decay to another unstable nuclide, resulting in a **decay series** in which each subsequent nuclide decays until a stable nuclide is finally produced. The decay series that starts from ^{238}U is of particular interest, since it produces the radioactive isotopes ^{226}Ra and ^{210}Po , which the Curies first discovered (see [\[link\]](#)). Radon gas is also produced (^{222}Rn in the series), an increasingly recognized naturally occurring hazard. Since radon is a noble gas, it emanates from materials, such as soil, containing even trace amounts of ^{238}U and can be inhaled. The decay of radon and its daughters produces internal damage. The ^{238}U decay series ends with ^{206}Pb , a stable isotope of lead.

Note that the daughters of α decay shown in [\[link\]](#) always have two fewer protons and two fewer neutrons than the parent. This seems reasonable, since we know that α decay is the emission of a ${}^4\text{He}$ nucleus, which has two protons and two neutrons. The daughters of β decay have one less neutron and one more proton than their parent. Beta decay is a little more subtle, as we shall see. No γ decays are shown in the figure, because they do not produce a daughter that differs from the parent.

Alpha Decay

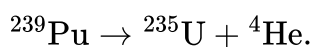
In **alpha decay**, a ${}^4\text{He}$ nucleus simply breaks away from the parent nucleus, leaving a daughter with two fewer protons and two fewer neutrons than the parent (see [\[link\]](#)). One example of α decay is shown in [\[link\]](#) for ${}^{238}\text{U}$. Another nuclide that undergoes α decay is ${}^{239}\text{Pu}$. The decay equations for these two nuclides are

Equation:



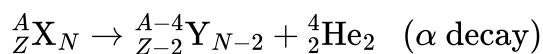
and

Equation:



Alpha decay is the separation of a ${}^4\text{He}$ nucleus from the parent. The daughter nucleus has two fewer protons and two fewer neutrons than the parent. Alpha decay occurs spontaneously only if the daughter and ${}^4\text{He}$ nucleus have less total mass than the parent.

If you examine the periodic table of the elements, you will find that Th has $Z = 90$, two fewer than U, which has $Z = 92$. Similarly, in the second **decay equation**, we see that U has two fewer protons than Pu, which has $Z = 94$. The general rule for α decay is best written in the format ${}^A_Z\text{X}_N$. If a certain nuclide is known to α decay (generally this information must be looked up in a table of isotopes, such as in [Appendix B](#)), its α **decay equation** is

Equation:

where Y is the nuclide that has two fewer protons than X, such as Th having two fewer than U. So if you were told that ${}^{239}\text{Pu}$ α decays and were asked to write the complete decay equation, you would first look up which element has two fewer protons (an atomic number two lower) and find that this is uranium. Then since four nucleons have broken away from the original 239, its atomic mass would be 235.

It is instructive to examine conservation laws related to α decay. You can see from the equation ${}_Z^AX_N \rightarrow {}_{Z-2}^{A-4}Y_{N-2} + {}_2^4\text{He}_2$ that total charge is conserved. Linear and angular momentum are conserved, too. Although conserved angular momentum is not of great consequence in this type of decay, conservation of linear momentum has interesting consequences. If the nucleus is at rest when it decays, its momentum is zero. In that case, the fragments must fly in opposite directions with equal-magnitude momenta so that total momentum remains zero. This results in the α particle carrying away most of the energy, as a bullet from a heavy rifle carries away most of the energy of the powder burned to shoot it. Total mass–energy is also conserved: the energy produced in the decay comes from conversion of a fraction of the original mass. As discussed in [Atomic Physics](#), the general relationship is

Equation:

$$E = (\Delta m)c^2.$$

Here, E is the **nuclear reaction energy** (the reaction can be nuclear decay or any other reaction), and Δm is the difference in mass between initial and final products. When the final products have less total mass, Δm is positive, and the reaction releases energy (is exothermic). When the products have greater total mass, the reaction is endothermic (Δm is negative) and must be induced with an energy input. For α decay to be spontaneous, the decay products must have smaller mass than the parent.

Example:**Alpha Decay Energy Found from Nuclear Masses**

Find the energy emitted in the α decay of ${}^{239}\text{Pu}$.

Strategy

Nuclear reaction energy, such as released in α decay, can be found using the equation $E = (\Delta m)c^2$. We must first find Δm , the difference in mass between the parent nucleus and the products of the decay. This is easily done using masses given in [Appendix A](#).

Solution

The decay equation was given earlier for ${}^{239}\text{Pu}$; it is

Equation:



Thus the pertinent masses are those of ^{239}Pu , ^{235}U , and the α particle or ^4He , all of which are listed in [Appendix A](#). The initial mass was $m(^{239}\text{Pu}) = 239.052157 \text{ u}$. The final mass is the sum $m(^{235}\text{U}) + m(^4\text{He}) = 235.043924 \text{ u} + 4.002602 \text{ u} = 239.046526 \text{ u}$. Thus,

Equation:

$$\begin{aligned}\Delta m &= m(^{239}\text{Pu}) - [m(^{235}\text{U}) + m(^4\text{He})] \\ &= 239.052157 \text{ u} - 239.046526 \text{ u} \\ &= 0.005631 \text{ u}.\end{aligned}$$

Now we can find E by entering Δm into the equation:

Equation:

$$E = (\Delta m)c^2 = (0.005631 \text{ u})c^2.$$

We know $1 \text{ u} = 931.5 \text{ MeV}/c^2$, and so

Equation:

$$E = (0.005631)(931.5 \text{ MeV}/c^2)(c^2) = 5.25 \text{ MeV}.$$

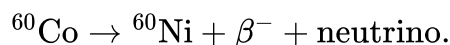
Discussion

The energy released in this α decay is in the MeV range, about 10^6 times as great as typical chemical reaction energies, consistent with many previous discussions. Most of this energy becomes kinetic energy of the α particle (or ^4He nucleus), which moves away at high speed. The energy carried away by the recoil of the ^{235}U nucleus is much smaller in order to conserve momentum. The ^{235}U nucleus can be left in an excited state to later emit photons (γ rays). This decay is spontaneous and releases energy, because the products have less mass than the parent nucleus. The question of why the products have less mass will be discussed in [Binding Energy](#). Note that the masses given in [Appendix A](#) are atomic masses of neutral atoms, including their electrons. The mass of the electrons is the same before and after α decay, and so their masses subtract out when finding Δm . In this case, there are 94 electrons before and after the decay.

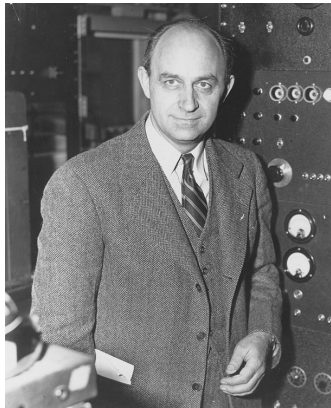
Beta Decay

There are actually *three* types of **beta decay**. The first discovered was “ordinary” beta decay and is called β^- decay or electron emission. The symbol β^- represents *an electron emitted in nuclear beta decay*. Cobalt-60 is a nuclide that β^- decays in the following manner:

Equation:



The **neutrino** is a particle emitted in beta decay that was unanticipated and is of fundamental importance. The neutrino was not even proposed in theory until more than 20 years after beta decay was known to involve electron emissions. Neutrinos are so difficult to detect that the first direct evidence of them was not obtained until 1953. Neutrinos are nearly massless, have no charge, and do not interact with nucleons via the strong nuclear force. Traveling approximately at the speed of light, they have little time to affect any nucleus they encounter. This is, owing to the fact that they have no charge (and they are not EM waves), they do not interact through the EM force. They do interact via the relatively weak and very short range weak nuclear force. Consequently, neutrinos escape almost any detector and penetrate almost any shielding. However, neutrinos do carry energy, angular momentum (they are fermions with half-integral spin), and linear momentum away from a beta decay. When accurate measurements of beta decay were made, it became apparent that energy, angular momentum, and linear momentum were not accounted for by the daughter nucleus and electron alone. Either a previously unsuspected particle was carrying them away, or three conservation laws were being violated. Wolfgang Pauli made a formal proposal for the existence of neutrinos in 1930. The Italian-born American physicist Enrico Fermi (1901–1954) gave neutrinos their name, meaning little neutral ones, when he developed a sophisticated theory of beta decay (see [link](#)). Part of Fermi's theory was the identification of the weak nuclear force as being distinct from the strong nuclear force and in fact responsible for beta decay.



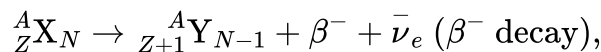
Enrico Fermi was
nearly unique
among 20th-
century physicists
—he made
significant
contributions both
as an
experimentalist and
a theorist. His
many contributions
to theoretical

physics included the identification of the weak nuclear force. The fermi (fm) is named after him, as are an entire class of subatomic particles (fermions), an element (Fermium), and a major research laboratory (Fermilab). His experimental work included studies of radioactivity, for which he won the 1938 Nobel Prize in physics, and creation of the first nuclear chain reaction. (credit: United States Department of Energy, Office of Public Affairs)

The neutrino also reveals a new conservation law. There are various families of particles, one of which is the electron family. We propose that the number of members of the electron family is constant in any process or any closed system. In our example of beta decay, there are no members of the electron family present before the decay, but after, there is an electron and a neutrino. So electrons are given an electron family number of +1. The neutrino in β^- decay is an **electron's antineutrino**, given the symbol $\bar{\nu}_e$, where ν is the Greek letter nu, and the subscript e means this neutrino is related to the electron. The bar indicates this is a particle of **antimatter**. (All particles have antimatter counterparts that are nearly identical except that they have the opposite charge. Antimatter is almost entirely absent on Earth, but it is found in nuclear decay and other nuclear and particle reactions as well as in outer space.) The electron's antineutrino $\bar{\nu}_e$, being antimatter, has an electron family number of -1 . The total is zero, before and after the decay. The new conservation law, obeyed in all circumstances, states that the *total electron family number is constant*. An electron cannot be created without also creating an antimatter family member. This law is analogous to the conservation of charge in a situation where total charge is originally zero, and equal amounts of positive and negative charge must be created in a reaction to keep the total zero.

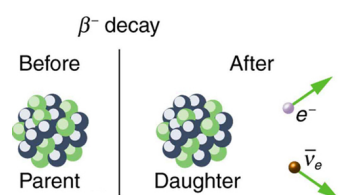
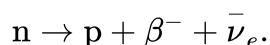
If a nuclide ${}^A_Z\text{X}_N$ is known to β^- decay, then its β^- decay equation is

Equation:



where Y is the nuclide having one more proton than X (see [\[link\]](#)). So if you know that a certain nuclide β^- decays, you can find the daughter nucleus by first looking up Z for the parent and then determining which element has atomic number $Z + 1$. In the example of the β^- decay of ${}^{60}\text{Co}$ given earlier, we see that $Z = 27$ for Co and $Z = 28$ is Ni. It is as if one of the neutrons in the parent nucleus decays into a proton, electron, and neutrino. In fact, neutrons outside of nuclei do just that—they live only an average of a few minutes and β^- decay in the following manner:

Equation:



In β^- decay, the parent nucleus emits an electron and an antineutrino.

The daughter nucleus has one more proton and one less neutron than its parent.

Neutrinos interact so weakly that they are almost never directly observed, but they play a fundamental role in particle physics.

We see that charge is conserved in β^- decay, since the total charge is Z before and after the decay. For example, in ^{60}Co decay, total charge is 27 before decay, since cobalt has $Z = 27$. After decay, the daughter nucleus is Ni, which has $Z = 28$, and there is an electron, so that the total charge is also $28 + (-1)$ or 27. Angular momentum is conserved, but not obviously (you have to examine the spins and angular momenta of the final products in detail to verify this). Linear momentum is also conserved, again imparting most of the decay energy to the electron and the antineutrino, since they are of low and zero mass, respectively. Another new conservation law is obeyed here and elsewhere in nature. *The total number of nucleons A is conserved.* In ^{60}Co decay, for example, there are 60 nucleons before and after the decay. Note that total A is also conserved in α decay. Also note that the total number of protons changes, as does the total number of neutrons, so that total Z and total N are *not* conserved in β^- decay, as they are in α decay. Energy released in β^- decay can be calculated given the masses of the parent and products.

Example:

β^- Decay Energy from Masses

Find the energy emitted in the β^- decay of ^{60}Co .

Strategy and Concept

As in the preceding example, we must first find Δm , the difference in mass between the parent nucleus and the products of the decay, using masses given in [Appendix A](#). Then the emitted energy is calculated as before, using $E = (\Delta m)c^2$. The initial mass is just that of the parent nucleus, and the final mass is that of the daughter nucleus and the electron created in the decay. The neutrino is massless, or nearly so. However, since the masses given in [Appendix A](#) are for neutral atoms, the daughter nucleus has one more electron than the parent, and so the extra electron mass that corresponds to the β^- is included in the atomic mass of Ni. Thus,

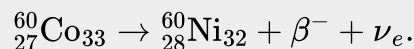
Equation:

$$\Delta m = m(^{60}\text{Co}) - m(^{60}\text{Ni}).$$

Solution

The β^- decay equation for ^{60}Co is

Equation:



As noticed,

Equation:

$$\Delta m = m(^{60}\text{Co}) - m(^{60}\text{Ni}).$$

Entering the masses found in [Appendix A](#) gives

Equation:

$$\Delta m = 59.933820 \text{ u} - 59.930789 \text{ u} = 0.003031 \text{ u}.$$

Thus,

Equation:

$$E = (\Delta m)c^2 = (0.003031 \text{ u})c^2.$$

Using $1 \text{ u} = 931.5 \text{ MeV}/c^2$, we obtain

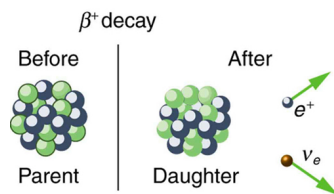
Equation:

$$E = (0.003031)(931.5 \text{ MeV}/c^2)(c^2) = 2.82 \text{ MeV}.$$

Discussion and Implications

Perhaps the most difficult thing about this example is convincing yourself that the β^- mass is included in the atomic mass of ^{60}Ni . Beyond that are other implications. Again the decay energy is in the MeV range. This energy is shared by all of the products of the decay. In many ^{60}Co decays, the daughter nucleus ^{60}Ni is left in an excited state and emits photons (γ rays). Most of the remaining energy goes to the electron and neutrino, since the recoil kinetic energy of the daughter nucleus is small. One final note: the electron emitted in β^- decay is created in the nucleus at the time of decay.

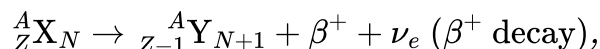
The second type of beta decay is less common than the first. It is β^+ decay. Certain nuclides decay by the emission of a *positive* electron. This is **antielectron** or **positron decay** (see [\[link\]](#)).



β^+ decay is the emission of a positron that eventually finds an electron to annihilate, characteristically producing gammas in opposite directions.

The antielectron is often represented by the symbol e^+ , but in beta decay it is written as β^+ to indicate the antielectron was emitted in a nuclear decay. Antielectrons are the antimatter counterpart to electrons, being nearly identical, having the same mass, spin, and so on, but having a positive charge and an electron family number of -1 . When a **positron** encounters an electron, there is a mutual annihilation in which all the mass of the antielectron-electron pair is converted into pure photon energy. (The reaction, $e^+ + e^- \rightarrow \gamma + \gamma$, conserves electron family number as well as all other conserved quantities.) If a nuclide ${}^A_Z\text{X}_N$ is known to β^+ decay, then its β^+ **decay equation** is

Equation:



where Y is the nuclide having one less proton than X (to conserve charge) and ν_e is the symbol for the **electron's neutrino**, which has an electron family number of $+1$. Since an antimatter member of the electron family (the β^+) is created in the decay, a matter member of the family (here the ν_e) must also be created. Given, for example, that ${}^{22}\text{Na}$ β^+ decays, you can write its full decay equation by first finding that $Z = 11$ for ${}^{22}\text{Na}$, so that the daughter nuclide will have $Z = 10$, the atomic number for neon. Thus the β^+ decay equation for ${}^{22}\text{Na}$ is

Equation:



In β^+ decay, it is as if one of the protons in the parent nucleus decays into a neutron, a positron, and a neutrino. Protons do not do this outside of the nucleus, and so the decay is due to the complexities of the nuclear force. Note again that the total number of nucleons is constant in this and any other reaction. To find the energy emitted in β^+ decay, you must again count the number of electrons in the neutral atoms, since atomic masses are used. The daughter has one less electron than the parent, and one electron mass is created in the decay. Thus, in β^+ decay,

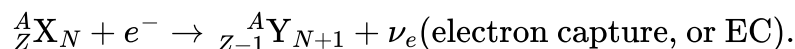
Equation:

$$\Delta m = m(\text{parent}) - [m(\text{daughter}) + 2m_e],$$

since we use the masses of neutral atoms.

Electron capture is the third type of beta decay. Here, a nucleus captures an inner-shell electron and undergoes a nuclear reaction that has the same effect as β^+ decay. Electron capture is sometimes denoted by the letters EC. We know that electrons cannot reside in the nucleus, but this is a nuclear reaction that consumes the electron and occurs spontaneously only when the products have less mass than the parent plus the electron. If a nuclide ${}^A_Z\text{X}_N$ is known to undergo electron capture, then its **electron capture equation** is

Equation:



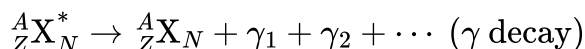
Any nuclide that can β^+ decay can also undergo electron capture (and often does both). The same conservation laws are obeyed for EC as for β^+ decay. It is good practice to confirm these for yourself.

All forms of beta decay occur because the parent nuclide is unstable and lies outside the region of stability in the chart of nuclides. Those nuclides that have relatively more neutrons than those in the region of stability will β^- decay to produce a daughter with fewer neutrons, producing a daughter nearer the region of stability. Similarly, those nuclides having relatively more protons than those in the region of stability will β^+ decay or undergo electron capture to produce a daughter with fewer protons, nearer the region of stability.

Gamma Decay

Gamma decay is the simplest form of nuclear decay—it is the emission of energetic photons by nuclei left in an excited state by some earlier process. Protons and neutrons in an excited nucleus are in higher orbitals, and they fall to lower levels by photon emission (analogous to electrons in excited atoms). Nuclear excited states have lifetimes typically of only about 10^{-14} s, an indication of the great strength of the forces pulling the nucleons to lower states. The γ decay equation is simply

Equation:



where the asterisk indicates the nucleus is in an excited state. There may be one or more γ s emitted, depending on how the nuclide de-excites. In radioactive decay, γ emission is common and is preceded by γ or β decay. For example, when ${}^{60}\text{Co}$ β^- decays, it most often leaves the daughter nucleus in an excited state, written ${}^{60}\text{Ni}^*$. Then the nickel nucleus quickly γ decays by the emission of two penetrating γ s:

Equation:



These are called cobalt γ rays, although they come from nickel—they are used for cancer therapy, for example. It is again constructive to verify the conservation laws for gamma decay. Finally, since γ decay does not change the nuclide to another species, it is not prominently featured in charts of decay series, such as that in [\[link\]](#).

There are other types of nuclear decay, but they occur less commonly than α , β , and γ decay. Spontaneous fission is the most important of the other forms of nuclear decay because of its applications in nuclear power and weapons. It is covered in the next chapter.

Section Summary

- When a parent nucleus decays, it produces a daughter nucleus following rules and conservation laws. There are three major types of nuclear decay, called alpha (α), beta (β), and gamma (γ). The α decay equation is

Equation:

$${}^A_Z\text{X}_N \rightarrow {}^{A-4}_{Z-2}\text{Y}_{N-2} + {}^4_2\text{He}_2.$$

- Nuclear decay releases an amount of energy E related to the mass destroyed Δm by

Equation:

$$E = (\Delta m)c^2.$$

- There are three forms of beta decay. The β^- decay equation is

Equation:

$${}^A_Z\text{X}_N \rightarrow {}^A_{Z+1}\text{Y}_{N-1} + \beta^- + \nu_e.$$

- The β^+ decay equation is

Equation:

$${}^A_Z\text{X}_N \rightarrow {}^A_{Z-1}\text{Y}_{N+1} + \beta^+ + \nu_e.$$

- The electron capture equation is

Equation:

$${}^A_Z\text{X}_N + e^- \rightarrow {}^A_{Z-1}\text{Y}_{N+1} + \nu_e.$$

- β^- is an electron, β^+ is an antielectron or positron, ν_e represents an electron's neutrino, and $\bar{\nu}_e$ is an electron's antineutrino. In addition to all previously known conservation laws, two new ones arise— conservation of electron family number and conservation of the total number of nucleons. The γ decay equation is

Equation:

$${}^A_Z\text{X}_N^* \rightarrow {}^A_Z\text{X}_N + \gamma_1 + \gamma_2 + \cdots$$

γ is a high-energy photon originating in a nucleus.

Conceptual Questions

Exercise:

Problem:

Star Trek fans have often heard the term “antimatter drive.” Describe how you could use a magnetic field to trap antimatter, such as produced by nuclear decay, and later combine it with matter to produce energy. Be specific about the type of antimatter, the need for vacuum storage, and the fraction of matter converted into energy.

Exercise:**Problem:**

What conservation law requires an electron’s neutrino to be produced in electron capture? Note that the electron no longer exists after it is captured by the nucleus.

Exercise:**Problem:**

Neutrinos are experimentally determined to have an extremely small mass. Huge numbers of neutrinos are created in a supernova at the same time as massive amounts of light are first produced. When the 1987A supernova occurred in the Large Magellanic Cloud, visible primarily in the Southern Hemisphere and some 100,000 light-years away from Earth, neutrinos from the explosion were observed at about the same time as the light from the blast. How could the relative arrival times of neutrinos and light be used to place limits on the mass of neutrinos?

Exercise:**Problem:**

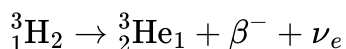
What do the three types of beta decay have in common that is distinctly different from alpha decay?

Problems & Exercises

In the following eight problems, write the complete decay equation for the given nuclide in the complete ${}^A_Z\text{X}_N$ notation. Refer to the periodic table for values of Z .

Exercise:**Problem:**

β^- decay of ${}^3\text{H}$ (tritium), a manufactured isotope of hydrogen used in some digital watch displays, and manufactured primarily for use in hydrogen bombs.

Solution:**Equation:**

Exercise:

Problem:

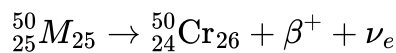
β^- decay of ^{40}K , a naturally occurring rare isotope of potassium responsible for some of our exposure to background radiation.

Exercise:

Problem: β^+ decay of ^{50}Mn .

Solution:

Equation:



Exercise:

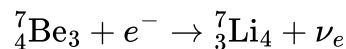
Problem: β^+ decay of ^{52}Fe .

Exercise:

Problem: Electron capture by ^7Be .

Solution:

Equation:



Exercise:

Problem: Electron capture by ^{106}In .

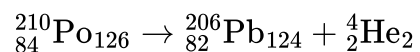
Exercise:

Problem:

α decay of ^{210}Po , the isotope of polonium in the decay series of ^{238}U that was discovered by the Curies. A favorite isotope in physics labs, since it has a short half-life and decays to a stable nuclide.

Solution:

Equation:



Exercise:**Problem:**

α decay of ^{226}Ra , another isotope in the decay series of ^{238}U , first recognized as a new element by the Curies. Poses special problems because its daughter is a radioactive noble gas.

In the following four problems, identify the parent nuclide and write the complete decay equation in the ${}^A_Z\text{X}_N$ notation. Refer to the periodic table for values of Z .

Exercise:**Problem:**

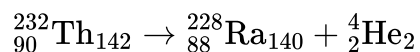
β^- decay producing ^{137}Ba . The parent nuclide is a major waste product of reactors and has chemistry similar to potassium and sodium, resulting in its concentration in your cells if ingested.

Solution:**Equation:****Exercise:****Problem:**

β^- decay producing ^{90}Y . The parent nuclide is a major waste product of reactors and has chemistry similar to calcium, so that it is concentrated in bones if ingested (^{90}Y is also radioactive.)

Exercise:**Problem:**

α decay producing ^{228}Ra . The parent nuclide is nearly 100% of the natural element and is found in gas lantern mantles and in metal alloys used in jets (^{228}Ra is also radioactive).

Solution:**Equation:****Exercise:**

Problem:

α decay producing ^{208}Pb . The parent nuclide is in the decay series produced by ^{232}Th , the only naturally occurring isotope of thorium.

Exercise:**Problem:**

When an electron and positron annihilate, both their masses are destroyed, creating two equal energy photons to preserve momentum. (a) Confirm that the annihilation equation $e^+ + e^- \rightarrow \gamma + \gamma$ conserves charge, electron family number, and total number of nucleons. To do this, identify the values of each before and after the annihilation. (b) Find the energy of each γ ray, assuming the electron and positron are initially nearly at rest. (c) Explain why the two γ rays travel in exactly opposite directions if the center of mass of the electron-positron system is initially at rest.

Solution:

(a)

charge: $(+1) + (-1) = 0$; electron family number: $(+1) + (-1) = 0$; A : $0 + 0 = 0$

(b) 0.511 MeV

(c) The two γ rays must travel in exactly opposite directions in order to conserve momentum, since initially there is zero momentum if the center of mass is initially at rest.

Exercise:**Problem:**

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for α decay given in the equation ${}_Z^AX_N \rightarrow {}_{Z-2}^{A-4}Y_{N-2} + {}_2^4\text{He}_2$. To do this, identify the values of each before and after the decay.

Exercise:**Problem:**

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for β^- decay given in the equation ${}_Z^AX_N \rightarrow {}_{Z+1}^AY_{N-1} + \beta^- + \nu_e$. To do this, identify the values of each before and after the decay.

Solution:**Equation:**

$$Z = (Z + 1) - 1; \quad A = A; \quad \text{efn} : 0 = (+1) + (-1)$$

Exercise:**Problem:**

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for β^- decay given in the equation ${}^A_Z\text{X}_N \rightarrow {}^A_{Z-1}\text{Y}_{N-1} + \beta^- + \nu_e$. To do this, identify the values of each before and after the decay.

Exercise:**Problem:**

Confirm that charge, electron family number, and the total number of nucleons are all conserved by the rule for electron capture given in the equation ${}^A_Z\text{X}_N + e^- \rightarrow {}^A_{Z-1}\text{Y}_{N+1} + \nu_e$. To do this, identify the values of each before and after the capture.

Solution:**Equation:**

$$Z - 1 = Z - 1; \quad A = A; \quad \text{efn} : (+1) = (+1)$$

Exercise:**Problem:**

A rare decay mode has been observed in which ${}^{222}\text{Ra}$ emits a ${}^{14}\text{C}$ nucleus. (a) The decay equation is ${}^{222}\text{Ra} \rightarrow {}^A\text{X} + {}^{14}\text{C}$. Identify the nuclide ${}^A\text{X}$. (b) Find the energy emitted in the decay. The mass of ${}^{222}\text{Ra}$ is 222.015353 u.

Exercise:

Problem: (a) Write the complete α decay equation for ${}^{226}\text{Ra}$.

(b) Find the energy released in the decay.

Solution:

(a) ${}^{226}_{88}\text{Ra}_{138} \rightarrow {}^{222}_{86}\text{Rn}_{136} + {}^4_2\text{He}_2$

(b) 4.87 MeV

Exercise:

Problem: (a) Write the complete α decay equation for ${}^{249}\text{Cf}$.

(b) Find the energy released in the decay.

Exercise:**Problem:**

(a) Write the complete β^- decay equation for the neutron. (b) Find the energy released in the decay.

Solution:

(a) $n \rightarrow p + \beta^- + \nu_e$

(b)) 0.783 MeV

Exercise:**Problem:**

(a) Write the complete β^- decay equation for ^{90}Sr , a major waste product of nuclear reactors. (b) Find the energy released in the decay.

Exercise:**Problem:**

Calculate the energy released in the β^+ decay of ^{22}Na , the equation for which is given in the text. The masses of ^{22}Na and ^{22}Ne are 21.994434 and 21.991383 u, respectively.

Solution:

1.82 MeV

Exercise:

Problem: (a) Write the complete β^+ decay equation for ^{11}C .

(b) Calculate the energy released in the decay. The masses of ^{11}C and ^{11}B are 11.011433 and 11.009305 u, respectively.

Exercise:

Problem: (a) Calculate the energy released in the α decay of ^{238}U .

(b) What fraction of the mass of a single ^{238}U is destroyed in the decay? The mass of ^{234}Th is 234.043593 u.

(c) Although the fractional mass loss is large for a single nucleus, it is difficult to observe for an entire macroscopic sample of uranium. Why is this?

Solution:

(a) 4.274 MeV

(b) 1.927×10^{-5}

(c) Since U-238 is a slowly decaying substance, only a very small number of nuclei decay on human timescales; therefore, although those nuclei that decay lose a noticeable fraction of their mass, the change in the total mass of the sample is not detectable for a macroscopic sample.

Exercise:

Problem: (a) Write the complete reaction equation for electron capture by ${}^7\text{Be}$.

(b) Calculate the energy released.

Exercise:

Problem: (a) Write the complete reaction equation for electron capture by ${}^{15}\text{O}$.

(b) Calculate the energy released.

Solution:

(a) ${}^{15}_8\text{O}_7 + e^- \rightarrow {}^{15}_7\text{N}_8 + \nu_e$

(b) 2.754 MeV

Glossary

parent

the original state of nucleus before decay

daughter

the nucleus obtained when parent nucleus decays and produces another nucleus following the rules and the conservation laws

positron

the particle that results from positive beta decay; also known as an antielectron

decay

the process by which an atomic nucleus of an unstable atom loses mass and energy by emitting ionizing particles

alpha decay

type of radioactive decay in which an atomic nucleus emits an alpha particle

beta decay

type of radioactive decay in which an atomic nucleus emits a beta particle

gamma decay

type of radioactive decay in which an atomic nucleus emits a gamma particle

decay equation

the equation to find out how much of a radioactive material is left after a given period of time

nuclear reaction energy

the energy created in a nuclear reaction

neutrino

an electrically neutral, weakly interacting elementary subatomic particle

electron's antineutrino

antiparticle of electron's neutrino

positron decay

type of beta decay in which a proton is converted to a neutron, releasing a positron and a neutrino

antielectron

another term for positron

decay series

process whereby subsequent nuclides decay until a stable nuclide is produced

electron's neutrino

a subatomic elementary particle which has no net electric charge

antimatter

composed of antiparticles

electron capture

the process in which a proton-rich nuclide absorbs an inner atomic electron and simultaneously emits a neutrino

electron capture equation

equation representing the electron capture

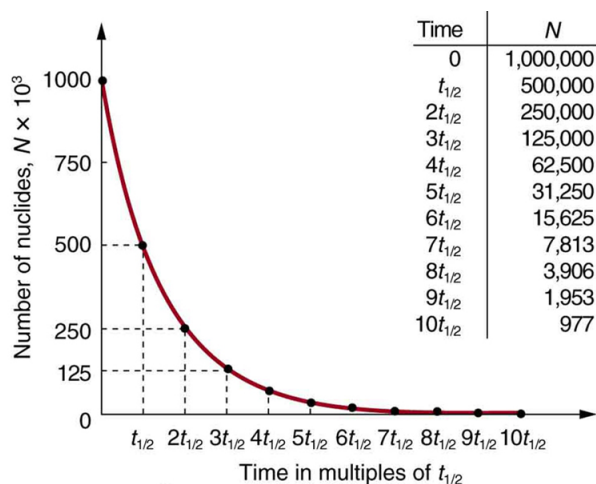
Half-Life and Activity

- Define half-life.
- Define dating.
- Calculate age of old objects by radioactive dating.

Unstable nuclei decay. However, some nuclides decay faster than others. For example, radium and polonium, discovered by the Curies, decay faster than uranium. This means they have shorter lifetimes, producing a greater rate of decay. In this section we explore half-life and activity, the quantitative terms for lifetime and rate of decay.

Half-Life

Why use a term like half-life rather than lifetime? The answer can be found by examining [\[link\]](#), which shows how the number of radioactive nuclei in a sample decreases with time. The *time in which half of the original number of nuclei decay* is defined as the **half-life**, $t_{1/2}$. Half of the remaining nuclei decay in the next half-life. Further, half of that amount decays in the following half-life. Therefore, the number of radioactive nuclei decreases from N to $N/2$ in one half-life, then to $N/4$ in the next, and to $N/8$ in the next, and so on. If N is a large number, then *many* half-lives (not just two) pass before all of the nuclei decay. Nuclear decay is an example of a purely statistical process. A more precise definition of half-life is that *each nucleus has a 50% chance of living for a time equal to one half-life $t_{1/2}$* . Thus, if N is reasonably large, half of the original nuclei decay in a time of one half-life. If an individual nucleus makes it through that time, it still has a 50% chance of surviving through another half-life. Even if it happens to make it through hundreds of half-lives, it still has a 50% chance of surviving through one more. The probability of decay is the same no matter when you start counting. This is like random coin flipping. The chance of heads is 50%, no matter what has happened before.



Radioactive decay reduces the number of radioactive nuclei over time. In one half-life $t_{1/2}$, the number decreases to half of its original value. Half of what remains decay in the next half-life, and half of those in the next, and so on. This is an exponential decay, as seen in the graph of the number of nuclei present as a function of time.

There is a tremendous range in the half-lives of various nuclides, from as short as 10^{-23} s for the most unstable, to more than 10^{16} y for the least unstable, or about 46 orders of magnitude. Nuclides with the shortest half-lives are those for which the nuclear forces are least attractive, an indication of the extent to which the nuclear force can depend on the particular combination of neutrons and protons. The concept of half-life is applicable to other subatomic particles, as will be discussed in [Particle Physics](#). It is also applicable to the decay of excited states in atoms and nuclei. The following equation gives the quantitative relationship between the original

number of nuclei present at time zero (N_0) and the number (N) at a later time t :

Equation:

$$N = N_0 e^{-\lambda t},$$

where $e = 2.71828\dots$ is the base of the natural logarithm, and λ is the **decay constant** for the nuclide. The shorter the half-life, the larger is the value of λ , and the faster the exponential $e^{-\lambda t}$ decreases with time. The relationship between the decay constant λ and the half-life $t_{1/2}$ is

Equation:

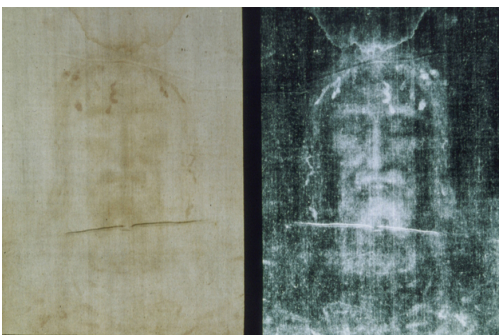
$$\lambda = \frac{\ln(2)}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}.$$

To see how the number of nuclei declines to half its original value in one half-life, let $t = t_{1/2}$ in the exponential in the equation $N = N_0 e^{-\lambda t}$. This gives $N = N_0 e^{-\lambda t} = N_0 e^{-0.693} = 0.500 N_0$. For integral numbers of half-lives, you can just divide the original number by 2 over and over again, rather than using the exponential relationship. For example, if ten half-lives have passed, we divide N by 2 ten times. This reduces it to $N/1024$. For an arbitrary time, not just a multiple of the half-life, the exponential relationship must be used.

Radioactive dating is a clever use of naturally occurring radioactivity. Its most famous application is **carbon-14 dating**. Carbon-14 has a half-life of 5730 years and is produced in a nuclear reaction induced when solar neutrinos strike ^{14}N in the atmosphere. Radioactive carbon has the same chemistry as stable carbon, and so it mixes into the ecosphere, where it is consumed and becomes part of every living organism. Carbon-14 has an abundance of 1.3 parts per trillion of normal carbon. Thus, if you know the number of carbon nuclei in an object (perhaps determined by mass and Avogadro's number), you multiply that number by 1.3×10^{-12} to find the number of ^{14}C nuclei in the object. When an organism dies, carbon exchange with the environment ceases, and ^{14}C is not replenished as it

decays. By comparing the abundance of ^{14}C in an artifact, such as mummy wrappings, with the normal abundance in living tissue, it is possible to determine the artifact's age (or time since death). Carbon-14 dating can be used for biological tissues as old as 50 or 60 thousand years, but is most accurate for younger samples, since the abundance of ^{14}C nuclei in them is greater. Very old biological materials contain no ^{14}C at all. There are instances in which the date of an artifact can be determined by other means, such as historical knowledge or tree-ring counting. These cross-references have confirmed the validity of carbon-14 dating and permitted us to calibrate the technique as well. Carbon-14 dating revolutionized parts of archaeology and is of such importance that it earned the 1960 Nobel Prize in chemistry for its developer, the American chemist Willard Libby (1908–1980).

One of the most famous cases of carbon-14 dating involves the Shroud of Turin, a long piece of fabric purported to be the burial shroud of Jesus (see [\[link\]](#)). This relic was first displayed in Turin in 1354 and was denounced as a fraud at that time by a French bishop. Its remarkable negative imprint of an apparently crucified body resembles the then-accepted image of Jesus, and so the shroud was never disregarded completely and remained controversial over the centuries. Carbon-14 dating was not performed on the shroud until 1988, when the process had been refined to the point where only a small amount of material needed to be destroyed. Samples were tested at three independent laboratories, each being given four pieces of cloth, with only one unidentified piece from the shroud, to avoid prejudice. All three laboratories found samples of the shroud contain 92% of the ^{14}C found in living tissues, allowing the shroud to be dated (see [\[link\]](#)).



Part of the Shroud of Turin, which shows a remarkable negative imprint likeness of Jesus complete with evidence of crucifixion wounds. The shroud first surfaced in the 14th century and was only recently carbon-14 dated. It has not been determined how the image was placed on the material. (credit: Butko, Wikimedia Commons)

Example:**How Old Is the Shroud of Turin?**

Calculate the age of the Shroud of Turin given that the amount of ^{14}C found in it is 92% of that in living tissue.

Strategy

Knowing that 92% of the ^{14}C remains means that $N/N_0 = 0.92$.

Therefore, the equation $N = N_0 e^{-\lambda t}$ can be used to find λt . We also know that the half-life of ^{14}C is 5730 y, and so once λt is known, we can use the equation $\lambda = \frac{0.693}{t_{1/2}}$ to find λ and then find t as requested. Here, we postulate that the decrease in ^{14}C is solely due to nuclear decay.

Solution

Solving the equation $N = N_0 e^{-\lambda t}$ for N/N_0 gives

Equation:

$$\frac{N}{N_0} = e^{-\lambda t}.$$

Thus,

Equation:

$$0.92 = e^{-\lambda t}.$$

Taking the natural logarithm of both sides of the equation yields

Equation:

$$\ln 0.92 = -\lambda t$$

so that

Equation:

$$-0.0834 = -\lambda t.$$

Rearranging to isolate t gives

Equation:

$$t = \frac{0.0834}{\lambda}.$$

Now, the equation $\lambda = \frac{0.693}{t_{1/2}}$ can be used to find λ for ^{14}C . Solving for λ and substituting the known half-life gives

Equation:

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5730 \text{ y}}.$$

We enter this value into the previous equation to find t :

Equation:

$$t = \frac{0.0834}{\frac{0.693}{5730 \text{ y}}} = 690 \text{ y}.$$

Discussion

This dates the material in the shroud to $1988 - 690 = \text{a.d. } 1300$. Our calculation is only accurate to two digits, so that the year is rounded to 1300. The values obtained at the three independent laboratories gave a

weighted average date of a.d. 1320 ± 60 . The uncertainty is typical of carbon-14 dating and is due to the small amount of ^{14}C in living tissues, the amount of material available, and experimental uncertainties (reduced by having three independent measurements). It is meaningful that the date of the shroud is consistent with the first record of its existence and inconsistent with the period in which Jesus lived.

There are other forms of radioactive dating. Rocks, for example, can sometimes be dated based on the decay of ^{238}U . The decay series for ^{238}U ends with ^{206}Pb , so that the ratio of these nuclides in a rock is an indication of how long it has been since the rock solidified. The original composition of the rock, such as the absence of lead, must be known with some confidence. However, as with carbon-14 dating, the technique can be verified by a consistent body of knowledge. Since ^{238}U has a half-life of 4.5×10^9 y, it is useful for dating only very old materials, showing, for example, that the oldest rocks on Earth solidified about 3.5×10^9 years ago.

Activity, the Rate of Decay

What do we mean when we say a source is highly radioactive? Generally, this means the number of decays per unit time is very high. We define **activity** R to be the **rate of decay** expressed in decays per unit time. In equation form, this is

Equation:

$$R = \frac{\Delta N}{\Delta t}$$

where ΔN is the number of decays that occur in time Δt . The SI unit for activity is one decay per second and is given the name **becquerel** (Bq) in honor of the discoverer of radioactivity. That is,

Equation:

$$1 \text{ Bq} = 1 \text{ decay/s.}$$

Activity R is often expressed in other units, such as decays per minute or decays per year. One of the most common units for activity is the **curie** (Ci), defined to be the activity of 1 g of ^{226}Ra , in honor of Marie Curie's work with radium. The definition of curie is

Equation:

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq,}$$

or 3.70×10^{10} decays per second. A curie is a large unit of activity, while a becquerel is a relatively small unit. $1 \text{ MBq} = 100$ microcuries (μCi). In countries like Australia and New Zealand that adhere more to SI units, most radioactive sources, such as those used in medical diagnostics or in physics laboratories, are labeled in Bq or megabecquerel (MBq).

Intuitively, you would expect the activity of a source to depend on two things: the amount of the radioactive substance present, and its half-life. The greater the number of radioactive nuclei present in the sample, the more will decay per unit of time. The shorter the half-life, the more decays per unit time, for a given number of nuclei. So activity R should be proportional to the number of radioactive nuclei, N , and inversely proportional to their half-life, $t_{1/2}$. In fact, your intuition is correct. It can be shown that the activity of a source is

Equation:

$$R = \frac{0.693N}{t_{1/2}}$$

where N is the number of radioactive nuclei present, having half-life $t_{1/2}$. This relationship is useful in a variety of calculations, as the next two examples illustrate.

Example:**How Great Is the ^{14}C Activity in Living Tissue?**

Calculate the activity due to ^{14}C in 1.00 kg of carbon found in a living organism. Express the activity in units of Bq and Ci.

Strategy

To find the activity R using the equation $R = \frac{0.693N}{t_{1/2}}$, we must know N and $t_{1/2}$. The half-life of ^{14}C can be found in [Appendix B](#), and was stated above as 5730 y. To find N , we first find the number of ^{12}C nuclei in 1.00 kg of carbon using the concept of a mole. As indicated, we then multiply by 1.3×10^{-12} (the abundance of ^{14}C in a carbon sample from a living organism) to get the number of ^{14}C nuclei in a living organism.

Solution

One mole of carbon has a mass of 12.0 g, since it is nearly pure ^{12}C . (A mole has a mass in grams equal in magnitude to A found in the periodic table.) Thus the number of carbon nuclei in a kilogram is

Equation:

$$N(^{12}\text{C}) = \frac{6.02 \times 10^{23} \text{ mol}^{-1}}{12.0 \text{ g/mol}} \times (1000 \text{ g}) = 5.02 \times 10^{25}.$$

So the number of ^{14}C nuclei in 1 kg of carbon is

Equation:

$$N(^{14}\text{C}) = (5.02 \times 10^{25})(1.3 \times 10^{-12}) = 6.52 \times 10^{13}.$$

Now the activity R is found using the equation $R = \frac{0.693N}{t_{1/2}}$.

Entering known values gives

Equation:

$$R = \frac{0.693(6.52 \times 10^{13})}{5730 \text{ y}} = 7.89 \times 10^9 \text{ y}^{-1},$$

or 7.89×10^9 decays per year. To convert this to the unit Bq, we simply convert years to seconds. Thus,

Equation:

$$R = (7.89 \times 10^9 \text{ y}^{-1}) \frac{1.00 \text{ y}}{3.16 \times 10^7 \text{ s}} = 250 \text{ Bq},$$

or 250 decays per second. To express R in curies, we use the definition of a curie,

Equation:

$$R = \frac{250 \text{ Bq}}{3.7 \times 10^{10} \text{ Bq/Ci}} = 6.76 \times 10^{-9} \text{ Ci}.$$

Thus,

Equation:

$$R = 6.76 \text{ nCi}.$$

Discussion

Our own bodies contain kilograms of carbon, and it is intriguing to think there are hundreds of ^{14}C decays per second taking place in us. Carbon-14 and other naturally occurring radioactive substances in our bodies contribute to the background radiation we receive. The small number of decays per second found for a kilogram of carbon in this example gives you some idea of how difficult it is to detect ^{14}C in a small sample of material. If there are 250 decays per second in a kilogram, then there are 0.25 decays per second in a gram of carbon in living tissue. To observe this, you must be able to distinguish decays from other forms of radiation, in order to reduce background noise. This becomes more difficult with an old tissue sample, since it contains less ^{14}C , and for samples more than 50 thousand years old, it is impossible.

Human-made (or artificial) radioactivity has been produced for decades and has many uses. Some of these include medical therapy for cancer, medical imaging and diagnostics, and food preservation by irradiation. Many applications as well as the biological effects of radiation are explored in [Medical Applications of Nuclear Physics](#), but it is clear that radiation is hazardous. A number of tragic examples of this exist, one of the most disastrous being the meltdown and fire at the Chernobyl reactor complex in

the Ukraine (see [\[link\]](#)). Several radioactive isotopes were released in huge quantities, contaminating many thousands of square kilometers and directly affecting hundreds of thousands of people. The most significant releases were of ^{131}I , ^{90}Sr , ^{137}Cs , ^{239}Pu , ^{238}U , and ^{235}U . Estimates are that the total amount of radiation released was about 100 million curies.

Human and Medical Applications



The Chernobyl reactor.
More than 100 people
died soon after its
meltdown, and there will
be thousands of deaths
from radiation-induced
cancer in the future.

While the accident was
due to a series of human
errors, the cleanup efforts
were heroic. Most of the
immediate fatalities were
firefighters and reactor
personnel. (credit: Elena
Filatova)

Example:**What Mass of ^{137}Cs Escaped Chernobyl?**

It is estimated that the Chernobyl disaster released 6.0 MCi of ^{137}Cs into the environment. Calculate the mass of ^{137}Cs released.

Strategy

We can calculate the mass released using Avogadro's number and the concept of a mole if we can first find the number of nuclei N released.

Since the activity R is given, and the half-life of ^{137}Cs is found in [Appendix B](#) to be 30.2 y, we can use the equation $R = \frac{0.693N}{t_{1/2}}$ to find N .

Solution

Solving the equation $R = \frac{0.693N}{t_{1/2}}$ for N gives

Equation:

$$N = \frac{Rt_{1/2}}{0.693}.$$

Entering the given values yields

Equation:

$$N = \frac{(6.0 \text{ MCi})(30.2 \text{ y})}{0.693}.$$

Converting curies to becquerels and years to seconds, we get

Equation:

$$\begin{aligned} N &= \frac{(6.0 \times 10^6 \text{ Ci})(3.7 \times 10^{10} \text{ Bq/Ci})(30.2 \text{ y})(3.16 \times 10^7 \text{ s/y})}{0.693} \\ &= 3.1 \times 10^{26}. \end{aligned}$$

One mole of a nuclide ^AX has a mass of A grams, so that one mole of ^{137}Cs has a mass of 137 g. A mole has 6.02×10^{23} nuclei. Thus the mass of ^{137}Cs released was

Equation:

$$\begin{aligned} m &= \left(\frac{137 \text{ g}}{6.02 \times 10^{23}} \right) (3.1 \times 10^{26}) = 70 \times 10^3 \text{ g} \\ &= 70 \text{ kg}. \end{aligned}$$

Discussion

While 70 kg of material may not be a very large mass compared to the amount of fuel in a power plant, it is extremely radioactive, since it only has a 30-year half-life. Six megacuries (6.0 MCi) is an extraordinary amount of activity but is only a fraction of what is produced in nuclear reactors. Similar amounts of the other isotopes were also released at Chernobyl. Although the chances of such a disaster may have seemed small, the consequences were extremely severe, requiring greater caution than was used. More will be said about safe reactor design in the next chapter, but it should be noted that Western reactors have a fundamentally safer design.

Activity R decreases in time, going to half its original value in one half-life, then to one-fourth its original value in the next half-life, and so on. Since $R = \frac{0.693N}{t_{1/2}}$, the activity decreases as the number of radioactive nuclei decreases. The equation for R as a function of time is found by combining the equations $N = N_0 e^{-\lambda t}$ and $R = \frac{0.693N}{t_{1/2}}$, yielding

Equation:

$$R = R_0 e^{-\lambda t},$$

where R_0 is the activity at $t = 0$. This equation shows exponential decay of radioactive nuclei. For example, if a source originally has a 1.00-mCi activity, it declines to 0.500 mCi in one half-life, to 0.250 mCi in two half-lives, to 0.125 mCi in three half-lives, and so on. For times other than whole half-lives, the equation $R = R_0 e^{-\lambda t}$ must be used to find R .

Note:

PhET Explorations: Alpha Decay

Watch alpha particles escape from a polonium nucleus, causing radioactive alpha decay. See how random decay times relate to the half life.

Section Summary

- Half-life $t_{1/2}$ is the time in which there is a 50% chance that a nucleus will decay. The number of nuclei N as a function of time is

Equation:

$$N = N_0 e^{-\lambda t},$$

where N_0 is the number present at $t = 0$, and λ is the decay constant, related to the half-life by

Equation:

$$\lambda = \frac{0.693}{t_{1/2}}.$$

- One of the applications of radioactive decay is radioactive dating, in which the age of a material is determined by the amount of radioactive decay that occurs. The rate of decay is called the activity R :

Equation:

$$R = \frac{\Delta N}{\Delta t}.$$

- The SI unit for R is the becquerel (Bq), defined by
- Equation:**

$$1 \text{ Bq} = 1 \text{ decay/s}.$$

- R is also expressed in terms of curies (Ci), where

Equation:

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}.$$

- The activity R of a source is related to N and $t_{1/2}$ by

Equation:

$$R = \frac{0.693N}{t_{1/2}}.$$

- Since N has an exponential behavior as in the equation $N = N_0 e^{-\lambda t}$, the activity also has an exponential behavior, given by

Equation:

$$R = R_0 e^{-\lambda t},$$

where R_0 is the activity at $t = 0$.

Conceptual Questions

Exercise:

Problem:

In a 3×10^9 -year-old rock that originally contained some ^{238}U , which has a half-life of 4.5×10^9 years, we expect to find some ^{238}U remaining in it. Why are ^{226}Ra , ^{222}Rn , and ^{210}Po also found in such a rock, even though they have much shorter half-lives (1600 years, 3.8 days, and 138 days, respectively)?

Exercise:

Problem:

Does the number of radioactive nuclei in a sample decrease to *exactly* half its original value in one half-life? Explain in terms of the statistical nature of radioactive decay.

Exercise:

Problem:

Radioactivity depends on the nucleus and not the atom or its chemical state. Why, then, is one kilogram of uranium more radioactive than one kilogram of uranium hexafluoride?

Exercise:**Problem:**

Explain how a bound system can have less mass than its components. Why is this not observed classically, say for a building made of bricks?

Exercise:**Problem:**

Spontaneous radioactive decay occurs only when the decay products have less mass than the parent, and it tends to produce a daughter that is more stable than the parent. Explain how this is related to the fact that more tightly bound nuclei are more stable. (Consider the binding energy per nucleon.)

Exercise:**Problem:**

To obtain the most precise value of BE from the equation $BE = [ZM(^1\text{H}) + Nm_n]c^2 - m(^A\text{X})c^2$, we should take into account the binding energy of the electrons in the neutral atoms. Will doing this produce a larger or smaller value for BE? Why is this effect usually negligible?

Exercise:**Problem:**

How does the finite range of the nuclear force relate to the fact that BE/A is greatest for A near 60?

Problems & Exercises

Data from the appendices and the periodic table may be needed for these problems.

Exercise:

Problem:

An old campfire is uncovered during an archaeological dig. Its charcoal is found to contain less than 1/1000 the normal amount of ^{14}C . Estimate the minimum age of the charcoal, noting that $2^{10} = 1024$.

Solution:

57,300 y

Exercise:

Problem:

A ^{60}Co source is labeled 4.00 mCi, but its present activity is found to be 1.85×10^7 Bq. (a) What is the present activity in mCi? (b) How long ago did it actually have a 4.00-mCi activity?

Exercise:

Problem:

(a) Calculate the activity R in curies of 1.00 g of ^{226}Ra . (b) Discuss why your answer is not exactly 1.00 Ci, given that the curie was originally supposed to be exactly the activity of a gram of radium.

Solution:

(a) 0.988 Ci

(b) The half-life of ^{226}Ra is now better known.

Exercise:

Problem:

Show that the activity of the ^{14}C in 1.00 g of ^{12}C found in living tissue is 0.250 Bq.

Exercise:**Problem:**

Mantles for gas lanterns contain thorium, because it forms an oxide that can survive being heated to incandescence for long periods of time. Natural thorium is almost 100% ^{232}Th , with a half-life of 1.405×10^{10} y. If an average lantern mantle contains 300 mg of thorium, what is its activity?

Solution:

$$1.22 \times 10^3 \text{ Bq}$$

Exercise:**Problem:**

Cow's milk produced near nuclear reactors can be tested for as little as 1.00 pCi of ^{131}I per liter, to check for possible reactor leakage. What mass of ^{131}I has this activity?

Exercise:**Problem:**

(a) Natural potassium contains ^{40}K , which has a half-life of 1.277×10^9 y. What mass of ^{40}K in a person would have a decay rate of 4140 Bq? (b) What is the fraction of ^{40}K in natural potassium, given that the person has 140 g in his body? (These numbers are typical for a 70-kg adult.)

Solution:

(a) 16.0 mg

(b) 0.0114%

Exercise:

Problem:

There is more than one isotope of natural uranium. If a researcher isolates 1.00 mg of the relatively scarce ^{235}U and finds this mass to have an activity of 80.0 Bq, what is its half-life in years?

Exercise:

Problem:

^{50}V has one of the longest known radioactive half-lives. In a difficult experiment, a researcher found that the activity of 1.00 kg of ^{50}V is 1.75 Bq. What is the half-life in years?

Solution:

$$1.48 \times 10^{17} \text{ y}$$

Exercise:

Problem:

You can sometimes find deep red crystal vases in antique stores, called uranium glass because their color was produced by doping the glass with uranium. Look up the natural isotopes of uranium and their half-lives, and calculate the activity of such a vase assuming it has 2.00 g of uranium in it. Neglect the activity of any daughter nuclides.

Exercise:

Problem:

A tree falls in a forest. How many years must pass before the ^{14}C activity in 1.00 g of the tree's carbon drops to 1.00 decay per hour?

Solution:

$$5.6 \times 10^4 \text{ y}$$

Exercise:**Problem:**

What fraction of the ^{40}K that was on Earth when it formed 4.5×10^9 years ago is left today?

Exercise:**Problem:**

A 5000-Ci ^{60}Co source used for cancer therapy is considered too weak to be useful when its activity falls to 3500 Ci. How long after its manufacture does this happen?

Solution:

2.71 y

Exercise:**Problem:**

Natural uranium is 0.7200% ^{235}U and 99.27% ^{238}U . What were the percentages of ^{235}U and ^{238}U in natural uranium when Earth formed 4.5×10^9 years ago?

Exercise:**Problem:**

The β^- particles emitted in the decay of ^3H (tritium) interact with matter to create light in a glow-in-the-dark exit sign. At the time of manufacture, such a sign contains 15.0 Ci of ^3H . (a) What is the mass of the tritium? (b) What is its activity 5.00 y after manufacture?

Solution:

(a) 1.56 mg

(b) 11.3 Ci

Exercise:**Problem:**

World War II aircraft had instruments with glowing radium-painted dials (see [\[link\]](#)). The activity of one such instrument was 1.0×10^5 Bq when new. (a) What mass of ^{226}Ra was present? (b) After some years, the phosphors on the dials deteriorated chemically, but the radium did not escape. What is the activity of this instrument 57.0 years after it was made?

Exercise:**Problem:**

(a) The ^{210}Po source used in a physics laboratory is labeled as having an activity of $1.0 \mu\text{Ci}$ on the date it was prepared. A student measures the radioactivity of this source with a Geiger counter and observes 1500 counts per minute. She notices that the source was prepared 120 days before her lab. What fraction of the decays is she observing with her apparatus? (b) Identify some of the reasons that only a fraction of the α s emitted are observed by the detector.

Solution:

(a) 1.23×10^{-3}

(b) Only part of the emitted radiation goes in the direction of the detector. Only a fraction of that causes a response in the detector. Some of the emitted radiation (mostly α particles) is observed within the source. Some is absorbed within the source, some is absorbed by the detector, and some does not penetrate the detector.

Exercise:

Problem:

Armor-piercing shells with depleted uranium cores are fired by aircraft at tanks. (The high density of the uranium makes them effective.) The uranium is called depleted because it has had its ^{235}U removed for reactor use and is nearly pure ^{238}U . Depleted uranium has been erroneously called non-radioactive. To demonstrate that this is wrong: (a) Calculate the activity of 60.0 g of pure ^{238}U . (b) Calculate the activity of 60.0 g of natural uranium, neglecting the ^{234}U and all daughter nuclides.

Exercise:**Problem:**

The ceramic glaze on a red-orange Fiestaware plate is U_2O_3 and contains 50.0 grams of ^{238}U , but very little ^{235}U . (a) What is the activity of the plate? (b) Calculate the total energy that will be released by the ^{238}U decay. (c) If energy is worth 12.0 cents per $\text{kW} \cdot \text{h}$, what is the monetary value of the energy emitted? (These plates went out of production some 30 years ago, but are still available as collectibles.)

Solution:

(a) $1.68 \times 10^{-5} \text{ Ci}$

(b) $8.65 \times 10^{10} \text{ J}$

(c) $\$2.9 \times 10^3$

Exercise:

Problem:

Large amounts of depleted uranium (^{238}U) are available as a by-product of uranium processing for reactor fuel and weapons. Uranium is very dense and makes good counter weights for aircraft. Suppose you have a 4000-kg block of ^{238}U . (a) Find its activity. (b) How many calories per day are generated by thermalization of the decay energy? (c) Do you think you could detect this as heat? Explain.

Exercise:**Problem:**

The *Galileo* space probe was launched on its long journey past several planets in 1989, with an ultimate goal of Jupiter. Its power source is 11.0 kg of ^{238}Pu , a by-product of nuclear weapons plutonium production. Electrical energy is generated thermoelectrically from the heat produced when the 5.59-MeV α particles emitted in each decay crash to a halt inside the plutonium and its shielding. The half-life of ^{238}Pu is 87.7 years. (a) What was the original activity of the ^{238}Pu in becquerel? (b) What power was emitted in kilowatts? (c) What power was emitted 12.0 y after launch? You may neglect any extra energy from daughter nuclides and any losses from escaping γ rays.

Solution:

(a) $6.97 \times 10^{15} \text{ Bq}$

(b) 6.24 kW

(c) 5.67 kW

Exercise:**Problem: Construct Your Own Problem**

Consider the generation of electricity by a radioactive isotope in a space probe, such as described in [\[link\]](#). Construct a problem in which you calculate the mass of a radioactive isotope you need in order to

supply power for a long space flight. Among the things to consider are the isotope chosen, its half-life and decay energy, the power needs of the probe and the length of the flight.

Exercise:

Problem: Unreasonable Results

A nuclear physicist finds $1.0\ \mu\text{g}$ of ^{236}U in a piece of uranium ore and assumes it is primordial since its half-life is $2.3 \times 10^7\ \text{y}$. (a) Calculate the amount of ^{236}U that would have had to have been on Earth when it formed $4.5 \times 10^9\ \text{y}$ ago for $1.0\ \mu\text{g}$ to be left today. (b) What is unreasonable about this result? (c) What assumption is responsible?

Exercise:

Problem: Unreasonable Results

(a) Repeat [\[link\]](#) but include the 0.0055% natural abundance of ^{234}U with its $2.45 \times 10^5\ \text{y}$ half-life. (b) What is unreasonable about this result? (c) What assumption is responsible? (d) Where does the ^{234}U come from if it is not primordial?

Exercise:

Problem: Unreasonable Results

The manufacturer of a smoke alarm decides that the smallest current of α radiation he can detect is $1.00\ \mu\text{A}$. (a) Find the activity in curies of an α emitter that produces a $1.00\ \mu\text{A}$ current of α particles. (b) What is unreasonable about this result? (c) What assumption is responsible?

Solution:

(a) 84.5 Ci

(b) An extremely large activity, many orders of magnitude greater than permitted for home use.

(c) The assumption of $1.00\ \mu\text{A}$ is unreasonably large. Other methods can detect much smaller decay rates.

Glossary

becquerel

SI unit for rate of decay of a radioactive material

half-life

the time in which there is a 50% chance that a nucleus will decay

radioactive dating

an application of radioactive decay in which the age of a material is determined by the amount of radioactivity of a particular type that occurs

decay constant

quantity that is inversely proportional to the half-life and that is used in equation for number of nuclei as a function of time

carbon-14 dating

a radioactive dating technique based on the radioactivity of carbon-14

activity

the rate of decay for radioactive nuclides

rate of decay

the number of radioactive events per unit time

curie

the activity of 1g of ^{226}Ra , equal to $3.70 \times 10^{10}\ \text{Bq}$

Binding Energy

- Define and discuss binding energy.
- Calculate the binding energy per nucleon of a particle.

The more tightly bound a system is, the stronger the forces that hold it together and the greater the energy required to pull it apart. We can therefore learn about nuclear forces by examining how tightly bound the nuclei are. We define the **binding energy** (BE) of a nucleus to be *the energy required to completely disassemble it into separate protons and neutrons*. We can determine the BE of a nucleus from its rest mass. The two are connected through Einstein's famous relationship $E = (\Delta m)c^2$. A bound system has a *smaller* mass than its separate constituents; the more tightly the nucleons are bound together, the smaller the mass of the nucleus.

Imagine pulling a nuclide apart as illustrated in [\[link\]](#). Work done to overcome the nuclear forces holding the nucleus together puts energy into the system. By definition, the energy input equals the binding energy BE. The pieces are at rest when separated, and so the energy put into them increases their total rest mass compared with what it was when they were glued together as a nucleus. That mass increase is thus $\Delta m = \text{BE}/c^2$. This difference in mass is known as *mass defect*. It implies that the mass of the nucleus is less than the sum of the masses of its constituent protons and neutrons. A nuclide ${}^A\text{X}$ has Z protons and N neutrons, so that the difference in mass is

Equation:

$$\Delta m = (Zm_p + Nm_n) - m_{\text{tot}}.$$

Thus,

Equation:

$$\text{BE} = (\Delta m)c^2 = [(Zm_p + Nm_n) - m_{\text{tot}}]c^2,$$

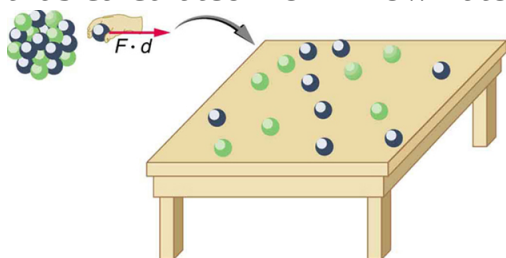
where m_{tot} is the mass of the nuclide ${}^A\text{X}$, m_p is the mass of a proton, and m_n is the mass of a neutron. Traditionally, we deal with the masses of

neutral atoms. To get atomic masses into the last equation, we first add Z electrons to m_{tot} , which gives $m(^A\text{X})$, the atomic mass of the nuclide. We then add Z electrons to the Z protons, which gives $Zm(^1\text{H})$, or Z times the mass of a hydrogen atom. Thus the binding energy of a nuclide ^AX is

Equation:

$$\text{BE} = \left\{ [Zm(^1\text{H}) + Nm_n] - m(^A\text{X}) \right\} c^2.$$

The atomic masses can be found in [Appendix A](#), most conveniently expressed in unified atomic mass units u ($1 u = 931.5 \text{ MeV}/c^2$). BE is thus calculated from known atomic masses.



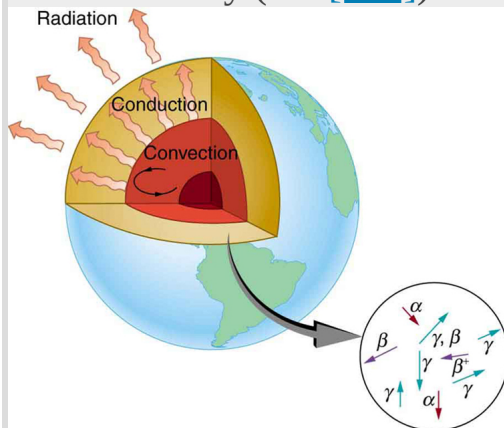
Work done to pull a nucleus apart into its constituent protons and neutrons increases the mass of the system. The work to disassemble the nucleus equals its binding energy BE. A bound system has less mass than the sum of its parts, especially noticeable in the nuclei, where forces and energies are very large.

Note:**Things Great and Small****Nuclear Decay Helps Explain Earth's Hot Interior**

A puzzle created by radioactive dating of rocks is resolved by radioactive heating of Earth's interior. This intriguing story is another example of how small-scale physics can explain large-scale phenomena.

Radioactive dating plays a role in determining the approximate age of the Earth. The oldest rocks on Earth solidified about 3.5×10^9 years ago—a number determined by uranium-238 dating. These rocks could only have solidified once the surface of the Earth had cooled sufficiently. The temperature of the Earth at formation can be estimated based on gravitational potential energy of the assemblage of pieces being converted to thermal energy. Using heat transfer concepts discussed in

[Thermodynamics](#) it is then possible to calculate how long it would take for the surface to cool to rock-formation temperatures. The result is about 10^9 years. The first rocks formed have been solid for 3.5×10^9 years, so that the age of the Earth is approximately 4.5×10^9 years. There is a large body of other types of evidence (both Earth-bound and solar system characteristics are used) that supports this age. The puzzle is that, given its age and initial temperature, the center of the Earth should be much cooler than it is today (see [\[link\]](#)).



The center of the Earth
cools by well-known heat
transfer methods.

Convection in the liquid
regions and conduction

move thermal energy to the surface, where it radiates into cold, dark space. Given the age of the Earth and its initial temperature, it should have cooled to a lower temperature by now. The blowup shows that nuclear decay releases energy in the Earth's interior. This energy has slowed the cooling process and is responsible for the interior still being molten.

We know from seismic waves produced by earthquakes that parts of the interior of the Earth are liquid. Shear or transverse waves cannot travel through a liquid and are not transmitted through the Earth's core. Yet compression or longitudinal waves can pass through a liquid and do go through the core. From this information, the temperature of the interior can be estimated. As noticed, the interior should have cooled more from its initial temperature in the 4.5×10^9 years since its formation. In fact, it should have taken no more than about 10^9 years to cool to its present temperature. What is keeping it hot? The answer seems to be radioactive decay of primordial elements that were part of the material that formed the Earth (see the blowup in [\[link\]](#)).

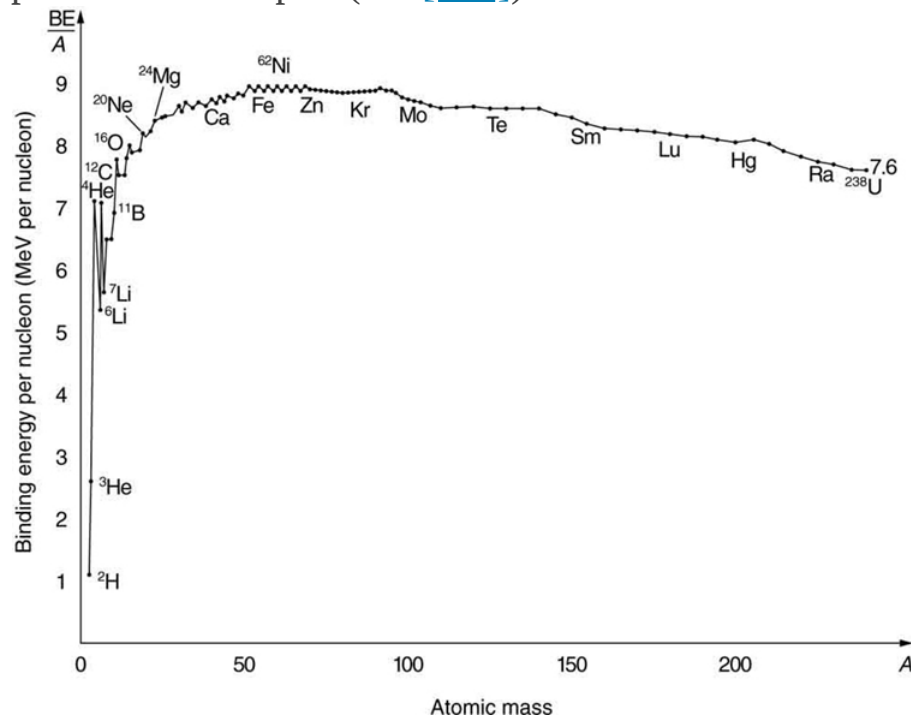
Nuclides such as ^{238}U and ^{40}K have half-lives similar to or longer than the age of the Earth, and their decay still contributes energy to the interior. Some of the primordial radioactive nuclides have unstable decay products that also release energy— ^{238}U has a long decay chain of these. Further, there were more of these primordial radioactive nuclides early in the life of the Earth, and thus the activity and energy contributed were greater then (perhaps by an order of magnitude). The amount of power created by these decays per cubic meter is very small. However, since a huge volume of

material lies deep below the surface, this relatively small amount of energy cannot escape quickly. The power produced near the surface has much less distance to go to escape and has a negligible effect on surface temperatures.

A final effect of this trapped radiation merits mention. Alpha decay produces helium nuclei, which form helium atoms when they are stopped and capture electrons. Most of the helium on Earth is obtained from wells and is produced in this manner. Any helium in the atmosphere will escape in geologically short times because of its high thermal velocity.

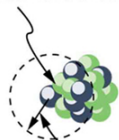
What patterns and insights are gained from an examination of the binding energy of various nuclides? First, we find that BE is approximately proportional to the number of nucleons A in any nucleus. About twice as much energy is needed to pull apart a nucleus like ^{24}Mg compared with pulling apart ^{12}C , for example. To help us look at other effects, we divide BE by A and consider the **binding energy per nucleon**, BE/A . The graph of BE/A in [\[link\]](#) reveals some very interesting aspects of nuclei. We see that the binding energy per nucleon averages about 8 MeV, but is lower for both the lightest and heaviest nuclei. This overall trend, in which nuclei with A equal to about 60 have the greatest BE/A and are thus the most tightly bound, is due to the combined characteristics of the attractive nuclear forces and the repulsive Coulomb force. It is especially important to note two things—the strong nuclear force is about 100 times stronger than the Coulomb force, *and* the nuclear forces are shorter in range compared to the Coulomb force. So, for low-mass nuclei, the nuclear attraction dominates and each added nucleon forms bonds with all others, causing progressively heavier nuclei to have progressively greater values of BE/A . This continues up to $A \approx 60$, roughly corresponding to the mass number of iron. Beyond that, new nucleons added to a nucleus will be too far from some others to feel their nuclear attraction. Added protons, however, feel the repulsion of all other protons, since the Coulomb force is longer in range. Coulomb repulsion grows for progressively heavier nuclei, but nuclear attraction remains about the same, and so BE/A becomes smaller. This is why stable nuclei heavier than $A \approx 40$ have more neutrons than

protons. Coulomb repulsion is reduced by having more neutrons to keep the protons farther apart (see [\[link\]](#)).



A graph of average binding energy per nucleon, BE/A , for stable nuclei. The most tightly bound nuclei are those with A near 60, where the attractive nuclear force has its greatest effect. At higher A s, the Coulomb repulsion progressively reduces the binding energy per nucleon, because the nuclear force is short ranged. The spikes on the curve are very tightly bound nuclides and indicate shell closures.

Nucleons inside range
feel nuclear force directly



Range of nuclear force

The nuclear force is

attractive and stronger than the Coulomb force, but it is short ranged. In low-mass nuclei, each nucleon feels the nuclear attraction of all others. In larger nuclei, the range of the nuclear force, shown for a single nucleon, is smaller than the size of the nucleus, but the Coulomb repulsion from all protons reaches all others. If the nucleus is large enough, the Coulomb repulsion can add to overcome the nuclear attraction.

There are some noticeable spikes on the BE/A graph, which represent particularly tightly bound nuclei. These spikes reveal further details of nuclear forces, such as confirming that closed-shell nuclei (those with magic numbers of protons or neutrons or both) are more tightly bound. The spikes also indicate that some nuclei with even numbers for Z and N , and with $Z = N$, are exceptionally tightly bound. This finding can be correlated with some of the cosmic abundances of the elements. The most common elements in the universe, as determined by observations of atomic spectra from outer space, are hydrogen, followed by ${}^4\text{He}$, with much smaller amounts of ${}^{12}\text{C}$ and other elements. It should be noted that the heavier elements are created in supernova explosions, while the lighter ones are produced by nuclear fusion during the normal life cycles of stars, as will be discussed in subsequent chapters. The most common elements have the

most tightly bound nuclei. It is also no accident that one of the most tightly bound light nuclei is ${}^4\text{He}$, emitted in α decay.

Example:

What Is BE/A for an Alpha Particle?

Calculate the binding energy per nucleon of ${}^4\text{He}$, the α particle.

Strategy

To find BE/A , we first find BE using the Equation

$\text{BE} = \{[Zm({}^1\text{H}) + Nm_n] - m({}^AX)\}c^2$ and then divide by A . This is straightforward once we have looked up the appropriate atomic masses in [Appendix A](#).

Solution

The binding energy for a nucleus is given by the equation

Equation:

$$\text{BE} = \{[Zm({}^1\text{H}) + Nm_n] - m({}^AX)\}c^2.$$

For ${}^4\text{He}$, we have $Z = N = 2$; thus,

Equation:

$$\text{BE} = \{[2m({}^1\text{H}) + 2m_n] - m({}^4\text{He})\}c^2.$$

[Appendix A](#) gives these masses as $m({}^4\text{He}) = 4.002602 \text{ u}$, $m({}^1\text{H}) = 1.007825 \text{ u}$, and $m_n = 1.008665 \text{ u}$. Thus,

Equation:

$$\text{BE} = (0.030378 \text{ u})c^2.$$

Noting that $1 \text{ u} = 931.5 \text{ MeV}/c^2$, we find

Equation:

$$\text{BE} = (0.030378)(931.5 \text{ MeV}/c^2)c^2 = 28.3 \text{ MeV}.$$

Since $A = 4$, we see that BE/A is this number divided by 4, or

Equation:

$$BE/A = 7.07 \text{ MeV/nucleon.}$$

Discussion

This is a large binding energy per nucleon compared with those for other low-mass nuclei, which have $BE/A \approx 3 \text{ MeV/nucleon}$. This indicates that ${}^4\text{He}$ is tightly bound compared with its neighbors on the chart of the nuclides. You can see the spike representing this value of BE/A for ${}^4\text{He}$ on the graph in [\[link\]](#). This is why ${}^4\text{He}$ is stable. Since ${}^4\text{He}$ is tightly bound, it has less mass than other $A = 4$ nuclei and, therefore, cannot spontaneously decay into them. The large binding energy also helps to explain why some nuclei undergo α decay. Smaller mass in the decay products can mean energy release, and such decays can be spontaneous. Further, it can happen that two protons and two neutrons in a nucleus can randomly find themselves together, experience the exceptionally large nuclear force that binds this combination, and act as a ${}^4\text{He}$ unit within the nucleus, at least for a while. In some cases, the ${}^4\text{He}$ escapes, and α decay has then taken place.

There is more to be learned from nuclear binding energies. The general trend in BE/A is fundamental to energy production in stars, and to fusion and fission energy sources on Earth, for example. This is one of the applications of nuclear physics covered in [Medical Applications of Nuclear Physics](#). The abundance of elements on Earth, in stars, and in the universe as a whole is related to the binding energy of nuclei and has implications for the continued expansion of the universe.

Problem-Solving Strategies

For Reaction And Binding Energies and Activity Calculations in Nuclear Physics

1. *Identify exactly what needs to be determined in the problem (identify the unknowns).* This will allow you to decide whether the energy of a decay or nuclear reaction is involved, for example, or whether the problem is primarily concerned with activity (rate of decay).

2. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).*
3. *For reaction and binding-energy problems, we use atomic rather than nuclear masses.* Since the masses of neutral atoms are used, you must count the number of electrons involved. If these do not balance (such as in β^+ decay), then an energy adjustment of 0.511 MeV per electron must be made. Also note that atomic masses may not be given in a problem; they can be found in tables.
4. *For problems involving activity, the relationship of activity to half-life, and the number of nuclei given in the equation $R = \frac{0.693N}{t_{1/2}}$ can be very useful.* Owing to the fact that number of nuclei is involved, you will also need to be familiar with moles and Avogadro's number.
5. *Perform the desired calculation; keep careful track of plus and minus signs as well as powers of 10.*
6. *Check the answer to see if it is reasonable: Does it make sense?*
Compare your results with worked examples and other information in the text. (Heeding the advice in Step 5 will also help you to be certain of your result.) You must understand the problem conceptually to be able to determine whether the numerical result is reasonable.

Note:**PhET Explorations: Nuclear Fission**

Start a chain reaction, or introduce non-radioactive isotopes to prevent one. Control energy production in a nuclear reactor!

<https://archive.cnx.org/specials/01caf0d0-116f-11e6-b891-abfdaa77b03b/nuclear-fission/#sim-one-nucleus>

Section Summary

- The binding energy (BE) of a nucleus is the energy needed to separate it into individual protons and neutrons. In terms of atomic masses,
Equation:

$$\text{BE} = \{[Zm(^1\text{H}) + Nm_n] - m(^A\text{X})\}c^2,$$

where $m(^1\text{H})$ is the mass of a hydrogen atom, $m(^A\text{X})$ is the atomic mass of the nuclide, and m_n is the mass of a neutron. Patterns in the binding energy per nucleon, BE/A , reveal details of the nuclear force. The larger the BE/A , the more stable the nucleus.

Conceptual Questions

Exercise:

Problem:

Why is the number of neutrons greater than the number of protons in stable nuclei having A greater than about 40, and why is this effect more pronounced for the heaviest nuclei?

Problems & Exercises

Exercise:

Problem:

^2H is a loosely bound isotope of hydrogen. Called deuterium or heavy hydrogen, it is stable but relatively rare—it is 0.015% of natural hydrogen. Note that deuterium has $Z = N$, which should tend to make it more tightly bound, but both are odd numbers. Calculate BE/A , the binding energy per nucleon, for ^2H and compare it with the approximate value obtained from the graph in [\[link\]](#).

Solution:

1.112 MeV, consistent with graph

Exercise:

Problem:

^{56}Fe is among the most tightly bound of all nuclides. It is more than 90% of natural iron. Note that ^{56}Fe has even numbers of both protons and neutrons. Calculate BE/A , the binding energy per nucleon, for ^{56}Fe and compare it with the approximate value obtained from the graph in [\[link\]](#).

Exercise:**Problem:**

^{209}Bi is the heaviest stable nuclide, and its BE/A is low compared with medium-mass nuclides. Calculate BE/A , the binding energy per nucleon, for ^{209}Bi and compare it with the approximate value obtained from the graph in [\[link\]](#).

Solution:

7.848 MeV, consistent with graph

Exercise:**Problem:**

(a) Calculate BE/A for ^{235}U , the rarer of the two most common uranium isotopes. (b) Calculate BE/A for ^{238}U . (Most of uranium is ^{238}U .) Note that ^{238}U has even numbers of both protons and neutrons. Is the BE/A of ^{238}U significantly different from that of ^{235}U ?

Exercise:**Problem:**

(a) Calculate BE/A for ^{12}C . Stable and relatively tightly bound, this nuclide is most of natural carbon. (b) Calculate BE/A for ^{14}C . Is the difference in BE/A between ^{12}C and ^{14}C significant? One is stable and common, and the other is unstable and rare.

Solution:

(a) 7.680 MeV, consistent with graph

(b) 7.520 MeV, consistent with graph. Not significantly different from value for ^{12}C , but sufficiently lower to allow decay into another nuclide that is more tightly bound.

Exercise:

Problem:

The fact that BE/A is greatest for A near 60 implies that the range of the nuclear force is about the diameter of such nuclides. (a) Calculate the diameter of an $A = 60$ nucleus. (b) Compare BE/A for ^{58}Ni and ^{90}Sr . The first is one of the most tightly bound nuclides, while the second is larger and less tightly bound.

Exercise:

Problem:

The purpose of this problem is to show in three ways that the binding energy of the electron in a hydrogen atom is negligible compared with the masses of the proton and electron. (a) Calculate the mass equivalent in u of the 13.6-eV binding energy of an electron in a hydrogen atom, and compare this with the mass of the hydrogen atom obtained from [Appendix A](#). (b) Subtract the mass of the proton given in [\[link\]](#) from the mass of the hydrogen atom given in [Appendix A](#). You will find the difference is equal to the electron's mass to three digits, implying the binding energy is small in comparison. (c) Take the ratio of the binding energy of the electron (13.6 eV) to the energy equivalent of the electron's mass (0.511 MeV). (d) Discuss how your answers confirm the stated purpose of this problem.

Solution:

(a) 1.46×10^{-8} u vs. 1.007825 u for ^1H

(b) 0.000549 u

(c) 2.66×10^{-5}

Exercise:

Problem: Unreasonable Results

A particle physicist discovers a neutral particle with a mass of 2.02733 u that he assumes is two neutrons bound together. (a) Find the binding energy. (b) What is unreasonable about this result? (c) What assumptions are unreasonable or inconsistent?

Solution:

(a) -9.315 MeV

(b) The negative binding energy implies an unbound system.

(c) This assumption that it is two bound neutrons is incorrect.

Glossary

binding energy

the energy needed to separate nucleus into individual protons and neutrons

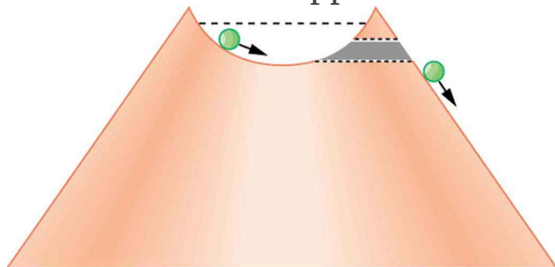
binding energy per nucleon

the binding energy calculated per nucleon; it reveals the details of the nuclear force—larger the BE/A , the more stable the nucleus

Tunneling

- Define and discuss tunneling.
- Define potential barrier.
- Explain quantum tunneling.

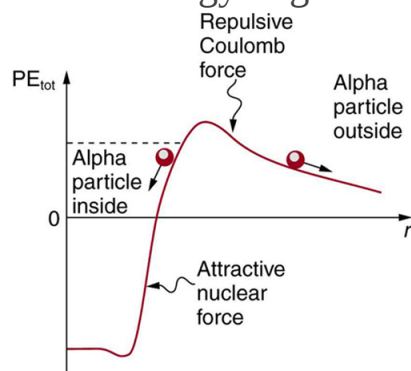
Protons and neutrons are *bound* inside nuclei, that means energy must be supplied to break them away. The situation is analogous to a marble in a bowl that can roll around but lacks the energy to get over the rim. It is bound inside the bowl (see [\[link\]](#)). If the marble could get over the rim, it would gain kinetic energy by rolling down outside. However classically, if the marble does not have enough kinetic energy to get over the rim, it remains forever trapped in its well.



The marble in this semicircular bowl at the top of a volcano has enough kinetic energy to get to the altitude of the dashed line, but not enough to get over the rim, so that it is trapped forever. If it could find a tunnel through the barrier, it would escape, roll downhill, and gain kinetic energy.

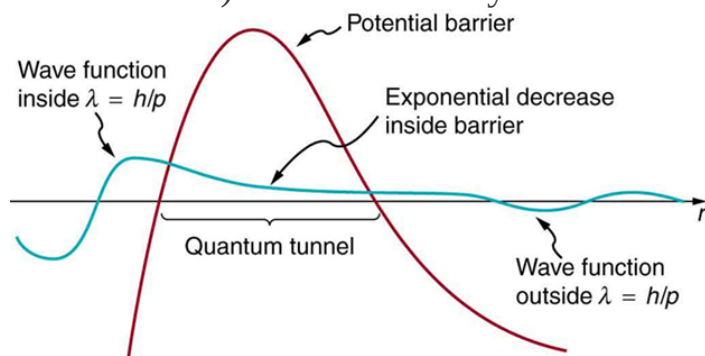
In a nucleus, the attractive nuclear potential is analogous to the bowl at the top of a volcano (where the “volcano” refers only to the shape). Protons and neutrons have kinetic energy, but it is about 8 MeV less than that needed to get out (see [\[link\]](#)). That is, they are bound by an average of 8 MeV per

nucleon. The slope of the hill outside the bowl is analogous to the repulsive Coulomb potential for a nucleus, such as for an α particle outside a positive nucleus. In α decay, two protons and two neutrons spontaneously break away as a ${}^4\text{He}$ unit. Yet the protons and neutrons do not have enough kinetic energy to get over the rim. So how does the α particle get out?



Nucleons within an atomic nucleus are bound or trapped by the attractive nuclear force, as shown in this simplified potential energy curve. An α particle outside the range of the nuclear force feels the repulsive Coulomb force. The α particle inside the nucleus does not have enough kinetic energy to get over the rim, yet it does manage to get out by quantum mechanical tunneling.

The answer was supplied in 1928 by the Russian physicist George Gamow (1904–1968). The α particle tunnels through a region of space it is forbidden to be in, and it comes out of the side of the nucleus. Like an electron making a transition between orbits around an atom, it travels from one point to another without ever having been in between. [\[link\]](#) indicates how this works. The wave function of a quantum mechanical particle varies smoothly, going from within an atomic nucleus (on one side of a potential energy barrier) to outside the nucleus (on the other side of the potential energy barrier). Inside the barrier, the wave function does not become zero but decreases exponentially, and we do not observe the particle inside the barrier. The probability of finding a particle is related to the square of its wave function, and so there is a small probability of finding the particle outside the barrier, which implies that the particle can tunnel through the barrier. This process is called **barrier penetration** or **quantum mechanical tunneling**. This concept was developed in theory by J. Robert Oppenheimer (who led the development of the first nuclear bombs during World War II) and was used by Gamow and others to describe α decay.

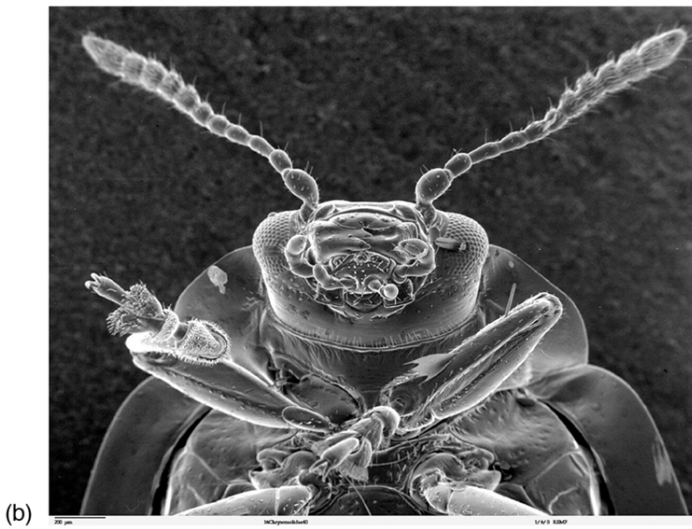
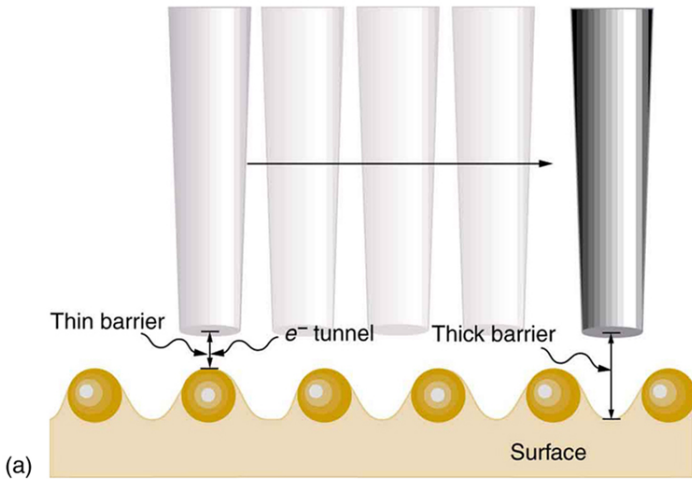


The wave function representing a quantum mechanical particle must vary smoothly, going from within the nucleus (to the left of the barrier) to outside the nucleus (to the right of the barrier). Inside the barrier, the wave function does not abruptly become zero; rather, it decreases exponentially. Outside the barrier, the

wave function is small but finite, and there it smoothly becomes sinusoidal. Owing to the fact that there is a small probability of finding the particle outside the barrier, the particle can tunnel through the barrier.

Good ideas explain more than one thing. In addition to qualitatively explaining how the four nucleons in an α particle can get out of the nucleus, the detailed theory also explains quantitatively the half-life of various nuclei that undergo α decay. This description is what Gamow and others devised, and it works for α decay half-lives that vary by 17 orders of magnitude. Experiments have shown that the more energetic the α decay of a particular nuclide is, the shorter is its half-life. **Tunneling** explains this in the following manner: For the decay to be more energetic, the nucleons must have more energy in the nucleus and should be able to ascend a little closer to the rim. The barrier is therefore not as thick for more energetic decay, and the exponential decrease of the wave function inside the barrier is not as great. Thus the probability of finding the particle outside the barrier is greater, and the half-life is shorter.

Tunneling as an effect also occurs in quantum mechanical systems other than nuclei. Electrons trapped in solids can tunnel from one object to another if the barrier between the objects is thin enough. The process is the same in principle as described for α decay. It is far more likely for a thin barrier than a thick one. Scanning tunneling electron microscopes function on this principle. The current of electrons that travels between a probe and a sample tunnels through a barrier and is very sensitive to its thickness, allowing detection of individual atoms as shown in [\[link\]](#).



(a) A scanning tunneling electron microscope can detect extremely small variations in dimensions, such as individual atoms. Electrons tunnel quantum mechanically between the probe and the sample. The probability of tunneling is extremely sensitive to barrier thickness, so that the electron current is a sensitive indicator of surface features. (b) Head and mouthparts of *Coleoptera Chrysomelidea* as seen through an electron microscope (credit: Louisa Howard, Dartmouth College)

Note:**PhET Explorations: Quantum Tunneling and Wave Packets**

Watch quantum "particles" tunnel through barriers. Explore the properties of the wave functions that describe these particles.

[Quantum
Tunnelin
g and
Wave
Packets](#)

Section Summary

- Tunneling is a quantum mechanical process of potential energy barrier penetration. The concept was first applied to explain α decay, but tunneling is found to occur in other quantum mechanical systems.

Conceptual Questions

Exercise:**Problem:**

A physics student caught breaking conservation laws is imprisoned. She leans against the cell wall hoping to tunnel out quantum mechanically. Explain why her chances are negligible. (This is so in any classical situation.)

Exercise:**Problem:**

When a nucleus α decays, does the α particle move continuously from inside the nucleus to outside? That is, does it travel each point along an imaginary line from inside to out? Explain.

Problems-Exercises**Exercise:****Problem:**

Derive an approximate relationship between the energy of α decay and half-life using the following data. It may be useful to graph the log of $t_{1/2}$ against E_α to find some straight-line relationship.

Nuclide	E_α (MeV)	$t_{1/2}$
^{216}Ra	9.5	0.18 μs
^{194}Po	7.0	0.7 s
^{240}Cm	6.4	27 d
^{226}Ra	4.91	1600 y
^{232}Th	4.1	1.4×10^{10} y

Energy and Half-Life for α Decay

Exercise:**Problem: Integrated Concepts**

A 2.00-T magnetic field is applied perpendicular to the path of charged particles in a bubble chamber. What is the radius of curvature of the path of a 10 MeV proton in this field? Neglect any slowing along its path.

Solution:

22.8 cm

Exercise:**Problem:**

(a) Write the decay equation for the α decay of ^{235}U . (b) What energy is released in this decay? The mass of the daughter nuclide is 231.036298 u. (c) Assuming the residual nucleus is formed in its ground state, how much energy goes to the α particle?

Solution:

(b) 4.679 MeV

(c) 4.599 MeV

Exercise:**Problem: Unreasonable Results**

The relatively scarce naturally occurring calcium isotope ^{48}Ca has a half-life of about 2×10^{16} y. (a) A small sample of this isotope is labeled as having an activity of 1.0 Ci. What is the mass of the ^{48}Ca in the sample? (b) What is unreasonable about this result? (c) What assumption is responsible?

Exercise:

Problem: Unreasonable Results

A physicist scatters γ rays from a substance and sees evidence of a nucleus 7.5×10^{-13} m in radius. (a) Find the atomic mass of such a nucleus. (b) What is unreasonable about this result? (c) What is unreasonable about the assumption?

Solution:

a) 2.4×10^8 u

(b) The greatest known atomic masses are about 260. This result found in (a) is extremely large.

(c) The assumed radius is much too large to be reasonable.

Exercise:

Problem: Unreasonable Results

A frazzled theoretical physicist reckons that all conservation laws are obeyed in the decay of a proton into a neutron, positron, and neutrino (as in β^+ decay of a nucleus) and sends a paper to a journal to announce the reaction as a possible end of the universe due to the spontaneous decay of protons. (a) What energy is released in this decay? (b) What is unreasonable about this result? (c) What assumption is responsible?

Solution:

(a) -1.805 MeV

(b) Negative energy implies energy input is necessary and the reaction cannot be spontaneous.

(c) Although all conservation laws are obeyed, energy must be supplied, so the assumption of spontaneous decay is incorrect.

Exercise:

Problem: Construct Your Own Problem

Consider the decay of radioactive substances in the Earth's interior. The energy emitted is converted to thermal energy that reaches the earth's surface and is radiated away into cold dark space. Construct a problem in which you estimate the activity in a cubic meter of earth rock? And then calculate the power generated. Calculate how much power must cross each square meter of the Earth's surface if the power is dissipated at the same rate as it is generated. Among the things to consider are the activity per cubic meter, the energy per decay, and the size of the Earth.

Glossary

barrier penetration

quantum mechanical effect whereby a particle has a nonzero probability to cross through a potential energy barrier despite not having sufficient energy to pass over the barrier; also called quantum mechanical tunneling

quantum mechanical tunneling

quantum mechanical effect whereby a particle has a nonzero probability to cross through a potential energy barrier despite not having sufficient energy to pass over the barrier; also called barrier penetration

tunneling

a quantum mechanical process of potential energy barrier penetration

Introduction to Applications of Nuclear Physics

class="introduction"

- Provide examples of various nuclear physics applications.

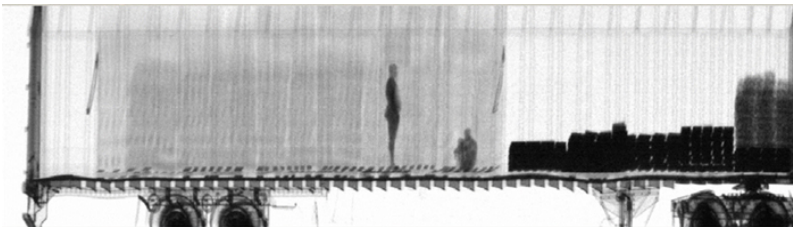
Tori Randall,
Ph.D., curator
for the
Department of
Physical
Anthropology
at the San
Diego Museum
of Man,
prepares a 550-
year-old
Peruvian child
mummy for a
CT scan at
Naval Medical
Center San
Diego. (credit:
U.S. Navy
photo by Mass
Communicatio
n Specialist 3rd
Class Samantha
A. Lewis)



Applications of nuclear physics have become an integral part of modern life. From the bone scan that detects a cancer to the radioiodine treatment that cures another, nuclear radiation has diagnostic and therapeutic effects on medicine. From the fission power reactor to the hope of controlled fusion, nuclear energy is now commonplace and is a part of our plans for the future. Yet, the destructive potential of nuclear weapons haunts us, as does the possibility of nuclear reactor accidents. Certainly, several applications of nuclear physics escape our view, as seen in [\[link\]](#). Not only has nuclear physics revealed secrets of nature, it has an inevitable impact based on its applications, as they are intertwined with human values. Because of its potential for alleviation of suffering, and its power as an ultimate destructor of life, nuclear physics is often viewed with ambivalence. But it provides perhaps the best example that applications can be good or evil, while knowledge itself is neither.



Customs officers inspect vehicles using neutron irradiation. Cars and trucks pass through portable x-ray machines that reveal their contents. (credit: Gerald L. Nino, CBP, U.S. Dept. of Homeland Security)

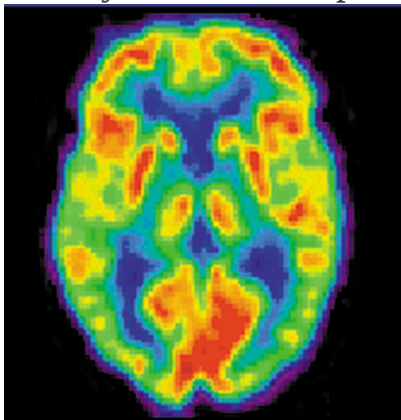


This image shows two stowaways caught illegally entering the United States from Canada. (credit: U.S. Customs and Border Protection)

Medical Imaging and Diagnostics

- Explain the working principle behind an anger camera.
- Describe the SPECT and PET imaging techniques.

A host of medical imaging techniques employ nuclear radiation. What makes nuclear radiation so useful? First, γ radiation can easily penetrate tissue; hence, it is a useful probe to monitor conditions inside the body. Second, nuclear radiation depends on the nuclide and not on the chemical compound it is in, so that a radioactive nuclide can be put into a compound designed for specific purposes. The compound is said to be **tagged**. A tagged compound used for medical purposes is called a **radiopharmaceutical**. Radiation detectors external to the body can determine the location and concentration of a radiopharmaceutical to yield medically useful information. For example, certain drugs are concentrated in inflamed regions of the body, and this information can aid diagnosis and treatment as seen in [\[link\]](#). Another application utilizes a radiopharmaceutical which the body sends to bone cells, particularly those that are most active, to detect cancerous tumors or healing points. Images can then be produced of such bone scans. Radioisotopes are also used to determine the functioning of body organs, such as blood flow, heart muscle activity, and iodine uptake in the thyroid gland.



A
radiopharmaceutica
l is used to produce
this brain image of
a patient with

Alzheimer's
disease. Certain
features are
computer enhanced.
(credit: National
Institutes of Health)

Medical Application

[\[link\]](#) lists certain medical diagnostic uses of radiopharmaceuticals, including isotopes and activities that are typically administered. Many organs can be imaged with a variety of nuclear isotopes replacing a stable element by a radioactive isotope. One common diagnostic employs iodine to image the thyroid, since iodine is concentrated in that organ. The most active thyroid cells, including cancerous cells, concentrate the most iodine and, therefore, emit the most radiation. Conversely, hypothyroidism is indicated by lack of iodine uptake. Note that there is more than one isotope that can be used for several types of scans. Another common nuclear diagnostic is the thallium scan for the cardiovascular system, particularly used to evaluate blockages in the coronary arteries and examine heart activity. The salt TlCl can be used, because it acts like NaCl and follows the blood. Gallium-67 accumulates where there is rapid cell growth, such as in tumors and sites of infection. Hence, it is useful in cancer imaging. Usually, the patient receives the injection one day and has a whole body scan 3 or 4 days later because it can take several days for the gallium to build up.

Procedure, isotope	Typical activity (mCi), where $1 \text{ mCi} = 3.7 \times 10^7 \text{ Bq}$
--------------------	---

Procedure, isotope	Typical activity (mCi), where $1 \text{ mCi} = 3.7 \times 10^7 \text{ Bq}$
<i>Brain scan</i>	
$^{99\text{m}}\text{Tc}$	7.5
$^{113\text{m}}\text{In}$	7.5
^{11}C (PET)	20
^{13}N (PET)	20
^{15}O (PET)	50
^{18}F (PET)	10
<i>Lung scan</i>	
$^{99\text{m}}\text{Tc}$	2

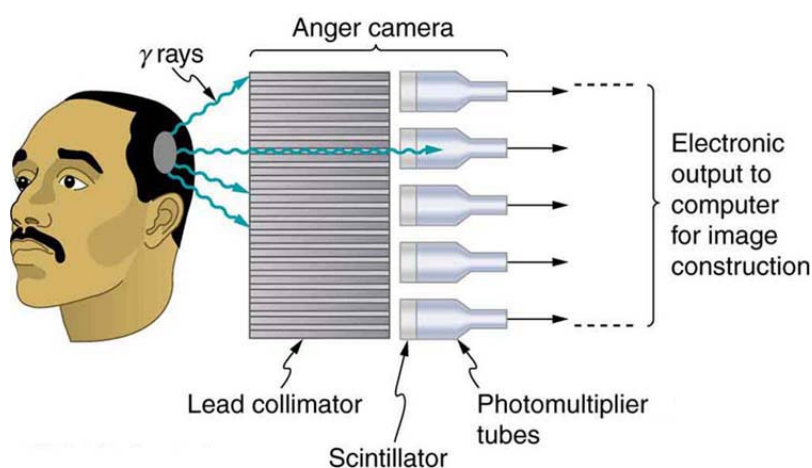
Procedure, isotope	Typical activity (mCi), where $1 \text{ mCi} = 3.7 \times 10^7 \text{ Bq}$
^{133}Xe	7.5
<i>Cardiovascular blood pool</i>	
^{131}I	0.2
$^{99\text{m}}\text{Tc}$	2
<i>Cardiovascular arterial flow</i>	
^{201}Tl	3
^{24}Na	7.5
<i>Thyroid scan</i>	
^{131}I	0.05
^{123}I	0.07

Procedure, isotope	Typical activity (mCi), where $1 \text{ mCi} = 3.7 \times 10^7 \text{ Bq}$
<i>Liver scan</i>	
^{198}Au (colloid)	0.1
$^{99\text{m}}\text{Tc}$ (colloid)	2
<i>Bone scan</i>	
^{85}Sr	0.1
$^{99\text{m}}\text{Tc}$	10
<i>Kidney scan</i>	
^{197}Hg	0.1
$^{99\text{m}}\text{Tc}$	1.5

Diagnostic Uses of Radiopharmaceuticals

Note that [\[link\]](#) lists many diagnostic uses for $^{99\text{m}}\text{Tc}$, where “m” stands for a metastable state of the technetium nucleus. Perhaps 80 percent of all radiopharmaceutical procedures employ $^{99\text{m}}\text{Tc}$ because of its many advantages. One is that the decay of its metastable state produces a single, easily identified 0.142-MeV γ ray. Additionally, the radiation dose to the patient is limited by the short 6.0-h half-life of $^{99\text{m}}\text{Tc}$. And, although its half-life is short, it is easily and continuously produced on site. The basic process for production is neutron activation of molybdenum, which quickly β decays into $^{99\text{m}}\text{Tc}$. Technetium-99m can be attached to many compounds to allow the imaging of the skeleton, heart, lungs, kidneys, etc.

[\[link\]](#) shows one of the simpler methods of imaging the concentration of nuclear activity, employing a device called an **Anger camera** or **gamma camera**. A piece of lead with holes bored through it collimates γ rays emerging from the patient, allowing detectors to receive γ rays from specific directions only. The computer analysis of detector signals produces an image. One of the disadvantages of this detection method is that there is no depth information (i.e., it provides a two-dimensional view of the tumor as opposed to a three-dimensional view), because radiation from any location under that detector produces a signal.



An Anger or gamma camera consists of a lead collimator and an array of detectors. Gamma rays produce light flashes in the

scintillators. The light output is converted to an electrical signal by the photomultipliers.

A computer constructs an image from the detector output.

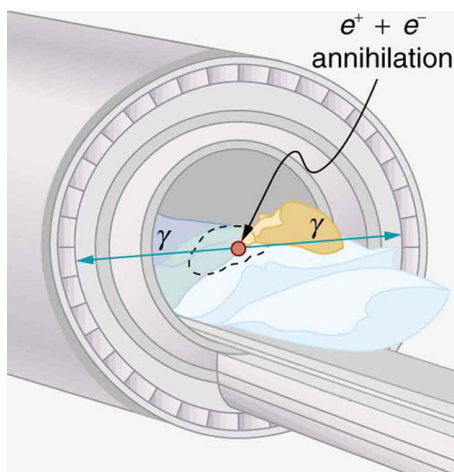
Imaging techniques much like those in x-ray computed tomography (CT) scans use nuclear activity in patients to form three-dimensional images.

[\[link\]](#) shows a patient in a circular array of detectors that may be stationary or rotated, with detector output used by a computer to construct a detailed image. This technique is called **single-photon-emission computed tomography(SPECT)** or sometimes simply SPET. The spatial resolution of this technique is poor, about 1 cm, but the contrast (i.e. the difference in visual properties that makes an object distinguishable from other objects and the background) is good.



SPECT uses a geometry similar to a CT scanner to form an image of the concentration of a radiopharmaceutical compound. (credit: Woldo, Wikimedia Commons)

Images produced by β^+ emitters have become important in recent years. When the emitted positron (β^+) encounters an electron, mutual annihilation occurs, producing two γ rays. These γ rays have identical 0.511-MeV energies (the energy comes from the destruction of an electron or positron mass) and they move directly away from one another, allowing detectors to determine their point of origin accurately, as shown in [\[link\]](#). The system is called **positron emission tomography (PET)**. It requires detectors on opposite sides to simultaneously (i.e., at the same time) detect photons of 0.511-MeV energy and utilizes computer imaging techniques similar to those in SPECT and CT scans. Examples of β^+ -emitting isotopes used in PET are ^{11}C , ^{13}N , ^{15}O , and ^{18}F , as seen in [\[link\]](#). This list includes C, N, and O, and so they have the advantage of being able to function as tags for natural body compounds. Its resolution of 0.5 cm is better than that of SPECT; the accuracy and sensitivity of PET scans make them useful for examining the brain's anatomy and function. The brain's use of oxygen and water can be monitored with ^{15}O . PET is used extensively for diagnosing brain disorders. It can note decreased metabolism in certain regions prior to a confirmation of Alzheimer's disease. PET can locate regions in the brain that become active when a person carries out specific activities, such as speaking, closing their eyes, and so on.



A PET system takes

advantage of the two identical γ -ray photons produced by positron-electron annihilation. These γ rays are emitted in opposite directions, so that the line along which each pair is emitted is determined.

Various events detected by several pairs of detectors are then analyzed by the computer to form an accurate image.

Note:

PhET Explorations: Simplified MRI

Is it a tumor? Magnetic Resonance Imaging (MRI) can tell. Your head is full of tiny radio transmitters (the nuclear spins of the hydrogen nuclei of your water molecules). In an MRI unit, these little radios can be made to broadcast their positions, giving a detailed picture of the inside of your head.

[Simplified MRI](#)

Section Summary

- Radiopharmaceuticals are compounds that are used for medical imaging and therapeutics.
- The process of attaching a radioactive substance is called tagging.
- [\[link\]](#) lists certain diagnostic uses of radiopharmaceuticals including the isotope and activity typically used in diagnostics.
- One common imaging device is the Anger camera, which consists of a lead collimator, radiation detectors, and an analysis computer.
- Tomography performed with γ -emitting radiopharmaceuticals is called SPECT and has the advantages of x-ray CT scans coupled with organ- and function-specific drugs.
- PET is a similar technique that uses β^+ emitters and detects the two annihilation γ rays, which aid to localize the source.

Conceptual Questions

Exercise:

Problem:

In terms of radiation dose, what is the major difference between medical diagnostic uses of radiation and medical therapeutic uses?

Exercise:

Problem:

One of the methods used to limit radiation dose to the patient in medical imaging is to employ isotopes with short half-lives. How would this limit the dose?

Problems & Exercises

Exercise:

Problem:

A neutron generator uses an α source, such as radium, to bombard beryllium, inducing the reaction ${}^4\text{He} + {}^9\text{Be} \rightarrow {}^{12}\text{C} + n$. Such neutron sources are called RaBe sources, or PuBe sources if they use plutonium to get the α s. Calculate the energy output of the reaction in MeV.

Solution:

5.701 MeV

Exercise:**Problem:**

Neutrons from a source (perhaps the one discussed in the preceding problem) bombard natural molybdenum, which is 24 percent ${}^{98}\text{Mo}$. What is the energy output of the reaction ${}^{98}\text{Mo} + n \rightarrow {}^{99}\text{Mo} + \gamma$? The mass of ${}^{98}\text{Mo}$ is given in [Appendix A: Atomic Masses](#), and that of ${}^{99}\text{Mo}$ is 98.907711 u.

Exercise:**Problem:**

The purpose of producing ${}^{99}\text{Mo}$ (usually by neutron activation of natural molybdenum, as in the preceding problem) is to produce ${}^{99\text{m}}\text{Tc}$. Using the rules, verify that the β^- decay of ${}^{99}\text{Mo}$ produces ${}^{99\text{m}}\text{Tc}$. (Most ${}^{99\text{m}}\text{Tc}$ nuclei produced in this decay are left in a metastable excited state denoted ${}^{99\text{m}}\text{Tc}$.)

Solution:**Exercise:**

Problem:

(a) Two annihilation γ rays in a PET scan originate at the same point and travel to detectors on either side of the patient. If the point of origin is 9.00 cm closer to one of the detectors, what is the difference in arrival times of the photons? (This could be used to give position information, but the time difference is small enough to make it difficult.)

(b) How accurately would you need to be able to measure arrival time differences to get a position resolution of 1.00 mm?

Exercise:**Problem:**

[\[link\]](#) indicates that 7.50 mCi of $^{99\text{m}}\text{Tc}$ is used in a brain scan. What is the mass of technetium?

Solution:

$$1.43 \times 10^{-9} \text{ g}$$

Exercise:**Problem:**

The activities of ^{131}I and ^{123}I used in thyroid scans are given in [\[link\]](#) to be 50 and 70 μCi , respectively. Find and compare the masses of ^{131}I and ^{123}I in such scans, given their respective half-lives are 8.04 d and 13.2 h. The masses are so small that the radioiodine is usually mixed with stable iodine as a carrier to ensure normal chemistry and distribution in the body.

Exercise:

Problem:

(a) Neutron activation of sodium, which is 100% ^{23}Na , produces ^{24}Na , which is used in some heart scans, as seen in [\[link\]](#). The equation for the reaction is $^{23}\text{Na} + n \rightarrow ^{24}\text{Na} + \gamma$. Find its energy output, given the mass of ^{24}Na is 23.990962 u.

(b) What mass of ^{24}Na produces the needed 5.0-mCi activity, given its half-life is 15.0 h?

Solution:

(a) 6.958 MeV

(b) 5.7×10^{-10} g

Glossary

Anger camera

a common medical imaging device that uses a scintillator connected to a series of photomultipliers

gamma camera

another name for an Anger camera

positron emission tomography (PET)

tomography technique that uses β^+ emitters and detects the two annihilation γ rays, aiding in source localization

radiopharmaceutical

compound used for medical imaging

single-photon-emission computed tomography (SPECT)

tomography performed with γ -emitting radiopharmaceuticals

tagged

process of attaching a radioactive substance to a chemical compound

Biological Effects of Ionizing Radiation

- Define various units of radiation.
- Describe RBE.

We hear many seemingly contradictory things about the biological effects of ionizing radiation. It can cause cancer, burns, and hair loss, yet it is used to treat and even cure cancer. How do we understand these effects? Once again, there is an underlying simplicity in nature, even in complicated biological organisms. All the effects of ionizing radiation on biological tissue can be understood by knowing that **ionizing radiation affects molecules within cells, particularly DNA molecules.**

Let us take a brief look at molecules within cells and how cells operate. Cells have long, double-helical DNA molecules containing chemical codes called genetic codes that govern the function and processes undertaken by the cell. It is for unraveling the double-helical structure of DNA that James Watson, Francis Crick, and Maurice Wilkins received the Nobel Prize. Damage to DNA consists of breaks in chemical bonds or other changes in the structural features of the DNA chain, leading to changes in the genetic code. In human cells, we can have as many as a million individual instances of damage to DNA per cell per day. It is remarkable that DNA contains codes that check whether the DNA is damaged or can repair itself. It is like an auto check and repair mechanism. This repair ability of DNA is vital for maintaining the integrity of the genetic code and for the normal functioning of the entire organism. It should be constantly active and needs to respond rapidly. The rate of DNA repair depends on various factors such as the cell type and age of the cell. A cell with a damaged ability to repair DNA, which could have been induced by ionizing radiation, can do one of the following:

- The cell can go into an irreversible state of dormancy, known as senescence.
- The cell can commit suicide, known as programmed cell death.
- The cell can go into unregulated cell division leading to tumors and cancers.

Since ionizing radiation damages the DNA, which is critical in cell reproduction, it has its greatest effect on cells that rapidly reproduce, including most types of cancer. Thus, cancer cells are more sensitive to radiation than normal cells and can be killed by it easily. Cancer is characterized by a malfunction of cell reproduction, and can also be caused by ionizing radiation. Without contradiction, ionizing radiation can be both a cure and a cause.

To discuss quantitatively the biological effects of ionizing radiation, we need a radiation dose unit that is directly related to those effects. All effects of radiation are assumed to be directly proportional to the amount of ionization produced in the biological organism. The amount of ionization is in turn proportional to the amount of deposited energy. Therefore, we define a **radiation dose unit** called the **rad**, as 1/100 of a joule of ionizing energy deposited per kilogram of tissue, which is

Equation:

$$1 \text{ rad} = 0.01 \text{ J/kg}.$$

For example, if a 50.0-kg person is exposed to ionizing radiation over her entire body and she absorbs 1.00 J, then her whole-body radiation dose is

Equation:

$$(1.00 \text{ J})/(50.0 \text{ kg}) = 0.0200 \text{ J/kg} = 2.00 \text{ rad}.$$

If the same 1.00 J of ionizing energy were absorbed in her 2.00-kg forearm alone, then the dose to the forearm would be

Equation:

$$(1.00 \text{ J})/(2.00 \text{ kg}) = 0.500 \text{ J/kg} = 50.0 \text{ rad},$$

and the unaffected tissue would have a zero rad dose. While calculating radiation doses, you divide the energy absorbed by the mass of affected tissue. You must specify the affected region, such as the whole body or forearm in addition to giving the numerical dose in rads. The SI unit for radiation dose is the **gray (Gy)**, which is defined to be

Equation:

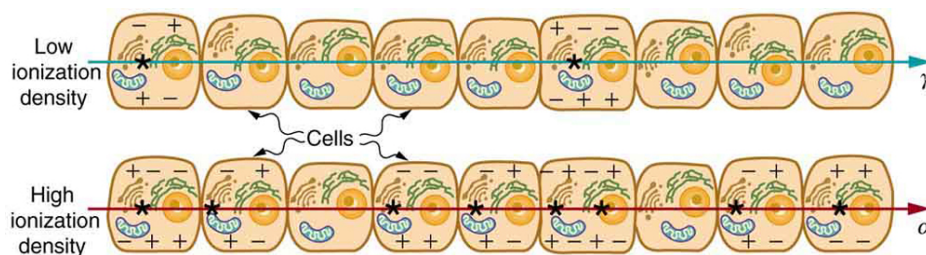
$$1 \text{ Gy} = 1 \text{ J/kg} = 100 \text{ rad}.$$

However, the rad is still commonly used. Although the energy per kilogram in 1 rad is small, it has significant effects since the energy causes ionization. The energy needed for a single ionization is a few eV, or less than 10^{-18} J. Thus, 0.01 J of ionizing energy can create a huge number of ion pairs and have an effect at the cellular level.

The effects of ionizing radiation may be directly proportional to the dose in rads, but they also depend on the type of radiation and the type of tissue. That is, for a given dose in rads, the effects depend on whether the radiation is α , β , γ , x-ray, or some other type of ionizing radiation. In the earlier discussion of the range of ionizing radiation, it was noted that energy is deposited in a series of ionizations and not in a single interaction. Each ion pair or ionization requires a certain amount of energy, so that the number of ion pairs is directly proportional to the amount of the deposited ionizing energy. But, if the range of the radiation is small, as it is for α s, then the ionization and the damage created is more concentrated and harder for the organism to repair, as seen in [\[link\]](#). Concentrated damage is more difficult for biological organisms to repair than damage that is spread out, so short-range particles have greater biological effects. The **relative biological effectiveness (RBE)** or **quality factor (QF)** is given in [\[link\]](#) for several types of ionizing radiation—the effect of the radiation is directly proportional to the RBE. A dose unit more closely related to effects in biological tissue is called the **roentgen equivalent man** or rem and is defined to be the dose in rads multiplied by the relative biological effectiveness.

Equation:

$$\text{rem} = \text{rad} \times \text{RBE}$$



The image shows ionization created in cells by α and γ radiation. Because of its shorter range, the ionization and damage created by α is more concentrated and harder for the organism to repair. Thus, the RBE for α s is greater than the RBE for γ s, even though they create the same amount of ionization at the same energy.

So, if a person had a whole-body dose of 2.00 rad of γ radiation, the dose in rem would be $(2.00 \text{ rad})(1) = 2.00 \text{ rem}$ whole body. If the person had a whole-body dose of 2.00 rad of α radiation, then the dose in rem would be $(2.00 \text{ rad})(20) = 40.0 \text{ rem}$ whole body. The α s would have 20 times the effect on the person than the γ s for the same deposited energy. The SI equivalent of the rem is the **sievert (Sv)**, defined to be $\text{Sv} = \text{Gy} \times \text{RBE}$, so that

Equation:

$$1 \text{ Sv} = 1 \text{ Gy} \times \text{RBE} = 100 \text{ rem}.$$

The RBEs given in [\[link\]](#) are approximate, but they yield certain insights. For example, the eyes are more sensitive to radiation, because the cells of the lens do not repair themselves. Neutrons cause more damage than γ rays, although both are neutral and have large ranges, because neutrons often cause secondary radiation when they are captured. Note that the RBEs are 1 for higher-energy β s, γ s, and x-rays, three of the most common types of radiation. For those types of radiation, the numerical values of the dose in rem and rad are identical. For example, 1 rad of γ radiation is also 1 rem. For that reason, rads are still widely quoted rather than rem. [\[link\]](#) summarizes the units that are used for radiation.

Note:

Misconception Alert: Activity vs. Dose

“Activity” refers to the radioactive source while “dose” refers to the amount of energy from the radiation that is deposited in a person or object.

A high level of activity doesn’t mean much if a person is far away from the source. The activity R of a source depends upon the quantity of material (kg) as well as the half-life. A short half-life will produce many more disintegrations per second. Recall that $R = \frac{0.693N}{t_{1/2}}$. Also, the activity decreases exponentially, which is seen in the equation $R = R_0 e^{-\lambda t}$.

Type and energy of radiation	RBE ^[footnote] Values approximate, difficult to determine.
X-rays	1
γ rays	1
β rays greater than 32 keV	1
β rays less than 32 keV	1.7
Neutrons, thermal to slow (<20 keV)	2–5
Neutrons, fast (1–10 MeV)	10 (body), 32 (eyes)
Protons (1–10 MeV)	10 (body), 32 (eyes)
α rays from radioactive decay	10–20
Heavy ions from accelerators	10–20

Relative Biological Effectiveness

Quantity	SI unit name	Definition	Former unit	Conversion
Activity	Becquerel (Bq)	decay/sec	Curie (Ci)	$1 \text{ Bq} = 2.7 \times 10^{-11} \text{ Ci}$
Absorbed dose	Gray (Gy)	1 J/kg	rad	$\text{Gy} = 100 \text{ rad}$
Dose Equivalent	Sievert (Sv)	$1 \text{ J/kg} \times \text{RBE}$	rem	$\text{Sv} = 100 \text{ rem}$

Units for Radiation

The large-scale effects of radiation on humans can be divided into two categories: immediate effects and long-term effects. [\[link\]](#) gives the immediate effects of whole-body exposures received in less than one day. If the radiation exposure is spread out over more time, greater doses are needed to cause the effects listed. This is due to the body's ability to partially repair the damage. Any dose less than 100 mSv (10 rem) is called a **low dose**, 0.1 Sv to 1 Sv (10 to 100 rem) is called a **moderate dose**, and anything greater than 1 Sv (100 rem) is called a **high dose**. There is no known way to determine after the fact if a person has been exposed to less than 10 mSv.

Dose in Sv [footnote] Multiply by 100 to obtain dose in rem.	Effect
0–0.10	No observable effect.
0.1 – 1	Slight to moderate decrease in white blood cell counts.
0.5	Temporary sterility; 0.35 for women, 0.50 for men.
1 – 2	Significant reduction in blood cell counts, brief nausea and vomiting. Rarely fatal.
2 – 5	Nausea, vomiting, hair loss, severe blood damage, hemorrhage, fatalities.

Dose in Sv [footnote] Multiply by 100 to obtain dose in rem.	Effect
4.5	LD50/32. Lethal to 50% of the population within 32 days after exposure if not treated.
5 – 20	Worst effects due to malfunction of small intestine and blood systems. Limited survival.
>20	Fatal within hours due to collapse of central nervous system.

Immediate Effects of Radiation (Adults, Whole Body, Single Exposure)

Immediate effects are explained by the effects of radiation on cells and the sensitivity of rapidly reproducing cells to radiation. The first clue that a person has been exposed to radiation is a change in blood count, which is not surprising since blood cells are the most rapidly reproducing cells in the body. At higher doses, nausea and hair loss are observed, which may be due to interference with cell reproduction. Cells in the lining of the digestive system also rapidly reproduce, and their destruction causes nausea. When the growth of hair cells slows, the hair follicles become thin and break off. High doses cause significant cell death in all systems, but the lowest doses that cause fatalities do so by weakening the immune system through the loss of white blood cells.

The two known long-term effects of radiation are cancer and genetic defects. Both are directly attributable to the interference of radiation with cell reproduction. For high doses of radiation, the risk of cancer is reasonably well known from studies of exposed groups. Hiroshima and Nagasaki survivors and a smaller number of people exposed by their occupation, such as radium dial painters, have been fully documented. Chernobyl victims will be studied for many decades, with some data already available. For example, a significant increase in childhood thyroid cancer has been observed. The risk of a radiation-induced cancer for low and moderate doses is generally *assumed* to be proportional to the risk known for high doses. Under this assumption, any dose of radiation, no matter how small, involves a risk to human health. This is called the **linear hypothesis** and it may be prudent, but it is controversial. There is some evidence that, unlike the immediate effects of radiation, the long-term effects are cumulative and there is little self-repair. This is analogous to the risk of skin cancer from UV exposure, which is known to be cumulative.

There is a latency period for the onset of radiation-induced cancer of about 2 years for leukemia and 15 years for most other forms. The person is at risk for at least 30 years after the latency period. Omitting many details, the overall risk of a radiation-induced cancer

death per year per rem of exposure is about 10 in a million, which can be written as $10/10^6 \text{ rem} \cdot \text{y}$.

If a person receives a dose of 1 rem, his risk each year of dying from radiation-induced cancer is 10 in a million and that risk continues for about 30 years. The lifetime risk is thus 300 in a million, or 0.03 percent. Since about 20 percent of all worldwide deaths are from cancer, the increase due to a 1 rem exposure is impossible to detect demographically. But 100 rem (1 Sv), which was the dose received by the average Hiroshima and Nagasaki survivor, causes a 3 percent risk, which can be observed in the presence of a 20 percent normal or natural incidence rate.

The incidence of genetic defects induced by radiation is about one-third that of cancer deaths, but is much more poorly known. The lifetime risk of a genetic defect due to a 1 rem exposure is about 100 in a million or $3.3/10^6 \text{ rem} \cdot \text{y}$, but the normal incidence is 60,000 in a million. Evidence of such a small increase, tragic as it is, is nearly impossible to obtain. For example, there is no evidence of increased genetic defects among the offspring of Hiroshima and Nagasaki survivors. Animal studies do not seem to correlate well with effects on humans and are not very helpful. For both cancer and genetic defects, the approach to safety has been to use the linear hypothesis, which is likely to be an overestimate of the risks of low doses. Certain researchers even claim that low doses are *beneficial*. **Hormesis** is a term used to describe generally favorable biological responses to low exposures of toxins or radiation. Such low levels may help certain repair mechanisms to develop or enable cells to adapt to the effects of the low exposures. Positive effects may occur at low doses that could be a problem at high doses.

Even the linear hypothesis estimates of the risks are relatively small, and the average person is not exposed to large amounts of radiation. [\[link\]](#) lists average annual background radiation doses from natural and artificial sources for Australia, the United States, Germany, and world-wide averages. Cosmic rays are partially shielded by the atmosphere, and the dose depends upon altitude and latitude, but the average is about 0.40 mSv/y. A good example of the variation of cosmic radiation dose with altitude comes from the airline industry. Monitored personnel show an average of 2 mSv/y. A 12-hour flight might give you an exposure of 0.02 to 0.03 mSv.

Doses from the Earth itself are mainly due to the isotopes of uranium, thorium, and potassium, and vary greatly by location. Some places have great natural concentrations of uranium and thorium, yielding doses ten times as high as the average value. Internal doses come from foods and liquids that we ingest. Fertilizers containing phosphates have potassium and uranium. So we are all a little radioactive. Carbon-14 has about 66 Bq/kg radioactivity whereas fertilizers may have more than 3000 Bq/kg radioactivity. Medical and dental diagnostic exposures are mostly from x-rays. It should be noted that x-ray doses tend to be localized and are becoming much smaller with improved techniques. [\[link\]](#) shows typical doses received during various diagnostic x-ray examinations. Note the large dose from a CT scan. While CT scans only account for less than 20 percent of

the x-ray procedures done today, they account for about 50 percent of the annual dose received.

Radon is usually more pronounced underground and in buildings with low air exchange with the outside world. Almost all soil contains some ^{226}Ra and ^{222}Rn , but radon is lower in mainly sedimentary soils and higher in granite soils. Thus, the exposure to the public can vary greatly, even within short distances. Radon can diffuse from the soil into homes, especially basements. The estimated exposure for ^{222}Rn is controversial. Recent studies indicate there is more radon in homes than had been realized, and it is speculated that radon may be responsible for 20 percent of lung cancers, being particularly hazardous to those who also smoke. Many countries have introduced limits on allowable radon concentrations in indoor air, often requiring the measurement of radon concentrations in a house prior to its sale. Ironically, it could be argued that the higher levels of radon exposure and their geographic variability, taken with the lack of demographic evidence of any effects, means that low-level radiation is *less* dangerous than previously thought.

Radiation Protection

Laws regulate radiation doses to which people can be exposed. The greatest occupational whole-body dose that is allowed depends upon the country and is about 20 to 50 mSv/y and is rarely reached by medical and nuclear power workers. Higher doses are allowed for the hands. Much lower doses are permitted for the reproductive organs and the fetuses of pregnant women. Inadvertent doses to the public are limited to 1/10 of occupational doses, except for those caused by nuclear power, which cannot legally expose the public to more than 1/1000 of the occupational limit or 0.05 mSv/y (5 mrem/y). This has been exceeded in the United States only at the time of the Three Mile Island (TMI) accident in 1979. Chernobyl is another story. Extensive monitoring with a variety of radiation detectors is performed to assure radiation safety. Increased ventilation in uranium mines has lowered the dose there to about 1 mSv/y.

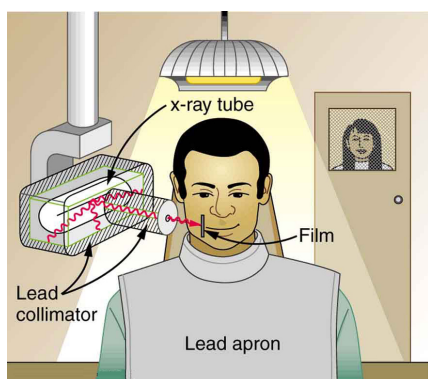
Source	Dose (mSv/y) [footnote] Multiply by 100 to obtain dose in mrem/y.			
Source	Australia	Germany	United States	World
Natural Radiation - external				

Source	Dose (mSv/y) [footnote]			
	Multiply by 100 to obtain dose in mrem/y.			
Cosmic Rays	0.30	0.28	0.30	0.39
Soil, building materials	0.40	0.40	0.30	0.48
Radon gas	0.90	1.1	2.0	1.2
Natural Radiation - internal				
^{40}K , ^{14}C , ^{226}Ra	0.24	0.28	0.40	0.29
Medical & Dental	0.80	0.90	0.53	0.40
TOTAL	2.6	3.0	3.5	2.8

Background Radiation Sources and Average Doses

To physically limit radiation doses, we use **shielding**, increase the **distance** from a source, and limit the **time of exposure**.

[\[link\]](#) illustrates how these are used to protect both the patient and the dental technician when an x-ray is taken. Shielding absorbs radiation and can be provided by any material, including sufficient air. The greater the distance from the source, the more the radiation spreads out. The less time a person is exposed to a given source, the smaller is the dose received by the person. Doses from most medical diagnostics have decreased in recent years due to faster films that require less exposure time.



A lead apron is placed over the dental patient and shielding surrounds the x-ray tube to limit exposure to tissue other than the tissue that is being imaged. Fast films limit the time needed to obtain images, reducing exposure to the imaged tissue. The technician stands a few meters away behind a lead-lined door with a lead glass window, reducing her occupational exposure.

Procedure	Effective dose (mSv)
Chest	0.02
Dental	0.01
Skull	0.07
Leg	0.02
Mammogram	0.40
Barium enema	7.0
Upper GI	3.0
CT head	2.0
CT abdomen	10.0

Typical Doses Received During Diagnostic X-ray Exams

Problem-Solving Strategy

You need to follow certain steps for dose calculations, which are

Step 1. *Examine the situation to determine that a person is exposed to ionizing radiation.*

Step 2. *Identify exactly what needs to be determined in the problem (identify the unknowns).* The most straightforward problems ask for a dose calculation.

Step 3. *Make a list of what is given or can be inferred from the problem as stated (identify the knowns).* Look for information on the type of radiation, the energy per event, the activity, and the mass of tissue affected.

Step 4. *For dose calculations, you need to determine the energy deposited.* This may take one or more steps, depending on the given information.

Step 5. *Divide the deposited energy by the mass of the affected tissue.* Use units of joules for energy and kilograms for mass. If a dose in Sv is involved, use the definition that $1 \text{ Sv} = 1 \text{ J/kg}$.

Step 6. *If a dose in mSv is involved, determine the RBE (QF) of the radiation.* Recall that $1 \text{ mSv} = 1 \text{ mGy} \times \text{RBE}$ (or $1 \text{ rem} = 1 \text{ rad} \times \text{RBE}$).

Step 7. *Check the answer to see if it is reasonable: Does it make sense?* The dose should be consistent with the numbers given in the text for diagnostic, occupational, and therapeutic exposures.

Example:

Dose from Inhaled Plutonium

Calculate the dose in rem/y for the lungs of a weapons plant employee who inhales and retains an activity of $1.00 \mu\text{Ci}$ of ^{239}Pu in an accident. The mass of affected lung tissue is 2.00 kg , the plutonium decays by emission of a 5.23-MeV α particle, and you may assume the higher value of the RBE for α s from [\[link\]](#).

Strategy

Dose in rem is defined by $1 \text{ rad} = 0.01 \text{ J/kg}$ and $\text{rem} = \text{rad} \times \text{RBE}$. The energy deposited is divided by the mass of tissue affected and then multiplied by the RBE. The latter two quantities are given, and so the main task in this example will be to find the energy deposited in one year. Since the activity of the source is given, we can calculate the number of decays, multiply by the energy per decay, and convert MeV to joules to get the total energy.

Solution

The activity $R = 1.00 \mu\text{Ci} = 3.70 \times 10^4 \text{ Bq} = 3.70 \times 10^4 \text{ decays/s}$. So, the number of decays per year is obtained by multiplying by the number of seconds in a year:

Equation:

$$(3.70 \times 10^4 \text{ decays/s})(3.16 \times 10^7 \text{ s}) = 1.17 \times 10^{12} \text{ decays.}$$

Thus, the ionizing energy deposited per year is

Equation:

$$E = (1.17 \times 10^{12} \text{ decays})(5.23 \text{ MeV/decay}) \times \left(\frac{1.60 \times 10^{-13} \text{ J}}{\text{MeV}} \right) = 0.978 \text{ J.}$$

Dividing by the mass of the affected tissue gives

Equation:

$$\frac{E}{\text{mass}} = \frac{0.978 \text{ J}}{2.00 \text{ kg}} = 0.489 \text{ J/kg.}$$

One Gray is 1.00 J/kg, and so the dose in Gy is

Equation:

$$\text{dose in Gy} = \frac{0.489 \text{ J/kg}}{1.00 (\text{J/kg})/\text{Gy}} = 0.489 \text{ Gy.}$$

Now, the dose in Sv is

Equation:

$$\text{dose in Sv} = \text{Gy} \times \text{RBE}$$

Equation:

$$= (0.489 \text{ Gy})(20) = 9.8 \text{ Sv.}$$

Discussion

First note that the dose is given to two digits, because the RBE is (at best) known only to two digits. By any standard, this yearly radiation dose is high and will have a devastating effect on the health of the worker. Worse yet, plutonium has a long radioactive half-life and is not readily eliminated by the body, and so it will remain in the lungs. Being an α emitter makes the effects 10 to 20 times worse than the same ionization produced by β s, γ rays, or x-rays. An activity of $1.00 \mu\text{Ci}$ is created by only $16 \mu\text{g}$ of ^{239}Pu (left as an end-of-chapter problem to verify), partly justifying claims that plutonium is the most toxic substance known. Its actual hazard depends on how likely it is to be spread out among a large population and then ingested. The Chernobyl disaster's deadly legacy, for example, has nothing to do with the plutonium it put into the environment.

Risk versus Benefit

Medical doses of radiation are also limited. Diagnostic doses are generally low and have further lowered with improved techniques and faster films. With the possible exception of routine dental x-rays, radiation is used diagnostically only when needed so that the low risk is justified by the benefit of the diagnosis. Chest x-rays give the lowest doses—about 0.1 mSv to the tissue affected, with less than 5 percent scattering into tissues that are not directly imaged. Other x-ray procedures range upward to about 10 mSv in a CT scan, and about 5 mSv (0.5 rem) per dental x-ray, again both only affecting the tissue imaged. Medical images with radiopharmaceuticals give doses ranging from 1 to 5 mSv, usually localized. One exception is the thyroid scan using ^{131}I . Because of its relatively long half-life, it exposes the thyroid to about 0.75 Sv. The isotope ^{123}I is more difficult to produce, but its short half-life limits thyroid exposure to about 15 mSv.

Note:

PhET Explorations: Alpha Decay

Watch alpha particles escape from a polonium nucleus, causing radioactive alpha decay. See how random decay times relate to the half life.

[Alpha
Decay](#)
y.

Section Summary

- The biological effects of ionizing radiation are due to two effects it has on cells: interference with cell reproduction, and destruction of cell function.
- A radiation dose unit called the rad is defined in terms of the ionizing energy deposited per kilogram of tissue:

Equation:

$$1 \text{ rad} = 0.01 \text{ J/kg}.$$

- The SI unit for radiation dose is the gray (Gy), which is defined to be $1 \text{ Gy} = 1 \text{ J/kg} = 100 \text{ rad}$.
- To account for the effect of the type of particle creating the ionization, we use the relative biological effectiveness (RBE) or quality factor (QF) given in [\[link\]](#) and define a unit called the roentgen equivalent man (rem) as

Equation:

$$\text{rem} = \text{rad} \times \text{RBE}.$$

- Particles that have short ranges or create large ionization densities have RBEs greater than unity. The SI equivalent of the rem is the sievert (Sv), defined to be

Equation:

$$\text{Sv} = \text{Gy} \times \text{RBE} \text{ and } 1 \text{ Sv} = 100 \text{ rem}.$$

- Whole-body, single-exposure doses of 0.1 Sv or less are low doses while those of 0.1 to 1 Sv are moderate, and those over 1 Sv are high doses. Some immediate radiation effects are given in [\[link\]](#). Effects due to low doses are not observed, but their risk is assumed to be directly proportional to those of high doses, an assumption known as the linear hypothesis. Long-term effects are cancer deaths at the rate of $10/10^6$ rem·y and genetic defects at roughly one-third this rate. Background radiation doses and sources are given in [\[link\]](#). World-wide average radiation exposure from natural sources, including radon, is about 3 mSv, or 300 mrem. Radiation protection utilizes shielding, distance, and time to limit exposure.

Conceptual Questions

Exercise:**Problem:**

Isotopes that emit α radiation are relatively safe outside the body and exceptionally hazardous inside. Yet those that emit γ radiation are hazardous outside and inside. Explain why.

Exercise:**Problem:**

Why is radon more closely associated with inducing lung cancer than other types of cancer?

Exercise:**Problem:**

The RBE for low-energy β s is 1.7, whereas that for higher-energy β s is only 1. Explain why, considering how the range of radiation depends on its energy.

Exercise:

Problem:

Which methods of radiation protection were used in the device shown in the first photo in [\[link\]](#)? Which were used in the situation shown in the second photo?

(a)



(a)



(b)

(a) This x-ray fluorescence machine is one of the thousands used in shoe stores to produce images of feet as a check on the fit of shoes. They are unshielded and remain on as long as the feet are in them, producing doses much greater than medical images. Children were fascinated with them. These machines were used in shoe stores until laws preventing such unwarranted radiation exposure were enacted in the 1950s. (credit: Andrew Kuchling) (b) Now that we know the effects of exposure to

radioactive material,
safety is a priority.
(credit: U.S. Navy)

Exercise:

Problem:

What radioisotope could be a problem in homes built of cinder blocks made from uranium mine tailings? (This is true of homes and schools in certain regions near uranium mines.)

Exercise:

Problem:

Are some types of cancer more sensitive to radiation than others? If so, what makes them more sensitive?

Exercise:

Problem:

Suppose a person swallows some radioactive material by accident. What information is needed to be able to assess possible damage?

Problems & Exercises

Exercise:

Problem:

What is the dose in mSv for: (a) a 0.1 Gy x-ray? (b) 2.5 mGy of neutron exposure to the eye? (c) 1.5 mGy of α exposure?

Solution:

(a) 100 mSv

(b) 80 mSv

(c) ~30 mSv

Exercise:

Problem:

Find the radiation dose in Gy for: (a) A 10-mSv fluoroscopic x-ray series. (b) 50 mSv of skin exposure by an α emitter. (c) 160 mSv of β^- and γ rays from the ^{40}K in your body.

Exercise:**Problem:**

How many Gy of exposure is needed to give a cancerous tumor a dose of 40 Sv if it is exposed to α activity?

Solution:

~2 Gy

Exercise:**Problem:**

What is the dose in Sv in a cancer treatment that exposes the patient to 200 Gy of γ rays?

Exercise:**Problem:**

One half the γ rays from $^{99\text{m}}\text{Tc}$ are absorbed by a 0.170-mm-thick lead shielding. Half of the γ rays that pass through the first layer of lead are absorbed in a second layer of equal thickness. What thickness of lead will absorb all but one in 1000 of these γ rays?

Solution:

1.69 mm

Exercise:**Problem:**

A plumber at a nuclear power plant receives a whole-body dose of 30 mSv in 15 minutes while repairing a crucial valve. Find the radiation-induced yearly risk of death from cancer and the chance of genetic defect from this maximum allowable exposure.

Exercise:

Problem:

In the 1980s, the term picowave was used to describe food irradiation in order to overcome public resistance by playing on the well-known safety of microwave radiation. Find the energy in MeV of a photon having a wavelength of a picometer.

Solution:

1.24 MeV

Exercise:

Problem: Find the mass of ^{239}Pu that has an activity of $1.00\ \mu\text{Ci}$.

Glossary

gray (Gy)

the SI unit for radiation dose which is defined to be $1\ \text{Gy} = 1\ \text{J/kg} = 100\ \text{rad}$

linear hypothesis

assumption that risk is directly proportional to risk from high doses

rad

the ionizing energy deposited per kilogram of tissue

sievert

the SI equivalent of the rem

relative biological effectiveness (RBE)

a number that expresses the relative amount of damage that a fixed amount of ionizing radiation of a given type can inflict on biological tissues

quality factor

same as relative biological effectiveness

roentgen equivalent man (rem)

a dose unit more closely related to effects in biological tissue

low dose

a dose less than 100 mSv (10 rem)

moderate dose

a dose from 0.1 Sv to 1 Sv (10 to 100 rem)

high dose

a dose greater than 1 Sv (100 rem)

hormesis

a term used to describe generally favorable biological responses to low exposures of toxins or radiation

shielding

a technique to limit radiation exposure

Therapeutic Uses of Ionizing Radiation

- Explain the concept of radiotherapy and list typical doses for cancer therapy.

Therapeutic applications of ionizing radiation, called radiation therapy or **radiotherapy**, have existed since the discovery of x-rays and nuclear radioactivity. Today, radiotherapy is used almost exclusively for cancer therapy, where it saves thousands of lives and improves the quality of life and longevity of many it cannot save. Radiotherapy may be used alone or in combination with surgery and chemotherapy (drug treatment) depending on the type of cancer and the response of the patient. A careful examination of all available data has established that radiotherapy's beneficial effects far outweigh its long-term risks.

Medical Application

The earliest uses of ionizing radiation on humans were mostly harmful, with many at the level of snake oil as seen in [\[link\]](#). Radium-doped cosmetics that glowed in the dark were used around the time of World War I. As recently as the 1950s, radon mine tours were promoted as healthful and rejuvenating—those who toured were exposed but gained no benefits. Radium salts were sold as health elixirs for many years. The gruesome death of a wealthy industrialist, who became psychologically addicted to the brew, alerted the unsuspecting to the dangers of radium salt elixirs. Most abuses finally ended after the legislation in the 1950s.

The Power of Radium at Your Disposal

Twenty-three years ago radium was unknown. Today, thanks to constant laboratory work, the power of this most unusual of elements is at your disposal. Through the medium of Undark, radium serves you safely and surely.

Does Undark really contain radium? Most assuredly. It is radium, combined in exactly the proper manner with zinc sulphide, which gives Undark its ability to shine continuously in the dark.

Manufacturers have been quick to recognize the value of Undark. They apply it to the dials of watches and clocks, to electric push buttons, to the buckles of bed room slippers, to house numbers, flashlights, compasses, gasoline gauges, autometers and many other articles which you frequently wish to see in the dark.

The next time you fumble for a lighting switch, bark your shins on furniture, wonder vainly what time it is because of the dark—remember Undark. It shines in the dark. Dealers can supply you with Undarked articles.

For interesting little folder telling of the production of radium and the uses of Undark address

RADIUM LUMINOUS MATERIAL CORPORATION
 35 FINE STREET NEW YORK CITY
 Factories: Glasgow, N. I. Miami, Colorado and Utah

UNDARK
Radium Luminous Material
Shines in the Dark

To Manufacturers

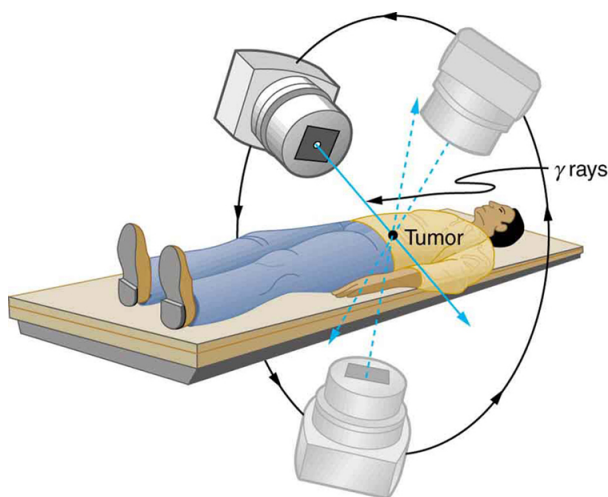
The number of manufactured articles to which Undark will add increased usefulness is manifold. From a sales standpoint, it has many obvious advantages. We gladly answer inquiries from manufacturers and, when it seems advisable, will carry on experimental work for them. Undark may be applied either at your plant, or at our own.

The application of Undark is simple. It is furnished as a powder, which is mixed with an adhesive. The paste thus formed is painted on with a brush. It adheres firmly to any surface.

The properties of radiation were once touted for far more than its modern use in cancer therapy. Until 1932, radium was advertised for a variety of uses, often with tragic results. (credit: Struthious Bandersnatch.)

Radiotherapy is effective against cancer because cancer cells reproduce rapidly and, consequently, are more sensitive to radiation. The central problem in radiotherapy is to make the dose for cancer cells as high as possible while limiting the dose for normal cells. The ratio of abnormal cells killed to normal cells killed is called the **therapeutic ratio**, and all radiotherapy techniques are designed to enhance this ratio. Radiation can be concentrated in cancerous tissue by a number of techniques. One of the most prevalent techniques for well-defined tumors is a geometric technique

shown in [\[link\]](#). A narrow beam of radiation is passed through the patient from a variety of directions with a common crossing point in the tumor. This concentrates the dose in the tumor while spreading it out over a large volume of normal tissue. The external radiation can be x-rays, ^{60}Co γ rays, or ionizing-particle beams produced by accelerators. Accelerator-produced beams of neutrons, π -mesons, and heavy ions such as nitrogen nuclei have been employed, and these can be quite effective. These particles have larger QFs or RBEs and sometimes can be better localized, producing a greater therapeutic ratio. But accelerator radiotherapy is much more expensive and less frequently employed than other forms.



The ^{60}Co source of γ -radiation is rotated around the patient so that the common crossing point is in the tumor, concentrating the dose there. This geometric technique works for well-defined tumors.

Another form of radiotherapy uses chemically inert radioactive implants. One use is for prostate cancer. Radioactive seeds (about 40 to 100 and the size of a grain of rice) are placed in the prostate region. The isotopes used

are usually ^{135}I (6-month half life) or ^{103}Pd (3-month half life). Alpha emitters have the dual advantages of a large QF and a small range for better localization.

Radiopharmaceuticals are used for cancer therapy when they can be localized well enough to produce a favorable therapeutic ratio. Thyroid cancer is commonly treated utilizing radioactive iodine. Thyroid cells concentrate iodine, and cancerous thyroid cells are more aggressive in doing this. An ingenious use of radiopharmaceuticals in cancer therapy tags antibodies with radioisotopes. Antibodies produced by a patient to combat his cancer are extracted, cultured, loaded with a radioisotope, and then returned to the patient. The antibodies are concentrated almost entirely in the tissue they developed to fight, thus localizing the radiation in abnormal tissue. The therapeutic ratio can be quite high for short-range radiation. There is, however, a significant dose for organs that eliminate radiopharmaceuticals from the body, such as the liver, kidneys, and bladder. As with most radiotherapy, the technique is limited by the tolerable amount of damage to the normal tissue.

[\[link\]](#) lists typical therapeutic doses of radiation used against certain cancers. The doses are large, but not fatal because they are localized and spread out in time. Protocols for treatment vary with the type of cancer and the condition and response of the patient. Three to five 200-rem treatments per week for a period of several weeks is typical. Time between treatments allows the body to repair normal tissue. This effect occurs because damage is concentrated in the abnormal tissue, and the abnormal tissue is more sensitive to radiation. Damage to normal tissue limits the doses. You will note that the greatest doses are given to any tissue that is not rapidly reproducing, such as in the adult brain. Lung cancer, on the other end of the scale, cannot ordinarily be cured with radiation because of the sensitivity of lung tissue and blood to radiation. But radiotherapy for lung cancer does alleviate symptoms and prolong life and is therefore justified in some cases.

Type of Cancer	Typical dose (Sv)
Lung	10–20
Hodgkin’s disease	40–45
Skin	40–50
Ovarian	50–75
Breast	50–80+
Brain	80+
Neck	80+
Bone	80+
Soft tissue	80+
Thyroid	80+

Cancer Radiotherapy

Finally, it is interesting to note that chemotherapy employs drugs that interfere with cell division and is, thus, also effective against cancer. It also has almost the same side effects, such as nausea and hair loss, and risks, such as the inducement of another cancer.

Section Summary

- Radiotherapy is the use of ionizing radiation to treat ailments, now limited to cancer therapy.
- The sensitivity of cancer cells to radiation enhances the ratio of cancer cells killed to normal cells killed, which is called the therapeutic ratio.

- Doses for various organs are limited by the tolerance of normal tissue for radiation. Treatment is localized in one region of the body and spread out in time.

Conceptual Questions

Exercise:

Problem:

Radiotherapy is more likely to be used to treat cancer in elderly patients than in young ones. Explain why. Why is radiotherapy used to treat young people at all?

Problems & Exercises

Exercise:

Problem:

A beam of 168-MeV nitrogen nuclei is used for cancer therapy. If this beam is directed onto a 0.200-kg tumor and gives it a 2.00-Sv dose, how many nitrogen nuclei were stopped? (Use an RBE of 20 for heavy ions.)

Solution:

$$7.44 \times 10^8$$

Exercise:

Problem:

(a) If the average molecular mass of compounds in food is 50.0 g, how many molecules are there in 1.00 kg of food? (b) How many ion pairs are created in 1.00 kg of food, if it is exposed to 1000 Sv and it takes 32.0 eV to create an ion pair? (c) Find the ratio of ion pairs to molecules. (d) If these ion pairs recombine into a distribution of 2000 new compounds, how many parts per billion is each?

Exercise:**Problem:**

Calculate the dose in Sv to the chest of a patient given an x-ray under the following conditions. The x-ray beam intensity is 1.50 W/m^2 , the area of the chest exposed is 0.0750 m^2 , 35.0% of the x-rays are absorbed in 20.0 kg of tissue, and the exposure time is 0.250 s.

Solution:

$$4.92 \times 10^{-4} \text{ Sv}$$

Exercise:**Problem:**

(a) A cancer patient is exposed to γ rays from a 5000-Ci ^{60}Co transillumination unit for 32.0 s. The γ rays are collimated in such a manner that only 1.00% of them strike the patient. Of those, 20.0% are absorbed in a tumor having a mass of 1.50 kg. What is the dose in rem to the tumor, if the average γ energy per decay is 1.25 MeV? None of the β s from the decay reach the patient. (b) Is the dose consistent with stated therapeutic doses?

Exercise:**Problem:**

What is the mass of ^{60}Co in a cancer therapy transillumination unit containing 5.00 kCi of ^{60}Co ?

Solution:

$$4.43 \text{ g}$$

Exercise:

Problem:

Large amounts of ^{65}Zn are produced in copper exposed to accelerator beams. While machining contaminated copper, a physicist ingests $50.0\ \mu\text{Ci}$ of ^{65}Zn . Each ^{65}Zn decay emits an average γ -ray energy of $0.550\ \text{MeV}$, 40.0% of which is absorbed in the scientist's 75.0-kg body. What dose in mSv is caused by this in one day?

Exercise:**Problem:**

Naturally occurring ^{40}K is listed as responsible for 16 mrem/y of background radiation. Calculate the mass of ^{40}K that must be inside the 55-kg body of a woman to produce this dose. Each ^{40}K decay emits a 1.32-MeV β , and 50% of the energy is absorbed inside the body.

Solution:

0.010 g

Exercise:**Problem:**

(a) Background radiation due to ^{226}Ra averages only 0.01 mSv/y, but it can range upward depending on where a person lives. Find the mass of ^{226}Ra in the 80.0-kg body of a man who receives a dose of 2.50-mSv/y from it, noting that each ^{226}Ra decay emits a 4.80-MeV α particle. You may neglect dose due to daughters and assume a constant amount, evenly distributed due to balanced ingestion and bodily elimination. (b) Is it surprising that such a small mass could cause a measurable radiation dose? Explain.

Exercise:

Problem:

The annual radiation dose from ^{14}C in our bodies is 0.01 mSv/y. Each ^{14}C decay emits a β^- averaging 0.0750 MeV. Taking the fraction of ^{14}C to be 1.3×10^{-12} N of normal ^{12}C , and assuming the body is 13% carbon, estimate the fraction of the decay energy absorbed. (The rest escapes, exposing those close to you.)

Solution:

95%

Exercise:**Problem:**

If everyone in Australia received an extra 0.05 mSv per year of radiation, what would be the increase in the number of cancer deaths per year? (Assume that time had elapsed for the effects to become apparent.) Assume that there are 200×10^{-4} deaths per Sv of radiation per year. What percent of the actual number of cancer deaths recorded is this?

Glossary

radiotherapy

the use of ionizing radiation to treat ailments

therapeutic ratio

the ratio of abnormal cells killed to normal cells killed

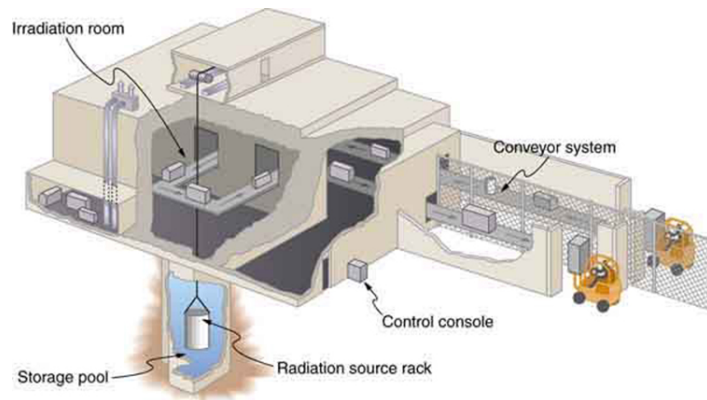
Food Irradiation

- Define food irradiation low dose, and free radicals.

Ionizing radiation is widely used to sterilize medical supplies, such as bandages, and consumer products, such as tampons. Worldwide, it is also used to irradiate food, an application that promises to grow in the future.

Food irradiation is the treatment of food with ionizing radiation. It is used to reduce pest infestation and to delay spoilage and prevent illness caused by microorganisms. Food irradiation is controversial. Proponents see it as superior to pasteurization, preservatives, and insecticides, supplanting dangerous chemicals with a more effective process. Opponents see its safety as unproven, perhaps leaving worse toxic residues as well as presenting an environmental hazard at treatment sites. In developing countries, food irradiation might increase crop production by 25.0% or more, and reduce food spoilage by a similar amount. It is used chiefly to treat spices and some fruits, and in some countries, red meat, poultry, and vegetables. Over 40 countries have approved food irradiation at some level.

Food irradiation exposes food to large doses of γ rays, x-rays, or electrons. These photons and electrons induce no nuclear reactions and thus create *no residual radioactivity*. (Some forms of ionizing radiation, such as neutron irradiation, cause residual radioactivity. These are not used for food irradiation.) The γ source is usually ^{60}Co or ^{137}Cs , the latter isotope being a major by-product of nuclear power. Cobalt-60 γ rays average 1.25 MeV, while those of ^{137}Cs are 0.67 MeV and are less penetrating. X-rays used for food irradiation are created with voltages of up to 5 million volts and, thus, have photon energies up to 5 MeV. Electrons used for food irradiation are accelerated to energies up to 10 MeV. The higher the energy per particle, the more penetrating the radiation is and the more ionization it can create. [\[link\]](#) shows a typical γ -irradiation plant.



A food irradiation plant has a conveyor system to pass items through an intense radiation field behind thick shielding walls. The γ source is lowered into a deep pool of water for safe storage when not in use.

Exposure times of up to an hour expose food to doses up to 10^4 Gy.

Owing to the fact that food irradiation seeks to destroy organisms such as insects and bacteria, much larger doses than those fatal to humans must be applied. Generally, the simpler the organism, the more radiation it can tolerate. (Cancer cells are a partial exception, because they are rapidly reproducing and, thus, more sensitive.) Current licensing allows up to 1000 Gy to be applied to fresh fruits and vegetables, called a *low dose* in food irradiation. Such a dose is enough to prevent or reduce the growth of many microorganisms, but about 10,000 Gy is needed to kill salmonella, and even more is needed to kill fungi. Doses greater than 10,000 Gy are considered to be high doses in food irradiation and product sterilization.

The effectiveness of food irradiation varies with the type of food. Spices and many fruits and vegetables have dramatically longer shelf lives. These also show no degradation in taste and no loss of food value or vitamins. If not for the mandatory labeling, such foods subjected to low-level irradiation (up to 1000 Gy) could not be distinguished from untreated foods in quality.

However, some foods actually spoil faster after irradiation, particularly those with high water content like lettuce and peaches. Others, such as milk, are given a noticeably unpleasant taste. High-level irradiation produces significant and chemically measurable changes in foods. It produces about a 15% loss of nutrients and a 25% loss of vitamins, as well as some change in taste. Such losses are similar to those that occur in ordinary freezing and cooking.

How does food irradiation work? Ionization produces a random assortment of broken molecules and ions, some with unstable oxygen- or hydrogen-containing molecules known as **free radicals**. These undergo rapid chemical reactions, producing perhaps four or five thousand different compounds called **radiolytic products**, some of which make cell function impossible by breaking cell membranes, fracturing DNA, and so on. How safe is the food afterward? Critics argue that the radiolytic products present a lasting hazard, perhaps being carcinogenic. However, the safety of irradiated food is not known precisely. We do know that low-level food irradiation produces no compounds in amounts that can be measured chemically. This is not surprising, since trace amounts of several thousand compounds may be created. We also know that there have been no observable negative short-term effects on consumers. Long-term effects may show up if large number of people consume large quantities of irradiated food, but no effects have appeared due to the small amounts of irradiated food that are consumed regularly. The case for safety is supported by testing of animal diets that were irradiated; no transmitted genetic effects have been observed. Food irradiation (at least up to a million rad) has been endorsed by the World Health Organization and the UN Food and Agricultural Organization. Finally, the hazard to consumers, if it exists, must be weighed against the benefits in food production and preservation. It must also be weighed against the very real hazards of existing insecticides and food preservatives.

Section Summary

- Food irradiation is the treatment of food with ionizing radiation.
- Irradiating food can destroy insects and bacteria by creating free radicals and radiolytic products that can break apart cell membranes.

- Food irradiation has produced no observable negative short-term effects for humans, but its long-term effects are unknown.

Conceptual Questions

Exercise:

Problem:

Does food irradiation leave the food radioactive? To what extent is the food altered chemically for low and high doses in food irradiation?

Exercise:

Problem:

Compare a low dose of radiation to a human with a low dose of radiation used in food treatment.

Exercise:

Problem:

Suppose one food irradiation plant uses a ^{137}Cs source while another uses an equal activity of ^{60}Co . Assuming equal fractions of the γ rays from the sources are absorbed, why is more time needed to get the same dose using the ^{137}Cs source?

Glossary

food irradiation

treatment of food with ionizing radiation

free radicals

ions with unstable oxygen- or hydrogen-containing molecules

radiolytic products

compounds produced due to chemical reactions of free radicals

Fusion

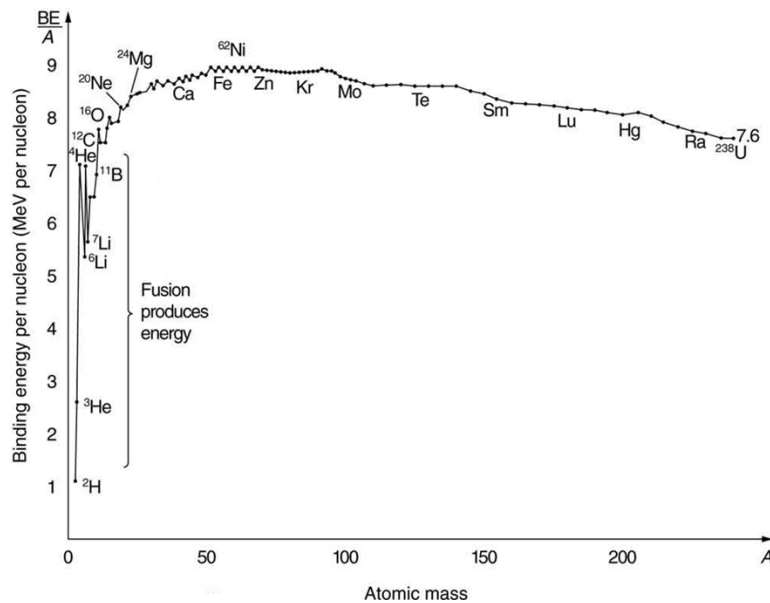
- Define nuclear fusion.
- Discuss processes to achieve practical fusion energy generation.

While basking in the warmth of the summer sun, a student reads of the latest breakthrough in achieving sustained thermonuclear power and vaguely recalls hearing about the cold fusion controversy. The three are connected. The Sun's energy is produced by nuclear fusion (see [\[link\]](#)). Thermonuclear power is the name given to the use of controlled nuclear fusion as an energy source. While research in the area of thermonuclear power is progressing, high temperatures and containment difficulties remain. The cold fusion controversy centered around unsubstantiated claims of practical fusion power at room temperatures.



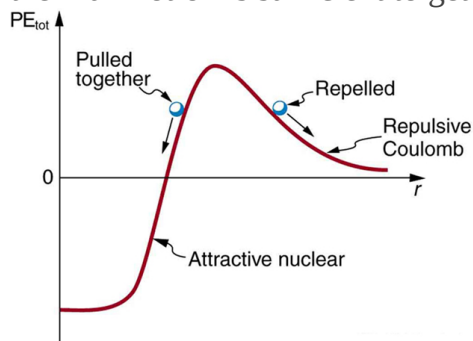
The Sun's energy is
produced by nuclear fusion.
(credit: Spiralz)

Nuclear fusion is a reaction in which two nuclei are combined, or *fused*, to form a larger nucleus. We know that all nuclei have less mass than the sum of the masses of the protons and neutrons that form them. The missing mass times c^2 equals the binding energy of the nucleus—the greater the binding energy, the greater the missing mass. We also know that BE/A , the binding energy per nucleon, is greater for medium-mass nuclei and has a maximum at Fe (iron). This means that if two low-mass nuclei can be fused together to form a larger nucleus, energy can be released. The larger nucleus has a greater binding energy and less mass per nucleon than the two that combined. Thus mass is destroyed in the fusion reaction, and energy is released (see [\[link\]](#)). On average, fusion of low-mass nuclei releases energy, but the details depend on the actual nuclides involved.



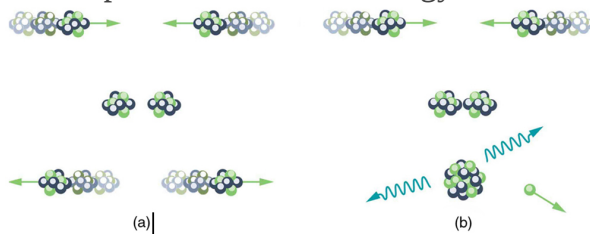
Fusion of light nuclei to form medium-mass nuclei destroys mass, because BE/A is greater for the product nuclei. The larger BE/A is, the less mass per nucleon, and so mass is converted to energy and released in these fusion reactions.

The major obstruction to fusion is the Coulomb repulsion between nuclei. Since the attractive nuclear force that can fuse nuclei together is short ranged, the repulsion of like positive charges must be overcome to get nuclei close enough to induce fusion. [\[link\]](#) shows an approximate graph of the potential energy between two nuclei as a function of the distance between their centers. The graph is analogous to a hill with a well in its center. A ball rolled from the right must have enough kinetic energy to get over the hump before it falls into the deeper well with a net gain in energy. So it is with fusion. If the nuclei are given enough kinetic energy to overcome the electric potential energy due to repulsion, then they can combine, release energy, and fall into a deep well. One way to accomplish this is to heat fusion fuel to high temperatures so that the kinetic energy of thermal motion is sufficient to get the nuclei together.



Potential energy between two light nuclei graphed as a function of distance between them. If the nuclei have enough kinetic energy to get over the Coulomb repulsion hump, they combine, release energy, and drop into a deep attractive well. Tunneling through the barrier is important in practice. The greater the kinetic energy and the higher the particles get up the barrier (or the lower the barrier), the more likely the tunneling.

You might think that, in the core of our Sun, nuclei are coming into contact and fusing. However, in fact, temperatures on the order of 10^8K are needed to actually get the nuclei in contact, exceeding the core temperature of the Sun. Quantum mechanical tunneling is what makes fusion in the Sun possible, and tunneling is an important process in most other practical applications of fusion, too. Since the probability of tunneling is extremely sensitive to barrier height and width, increasing the temperature greatly increases the rate of fusion. The closer reactants get to one another, the more likely they are to fuse (see [\[link\]](#)). Thus most fusion in the Sun and other stars takes place at their centers, where temperatures are highest. Moreover, high temperature is needed for thermonuclear power to be a practical source of energy.

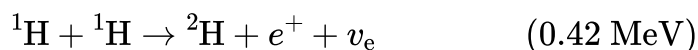


- (a) Two nuclei heading toward each other slow down, then stop, and then fly away without touching or fusing.
- (b) At higher energies, the two nuclei approach close enough for fusion via tunneling. The probability of tunneling increases as they approach,

but they do not have to touch for the reaction to occur.

The Sun produces energy by fusing protons or hydrogen nuclei ^1H (by far the Sun's most abundant nuclide) into helium nuclei ^4He . The principal sequence of fusion reactions forms what is called the **proton-proton cycle**:

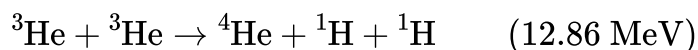
Equation:



Equation:

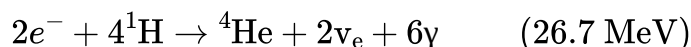


Equation:

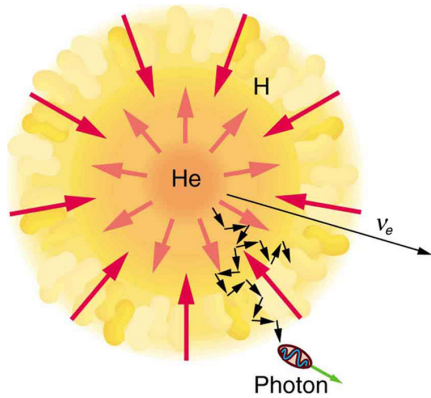


where e^+ stands for a positron and ν_e is an electron neutrino. (The energy in parentheses is *released* by the reaction.) Note that the first two reactions must occur twice for the third to be possible, so that the cycle consumes six protons (^1H) but gives back two. Furthermore, the two positrons produced will find two electrons and annihilate to form four more γ rays, for a total of six. The overall effect of the cycle is thus

Equation:

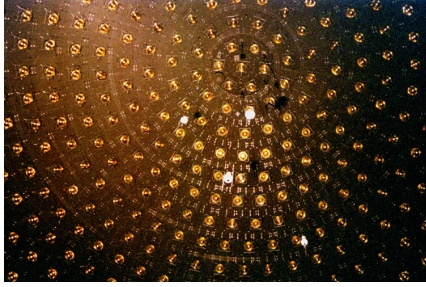


where the 26.7 MeV includes the annihilation energy of the positrons and electrons and is distributed among all the reaction products. The solar interior is dense, and the reactions occur deep in the Sun where temperatures are highest. It takes about 32,000 years for the energy to diffuse to the surface and radiate away. However, the neutrinos escape the Sun in less than two seconds, carrying their energy with them, because they interact so weakly that the Sun is transparent to them. Negative feedback in the Sun acts as a thermostat to regulate the overall energy output. For instance, if the interior of the Sun becomes hotter than normal, the reaction rate increases, producing energy that expands the interior. This cools it and lowers the reaction rate. Conversely, if the interior becomes too cool, it contracts, increasing the temperature and reaction rate (see [\[link\]](#)). Stars like the Sun are stable for billions of years, until a significant fraction of their hydrogen has been depleted. What happens then is discussed in [Introduction to Frontiers of Physics](#).

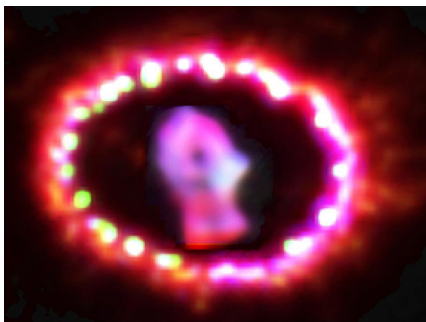


Nuclear fusion in the Sun converts hydrogen nuclei into helium; fusion occurs primarily at the boundary of the helium core, where temperature is highest and sufficient hydrogen remains. Energy released diffuses slowly to the surface, with the exception of neutrinos, which escape immediately. Energy production remains stable because of negative feedback effects.

Theories of the proton-proton cycle (and other energy-producing cycles in stars) were pioneered by the German-born, American physicist Hans Bethe (1906–2005), starting in 1938. He was awarded the 1967 Nobel Prize in physics for this work, and he has made many other contributions to physics and society. Neutrinos produced in these cycles escape so readily that they provide us an excellent means to test these theories and study stellar interiors. Detectors have been constructed and operated for more than four decades now to measure solar neutrinos (see [\[link\]](#)). Although solar neutrinos are detected and neutrinos were observed from Supernova 1987A ([\[link\]](#)), too few solar neutrinos were observed to be consistent with predictions of solar energy production. After many years, this solar neutrino problem was resolved with a blend of theory and experiment that showed that the neutrino does indeed have mass. It was also found that there are three types of neutrinos, each associated with a different type of nuclear decay.



This array of photomultiplier tubes is part of the large solar neutrino detector at the Fermi National Accelerator Laboratory in Illinois. In these experiments, the neutrinos interact with heavy water and produce flashes of light, which are detected by the photomultiplier tubes. In spite of its size and the huge flux of neutrinos that strike it, very few are detected each day since they interact so weakly. This, of course, is the same reason they escape the Sun so readily.
(credit: Fred Ullrich)

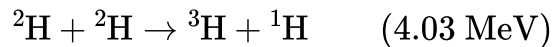


Supernovas are the source of elements heavier than

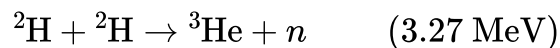
iron. Energy released powers nucleosynthesis. Spectroscopic analysis of the ring of material ejected by Supernova 1987A observable in the southern hemisphere, shows evidence of heavy elements. The study of this supernova also provided indications that neutrinos might have mass. (credit: NASA, ESA, and P. Challis)

The proton-proton cycle is not a practical source of energy on Earth, in spite of the great abundance of hydrogen (^1H). The reaction $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^+ + \nu_e$ has a very low probability of occurring. (This is why our Sun will last for about ten billion years.) However, a number of other fusion reactions are easier to induce. Among them are:

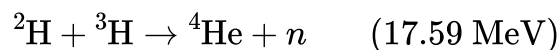
Equation:



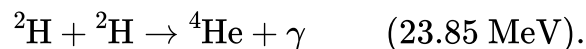
Equation:



Equation:

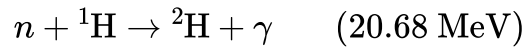


Equation:



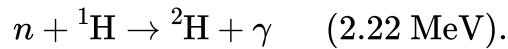
Deuterium (^2H) is about 0.015% of natural hydrogen, so there is an immense amount of it in sea water alone. In addition to an abundance of deuterium fuel, these fusion reactions produce large energies per reaction (in parentheses), but they do not produce much radioactive waste. Tritium (^3H) is radioactive, but it is consumed as a fuel (the reaction $^2\text{H} + ^3\text{H} \rightarrow ^4\text{He} + n$), and the neutrons and γ s can be shielded. The neutrons produced can also be used to create more energy and fuel in reactions like

Equation:



and

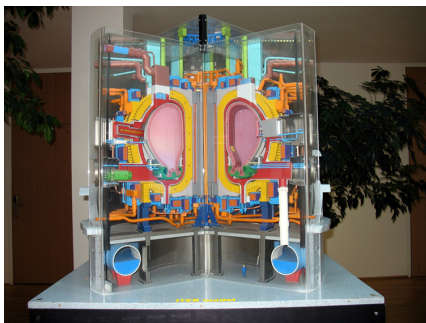
Equation:



Note that these last two reactions, and ${}^2\text{H} + {}^2\text{H} \rightarrow {}^4\text{He} + \gamma$, put most of their energy output into the γ ray, and such energy is difficult to utilize.

The three keys to practical fusion energy generation are to achieve the temperatures necessary to make the reactions likely, to raise the density of the fuel, and to confine it long enough to produce large amounts of energy. These three factors—temperature, density, and time—complement one another, and so a deficiency in one can be compensated for by the others. **Ignition** is defined to occur when the reactions produce enough energy to be self-sustaining after external energy input is cut off. This goal, which must be reached before commercial plants can be a reality, has not been achieved. Another milestone, called **break-even**, occurs when the fusion power produced equals the heating power input. Break-even has nearly been reached and gives hope that ignition and commercial plants may become a reality in a few decades.

Two techniques have shown considerable promise. The first of these is called **magnetic confinement** and uses the property that charged particles have difficulty crossing magnetic field lines. The tokamak, shown in [\[link\]](#), has shown particular promise. The tokamak's toroidal coil confines charged particles into a circular path with a helical twist due to the circulating ions themselves. In 1995, the Tokamak Fusion Test Reactor at Princeton in the US achieved world-record plasma temperatures as high as 500 million degrees Celsius. This facility operated between 1982 and 1997. A joint international effort is underway in France to build a tokamak-type reactor that will be the stepping stone to commercial power. ITER, as it is called, will be a full-scale device that aims to demonstrate the feasibility of fusion energy. It will generate 500 MW of power for extended periods of time and will achieve break-even conditions. It will study plasmas in conditions similar to those expected in a fusion power plant. Completion is scheduled for 2018.



(a) Artist's rendition of ITER, a tokamak-type fusion reactor being built in southern France. It is hoped that this gigantic machine will reach the break-even point. Completion is scheduled for 2018. (credit: Stephan Mosel, Flickr)

The second promising technique aims multiple lasers at tiny fuel pellets filled with a mixture of deuterium and tritium. Huge power input heats the fuel, evaporating the confining pellet and crushing the fuel to high density with the expanding hot plasma produced. This technique is called **inertial confinement**, because the fuel's inertia prevents it from escaping before significant fusion can take place. Higher densities have been reached than with tokamaks, but with smaller confinement times. In 2009, the Lawrence Livermore Laboratory (CA) completed a laser fusion device with 192 ultraviolet laser beams that are focused upon a D-T pellet (see [\[link\]](#)).



National Ignition Facility (CA). This image shows a laser bay where 192 laser beams will focus onto a small D-T target, producing fusion. (credit: Lawrence Livermore National Laboratory, Lawrence Livermore National Security, LLC, and the Department of Energy)

Example:**Calculating Energy and Power from Fusion**

(a) Calculate the energy released by the fusion of a 1.00-kg mixture of deuterium and tritium, which produces helium. There are equal numbers of deuterium and tritium nuclei in the mixture.

(b) If this takes place continuously over a period of a year, what is the average power output?

Strategy

According to ${}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n$, the energy per reaction is 17.59 MeV. To find the total energy released, we must find the number of deuterium and tritium atoms in a kilogram. Deuterium has an atomic mass of about 2 and tritium has an atomic mass of about 3, for a total of about 5 g per mole of reactants or about 200 mol in 1.00 kg. To get a more precise figure, we will use the atomic masses from Appendix A. The power output is best expressed in watts, and so the energy output needs to be calculated in joules and then divided by the number of seconds in a year.

Solution for (a)

The atomic mass of deuterium (${}^2\text{H}$) is 2.014102 u, while that of tritium (${}^3\text{H}$) is 3.016049 u, for a total of 5.032151 u per reaction. So a mole of reactants has a mass of 5.03 g, and in 1.00 kg there are $(1000 \text{ g})/(5.03 \text{ g/mol}) = 198.8 \text{ mol}$ of reactants. The number of reactions that take place is therefore

Equation:

$$(198.8 \text{ mol})(6.02 \times 10^{23} \text{ mol}^{-1}) = 1.20 \times 10^{26} \text{ reactions.}$$

The total energy output is the number of reactions times the energy per reaction:

Equation:

$$\begin{aligned} E &= (1.20 \times 10^{26} \text{ reactions})(17.59 \text{ MeV/reaction})(1.602 \times 10^{-13} \text{ J/MeV}) \\ &= 3.37 \times 10^{14} \text{ J.} \end{aligned}$$

Solution for (b)

Power is energy per unit time. One year has $3.16 \times 10^7 \text{ s}$, so

Equation:

$$\begin{aligned} P &= \frac{E}{t} = \frac{3.37 \times 10^{14} \text{ J}}{3.16 \times 10^7 \text{ s}} \\ &= 1.07 \times 10^7 \text{ W} = 10.7 \text{ MW.} \end{aligned}$$

Discussion

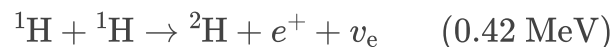
By now we expect nuclear processes to yield large amounts of energy, and we are not disappointed here. The energy output of $3.37 \times 10^{14} \text{ J}$ from fusing 1.00 kg of deuterium

and tritium is equivalent to 2.6 million gallons of gasoline and about eight times the energy output of the bomb that destroyed Hiroshima. Yet the average backyard swimming pool has about 6 kg of deuterium in it, so that fuel is plentiful if it can be utilized in a controlled manner. The average power output over a year is more than 10 MW, impressive but a bit small for a commercial power plant. About 32 times this power output would allow generation of 100 MW of electricity, assuming an efficiency of one-third in converting the fusion energy to electrical energy.

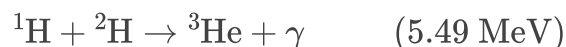
Section Summary

- Nuclear fusion is a reaction in which two nuclei are combined to form a larger nucleus. It releases energy when light nuclei are fused to form medium-mass nuclei.
- Fusion is the source of energy in stars, with the proton-proton cycle,

Equation:



Equation:



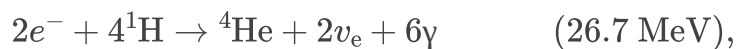
Equation:



being the principal sequence of energy-producing reactions in our Sun.

- The overall effect of the proton-proton cycle is

Equation:



where the 26.7 MeV includes the energy of the positrons emitted and annihilated.

- Attempts to utilize controlled fusion as an energy source on Earth are related to deuterium and tritium, and the reactions play important roles.
- Ignition is the condition under which controlled fusion is self-sustaining; it has not yet been achieved. Break-even, in which the fusion energy output is as great as the external energy input, has nearly been achieved.
- Magnetic confinement and inertial confinement are the two methods being developed for heating fuel to sufficiently high temperatures, at sufficient density, and for sufficiently long times to achieve ignition. The first method uses magnetic fields

and the second method uses the momentum of impinging laser beams for confinement.

Conceptual Questions

Exercise:

Problem: Why does the fusion of light nuclei into heavier nuclei release energy?

Exercise:

Problem:

Energy input is required to fuse medium-mass nuclei, such as iron or cobalt, into more massive nuclei. Explain why.

Exercise:

Problem:

In considering potential fusion reactions, what is the advantage of the reaction ${}^2\text{H} + {}^3\text{H} \rightarrow {}^4\text{He} + n$ over the reaction ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + n$?

Exercise:

Problem:

Give reasons justifying the contention made in the text that energy from the fusion reaction ${}^2\text{H} + {}^2\text{H} \rightarrow {}^4\text{He} + \gamma$ is relatively difficult to capture and utilize.

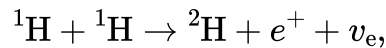
Problems & Exercises

Exercise:

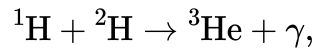
Problem:

Verify that the total number of nucleons, total charge, and electron family number are conserved for each of the fusion reactions in the proton-proton cycle in

Equation:

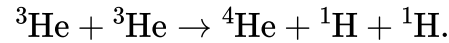


Equation:



and

Equation:



(List the value of each of the conserved quantities before and after each of the reactions.)

Solution:

(a) $A=1+1=2$, $Z=1+1=1+1$, $\text{efn} = 0 = -1 + 1$

(b) $A=1+2=3$, $Z=1+1=2$, $\text{efn}=0=0$

(c) $A=3+3=4+1+1$, $Z=2+2=2+1+1$, $\text{efn}=0=0$

Exercise:

Problem:

Calculate the energy output in each of the fusion reactions in the proton-proton cycle, and verify the values given in the above summary.

Exercise:

Problem:

Show that the total energy released in the proton-proton cycle is 26.7 MeV, considering the overall effect in ${}^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{H} + e^+ + \nu_e$, ${}^1\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + \gamma$, and ${}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + {}^1\text{H} + {}^1\text{H}$ and being certain to include the annihilation energy.

Solution:

$$\begin{aligned} E &= (m_i - m_f)c^2 \\ &= [4m({}^1\text{H}) - m({}^4\text{He})]c^2 \\ &= [4(1.007825) - 4.002603](931.5 \text{ MeV}) \\ &= 26.73 \text{ MeV} \end{aligned}$$

Exercise:

Problem:

Verify by listing the number of nucleons, total charge, and electron family number before and after the cycle that these quantities are conserved in the overall proton-proton cycle in $2e^- + 4{}^1\text{H} \rightarrow {}^4\text{He} + 2\nu_e + 6\gamma$.

Exercise:**Problem:**

The energy produced by the fusion of a 1.00-kg mixture of deuterium and tritium was found in Example [Calculating Energy and Power from Fusion](#). Approximately how many kilograms would be required to supply the annual energy use in the United States?

Solution:

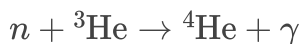
$$3.12 \times 10^5 \text{ kg (about 200 tons)}$$

Exercise:**Problem:**

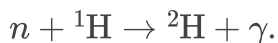
Tritium is naturally rare, but can be produced by the reaction $n + {}^2\text{H} \rightarrow {}^3\text{H} + \gamma$. How much energy in MeV is released in this neutron capture?

Exercise:

Problem: Two fusion reactions mentioned in the text are



and



Both reactions release energy, but the second also creates more fuel. Confirm that the energies produced in the reactions are 20.58 and 2.22 MeV, respectively. Comment on which product nuclide is most tightly bound, ${}^4\text{He}$ or ${}^2\text{H}$.

Solution:

$$E = (m_i - m_f)c^2$$

$$\begin{aligned} E_1 &= (1.008665 + 3.016030 - 4.002603)(931.5 \text{ MeV}) \\ &= 20.58 \text{ MeV} \end{aligned}$$

$$\begin{aligned} E_2 &= (1.008665 + 1.007825 - 2.014102)(931.5 \text{ MeV}) \\ &= 2.224 \text{ MeV} \end{aligned}$$

${}^4\text{He}$ is more tightly bound, since this reaction gives off more energy per nucleon.

Exercise:

Problem:

- (a) Calculate the number of grams of deuterium in an 80,000-L swimming pool, given deuterium is 0.0150% of natural hydrogen.
- (b) Find the energy released in joules if this deuterium is fused via the reaction ${}^2\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + n$.
- (c) Could the neutrons be used to create more energy?
- (d) Discuss the amount of this type of energy in a swimming pool as compared to that in, say, a gallon of gasoline, also taking into consideration that water is far more abundant.

Exercise:**Problem:**

How many kilograms of water are needed to obtain the 198.8 mol of deuterium, assuming that deuterium is 0.01500% (by number) of natural hydrogen?

Solution:

$$1.19 \times 10^4 \text{ kg}$$

Exercise:

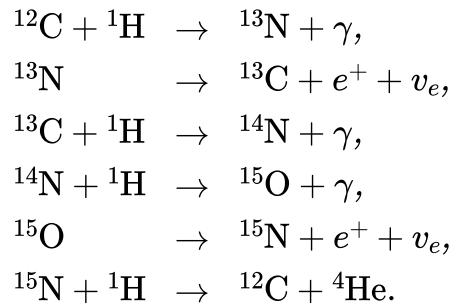
Problem: The power output of the Sun is $4 \times 10^{26} \text{ W}$.

- (a) If 90% of this is supplied by the proton-proton cycle, how many protons are consumed per second?
- (b) How many neutrinos per second should there be per square meter at the Earth from this process? This huge number is indicative of how rarely a neutrino interacts, since large detectors observe very few per day.

Exercise:**Problem:**

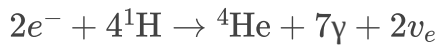
Another set of reactions that result in the fusing of hydrogen into helium in the Sun and especially in hotter stars is called the carbon cycle. It is

Equation:



Write down the overall effect of the carbon cycle (as was done for the proton-proton cycle in $2e^- + 4^1\text{H} \rightarrow ^4\text{He} + 2\nu_e + 6\gamma$). Note the number of protons (^1H) required and assume that the positrons (e^+) annihilate electrons to form more γ rays.

Solution:



Exercise:

Problem:

- Find the total energy released in MeV in each carbon cycle (elaborated in the above problem) including the annihilation energy.
- How does this compare with the proton-proton cycle output?

Exercise:

Problem:

Verify that the total number of nucleons, total charge, and electron family number are conserved for each of the fusion reactions in the carbon cycle given in the above problem. (List the value of each of the conserved quantities before and after each of the reactions.)

Solution:

- $A=12+1=13$, $Z=6+1=7$, $\text{efn} = 0 = 0$
- $A=13=13$, $Z=7=6+1$, $\text{efn} = 0 = -1 + 1$
- $A=13 + 1=14$, $Z=6+1=7$, $\text{efn} = 0 = 0$
- $A=14 + 1=15$, $Z=7+1=8$, $\text{efn} = 0 = 0$
- $A=15=15$, $Z=8=7+1$, $\text{efn} = 0 = -1 + 1$

(f) $A=15 + 1=12 + 4$, $Z=7+1=6 + 2$, $e_{fn} = 0 = 0$

Exercise:

Problem: Integrated Concepts

The laser system tested for inertial confinement can produce a 100-kJ pulse only 1.00 ns in duration. (a) What is the power output of the laser system during the brief pulse?

(b) How many photons are in the pulse, given their wavelength is $1.06 \mu\text{m}$?

(c) What is the total momentum of all these photons?

(d) How does the total photon momentum compare with that of a single 1.00 MeV deuterium nucleus?

Exercise:

Problem: Integrated Concepts

Find the amount of energy given to the ${}^4\text{He}$ nucleus and to the γ ray in the reaction $n + {}^3\text{He} \rightarrow {}^4\text{He} + \gamma$, using the conservation of momentum principle and taking the reactants to be initially at rest. This should confirm the contention that most of the energy goes to the γ ray.

Solution:

$$E_{\gamma} = 20.6 \text{ MeV}$$

$$E_{{}^4\text{He}} = 5.68 \times 10^{-2} \text{ MeV}$$

Exercise:

Problem: Integrated Concepts

(a) What temperature gas would have atoms moving fast enough to bring two ${}^3\text{He}$ nuclei into contact? Note that, because both are moving, the average kinetic energy only needs to be half the electric potential energy of these doubly charged nuclei when just in contact with one another.

(b) Does this high temperature imply practical difficulties for doing this in controlled fusion?

Exercise:

Problem: Integrated Concepts

(a) Estimate the years that the deuterium fuel in the oceans could supply the energy needs of the world. Assume world energy consumption to be ten times that of the United States which is 8×10^{19} J/y and that the deuterium in the oceans could be converted to energy with an efficiency of 32%. You must estimate or look up the amount of water in the oceans and take the deuterium content to be 0.015% of natural hydrogen to find the mass of deuterium available. Note that approximate energy yield of deuterium is 3.37×10^{14} J/kg.

(b) Comment on how much time this is by any human measure. (It is not an unreasonable result, only an impressive one.)

Solution:

(a) 3×10^9 y

(b) This is approximately half the lifetime of the Earth.

Glossary

break-even

when fusion power produced equals the heating power input

ignition

when a fusion reaction produces enough energy to be self-sustaining after external energy input is cut off

inertial confinement

a technique that aims multiple lasers at tiny fuel pellets evaporating and crushing them to high density

magnetic confinement

a technique in which charged particles are trapped in a small region because of difficulty in crossing magnetic field lines

nuclear fusion

a reaction in which two nuclei are combined, or fused, to form a larger nucleus

proton-proton cycle

the combined reactions ${}^1\text{H}+{}^1\text{H} \rightarrow {}^2\text{H}+e^++\nu_e$, ${}^1\text{H}+{}^2\text{H} \rightarrow {}^3\text{He}+\gamma$, and ${}^3\text{He}+{}^3\text{He} \rightarrow {}^4\text{He}+{}^1\text{H}+{}^1\text{H}$

Fission

- Define nuclear fission.
- Discuss how fission fuel reacts and describe what it produces.
- Describe controlled and uncontrolled chain reactions.

Nuclear fission is a reaction in which a nucleus is split (or *fissured*). Controlled fission is a reality, whereas controlled fusion is a hope for the future. Hundreds of nuclear fission power plants around the world attest to the fact that controlled fission is practical and, at least in the short term, economical, as seen in [\[link\]](#). Whereas nuclear power was of little interest for decades following TMI and Chernobyl (and now Fukushima Daiichi), growing concerns over global warming has brought nuclear power back on the table as a viable energy alternative. By the end of 2009, there were 442 reactors operating in 30 countries, providing 15% of the world's electricity. France provides over 75% of its electricity with nuclear power, while the US has 104 operating reactors providing 20% of its electricity. Australia and New Zealand have none. China is building nuclear power plants at the rate of one start every month.



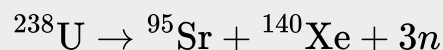
The people living near this nuclear power plant have no measurable exposure to radiation that is traceable to the plant. About 16% of the world's electrical power is generated by controlled nuclear fission in such plants. The cooling towers are the most prominent features but are not unique to nuclear

power. The reactor is in the small domed building to the left of the towers.
(credit: Kalmthouts)

Fission is the opposite of fusion and releases energy only when heavy nuclei are split. As noted in [Fusion](#), energy is released if the products of a nuclear reaction have a greater binding energy per nucleon (BE/A) than the parent nuclei. [\[link\]](#) shows that BE/A is greater for medium-mass nuclei than heavy nuclei, implying that when a heavy nucleus is split, the products have less mass per nucleon, so that mass is destroyed and energy is released in the reaction. The amount of energy per fission reaction can be large, even by nuclear standards. The graph in [\[link\]](#) shows BE/A to be about 7.6 MeV/nucleon for the heaviest nuclei (A about 240), while BE/A is about 8.6 MeV/nucleon for nuclei having A about 120. Thus, if a heavy nucleus splits in half, then about 1 MeV per nucleon, or approximately 240 MeV per fission, is released. This is about 10 times the energy per fusion reaction, and about 100 times the energy of the average α , β , or γ decay.

Example:**Calculating Energy Released by Fission**

Calculate the energy released in the following spontaneous fission reaction:

Equation:

given the atomic masses to be $m(^{238}\text{U}) = 238.050784$ u, $m(^{95}\text{Sr}) = 94.919388$ u, $m(^{140}\text{Xe}) = 139.921610$ u, and $m(n) = 1.008665$ u.

Strategy

As always, the energy released is equal to the mass destroyed times c^2 , so we must find the difference in mass between the parent ^{238}U and the fission products.

Solution

The products have a total mass of

Equation:

$$\begin{aligned}
 m_{\text{products}} &= 94.919388 \text{ u} + 139.921610 \text{ u} + 3(1.008665 \text{ u}) \\
 &= 237.866993 \text{ u}.
 \end{aligned}$$

The mass lost is the mass of ^{238}U minus m_{products} , or

Equation:

$$\Delta m = 238.050784 \text{ u} - 237.8669933 \text{ u} = 0.183791 \text{ u},$$

so the energy released is

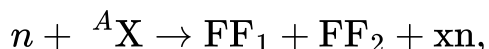
Equation:

$$\begin{aligned}
 E &= (\Delta m)c^2 \\
 &= (0.183791 \text{ u}) \frac{931.5 \text{ MeV}/c^2}{\text{u}} c^2 = 171.2 \text{ MeV}.
 \end{aligned}$$

Discussion

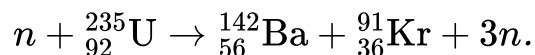
A number of important things arise in this example. The 171-MeV energy released is large, but a little less than the earlier estimated 240 MeV. This is because this fission reaction produces neutrons and does not split the nucleus into two equal parts. Fission of a given nuclide, such as ^{238}U , does not always produce the same products. Fission is a statistical process in which an entire range of products are produced with various probabilities. Most fission produces neutrons, although the number varies with each fission. This is an extremely important aspect of fission, because *neutrons can induce more fission*, enabling self-sustaining chain reactions.

Spontaneous fission can occur, but this is usually not the most common decay mode for a given nuclide. For example, ^{238}U can spontaneously fission, but it decays mostly by α emission. Neutron-induced fission is crucial as seen in [\[link\]](#). Being chargeless, even low-energy neutrons can strike a nucleus and be absorbed once they feel the attractive nuclear force. Large nuclei are described by a **liquid drop model** with surface tension and oscillation modes, because the large number of nucleons act like atoms in a drop. The neutron is attracted and thus, deposits energy, causing the nucleus to deform as a liquid drop. If stretched enough, the nucleus narrows in the middle. The number of nucleons in contact and the strength of the nuclear force binding the nucleus together are reduced. Coulomb repulsion between the two ends then succeeds in fissioning the nucleus, which pops like a water drop into two large pieces and a few neutrons. **Neutron-induced fission** can be written as

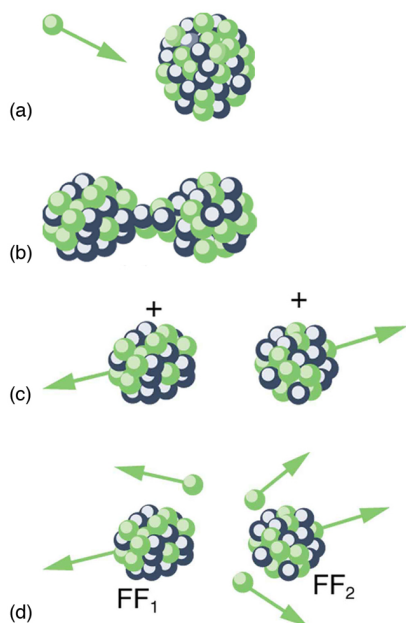
Equation:

where FF_1 and FF_2 are the two daughter nuclei, called **fission fragments**, and x is the number of neutrons produced. Most often, the masses of the fission fragments are not the same. Most of the released energy goes into the kinetic energy of the fission fragments, with the remainder going into the neutrons and excited states of the fragments. Since neutrons can induce fission, a self-sustaining chain reaction is possible, provided more than one neutron is produced on average — that is, if $x > 1$ in $n + {}^AX \rightarrow FF_1 + FF_2 + xn$. This can also be seen in [\[link\]](#).

An example of a typical neutron-induced fission reaction is

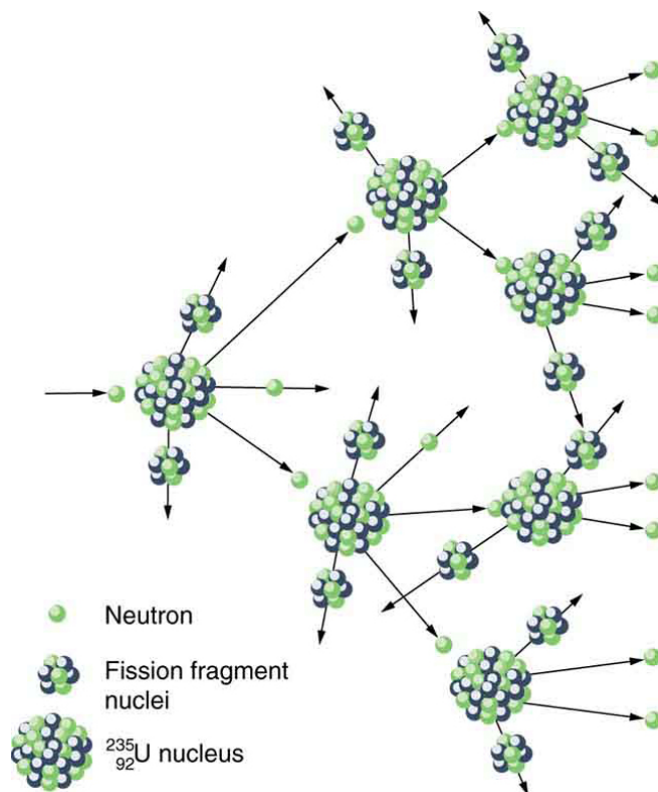
Equation:

Note that in this equation, the total charge remains the same (is conserved): $92 + 0 = 56 + 36$. Also, as far as whole numbers are concerned, the mass is constant: $1 + 235 = 142 + 91 + 3$. This is not true when we consider the masses out to 6 or 7 significant places, as in the previous example.



Neutron-induced

fission is shown. First, energy is put into this large nucleus when it absorbs a neutron. Acting like a struck liquid drop, the nucleus deforms and begins to narrow in the middle. Since fewer nucleons are in contact, the repulsive Coulomb force is able to break the nucleus into two parts with some neutrons also flying away.



A chain reaction can produce self-sustained fission if each fission

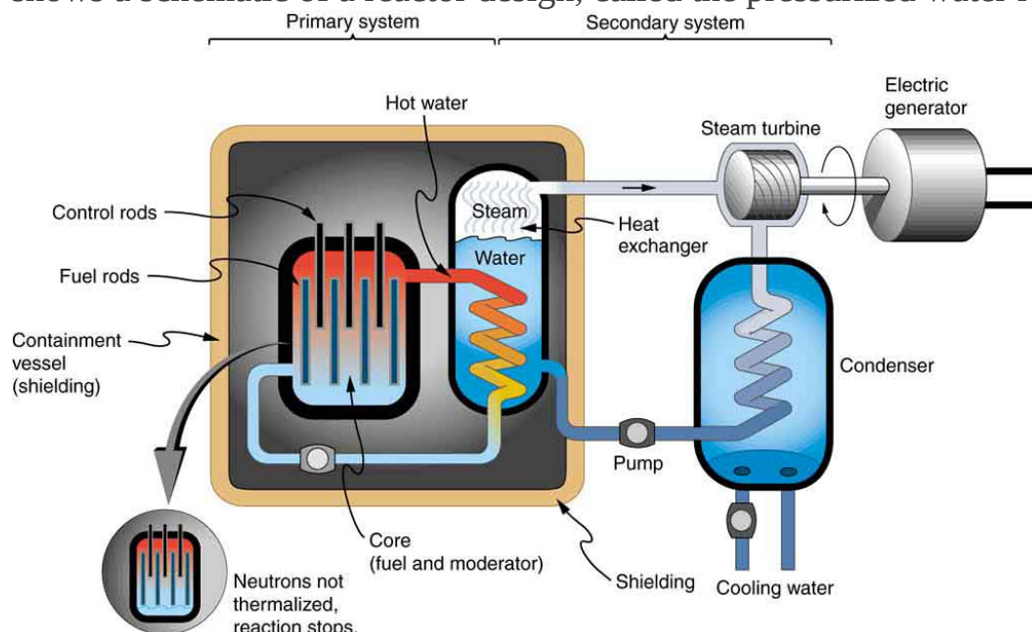
produces enough neutrons to induce at least one more fission. This depends on several factors, including how many neutrons are produced in an average fission and how easy it is to make a particular type of nuclide fission.

Not every neutron produced by fission induces fission. Some neutrons escape the fissionable material, while others interact with a nucleus without making it fission. We can enhance the number of fissions produced by neutrons by having a large amount of fissionable material. The minimum amount necessary for self-sustained fission of a given nuclide is called its **critical mass**. Some nuclides, such as ^{239}Pu , produce more neutrons per fission than others, such as ^{235}U . Additionally, some nuclides are easier to make fission than others. In particular, ^{235}U and ^{239}Pu are easier to fission than the much more abundant ^{238}U . Both factors affect critical mass, which is smallest for ^{239}Pu .

The reason ^{235}U and ^{239}Pu are easier to fission than ^{238}U is that the nuclear force is more attractive for an even number of neutrons in a nucleus than for an odd number. Consider that $^{235}_{92}\text{U}_{143}$ has 143 neutrons, and $^{239}_{94}\text{Pu}_{145}$ has 145 neutrons, whereas $^{238}_{92}\text{U}_{146}$ has 146. When a neutron encounters a nucleus with an odd number of neutrons, the nuclear force is more attractive, because the additional neutron will make the number even. About 2-MeV more energy is deposited in the resulting nucleus than would be the case if the number of neutrons was already even. This extra energy produces greater deformation, making fission more likely. Thus, ^{235}U and ^{239}Pu are superior fission fuels. The isotope ^{235}U is only 0.72 % of natural uranium, while ^{238}U is 99.27%, and ^{239}Pu does not exist in nature. Australia has the largest deposits of uranium in the world, standing at 28% of the total. This is followed by Kazakhstan and Canada. The US has only 3% of global reserves.

Most fission reactors utilize ^{235}U , which is separated from ^{238}U at some expense. This is called enrichment. The most common separation method is gaseous diffusion of uranium hexafluoride (UF_6) through membranes. Since ^{235}U has less mass than ^{238}U , its UF_6 molecules have higher average velocity at the same temperature and diffuse faster. Another interesting characteristic of ^{235}U is that it preferentially absorbs very slow moving neutrons (with energies a

fraction of an eV), whereas fission reactions produce fast neutrons with energies in the order of an MeV. To make a self-sustained fission reactor with ^{235}U , it is thus necessary to slow down (“thermalize”) the neutrons. Water is very effective, since neutrons collide with protons in water molecules and lose energy. [\[link\]](#) shows a schematic of a reactor design, called the pressurized water reactor.



A pressurized water reactor is cleverly designed to control the fission of large amounts of ^{235}U , while using the heat produced in the fission reaction to create steam for generating electrical energy. Control rods adjust neutron flux so that criticality is obtained, but not exceeded. In case the reactor overheats and boils the water away, the chain reaction terminates, because water is needed to thermalize the neutrons. This inherent safety feature can be overwhelmed in extreme circumstances.

Control rods containing nuclides that very strongly absorb neutrons are used to adjust neutron flux. To produce large power, reactors contain hundreds to thousands of critical masses, and the chain reaction easily becomes self-sustaining, a condition called **criticality**. Neutron flux should be carefully regulated to avoid an exponential increase in fissions, a condition called **supercriticality**. Control rods help prevent overheating, perhaps even a meltdown or explosive disassembly. The water that is used to thermalize

neutrons, necessary to get them to induce fission in ^{235}U , and achieve criticality, provides a negative feedback for temperature increases. In case the reactor overheats and boils the water to steam or is breached, the absence of water kills the chain reaction. Considerable heat, however, can still be generated by the reactor's radioactive fission products. Other safety features, thus, need to be incorporated in the event of a *loss of coolant* accident, including auxiliary cooling water and pumps.

Example:

Calculating Energy from a Kilogram of Fissionable Fuel

Calculate the amount of energy produced by the fission of 1.00 kg of ^{235}U , given the average fission reaction of ^{235}U produces 200 MeV.

Strategy

The total energy produced is the number of ^{235}U atoms times the given energy per ^{235}U fission. We should therefore find the number of ^{235}U atoms in 1.00 kg.

Solution

The number of ^{235}U atoms in 1.00 kg is Avogadro's number times the number of moles. One mole of ^{235}U has a mass of 235.04 g; thus, there are $(1000 \text{ g})/(235.04 \text{ g/mol}) = 4.25 \text{ mol}$. The number of ^{235}U atoms is therefore,

Equation:

$$(4.25 \text{ mol})(6.02 \times 10^{23} \text{ }^{235}\text{U}/\text{mol}) = 2.56 \times 10^{24} \text{ }^{235}\text{U}.$$

So the total energy released is

Equation:

$$\begin{aligned} E &= (2.56 \times 10^{24} \text{ }^{235}\text{U}) \left(\frac{200 \text{ MeV}}{^{235}\text{U}} \right) \left(\frac{1.60 \times 10^{-13} \text{ J}}{\text{MeV}} \right) \\ &= 8.21 \times 10^{13} \text{ J}. \end{aligned}$$

Discussion

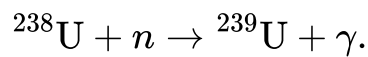
This is another impressively large amount of energy, equivalent to about 14,000 barrels of crude oil or 600,000 gallons of gasoline. But, it is only one-fourth the energy produced by the fusion of a kilogram mixture of deuterium and tritium as seen in [\[link\]](#). Even though each fission reaction yields about ten times the energy of a fusion reaction, the energy per kilogram of fission fuel is less, because there are far fewer moles per kilogram of the heavy nuclides. Fission

fuel is also much more scarce than fusion fuel, and less than 1% of uranium (the ^{235}U) is readily usable.

One nuclide already mentioned is ^{239}Pu , which has a 24,120-y half-life and does not exist in nature. Plutonium-239 is manufactured from ^{238}U in reactors, and it provides an opportunity to utilize the other 99% of natural uranium as an energy source. The following reaction sequence, called **breeding**, produces ^{239}Pu .

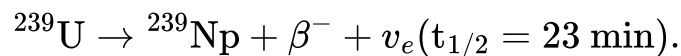
Breeding begins with neutron capture by ^{238}U :

Equation:



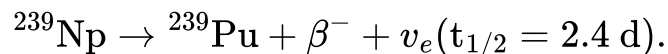
Uranium-239 then β^- decays:

Equation:



Neptunium-239 also β^- decays:

Equation:



Plutonium-239 builds up in reactor fuel at a rate that depends on the probability of neutron capture by ^{238}U (all reactor fuel contains more ^{238}U than ^{235}U). Reactors designed specifically to make plutonium are called **breeder reactors**. They seem to be inherently more hazardous than conventional reactors, but it remains unknown whether their hazards can be made economically acceptable. The four reactors at Chernobyl, including the one that was destroyed, were built to breed plutonium and produce electricity. These reactors had a design that was significantly different from the pressurized water reactor illustrated above.

Plutonium-239 has advantages over ^{235}U as a reactor fuel — it produces more neutrons per fission on average, and it is easier for a thermal neutron to cause it to fission. It is also chemically different from uranium, so it is inherently easier to separate from uranium ore. This means ^{239}Pu has a particularly small critical mass, an advantage for nuclear weapons.

Note:**PhET Explorations: Nuclear Fission**

Start a chain reaction, or introduce non-radioactive isotopes to prevent one.
Control energy production in a nuclear reactor!

<https://archive.cnx.org/specials/01caf0d0-116f-11e6-b891-abfdaa77b03b/nuclear-fission/#sim-one-nucleus>

Section Summary

- Nuclear fission is a reaction in which a nucleus is split.
- Fission releases energy when heavy nuclei are split into medium-mass nuclei.
- Self-sustained fission is possible, because neutron-induced fission also produces neutrons that can induce other fissions,
$$n + {}^A X \rightarrow \text{FF}_1 + \text{FF}_2 + x n$$
where FF_1 and FF_2 are the two daughter nuclei, or fission fragments, and x is the number of neutrons produced.
- A minimum mass, called the critical mass, should be present to achieve criticality.
- More than a critical mass can produce supercriticality.
- The production of new or different isotopes (especially ${}^{239}\text{Pu}$) by nuclear transformation is called breeding, and reactors designed for this purpose are called breeder reactors.

Conceptual Questions

Exercise:**Problem:**

Explain why the fission of heavy nuclei releases energy. Similarly, why is it that energy input is required to fission light nuclei?

Exercise:

Problem:

Explain, in terms of conservation of momentum and energy, why collisions of neutrons with protons will thermalize neutrons better than collisions with oxygen.

Exercise:

Problem:

The ruins of the Chernobyl reactor are enclosed in a huge concrete structure built around it after the accident. Some rain penetrates the building in winter, and radioactivity from the building increases. What does this imply is happening inside?

Exercise:

Problem:

Since the uranium or plutonium nucleus fissions into several fission fragments whose mass distribution covers a wide range of pieces, would you expect more residual radioactivity from fission than fusion? Explain.

Exercise:

Problem:

The core of a nuclear reactor generates a large amount of thermal energy from the decay of fission products, even when the power-producing fission chain reaction is turned off. Would this residual heat be greatest after the reactor has run for a long time or short time? What if the reactor has been shut down for months?

Exercise:

Problem:

How can a nuclear reactor contain many critical masses and not go supercritical? What methods are used to control the fission in the reactor?

Exercise:

Problem:

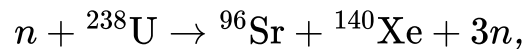
Why can heavy nuclei with odd numbers of neutrons be induced to fission with thermal neutrons, whereas those with even numbers of neutrons require more energy input to induce fission?

Exercise:**Problem:**

Why is a conventional fission nuclear reactor not able to explode as a bomb?

Problem Exercises**Exercise:****Problem:**

(a) Calculate the energy released in the neutron-induced fission (similar to the spontaneous fission in [\[link\]](#))

Equation:

given $m({}^{96}\text{Sr}) = 95.921750 \text{ u}$ and $m({}^{140}\text{Xe}) = 139.92164$. (b) This result is about 6 MeV greater than the result for spontaneous fission. Why? (c) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Solution:

(a) 177.1 MeV

(b) Because the gain of an external neutron yields about 6 MeV, which is the average BE/A for heavy nuclei.

(c)

$$A = 1 + 238 = 96 + 140 + 1 + 1 + 1, Z = 92 = 38 + 53, \text{efn} = 0 = 0$$

Exercise:

Problem:

(a) Calculate the energy released in the neutron-induced fission reaction

Equation:

given $m({}^{92}\text{Kr}) = 91.926269 \text{ u}$ and $m({}^{142}\text{Ba}) = 141.916361 \text{ u}$.

(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Exercise:**Problem:**

(a) Calculate the energy released in the neutron-induced fission reaction

Equation:

given $m({}^{96}\text{Sr}) = 95.921750 \text{ u}$ and $m({}^{140}\text{Ba}) = 139.910581 \text{ u}$.

(b) Confirm that the total number of nucleons and total charge are conserved in this reaction.

Solution:

(a) 180.6 MeV

(b)

$$A = 1 + 239 = 96 + 140 + 1 + 1 + 1 + 1, Z = 94 = 38 + 56, \text{efn} = 0 = 0$$

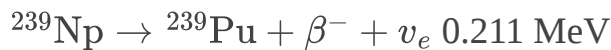
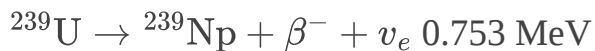
Exercise:**Problem:**

Confirm that each of the reactions listed for plutonium breeding just following [\[link\]](#) conserves the total number of nucleons, the total charge, and electron family number.

Exercise:

Problem:

Breeding plutonium produces energy even before any plutonium is fissioned. (The primary purpose of the four nuclear reactors at Chernobyl was breeding plutonium for weapons. Electrical power was a by-product used by the civilian population.) Calculate the energy produced in each of the reactions listed for plutonium breeding just following [\[link\]](#). The pertinent masses are $m(^{239}\text{U}) = 239.054289 \text{ u}$, $m(^{239}\text{Np}) = 239.052932 \text{ u}$, and $m(^{239}\text{Pu}) = 239.052157 \text{ u}$.

Solution:**Exercise:****Problem:**

The naturally occurring radioactive isotope ^{232}Th does not make good fission fuel, because it has an even number of neutrons; however, it can be bred into a suitable fuel (much as ^{238}U is bred into ^{239}Pu).

- (a) What are Z and N for ^{232}Th ?
- (b) Write the reaction equation for neutron captured by ^{232}Th and identify the nuclide ^AX produced in $n + ^{232}\text{Th} \rightarrow ^AX + \gamma$.
- (c) The product nucleus β^- decays, as does its daughter. Write the decay equations for each, and identify the final nucleus.
- (d) Confirm that the final nucleus has an odd number of neutrons, making it a better fission fuel.
- (e) Look up the half-life of the final nucleus to see if it lives long enough to be a useful fuel.

Exercise:

Problem:

The electrical power output of a large nuclear reactor facility is 900 MW. It has a 35.0% efficiency in converting nuclear power to electrical.

- (a) What is the thermal nuclear power output in megawatts?
 - (b) How many ^{235}U nuclei fission each second, assuming the average fission produces 200 MeV?
 - (c) What mass of ^{235}U is fissioned in one year of full-power operation?
-

Solution:

- (a) 2.57×10^3 MW
- (b) 8.03×10^{19} fission/s
- (c) 991 kg

Exercise:**Problem:**

A large power reactor that has been in operation for some months is turned off, but residual activity in the core still produces 150 MW of power. If the average energy per decay of the fission products is 1.00 MeV, what is the core activity in curies?

Glossary

breeder reactors

reactors that are designed specifically to make plutonium

breeding

reaction process that produces ^{239}Pu

criticality

condition in which a chain reaction easily becomes self-sustaining

critical mass

minimum amount necessary for self-sustained fission of a given nuclide

fission fragments

a daughter nuclei

liquid drop model

a model of nucleus (only to understand some of its features) in which nucleons in a nucleus act like atoms in a drop

nuclear fission

reaction in which a nucleus splits

neutron-induced fission

fission that is initiated after the absorption of neutron

supercriticality

an exponential increase in fissions

Nuclear Weapons

- Discuss different types of fission and thermonuclear bombs.
- Explain the ill effects of nuclear explosion.

The world was in turmoil when fission was discovered in 1938. The discovery of fission, made by two German physicists, Otto Hahn and Fritz Strassman, was quickly verified by two Jewish refugees from Nazi Germany, Lise Meitner and her nephew Otto Frisch. Fermi, among others, soon found that not only did neutrons induce fission; more neutrons were produced during fission. The possibility of a self-sustained chain reaction was immediately recognized by leading scientists the world over. The enormous energy known to be in nuclei, but considered inaccessible, now seemed to be available on a large scale.

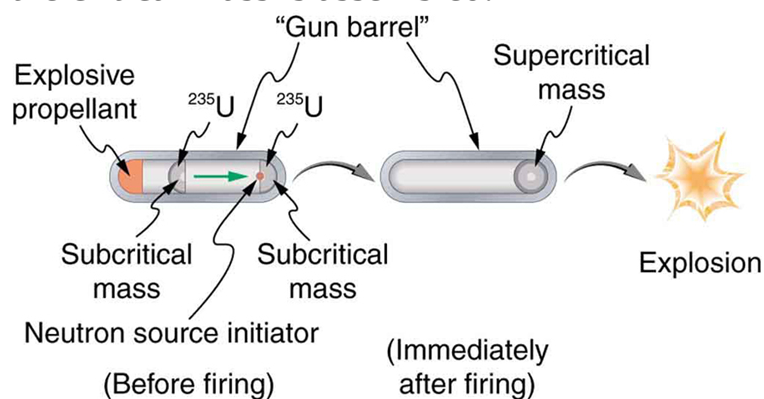
Within months after the announcement of the discovery of fission, Adolf Hitler banned the export of uranium from newly occupied Czechoslovakia. It seemed that the military value of uranium had been recognized in Nazi Germany, and that a serious effort to build a nuclear bomb had begun.

Alarmed scientists, many of them who fled Nazi Germany, decided to take action. None was more famous or revered than Einstein. It was felt that his help was needed to get the American government to make a serious effort at nuclear weapons as a matter of survival. Leo Szilard, an escaped Hungarian physicist, took a draft of a letter to Einstein, who, although pacifistic, signed the final version. The letter was for President Franklin Roosevelt, warning of the German potential to build extremely powerful bombs of a new type. It was sent in August of 1939, just before the German invasion of Poland that marked the start of World War II.

It was not until December 6, 1941, the day before the Japanese attack on Pearl Harbor, that the United States made a massive commitment to building a nuclear bomb. The top secret Manhattan Project was a crash program aimed at beating the Germans. It was carried out in remote locations, such as Los Alamos, New Mexico, whenever possible, and eventually came to cost billions of dollars and employ the efforts of more than 100,000 people. J. Robert Oppenheimer (1904–1967), whose talent and ambitions made him ideal, was chosen to head the project. The first

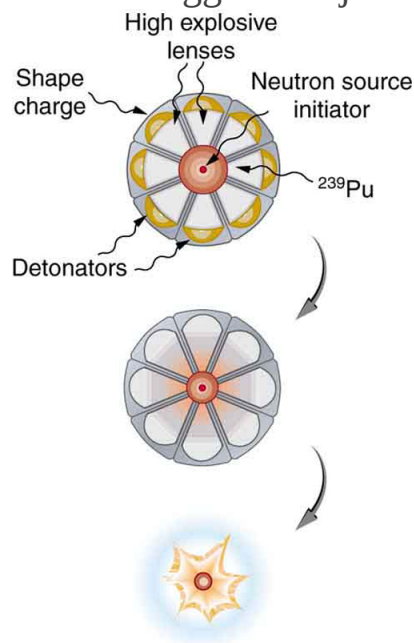
major step was made by Enrico Fermi and his group in December 1942, when they achieved the first self-sustained nuclear reactor. This first “atomic pile”, built in a squash court at the University of Chicago, used carbon blocks to thermalize neutrons. It not only proved that the chain reaction was possible, it began the era of nuclear reactors. Glenn Seaborg, an American chemist and physicist, received the Nobel Prize in physics in 1951 for discovery of several transuranic elements, including plutonium. Carbon-moderated reactors are relatively inexpensive and simple in design and are still used for breeding plutonium, such as at Chernobyl, where two such reactors remain in operation.

Plutonium was recognized as easier to fission with neutrons and, hence, a superior fission material very early in the Manhattan Project. Plutonium availability was uncertain, and so a uranium bomb was developed simultaneously. [\[link\]](#) shows a gun-type bomb, which takes two subcritical uranium masses and blows them together. To get an appreciable yield, the critical mass must be held together by the explosive charges inside the cannon barrel for a few microseconds. Since the buildup of the uranium chain reaction is relatively slow, the device to hold the critical mass together can be relatively simple. Owing to the fact that the rate of spontaneous fission is low, a neutron source is triggered at the same time the critical mass is assembled.



A gun-type fission bomb for ^{235}U utilizes two subcritical masses forced together by explosive charges inside a cannon barrel. The energy yield depends on the amount of uranium and the time it can be held together before it disassembles itself.

Plutonium's special properties necessitated a more sophisticated critical mass assembly, shown schematically in [\[link\]](#). A spherical mass of plutonium is surrounded by shape charges (high explosives that release most of their blast in one direction) that implode the plutonium, crushing it into a smaller volume to form a critical mass. The implosion technique is faster and more effective, because it compresses three-dimensionally rather than one-dimensionally as in the gun-type bomb. Again, a neutron source must be triggered at just the correct time to initiate the chain reaction.

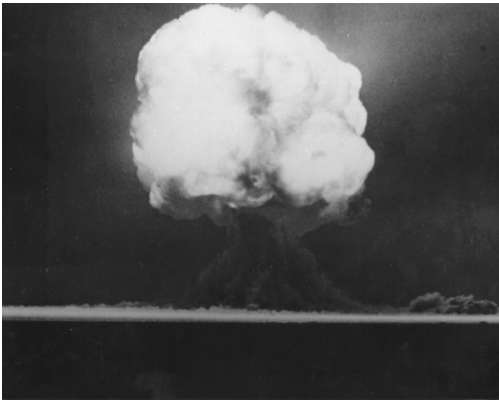


An implosion
created by high
explosives
compresses a
sphere of ^{239}Pu
into a critical mass.

The superior
fissionability of
plutonium has
made it the

universal bomb
material.

Owing to its complexity, the plutonium bomb needed to be tested before there could be any attempt to use it. On July 16, 1945, the test named Trinity was conducted in the isolated Alamogordo Desert about 200 miles south of Los Alamos (see [\[link\]](#)). A new age had begun. The yield of this device was about 10 kilotons (kT), the equivalent of 5000 of the largest conventional bombs.



Trinity test (1945), the
first nuclear bomb (credit:
United States Department
of Energy)

Although Germany surrendered on May 7, 1945, Japan had been steadfastly refusing to surrender for many months, forcing large casualties. Invasion plans by the Allies estimated a million casualties of their own and untold losses of Japanese lives. The bomb was viewed as a way to end the war. The first was a uranium bomb dropped on Hiroshima on August 6. Its yield of about 15 kT destroyed the city and killed an estimated 80,000 people, with 100,000 more being seriously injured (see [\[link\]](#)). The second was a plutonium bomb dropped on Nagasaki only three days later, on August 9. Its 20 kT yield killed at least 50,000 people, something less than Hiroshima because of the hilly terrain and the fact that it was a few kilometers off target. The Japanese were told that one bomb a week would be dropped

until they surrendered unconditionally, which they did on August 14. In actuality, the United States had only enough plutonium for one more and as yet unassembled bomb.

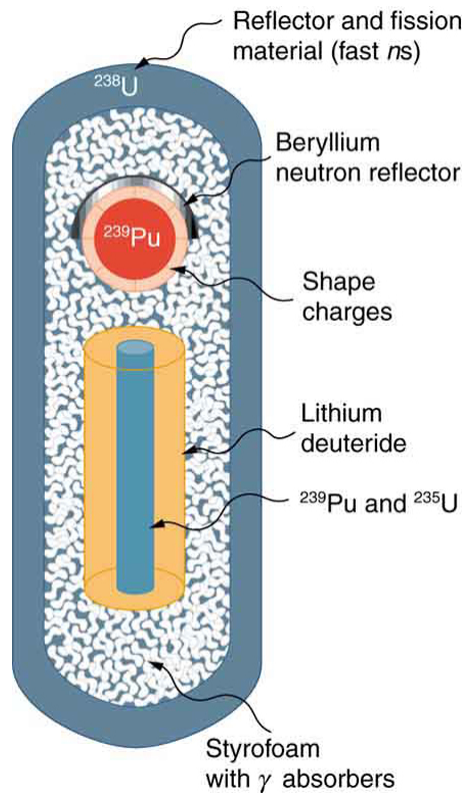


Destruction in Hiroshima
(credit: United States Federal
Government)

Knowing that fusion produces several times more energy per kilogram of fuel than fission, some scientists pushed the idea of a fusion bomb starting very early on. Calling this bomb the Super, they realized that it could have another advantage over fission—high-energy neutrons would aid fusion, while they are ineffective in ^{239}Pu fission. Thus the fusion bomb could be virtually unlimited in energy release. The first such bomb was detonated by the United States on October 31, 1952, at Eniwetok Atoll with a yield of 10 megatons (MT), about 670 times that of the fission bomb that destroyed Hiroshima. The Soviets followed with a fusion device of their own in August 1953, and a weapons race, beyond the aim of this text to discuss, continued until the end of the Cold War.

[\[link\]](#) shows a simple diagram of how a thermonuclear bomb is constructed. A fission bomb is exploded next to fusion fuel in the solid form of lithium deuteride. Before the shock wave blows it apart, γ rays heat and compress the fuel, and neutrons create tritium through the reaction $n + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^4\text{He}$. Additional fusion and fission fuels are enclosed in a dense shell of ^{238}U . The shell reflects some of the neutrons back into the fuel to enhance its fusion, but at high internal temperatures fast neutrons are

created that also cause the plentiful and inexpensive ^{238}U to fission, part of what allows thermonuclear bombs to be so large.

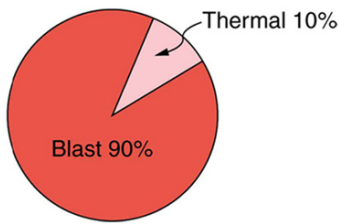


This schematic of a fusion bomb (H-bomb) gives some idea of how the ^{239}Pu fission trigger is used to ignite fusion fuel. Neutrons and γ rays transmit energy to the fusion fuel, create tritium from deuterium, and heat and compress the fusion fuel. The outer shell of ^{238}U serves to reflect some neutrons back into the fuel, causing more fusion,

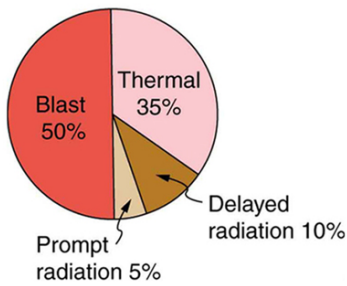
and it boosts the
energy output by
fissioning itself when
neutron energies
become high enough.

The energy yield and the types of energy produced by nuclear bombs can be varied. Energy yields in current arsenals range from about 0.1 kT to 20 MT, although the Soviets once detonated a 67 MT device. Nuclear bombs differ from conventional explosives in more than size. [\[link\]](#) shows the approximate fraction of energy output in various forms for conventional explosives and for two types of nuclear bombs. Nuclear bombs put a much larger fraction of their output into thermal energy than do conventional bombs, which tend to concentrate the energy in blast. Another difference is the immediate and residual radiation energy from nuclear weapons. This can be adjusted to put more energy into radiation (the so-called neutron bomb) so that the bomb can be used to irradiate advancing troops without killing friendly troops with blast and heat.

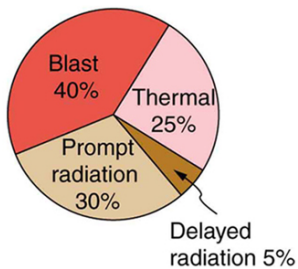
(a) Conventional chemical bomb



(b) Conventional nuclear bomb



(c) Radiation-enhanced nuclear bomb (neutron bomb)



Approximate fractions of energy output by conventional and two types of nuclear weapons. In addition to yielding more energy than conventional weapons, nuclear

bombs put a much larger fraction into thermal energy.

This can be adjusted to enhance the radiation output to be more effective against troops. An enhanced radiation bomb is also called a neutron bomb.

At its peak in 1986, the combined arsenals of the United States and the Soviet Union totaled about 60,000 nuclear warheads. In addition, the British, French, and Chinese each have several hundred bombs of various sizes, and a few other countries have a small number. Nuclear weapons are generally divided into two categories. Strategic nuclear weapons are those intended for military targets, such as bases and missile complexes, and moderate to large cities. There were about 20,000 strategic weapons in 1988. Tactical weapons are intended for use in smaller battles. Since the collapse of the Soviet Union and the end of the Cold War in 1989, most of the 32,000 tactical weapons (including Cruise missiles, artillery shells, land mines, torpedoes, depth charges, and backpacks) have been demobilized, and parts of the strategic weapon systems are being dismantled with warheads and missiles being disassembled. According to the Treaty of Moscow of 2002, Russia and the United States have been required to reduce their strategic nuclear arsenal down to about 2000 warheads each.

A few small countries have built or are capable of building nuclear bombs, as are some terrorist groups. Two things are needed—a minimum level of technical expertise and sufficient fissionable material. The first is easy. Fissionable material is controlled but is also available. There are international agreements and organizations that attempt to control nuclear proliferation, but it is increasingly difficult given the availability of

fissionable material and the small amount needed for a crude bomb. The production of fissionable fuel itself is technologically difficult. However, the presence of large amounts of such material worldwide, though in the hands of a few, makes control and accountability crucial.

Section Summary

- There are two types of nuclear weapons—fission bombs use fission alone, whereas thermonuclear bombs use fission to ignite fusion.
- Both types of weapons produce huge numbers of nuclear reactions in a very short time.
- Energy yields are measured in kilotons or megatons of equivalent conventional explosives and range from 0.1 kT to more than 20 MT.
- Nuclear bombs are characterized by far more thermal output and nuclear radiation output than conventional explosives.

Conceptual Questions

Exercise:

Problem:

What are some of the reasons that plutonium rather than uranium is used in all fission bombs and as the trigger in all fusion bombs?

Exercise:

Problem:

Use the laws of conservation of momentum and energy to explain how a shape charge can direct most of the energy released in an explosion in a specific direction. (Note that this is similar to the situation in guns and cannons—most of the energy goes into the bullet.)

Exercise:

Problem:

How does the lithium deuteride in the thermonuclear bomb shown in [\[link\]](#) supply tritium (^3H) as well as deuterium (^2H)?

Exercise:**Problem:**

Fallout from nuclear weapons tests in the atmosphere is mainly ^{90}Sr and ^{137}Cs , which have 28.6- and 32.2-y half-lives, respectively. Atmospheric tests were terminated in most countries in 1963, although China only did so in 1980. It has been found that environmental activities of these two isotopes are decreasing faster than their half-lives. Why might this be?

Problems & Exercises**Exercise:**

Problem: Find the mass converted into energy by a 12.0-kT bomb.

Solution:

0.56 g

Exercise:

Problem: What mass is converted into energy by a 1.00-MT bomb?

Exercise:**Problem:**

Fusion bombs use neutrons from their fission trigger to create tritium fuel in the reaction $n + {}^6\text{Li} \rightarrow {}^3\text{H} + {}^4\text{He}$. What is the energy released by this reaction in MeV?

Solution:

4.781 MeV

Exercise:

Problem:

It is estimated that the total explosive yield of all the nuclear bombs in existence currently is about 4,000 MT.

(a) Convert this amount of energy to kilowatt-hours, noting that $1 \text{ kW} \cdot \text{h} = 3.60 \times 10^6 \text{ J}$.

(b) What would the monetary value of this energy be if it could be converted to electricity costing 10 cents per kW·h?

Exercise:**Problem:**

A radiation-enhanced nuclear weapon (or neutron bomb) can have a smaller total yield and still produce more prompt radiation than a conventional nuclear bomb. This allows the use of neutron bombs to kill nearby advancing enemy forces with radiation without blowing up your own forces with the blast. For a 0.500-kT radiation-enhanced weapon and a 1.00-kT conventional nuclear bomb: (a) Compare the blast yields. (b) Compare the prompt radiation yields.

Solution:

(a) Blast yields $2.1 \times 10^{12} \text{ J}$ to $8.4 \times 10^{11} \text{ J}$, or 2.5 to 1, conventional to radiation enhanced.

(b) Prompt radiation yields $6.3 \times 10^{11} \text{ J}$ to $2.1 \times 10^{11} \text{ J}$, or 3 to 1, radiation enhanced to conventional.

Exercise:**Problem:**

(a) How many ^{239}Pu nuclei must fission to produce a 20.0-kT yield, assuming 200 MeV per fission? (b) What is the mass of this much ^{239}Pu ?

Exercise:

Problem:

Assume one-fourth of the yield of a typical 320-kT strategic bomb comes from fission reactions averaging 200 MeV and the remainder from fusion reactions averaging 20 MeV.

- (a) Calculate the number of fissions and the approximate mass of uranium and plutonium fissioned, taking the average atomic mass to be 238.
 - (b) Find the number of fusions and calculate the approximate mass of fusion fuel, assuming an average total atomic mass of the two nuclei in each reaction to be 5.
 - (c) Considering the masses found, does it seem reasonable that some missiles could carry 10 warheads? Discuss, noting that the nuclear fuel is only a part of the mass of a warhead.
-

Solution:

(a) 1.1×10^{25} fissions , 4.4 kg

(b) 3.2×10^{26} fusions , 2.7 kg

(c) The nuclear fuel totals only 6 kg, so it is quite reasonable that some missiles carry 10 warheads. The mass of the fuel would only be 60 kg and therefore the mass of the 10 warheads, weighing about 10 times the nuclear fuel, would be only 1500 lbs. If the fuel for the missiles weighs 5 times the total weight of the warheads, the missile would weigh about 9000 lbs or 4.5 tons. This is not an unreasonable weight for a missile.

Exercise:

Problem:

This problem gives some idea of the magnitude of the energy yield of a small tactical bomb. Assume that half the energy of a 1.00-kT nuclear depth charge set off under an aircraft carrier goes into lifting it out of the water—that is, into gravitational potential energy. How high is the carrier lifted if its mass is 90,000 tons?

Exercise:**Problem:**

It is estimated that weapons tests in the atmosphere have deposited approximately 9 MCi of ^{90}Sr on the surface of the earth. Find the mass of this amount of ^{90}Sr .

Solution:

$$7 \times 10^4 \text{ g}$$

Exercise:**Problem:**

A 1.00-MT bomb exploded a few kilometers above the ground deposits 25.0% of its energy into radiant heat.

(a) Find the calories per cm^2 at a distance of 10.0 km by assuming a uniform distribution over a spherical surface of that radius.

(b) If this heat falls on a person's body, what temperature increase does it cause in the affected tissue, assuming it is absorbed in a layer 1.00-cm deep?

Exercise:**Problem: Integrated Concepts**

One scheme to put nuclear weapons to nonmilitary use is to explode them underground in a geologically stable region and extract the

geothermal energy for electricity production. There was a total yield of about 4,000 MT in the combined arsenals in 2006. If 1.00 MT per day could be converted to electricity with an efficiency of 10.0%:

- (a) What would the average electrical power output be?
 - (b) How many years would the arsenal last at this rate?
-

Solution:

(a) $4.86 \times 10^9 \text{ W}$

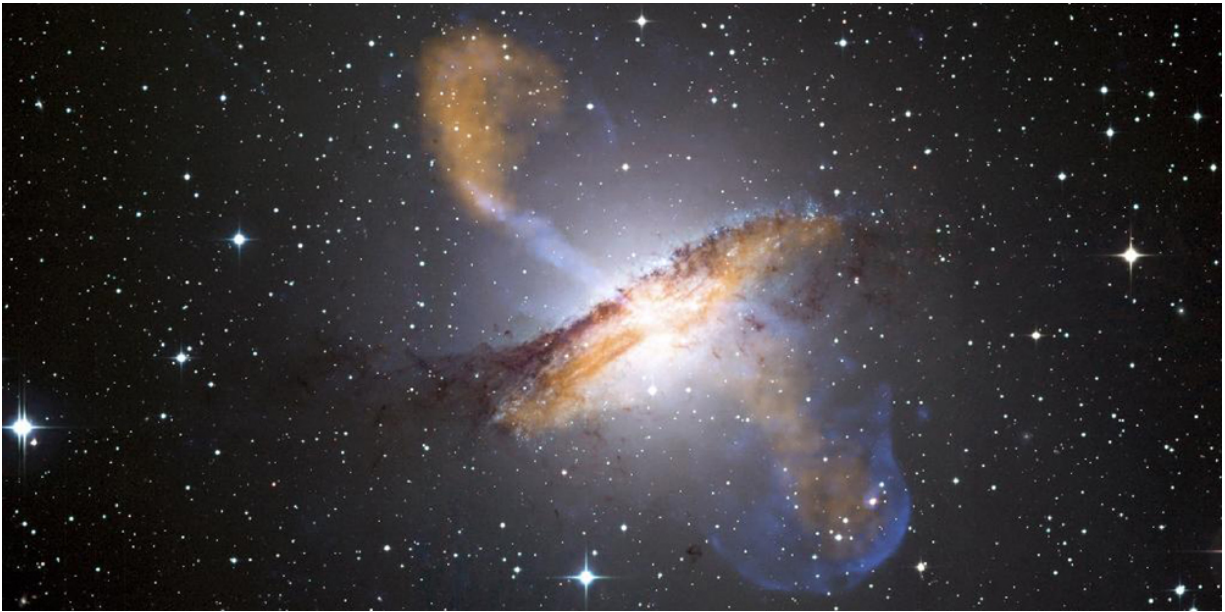
(b) 11.0 y

Introduction to Frontiers of Physics

class="introduction"

This galaxy is
ejecting huge jets of
matter, powered by
an immensely
massive black hole
at its center. (credit:

X-ray:
NASA/CXC/CfA/R.
Kraft et al.)



Frontiers are exciting. There is mystery, surprise, adventure, and discovery. The satisfaction of finding the answer to a question is made keener by the fact that the answer always leads to a new question. The picture of nature becomes more complete, yet nature retains its sense of mystery and never loses its ability to awe us. The view of physics is beautiful looking both backward and forward in time. What marvelous patterns we have discovered. How clever nature seems in its rules and connections. How awesome. And we continue looking ever deeper and ever further, probing

the basic structure of matter, energy, space, and time and wondering about the scope of the universe, its beginnings and future.

You are now in a wonderful position to explore the forefronts of physics, both the new discoveries and the unanswered questions. With the concepts, qualitative and quantitative, the problem-solving skills, the feeling for connections among topics, and all the rest you have mastered, you can more deeply appreciate and enjoy the brief treatments that follow. Years from now you will still enjoy the quest with an insight all the greater for your efforts.

Cosmology and Particle Physics

- Discuss the expansion of the universe.
- Explain the Big Bang.

Look at the sky on some clear night when you are away from city lights. There you will see thousands of individual stars and a faint glowing background of millions more. The Milky Way, as it has been called since ancient times, is an arm of our galaxy of stars—the word *galaxy* coming from the Greek word *galaxias*, meaning milky. We know a great deal about our Milky Way galaxy and of the billions of other galaxies beyond its fringes. But they still provoke wonder and awe (see [\[link\]](#)). And there are still many questions to be answered. Most remarkable when we view the universe on the large scale is that once again explanations of its character and evolution are tied to the very small scale. Particle physics and the questions being asked about the very small scales may also have their answers in the very large scales.

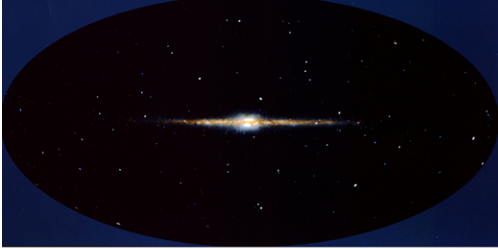


Take a moment to contemplate these clusters of galaxies, photographed by the Hubble Space Telescope. Trillions of stars linked by gravity in fantastic forms, glowing with light and showing evidence of undiscovered matter. What are they like, these myriad stars? How did they evolve? What can

they tell us of matter, energy,
space, and time? (credit: NASA,
ESA, K. Sharon (Tel Aviv
University) and E. Ofek
(Caltech))

As has been noted in numerous Things Great and Small vignettes, this is not the first time the large has been explained by the small and vice versa. Newton realized that the nature of gravity on Earth that pulls an apple to the ground could explain the motion of the moon and planets so much farther away. Minute atoms and molecules explain the chemistry of substances on a much larger scale. Decays of tiny nuclei explain the hot interior of the Earth. Fusion of nuclei likewise explains the energy of stars. Today, the patterns in particle physics seem to be explaining the evolution and character of the universe. And the nature of the universe has implications for unexplored regions of particle physics.

Cosmology is the study of the character and evolution of the universe. What are the major characteristics of the universe as we know them today? First, there are approximately 10^{11} galaxies in the observable part of the universe. An average galaxy contains more than 10^{11} stars, with our Milky Way galaxy being larger than average, both in its number of stars and its dimensions. Ours is a spiral-shaped galaxy with a diameter of about 100,000 light years and a thickness of about 2000 light years in the arms with a central bulge about 10,000 light years across. The Sun lies about 30,000 light years from the center near the galactic plane. There are significant clouds of gas, and there is a halo of less-dense regions of stars surrounding the main body. (See [\[link\]](#).) Evidence strongly suggests the existence of a large amount of additional matter in galaxies that does not produce light—the mysterious dark matter we shall later discuss.



(a)



(b)



(c)

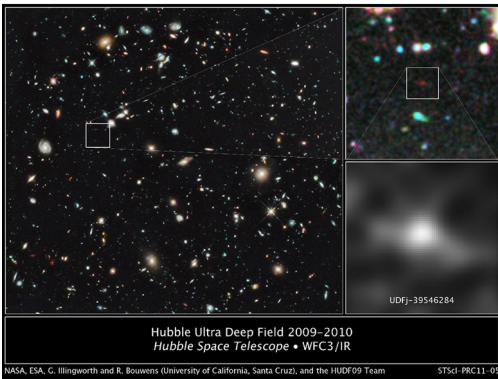
The Milky Way galaxy is typical of large spiral galaxies in its size, its shape, and the presence of gas and dust. We are fortunate to be in a location where we can see out of the galaxy and observe the vastly larger and fascinating universe

around us. (a) Side view.
(b) View from above. (c)
The Milky Way as seen
from Earth. (credits: (a)
NASA, (b) Nick Risinger,
(c) Andy)

Distances are great even within our galaxy and are measured in light years (the distance traveled by light in one year). The average distance between galaxies is on the order of a million light years, but it varies greatly with galaxies forming clusters such as shown in [\[link\]](#). The Magellanic Clouds, for example, are small galaxies close to our own, some 160,000 light years from Earth. The Andromeda galaxy is a large spiral galaxy like ours and lies 2 million light years away. It is just visible to the naked eye as an extended glow in the Andromeda constellation. Andromeda is the closest large galaxy in our local group, and we can see some individual stars in it with our larger telescopes. The most distant known galaxy is 14 billion light years from Earth—a truly incredible distance. (See [\[link\]](#).)



(a)



(b)

(a) Andromeda is the closest large galaxy, at 2 million light years distance, and is very similar to our Milky Way. The blue regions harbor young and emerging stars, while dark streaks are vast clouds of gas and dust. A smaller satellite galaxy is clearly visible.

(b) The box indicates what may be the most distant known galaxy, estimated to be 13 billion light years from us. It exists in a much older part of the universe.

(credit: NASA, ESA, G.

Illingworth (University of
California, Santa Cruz),
R. Bouwens (University
of California, Santa Cruz
and Leiden University),
and the HUDF09 Team)

Consider the fact that the light we receive from these vast distances has been on its way to us for a long time. In fact, the time in years is the same as the distance in light years. For example, the Andromeda galaxy is 2 million light years away, so that the light now reaching us left it 2 million years ago. If we could be there now, Andromeda would be different. Similarly, light from the most distant galaxy left it 14 billion years ago. We have an incredible view of the past when looking great distances. We can try to see if the universe was different then—if distant galaxies are more tightly packed or have younger-looking stars, for example, than closer galaxies, in which case there has been an evolution in time. But the problem is that the uncertainties in our data are great. Cosmology is almost typified by these large uncertainties, so that we must be especially cautious in drawing conclusions. One consequence is that there are more questions than answers, and so there are many competing theories. Another consequence is that any hard data produce a major result. Discoveries of some importance are being made on a regular basis, the hallmark of a field in its golden age.

Perhaps the most important characteristic of the universe is that all galaxies except those in our local cluster seem to be moving away from us at speeds proportional to their distance from our galaxy. It looks as if a gigantic explosion, universally called the **Big Bang**, threw matter out some billions of years ago. This amazing conclusion is based on the pioneering work of Edwin Hubble (1889–1953), the American astronomer. In the 1920s, Hubble first demonstrated conclusively that other galaxies, many previously called nebulae or clouds of stars, were outside our own. He then found that all but the closest galaxies have a red shift in their hydrogen spectra that is proportional to their distance. The explanation is that there is a **cosmological red shift** due to the expansion of space itself. The photon

wavelength is stretched in transit from the source to the observer. Double the distance, and the red shift is doubled. While this cosmological red shift is often called a Doppler shift, it is not—space itself is expanding. There is no center of expansion in the universe. All observers see themselves as stationary; the other objects in space appear to be moving away from them. Hubble was directly responsible for discovering that the universe was much larger than had previously been imagined and that it had this amazing characteristic of rapid expansion.

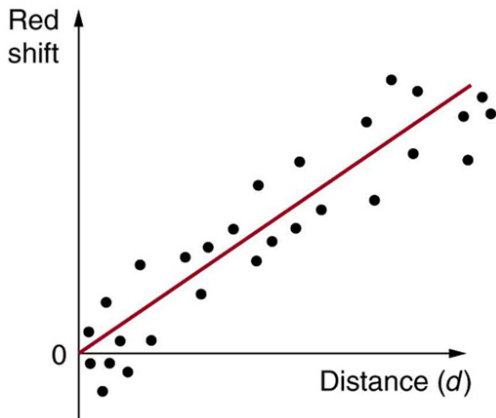
Universal expansion on the scale of galactic clusters (that is, galaxies at smaller distances are not uniformly receding from one another) is an integral part of modern cosmology. For galaxies farther away than about 50 Mly (50 million light years), the expansion is uniform with variations due to local motions of galaxies within clusters. A representative recession velocity v can be obtained from the simple formula

Equation:

$$v = H_0 d,$$

where d is the distance to the galaxy and H_0 is the **Hubble constant**. The Hubble constant is a central concept in cosmology. Its value is determined by taking the slope of a graph of velocity versus distance, obtained from red shift measurements, such as shown in [\[link\]](#). We shall use an approximate value of $H_0 = 20 \text{ km/s} \cdot \text{Mly}$. Thus, $v = H_0 d$ is an average behavior for all but the closest galaxies. For example, a galaxy 100 Mly away (as determined by its size and brightness) typically moves away from us at a speed of $v = (20 \text{ km/s} \cdot \text{Mly})(100 \text{ Mly}) = 2000 \text{ km/s}$. There can be variations in this speed due to so-called local motions or interactions with neighboring galaxies. Conversely, if a galaxy is found to be moving away from us at speed of 100,000 km/s based on its red shift, it is at a distance

$d = v/H_0 = (10,000 \text{ km/s})/(20 \text{ km/s} \cdot \text{Mly}) = 5000 \text{ Mly} = 5 \text{ Gly}$ or $5 \times 10^9 \text{ ly}$. This last calculation is approximate, because it assumes the expansion rate was the same 5 billion years ago as now. A similar calculation in Hubble's measurement changed the notion that the universe is in a steady state.

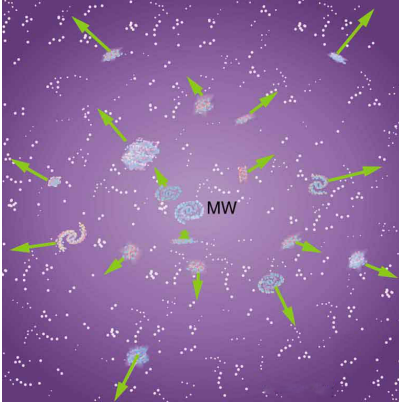


This graph of red shift versus distance for galaxies shows a linear relationship, with larger red shifts at greater distances, implying an expanding universe. The slope gives an approximate value for the expansion rate. (credit: John Cub).

One of the most intriguing developments recently has been the discovery that the expansion of the universe may be *faster now* than in the past, rather than slowing due to gravity as expected. Various groups have been looking, in particular, at supernovas in moderately distant galaxies (less than 1 Gly) to get improved distance measurements. Those distances are larger than expected for the observed galactic red shifts, implying the expansion was slower when that light was emitted. This has cosmological consequences that are discussed in [Dark Matter and Closure](#). The first results, published in 1999, are only the beginning of emerging data, with astronomy now entering a data-rich era.

[\[link\]](#) shows how the recession of galaxies looks like the remnants of a gigantic explosion, the famous Big Bang. Extrapolating backward in time,

the Big Bang would have occurred between 13 and 15 billion years ago when all matter would have been at a point. Questions instantly arise. What caused the explosion? What happened before the Big Bang? Was there a before, or did time start then? Will the universe expand forever, or will gravity reverse it into a Big Crunch? And is there other evidence of the Big Bang besides the well-documented red shifts?

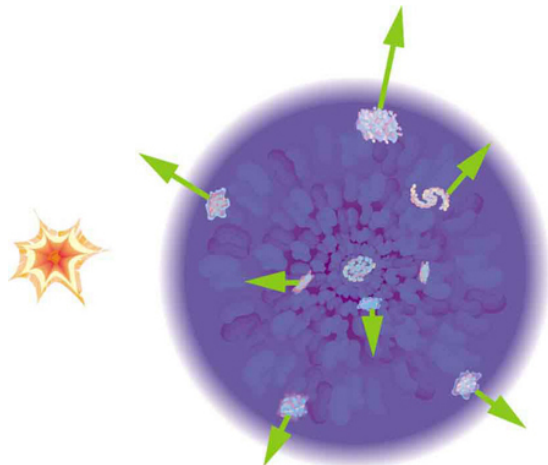


Galaxies are flying
apart from one
another, with the
more distant
moving faster as if
a primordial
explosion expelled
the matter from
which they formed.
The most distant
known galaxies
move nearly at the
speed of light
relative to us.

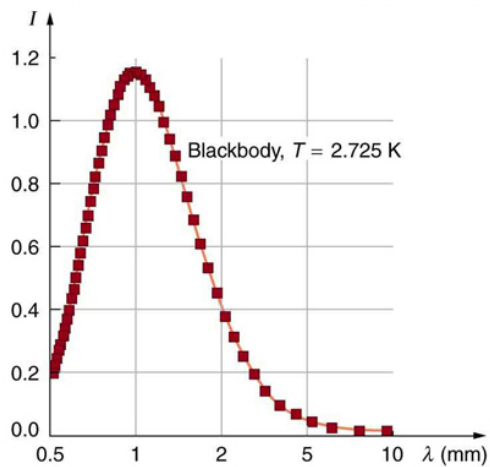
The Russian-born American physicist George Gamow (1904–1968) was among the first to note that, if there was a Big Bang, the remnants of the primordial fireball should still be evident and should be blackbody radiation. Since the radiation from this fireball has been traveling to us

since shortly after the Big Bang, its wavelengths should be greatly stretched. It will look as if the fireball has cooled in the billions of years since the Big Bang. Gamow and collaborators predicted in the late 1940s that there should be blackbody radiation from the explosion filling space with a characteristic temperature of about 7 K. Such blackbody radiation would have its peak intensity in the microwave part of the spectrum. (See [\[link\]](#).) In 1964, Arno Penzias and Robert Wilson, two American scientists working with Bell Telephone Laboratories on a low-noise radio antenna, detected the radiation and eventually recognized it for what it is.

[\[link\]](#)(b) shows the spectrum of this microwave radiation that permeates space and is of cosmic origin. It is the most perfect blackbody spectrum known, and the temperature of the fireball remnant is determined from it to be 2.725 ± 0.002 K. The detection of what is now called the **cosmic microwave background** (CMBR) was so important (generally considered as important as Hubble's detection that the galactic red shift is proportional to distance) that virtually every scientist has accepted the expansion of the universe as fact. Penzias and Wilson shared the 1978 Nobel Prize in Physics for their discovery.



(a)



(b)

(a) The Big Bang is used to explain the present observed expansion of the universe. It was an incredibly energetic explosion some 10 to 20 billion years ago. After expanding and cooling, galaxies form inside the now-cold remnants of the primordial fireball. (b) The spectrum of cosmic microwave radiation is the most perfect blackbody

spectrum ever detected. It is characteristic of a temperature of 2.725 K, the expansion-cooled temperature of the Big Bang's remnant. This radiation can be measured coming from any direction in space not obscured by some other source. It is compelling evidence of the creation of the universe in a gigantic explosion, already indicated by galactic red shifts.

Note:

Making Connections: Cosmology and Particle Physics

There are many connections of cosmology—by definition involving physics on the largest scale—with particle physics—by definition physics on the smallest scale. Among these are the dominance of matter over antimatter, the nearly perfect uniformity of the cosmic microwave background, and the mere existence of galaxies.

Matter versus antimatter

We know from direct observation that antimatter is rare. The Earth and the solar system are nearly pure matter. Space probes and cosmic rays give direct evidence—the landing of the Viking probes on Mars would have been spectacular explosions of mutual annihilation energy if Mars were antimatter. We also know that most of the universe is dominated by matter. This is proven by the lack of annihilation radiation coming to us from space, particularly the relative absence of 0.511-MeV γ rays created by the

mutual annihilation of electrons and positrons. It seemed possible that there could be entire solar systems or galaxies made of antimatter in perfect symmetry with our matter-dominated systems. But the interactions between stars and galaxies would sometimes bring matter and antimatter together in large amounts. The annihilation radiation they would produce is simply not observed. Antimatter in nature is created in particle collisions and in β^+ decays, but only in small amounts that quickly annihilate, leaving almost pure matter surviving.

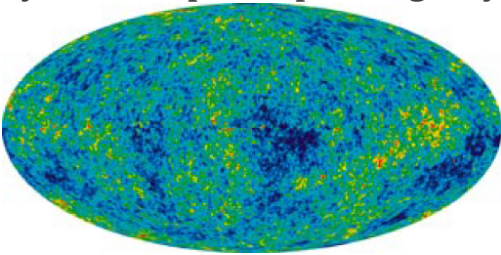
Particle physics seems symmetric in matter and antimatter. Why isn't the cosmos? The answer is that particle physics is not quite perfectly symmetric in this regard. The decay of one of the neutral K -mesons, for example, preferentially creates more matter than antimatter. This is caused by a fundamental small asymmetry in the basic forces. This small asymmetry produced slightly more matter than antimatter in the early universe. If there was only one part in 10^9 more matter (a small asymmetry), the rest would annihilate pair for pair, leaving nearly pure matter to form the stars and galaxies we see today. So the vast number of stars we observe may be only a tiny remnant of the original matter created in the Big Bang. Here at last we see a very real and important asymmetry in nature. Rather than be disturbed by an asymmetry, most physicists are impressed by how small it is. Furthermore, if the universe were completely symmetric, the mutual annihilation would be more complete, leaving far less matter to form us and the universe we know.

How can something so old have so few wrinkles?

A troubling aspect of cosmic microwave background radiation (CMBR) was soon recognized. True, the CMBR verified the Big Bang, had the correct temperature, and had a blackbody spectrum as expected. But the CMBR was *too* smooth—it looked identical in every direction. Galaxies and other similar entities could not be formed without the existence of fluctuations in the primordial stages of the universe and so there should be hot and cool spots in the CMBR, nicknamed wrinkles, corresponding to dense and sparse regions of gas caused by turbulence or early fluctuations. Over time, dense regions would contract under gravity and form stars and galaxies. Why aren't the fluctuations there? (This is a good example of an answer producing more questions.) Furthermore, galaxies are observed very

far from us, so that they formed very long ago. The problem was to explain how galaxies could form so early and so quickly after the Big Bang if its remnant fingerprint is perfectly smooth. The answer is that if you look very closely, the CMBR is not perfectly smooth, only extremely smooth.

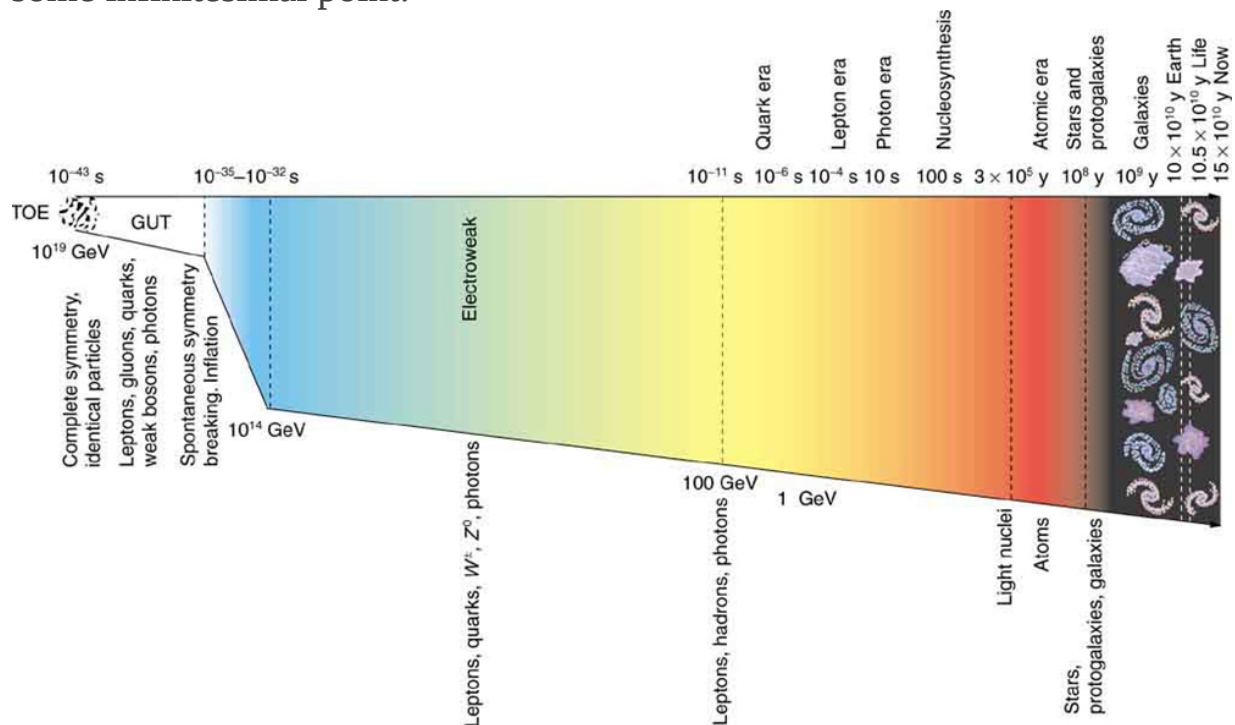
A satellite called the Cosmic Background Explorer (COBE) carried an instrument that made very sensitive and accurate measurements of the CMBR. In April of 1992, there was extraordinary publicity of COBE's first results—there were small fluctuations in the CMBR. Further measurements were carried out by experiments including NASA's Wilkinson Microwave Anisotropy Probe (WMAP), which launched in 2001. Data from WMAP provided a much more detailed picture of the CMBR fluctuations. (See [\[link\]](#).) These amount to temperature fluctuations of only $200\ \mu\text{K}$ out of $2.7\ \text{K}$, better than one part in 1000. The WMAP experiment will be followed up by the European Space Agency's Planck Surveyor, which launched in 2009.



This map of the sky uses color to show fluctuations, or wrinkles, in the cosmic microwave background observed with the WMAP spacecraft. The Milky Way has been removed for clarity. Red represents higher temperature and higher density, while blue is lower temperature and density. The fluctuations are small, less than one part in 1000, but these are still thought to be the

cause of the eventual
formation of galaxies.
(credit: NASA/WMAP
Science Team)

Let us now examine the various stages of the overall evolution of the universe from the Big Bang to the present, illustrated in [\[link\]](#). Note that scientific notation is used to encompass the many orders of magnitude in time, energy, temperature, and size of the universe. Going back in time, the two lines approach but do not cross (there is no zero on an exponential scale). Rather, they extend indefinitely in ever-smaller time intervals to some infinitesimal point.



The evolution of the universe from the Big Bang onward is intimately tied to the laws of physics, especially those of particle physics at the earliest stages. The universe is relativistic throughout its history. Theories of the unification of forces at high energies may be verified by their shaping of the universe and its evolution.

Going back in time is equivalent to what would happen if expansion stopped and gravity pulled all the galaxies together, compressing and heating all matter. At a time long ago, the temperature and density were too high for stars and galaxies to exist. Before then, there was a time when the temperature was too great for atoms to exist. And farther back yet, there was a time when the temperature and density were so great that nuclei could not exist. Even farther back in time, the temperature was so high that average kinetic energy was great enough to create short-lived particles, and the density was high enough to make this likely. When we extrapolate back to the point of $W^\pm$ and Z^0 production (thermal energies reaching 1 TeV, or a temperature of about 10^{15} K), we reach the limits of what we know directly about particle physics. This is at a time about 10^{-12} s after the Big Bang. While 10^{-12} s may seem to be negligibly close to the instant of creation, it is not. There are important stages before this time that are tied to the unification of forces. At those stages, the universe was at extremely high energies and average particle separations were smaller than we can achieve with accelerators. What happened in the early stages before 10^{-12} s is crucial to all later stages and is possibly discerned by observing present conditions in the universe. One of these is the smoothness of the CMBR.

Names are given to early stages representing key conditions. The stage before 10^{-11} s back to 10^{-34} s is called the **electroweak epoch**, because the electromagnetic and weak forces become identical for energies above about 100 GeV. As discussed earlier, theorists expect that the strong force becomes identical to and thus unified with the electroweak force at energies of about 10^{14} GeV. The average particle energy would be this great at 10^{-34} s after the Big Bang, if there are no surprises in the unknown physics at energies above about 1 TeV. At the immense energy of 10^{14} GeV (corresponding to a temperature of about 10^{26} K), the $W^\pm$ and Z^0 carrier particles would be transformed into massless gauge bosons to accomplish the unification. Before 10^{-34} s back to about 10^{-43} s, we have Grand Unification in the **GUT epoch**, in which all forces except gravity are identical. At 10^{-43} s, the average energy reaches the immense 10^{19} GeV needed to unify gravity with the other forces in TOE, the Theory of Everything. Before that time is the **TOE epoch**, but we have almost no idea

as to the nature of the universe then, since we have no workable theory of quantum gravity. We call the hypothetical unified force **superforce**.

Now let us imagine starting at TOE and moving forward in time to see what type of universe is created from various events along the way. As temperatures and average energies decrease with expansion, the universe reaches the stage where average particle separations are large enough to see differences between the strong and electroweak forces (at about 10^{-35} s). After this time, the forces become distinct in almost all interactions—they are no longer unified or symmetric. This transition from GUT to electroweak is an example of **spontaneous symmetry breaking**, in which conditions spontaneously evolved to a point where the forces were no longer unified, breaking that symmetry. This is analogous to a phase transition in the universe, and a clever proposal by American physicist Alan Guth in the early 1980s ties it to the smoothness of the CMBR. Guth proposed that spontaneous symmetry breaking (like a phase transition during cooling of normal matter) released an immense amount of energy that caused the universe to expand extremely rapidly for the brief time from 10^{-35} s to about 10^{-32} s. This expansion may have been by an incredible factor of 10^{50} or more in the size of the universe and is thus called the **inflationary scenario**. One result of this inflation is that it would stretch the wrinkles in the universe nearly flat, leaving an extremely smooth CMBR. While speculative, there is as yet no other plausible explanation for the smoothness of the CMBR. Unless the CMBR is not really cosmic but local in origin, the distances between regions of similar temperatures are too great for any coordination to have caused them, since any coordination mechanism must travel at the speed of light. Again, particle physics and cosmology are intimately entwined. There is little hope that we may be able to test the inflationary scenario directly, since it occurs at energies near 10^{14} GeV, vastly greater than the limits of modern accelerators. But the idea is so attractive that it is incorporated into most cosmological theories.

Characteristics of the present universe may help us determine the validity of this intriguing idea. Additionally, the recent indications that the universe's expansion rate may be *increasing* (see [Dark Matter and Closure](#)) could even imply that we are *in* another inflationary epoch.

It is important to note that, if conditions such as those found in the early universe could be created in the laboratory, we would see the unification of forces directly today. The forces have not changed in time, but the average energy and separation of particles in the universe have. As discussed in [The Four Basic Forces](#), the four basic forces in nature are distinct under most circumstances found today. The early universe and its remnants provide evidence from times when they were unified under most circumstances.

Section Summary

- Cosmology is the study of the character and evolution of the universe.
- The two most important features of the universe are the cosmological red shifts of its galaxies being proportional to distance and its cosmic microwave background (CMBR). Both support the notion that there was a gigantic explosion, known as the Big Bang that created the universe.
- Galaxies farther away than our local group have, on an average, a recessional velocity given by

Equation:

$$v = H_0 d,$$

where d is the distance to the galaxy and H_0 is the Hubble constant, taken to have the average value $H_0 = 20 \text{ km/s} \cdot \text{Mly}$.

- Explanations of the large-scale characteristics of the universe are intimately tied to particle physics.
- The dominance of matter over antimatter and the smoothness of the CMBR are two characteristics that are tied to particle physics.
- The epochs of the universe are known back to very shortly after the Big Bang, based on known laws of physics.
- The earliest epochs are tied to the unification of forces, with the electroweak epoch being partially understood, the GUT epoch being speculative, and the TOE epoch being highly speculative since it involves an unknown single superforce.
- The transition from GUT to electroweak is called spontaneous symmetry breaking. It released energy that caused the inflationary

scenario, which in turn explains the smoothness of the CMBR.

Conceptual Questions

Exercise:

Problem:

Explain why it only *appears* that we are at the center of expansion of the universe and why an observer in another galaxy would see the same relative motion of all but the closest galaxies away from her.

Exercise:

Problem:

If there is no observable edge to the universe, can we determine where its center of expansion is? Explain.

Exercise:

Problem: If the universe is infinite, does it have a center? Discuss.

Exercise:

Problem:

Another known cause of red shift in light is the source being in a high gravitational field. Discuss how this can be eliminated as the source of galactic red shifts, given that the shifts are proportional to distance and not to the size of the galaxy.

Exercise:

Problem:

If some unknown cause of red shift—such as light becoming “tired” from traveling long distances through empty space—is discovered, what effect would there be on cosmology?

Exercise:

Problem:

Olbers's paradox poses an interesting question: If the universe is infinite, then any line of sight should eventually fall on a star's surface. Why then is the sky dark at night? Discuss the commonly accepted evolution of the universe as a solution to this paradox.

Exercise:**Problem:**

If the cosmic microwave background radiation (CMBR) is the remnant of the Big Bang's fireball, we expect to see hot and cold regions in it. What are two causes of these wrinkles in the CMBR? Are the observed temperature variations greater or less than originally expected?

Exercise:**Problem:**

The decay of one type of K -meson is cited as evidence that nature favors matter over antimatter. Since mesons are composed of a quark and an antiquark, is it surprising that they would preferentially decay to one type over another? Is this an asymmetry in nature? Is the predominance of matter over antimatter an asymmetry?

Exercise:**Problem:**

Distances to local galaxies are determined by measuring the brightness of stars, called Cepheid variables, that can be observed individually and that have absolute brightnesses at a standard distance that are well known. Explain how the measured brightness would vary with distance as compared with the absolute brightness.

Exercise:

Problem:

Distances to very remote galaxies are estimated based on their apparent type, which indicate the number of stars in the galaxy, and their measured brightness. Explain how the measured brightness would vary with distance. Would there be any correction necessary to compensate for the red shift of the galaxy (all distant galaxies have significant red shifts)? Discuss possible causes of uncertainties in these measurements.

Exercise:**Problem:**

If the smallest meaningful time interval is greater than zero, will the lines in [\[link\]](#) ever meet?

Problems & Exercises**Exercise:****Problem:**

Find the approximate mass of the luminous matter in the Milky Way galaxy, given it has approximately 10^{11} stars of average mass 1.5 times that of our Sun.

Solution:

$$3 \times 10^{41} \text{ kg}$$

Exercise:

Problem:

Find the approximate mass of the dark and luminous matter in the Milky Way galaxy. Assume the luminous matter is due to approximately 10^{11} stars of average mass 1.5 times that of our Sun, and take the dark matter to be 10 times as massive as the luminous matter.

Exercise:**Problem:**

(a) Estimate the mass of the luminous matter in the known universe, given there are 10^{11} galaxies, each containing 10^{11} stars of average mass 1.5 times that of our Sun. (b) How many protons (the most abundant nuclide) are there in this mass? (c) Estimate the total number of particles in the observable universe by multiplying the answer to (b) by two, since there is an electron for each proton, and then by 10^9 , since there are far more particles (such as photons and neutrinos) in space than in luminous matter.

Solution:

(a) $3 \times 10^{52} \text{ kg}$

(b) 2×10^{79}

(c) 4×10^{88}

Exercise:**Problem:**

If a galaxy is 500 Mly away from us, how fast do we expect it to be moving and in what direction?

Exercise:

Problem:

On average, how far away are galaxies that are moving away from us at 2.0% of the speed of light?

Solution:

0.30 Gly

Exercise:**Problem:**

Our solar system orbits the center of the Milky Way galaxy. Assuming a circular orbit 30,000 ly in radius and an orbital speed of 250 km/s, how many years does it take for one revolution? Note that this is approximate, assuming constant speed and circular orbit, but it is representative of the time for our system and local stars to make one revolution around the galaxy.

Exercise:**Problem:**

(a) What is the approximate speed relative to us of a galaxy near the edge of the known universe, some 10 Gly away? (b) What fraction of the speed of light is this? Note that we have observed galaxies moving away from us at greater than $0.9c$.

Solution:

(a) 2.0×10^5 km/s

(b) $0.67c$

Exercise:

Problem:

(a) Calculate the approximate age of the universe from the average value of the Hubble constant, $H_0 = 20 \text{ km/s} \cdot \text{Mly}$. To do this, calculate the time it would take to travel 1 Mly at a constant expansion rate of 20 km/s. (b) If deceleration is taken into account, would the actual age of the universe be greater or less than that found here? Explain.

Exercise:**Problem:**

Assuming a circular orbit for the Sun about the center of the Milky Way galaxy, calculate its orbital speed using the following information: The mass of the galaxy is equivalent to a single mass 1.5×10^{11} times that of the Sun (or $3 \times 10^{41} \text{ kg}$), located 30,000 ly away.

Solution:

$$2.7 \times 10^5 \text{ m/s}$$

Exercise:**Problem:**

(a) What is the approximate force of gravity on a 70-kg person due to the Andromeda galaxy, assuming its total mass is 10^{13} that of our Sun and acts like a single mass 2 Mly away? (b) What is the ratio of this force to the person's weight? Note that Andromeda is the closest large galaxy.

Exercise:**Problem:**

Andromeda galaxy is the closest large galaxy and is visible to the naked eye. Estimate its brightness relative to the Sun, assuming it has luminosity 10^{12} times that of the Sun and lies 2 Mly away.

Solution:

6×10^{-11} (an overestimate, since some of the light from Andromeda is blocked by gas and dust within that galaxy)

Exercise:**Problem:**

(a) A particle and its antiparticle are at rest relative to an observer and annihilate (completely destroying both masses), creating two γ rays of equal energy. What is the characteristic γ -ray energy you would look for if searching for evidence of proton-antiproton annihilation? (The fact that such radiation is rarely observed is evidence that there is very little antimatter in the universe.) (b) How does this compare with the 0.511-MeV energy associated with electron-positron annihilation?

Exercise:**Problem:**

The average particle energy needed to observe unification of forces is estimated to be 10^{19} GeV. (a) What is the rest mass in kilograms of a particle that has a rest mass of 10^{19} GeV/ c^2 ? (b) How many times the mass of a hydrogen atom is this?

Solution:

(a) 2×10^{-8} kg

(b) 1×10^{19}

Exercise:

Problem:

The peak intensity of the CMBR occurs at a wavelength of 1.1 mm. (a) What is the energy in eV of a 1.1-mm photon? (b) There are approximately 10^9 photons for each massive particle in deep space. Calculate the energy of 10^9 such photons. (c) If the average massive particle in space has a mass half that of a proton, what energy would be created by converting its mass to energy? (d) Does this imply that space is “matter dominated”? Explain briefly.

Exercise:**Problem:**

(a) What Hubble constant corresponds to an approximate age of the universe of 10^{10} y? To get an approximate value, assume the expansion rate is constant and calculate the speed at which two galaxies must move apart to be separated by 1 Mly (present average galactic separation) in a time of 10^{10} y. (b) Similarly, what Hubble constant corresponds to a universe approximately 2×10^{10} -y old?

Solution:

(a) $30 \text{ km/s} \cdot \text{Mly}$

(b) $15 \text{ km/s} \cdot \text{Mly}$

Exercise:**Problem:**

Show that the velocity of a star orbiting its galaxy in a circular orbit is inversely proportional to the square root of its orbital radius, assuming the mass of the stars inside its orbit acts like a single mass at the center of the galaxy. You may use an equation from a previous chapter to support your conclusion, but you must justify its use and define all terms used.

Exercise:

Problem:

The core of a star collapses during a supernova, forming a neutron star. Angular momentum of the core is conserved, and so the neutron star spins rapidly. If the initial core radius is 5.0×10^5 km and it collapses to 10.0 km, find the neutron star's angular velocity in revolutions per second, given the core's angular velocity was originally 1 revolution per 30.0 days.

Solution:

960 rev/s

Exercise:**Problem:**

Using data from the previous problem, find the increase in rotational kinetic energy, given the core's mass is 1.3 times that of our Sun. Where does this increase in kinetic energy come from?

Exercise:**Problem:**

Distances to the nearest stars (up to 500 ly away) can be measured by a technique called parallax, as shown in [\[link\]](#). What are the angles θ_1 and θ_2 relative to the plane of the Earth's orbit for a star 4.0 ly directly above the Sun?

Solution:

89.999773° (many digits are used to show the difference between 90°)

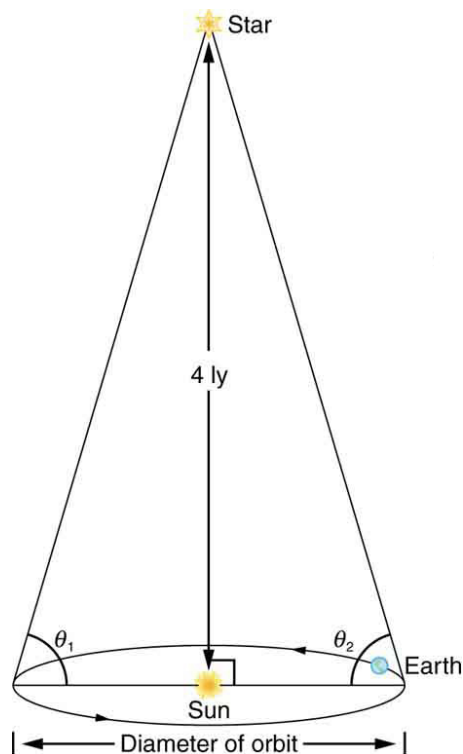
Exercise:

Problem:

(a) Use the Heisenberg uncertainty principle to calculate the uncertainty in energy for a corresponding time interval of 10^{-43} s. (b) Compare this energy with the 10^{19} GeV unification-of-forces energy and discuss why they are similar.

Exercise:**Problem: Construct Your Own Problem**

Consider a star moving in a circular orbit at the edge of a galaxy. Construct a problem in which you calculate the mass of that galaxy in kg and in multiples of the solar mass based on the velocity of the star and its distance from the center of the galaxy.



Distances to nearby stars are measured using triangulation, also called the parallax method. The angle of

line of sight to the star
is measured at
intervals six months
apart, and the distance
is calculated by using
the known diameter of
the Earth's orbit. This
can be done for stars
up to about 500 ly
away.

Glossary

Big Bang

a gigantic explosion that threw out matter a few billion years ago

cosmic microwave background

the spectrum of microwave radiation of cosmic origin

cosmological red shift

the photon wavelength is stretched in transit from the source to the observer because of the expansion of space itself

cosmology

the study of the character and evolution of the universe

electroweak epoch

the stage before 10^{-11} back to 10^{-34} after the Big Bang

GUT epoch

the time period from 10^{-43} to 10^{-34} after the Big Bang, when Grand Unification Theory, in which all forces except gravity are identical, governed the universe

Hubble constant

a central concept in cosmology whose value is determined by taking the slope of a graph of velocity versus distance, obtained from red shift measurements

inflationary scenario

the rapid expansion of the universe by an incredible factor of 10^{-50} for the brief time from 10^{-35} to about 10^{-32} s

spontaneous symmetry breaking

the transition from GUT to electroweak where the forces were no longer unified

superforce

hypothetical unified force in TOE epoch

TOE epoch

before 10^{-43} after the Big Bang

General Relativity and Quantum Gravity

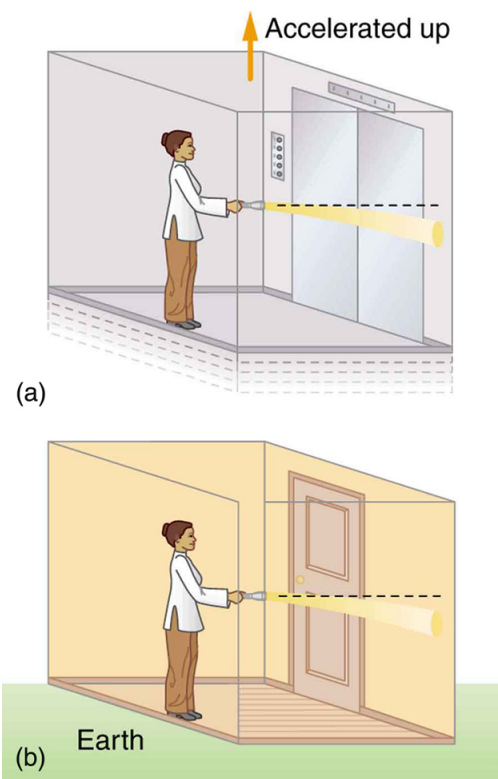
- Explain the effect of gravity on light.
- Discuss black hole.
- Explain quantum gravity.

When we talk of black holes or the unification of forces, we are actually discussing aspects of general relativity and quantum gravity. We know from [Special Relativity](#) that relativity is the study of how different observers measure the same event, particularly if they move relative to one another. Einstein's theory of **general relativity** describes all types of relative motion including accelerated motion and the effects of gravity. General relativity encompasses special relativity and classical relativity in situations where acceleration is zero and relative velocity is small compared with the speed of light. Many aspects of general relativity have been verified experimentally, some of which are better than science fiction in that they are bizarre but true. **Quantum gravity** is the theory that deals with particle exchange of gravitons as the mechanism for the force, and with extreme conditions where quantum mechanics and general relativity must both be used. A good theory of quantum gravity does not yet exist, but one will be needed to understand how all four forces may be unified. If we are successful, the theory of quantum gravity will encompass all others, from classical physics to relativity to quantum mechanics—truly a Theory of Everything (TOE).

General Relativity

Einstein first considered the case of no observer acceleration when he developed the revolutionary special theory of relativity, publishing his first work on it in 1905. By 1916, he had laid the foundation of general relativity, again almost on his own. Much of what Einstein did to develop his ideas was to mentally analyze certain carefully and clearly defined situations—doing this is to perform a **thought experiment**. [\[link\]](#) illustrates a thought experiment like the ones that convinced Einstein that light must fall in a gravitational field. Think about what a person feels in an elevator that is accelerated upward. It is identical to being in a stationary elevator in a gravitational field. The feet of a person are pressed against the floor, and

objects released from hand fall with identical accelerations. In fact, it is not possible, without looking outside, to know what is happening—acceleration upward or gravity. This led Einstein to correctly postulate that acceleration and gravity will produce identical effects in all situations. So, if acceleration affects light, then gravity will, too. [\[link\]](#) shows the effect of acceleration on a beam of light shone horizontally at one wall. Since the accelerated elevator moves up during the time light travels across the elevator, the beam of light strikes low, seeming to the person to bend down. (Normally a tiny effect, since the speed of light is so great.) The same effect must occur due to gravity, Einstein reasoned, since there is no way to tell the effects of gravity acting downward from acceleration of the elevator upward. Thus gravity affects the path of light, even though we think of gravity as acting between masses and photons are massless.



(a) A beam of light emerges from a flashlight in an upward-accelerating

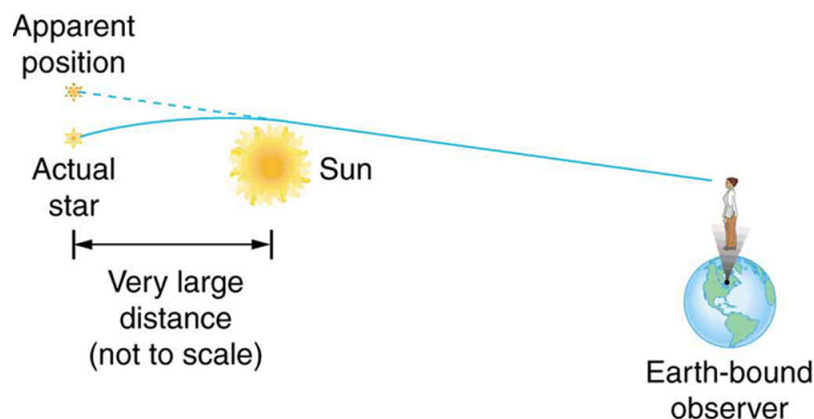
elevator. Since the elevator moves up during the time the light takes to reach the wall, the beam strikes lower than it would if the elevator were not accelerated. (b) Gravity has the same effect on light, since it is not possible to tell whether the elevator is accelerating upward or acted upon by gravity.

Einstein's theory of general relativity got its first verification in 1919 when starlight passing near the Sun was observed during a solar eclipse. (See [\[link\]](#).) During an eclipse, the sky is darkened and we can briefly see stars. Those in a line of sight nearest the Sun should have a shift in their apparent positions. Not only was this shift observed, but it agreed with Einstein's predictions well within experimental uncertainties. This discovery created a scientific and public sensation. Einstein was now a folk hero as well as a very great scientist. The bending of light by matter is equivalent to a bending of space itself, with light following the curve. This is another radical change in our concept of space and time. It is also another connection that any particle with mass or energy (massless photons) is affected by gravity.

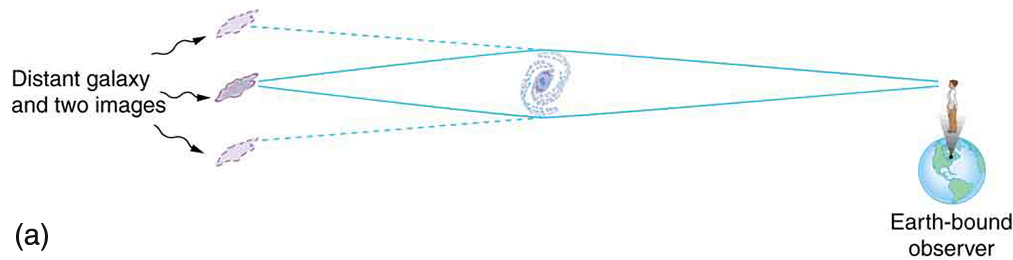
There are several current forefront efforts related to general relativity. One is the observation and analysis of gravitational lensing of light. Another is analysis of the definitive proof of the existence of black holes. Direct observation of gravitational waves or moving wrinkles in space is being searched for. Theoretical efforts are also being aimed at the possibility of time travel and wormholes into other parts of space due to black holes.

Gravitational lensing

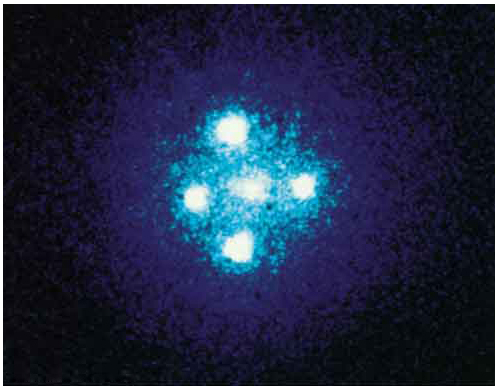
As you can see in [\[link\]](#), light is bent toward a mass, producing an effect much like a converging lens (large masses are needed to produce observable effects). On a galactic scale, the light from a distant galaxy could be “lensed” into several images when passing close by another galaxy on its way to Earth. Einstein predicted this effect, but he considered it unlikely that we would ever observe it. A number of cases of this effect have now been observed; one is shown in [\[link\]](#). This effect is a much larger scale verification of general relativity. But such gravitational lensing is also useful in verifying that the red shift is proportional to distance. The red shift of the intervening galaxy is always less than that of the one being lensed, and each image of the lensed galaxy has the same red shift. This verification supplies more evidence that red shift is proportional to distance. Confidence that the multiple images are not different objects is bolstered by the observations that if one image varies in brightness over time, the others also vary in the same manner.



This schematic shows how light passing near a massive body like the Sun is curved toward it. The light that reaches the Earth then seems to be coming from different locations than the known positions of the originating stars. Not only was this effect observed, the amount of bending was precisely what Einstein predicted in his general theory of relativity.



(a)



(b)

(a) Light from a distant galaxy can travel different paths to the Earth because it is bent around an intermediary galaxy by gravity. This produces several images of the more distant galaxy. (b) The images around the central galaxy are produced by gravitational lensing. Each image has the same spectrum and a larger red shift than the intermediary. (credit: NASA, ESA, and STScI)

Black holes

Black holes are objects having such large gravitational fields that things can fall in, but nothing, not even light, can escape. Bodies, like the Earth or the Sun, have what is called an **escape velocity**. If an object moves straight up from the body, starting at the escape velocity, it will just be able to escape the gravity of the body. The greater the acceleration of gravity on the body, the greater is the escape velocity. As long ago as the late 1700s, it was proposed that if the escape velocity is greater than the speed of light, then

light cannot escape. Simon Laplace (1749–1827), the French astronomer and mathematician, even incorporated this idea of a dark star into his writings. But the idea was dropped after Young’s double slit experiment showed light to be a wave. For some time, light was thought not to have particle characteristics and, thus, could not be acted upon by gravity. The idea of a black hole was very quickly reincarnated in 1916 after Einstein’s theory of general relativity was published. It is now thought that black holes can form in the supernova collapse of a massive star, forming an object perhaps 10 km across and having a mass greater than that of our Sun. It is interesting that several prominent physicists who worked on the concept, including Einstein, firmly believed that nature would find a way to prohibit such objects.

Black holes are difficult to observe directly, because they are small and no light comes directly from them. In fact, no light comes from inside the **event horizon**, which is defined to be at a distance from the object at which the escape velocity is exactly the speed of light. The radius of the event horizon is known as the **Schwarzschild radius** R_S and is given by

Equation:

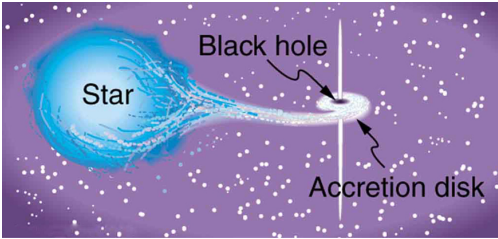
$$R_S = \frac{2GM}{c^2},$$

where G is the universal gravitational constant, M is the mass of the body, and c is the speed of light. The event horizon is the edge of the black hole and R_S is its radius (that is, the size of a black hole is twice R_S). Since G is small and c^2 is large, you can see that black holes are extremely small, only a few kilometers for masses a little greater than the Sun’s. The object itself is inside the event horizon.

Physics near a black hole is fascinating. Gravity increases so rapidly that, as you approach a black hole, the tidal effects tear matter apart, with matter closer to the hole being pulled in with much more force than that only slightly farther away. This can pull a companion star apart and heat inflowing gases to the point of producing X rays. (See [\[link\]](#).) We have observed X rays from certain binary star systems that are consistent with such a picture. This is not quite proof of black holes, because the X rays

could also be caused by matter falling onto a neutron star. These objects were first discovered in 1967 by the British astrophysicists, Jocelyn Bell and Anthony Hewish. **Neutron stars** are literally a star composed of neutrons. They are formed by the collapse of a star's core in a supernova, during which electrons and protons are forced together to form neutrons (the reverse of neutron β decay). Neutron stars are slightly larger than a black hole of the same mass and will not collapse further because of resistance by the strong force. However, neutron stars cannot have a mass greater than about eight solar masses or they must collapse to a black hole. With recent improvements in our ability to resolve small details, such as with the orbiting Chandra X-ray Observatory, it has become possible to measure the masses of X-ray-emitting objects by observing the motion of companion stars and other matter in their vicinity. What has emerged is a plethora of X-ray-emitting objects too massive to be neutron stars. This evidence is considered conclusive and the existence of black holes is widely accepted. These black holes are concentrated near galactic centers.

We also have evidence that supermassive black holes may exist at the cores of many galaxies, including the Milky Way. Such a black hole might have a mass millions or even billions of times that of the Sun, and it would probably have formed when matter first coalesced into a galaxy billions of years ago. Supporting this is the fact that very distant galaxies are more likely to have abnormally energetic cores. Some of the moderately distant galaxies, and hence among the younger, are known as **quasars** and emit as much or more energy than a normal galaxy but from a region less than a light year across. Quasar energy outputs may vary in times less than a year, so that the energy-emitting region must be less than a light year across. The best explanation of quasars is that they are young galaxies with a supermassive black hole forming at their core, and that they become less energetic over billions of years. In closer superactive galaxies, we observe tremendous amounts of energy being emitted from very small regions of space, consistent with stars falling into a black hole at the rate of one or more a month. The Hubble Space Telescope (1994) observed an accretion disk in the galaxy M87 rotating rapidly around a region of extreme energy emission. (See [\[link\]](#).) A jet of material being ejected perpendicular to the plane of rotation gives further evidence of a supermassive black hole as the engine.



A black hole is shown pulling matter away from a companion star, forming a superheated accretion disk where X rays are emitted before the matter disappears forever into the hole. The in-fall energy also ejects some material, forming the two vertical spikes. (See also the photograph in [Introduction to Frontiers of Physics](#).) There are several X-ray-emitting objects in space that are consistent with this picture and are likely to be black holes.

Gravitational waves

If a massive object distorts the space around it, like the foot of a water bug on the surface of a pond, then movement of the massive object should create waves in space like those on a pond. **Gravitational waves** are mass-created distortions in space that propagate at the speed of light and are predicted by general relativity. Since gravity is by far the weakest force, extreme conditions are needed to generate significant gravitational waves. Gravity near binary neutron star systems is so great that significant gravitational wave energy is radiated as the two neutron stars orbit one

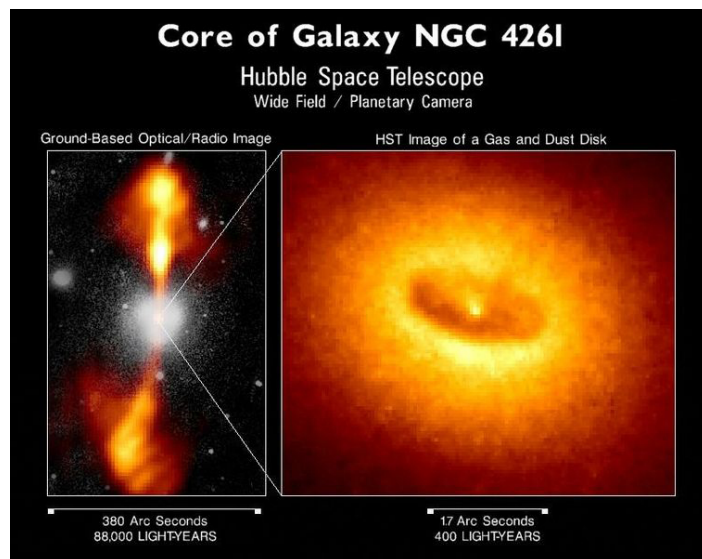
another. American astronomers, Joseph Taylor and Russell Hulse, measured changes in the orbit of such a binary neutron star system. They found its orbit to change precisely as predicted by general relativity, a strong indication of gravitational waves, and were awarded the 1993 Nobel Prize. But direct detection of gravitational waves on Earth would be conclusive. For many years, various attempts have been made to detect gravitational waves by observing vibrations induced in matter distorted by these waves. American physicist Joseph Weber pioneered this field in the 1960s, but no conclusive events have been observed. (No gravity wave detectors were in operation at the time of the 1987A supernova, unfortunately.) There are now several ambitious systems of gravitational wave detectors in use around the world. These include the LIGO (Laser Interferometer Gravitational Wave Observatory) system with two laser interferometer detectors, one in the state of Washington and another in Louisiana (See [\[link\]](#)) and the VIRGO (Variability of Irradiance and Gravitational Oscillations) facility in Italy with a single detector.

Quantum Gravity

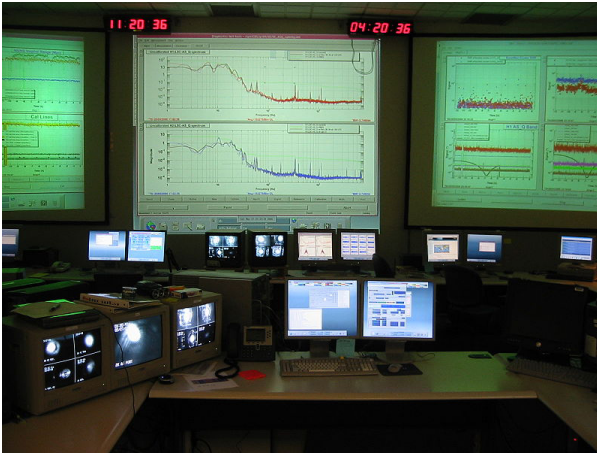
Black holes radiate

Quantum gravity is important in those situations where gravity is so extremely strong that it has effects on the quantum scale, where the other forces are ordinarily much stronger. The early universe was such a place, but black holes are another. The first significant connection between gravity and quantum effects was made by the Russian physicist Yakov Zel'dovich in 1971, and other significant advances followed from the British physicist Stephen Hawking. (See [\[link\]](#).) These two showed that black holes could radiate away energy by quantum effects just outside the event horizon (nothing can escape from inside the event horizon). Black holes are, thus, expected to radiate energy and shrink to nothing, although extremely slowly for most black holes. The mechanism is the creation of a particle-antiparticle pair from energy in the extremely strong gravitational field near the event horizon. One member of the pair falls into the hole and the other escapes, conserving momentum. (See [\[link\]](#).) When a black hole loses energy and, hence, rest mass, its event horizon shrinks, creating an even greater gravitational field. This increases the rate of pair production so that the process grows exponentially until the black hole is nuclear in size. A

final burst of particles and γ rays ensues. This is an extremely slow process for black holes about the mass of the Sun (produced by supernovas) or larger ones (like those thought to be at galactic centers), taking on the order of 10^{67} years or longer! Smaller black holes would evaporate faster, but they are only speculated to exist as remnants of the Big Bang. Searches for characteristic γ -ray bursts have produced events attributable to more mundane objects like neutron stars accreting matter.



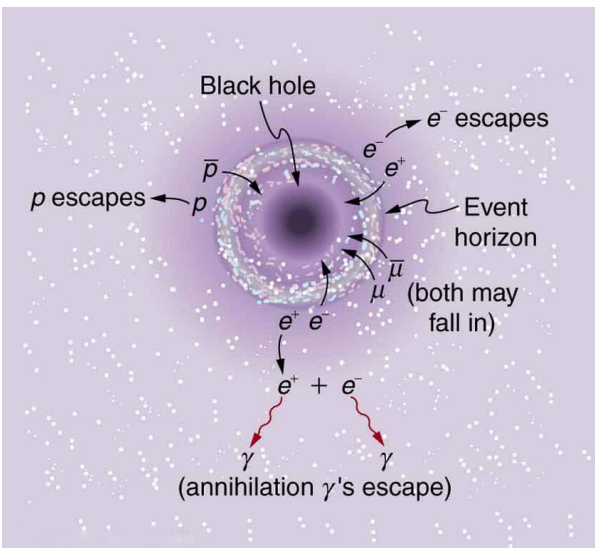
This Hubble Space Telescope photograph shows the extremely energetic core of the NGC 4261 galaxy. With the superior resolution of the orbiting telescope, it has been possible to observe the rotation of an accretion disk around the energy-producing object as well as to map jets of material being ejected from the object. A supermassive black hole is consistent with these observations, but other possibilities are not quite eliminated. (credit: NASA and ESA)



The control room of the LIGO gravitational wave detector. Gravitational waves will cause extremely small vibrations in a mass in this detector, which will be detected by laser interferometer techniques. Such detection in coincidence with other detectors and with astronomical events, such as supernovas, would provide direct evidence of gravitational waves. (credit: Tobin Fricke)



Stephen Hawking (b. 1942) has made many contributions to the theory of quantum gravity. Hawking is a long-time survivor of ALS and has produced popular books on general relativity, cosmology, and quantum gravity. (credit: Lwp Kommunikáció)



Gravity and quantum mechanics

come into play when a black hole creates a particle-antiparticle pair from the energy in its gravitational field. One member of the pair falls into the hole while the other escapes, removing energy and shrinking the black hole. The search is on for the characteristic energy.

Wormholes and time travel

The subject of time travel captures the imagination. Theoretical physicists, such as the American Kip Thorne, have treated the subject seriously, looking into the possibility that falling into a black hole could result in popping up in another time and place—a trip through a so-called wormhole. Time travel and wormholes appear in innumerable science fiction dramatizations, but the consensus is that time travel is not possible in theory. While still debated, it appears that quantum gravity effects inside a black hole prevent time travel due to the creation of particle pairs. Direct evidence is elusive.

The shortest time

Theoretical studies indicate that, at extremely high energies and correspondingly early in the universe, quantum fluctuations may make time intervals meaningful only down to some finite time limit. Early work indicated that this might be the case for times as long as 10^{-43} s, the time at which all forces were unified. If so, then it would be meaningless to consider the universe at times earlier than this. Subsequent studies indicate that the crucial time may be as short as 10^{-95} s. But the point remains—quantum gravity seems to imply that there is no such thing as a vanishingly short time. Time may, in fact, be grainy with no meaning to time intervals shorter than some tiny but finite size.

The future of quantum gravity

Not only is quantum gravity in its infancy, no one knows how to get started on a theory of gravitons and unification of forces. The energies at which TOE should be valid may be so high (at least 10^{19} GeV) and the necessary particle separation so small (less than 10^{-35} m) that only indirect evidence can provide clues. For some time, the common lament of theoretical physicists was one so familiar to struggling students—how do you even get started? But Hawking and others have made a start, and the approach many theorists have taken is called Superstring theory, the topic of the [Superstrings](#).

Section Summary

- Einstein's theory of general relativity includes accelerated frames and, thus, encompasses special relativity and gravity. Created by use of careful thought experiments, it has been repeatedly verified by real experiments.
- One direct result of this behavior of nature is the gravitational lensing of light by massive objects, such as galaxies, also seen in the microlensing of light by smaller bodies in our galaxy.
- Another prediction is the existence of black holes, objects for which the escape velocity is greater than the speed of light and from which nothing can escape.
- The event horizon is the distance from the object at which the escape velocity equals the speed of light c . It is called the Schwarzschild radius R_S and is given by

Equation:

$$R_S = \frac{2GM}{c^2},$$

where G is the universal gravitational constant, and M is the mass of the body.

- Physics is unknown inside the event horizon, and the possibility of wormholes and time travel are being studied.
- Candidates for black holes may power the extremely energetic emissions of quasars, distant objects that seem to be early stages of

galactic evolution.

- Neutron stars are stellar remnants, having the density of a nucleus, that hint that black holes could form from supernovas, too.
- Gravitational waves are wrinkles in space, predicted by general relativity but not yet observed, caused by changes in very massive objects.
- Quantum gravity is an incompletely developed theory that strives to include general relativity, quantum mechanics, and unification of forces (thus, a TOE).
- One unconfirmed connection between general relativity and quantum mechanics is the prediction of characteristic radiation from just outside black holes.

Conceptual Questions

Exercise:

Problem:

Quantum gravity, if developed, would be an improvement on both general relativity and quantum mechanics, but more mathematically difficult. Under what circumstances would it be necessary to use quantum gravity? Similarly, under what circumstances could general relativity be used? When could special relativity, quantum mechanics, or classical physics be used?

Exercise:

Problem:

Does observed gravitational lensing correspond to a converging or diverging lens? Explain briefly.

Exercise:

Problem:

Suppose you measure the red shifts of all the images produced by gravitational lensing, such as in [\[link\]](#). You find that the central image has a red shift less than the outer images, and those all have the same red shift. Discuss how this not only shows that the images are of the same object, but also implies that the red shift is not affected by taking different paths through space. Does it imply that cosmological red shifts are not caused by traveling through space (light getting tired, perhaps)?

Exercise:**Problem:**

What are gravitational waves, and have they yet been observed either directly or indirectly?

Exercise:**Problem:**

Is the event horizon of a black hole the actual physical surface of the object?

Exercise:**Problem:**

Suppose black holes radiate their mass away and the lifetime of a black hole created by a supernova is about 10^{67} years. How does this lifetime compare with the accepted age of the universe? Is it surprising that we do not observe the predicted characteristic radiation?

Problems & Exercises**Exercise:**

Problem:

What is the Schwarzschild radius of a black hole that has a mass eight times that of our Sun? Note that stars must be more massive than the Sun to form black holes as a result of a supernova.

Solution:

23.6 km

Exercise:**Problem:**

Black holes with masses smaller than those formed in supernovas may have been created in the Big Bang. Calculate the radius of one that has a mass equal to the Earth's.

Exercise:**Problem:**

Supermassive black holes are thought to exist at the center of many galaxies.

- (a) What is the radius of such an object if it has a mass of 10^9 Suns?
 - (b) What is this radius in light years?
-

Solution:

(a) 2.95×10^{12} m

(b) 3.12×10^{-4} ly

Exercise:**Problem: Construct Your Own Problem**

Consider a supermassive black hole near the center of a galaxy. Calculate the radius of such an object based on its mass. You must consider how much mass is reasonable for these large objects, and which is now nearly directly observed. (Information on black holes posted on the Web by NASA and other agencies is reliable, for example.)

Glossary

black holes

objects having such large gravitational fields that things can fall in, but nothing, not even light, can escape

general relativity

Einstein's theory that describes all types of relative motion including accelerated motion and the effects of gravity

gravitational waves

mass-created distortions in space that propagate at the speed of light and that are predicted by general relativity

escape velocity

takeoff velocity when kinetic energy just cancels gravitational potential energy

event horizon

the distance from the object at which the escape velocity is exactly the speed of light

neutron stars

literally a star composed of neutrons

Schwarzschild radius

the radius of the event horizon

thought experiment

mental analysis of certain carefully and clearly defined situations to develop an idea

quasars

the moderately distant galaxies that emit as much or more energy than a normal galaxy

Quantum gravity

the theory that deals with particle exchange of gravitons as the mechanism for the force

Complexity and Chaos

- Explain complex systems.
- Discuss chaotic behavior of different systems.

Much of what impresses us about physics is related to the underlying connections and basic simplicity of the laws we have discovered. The language of physics is precise and well defined because many basic systems we study are simple enough that we can perform controlled experiments and discover unambiguous relationships. Our most spectacular successes, such as the prediction of previously unobserved particles, come from the simple underlying patterns we have been able to recognize. But there are systems of interest to physicists that are inherently complex. The simple laws of physics apply, of course, but complex systems may reveal patterns that simple systems do not. The emerging field of **complexity** is devoted to the study of complex systems, including those outside the traditional bounds of physics. Of particular interest is the ability of complex systems to adapt and evolve.

What are some examples of complex adaptive systems? One is the primordial ocean. When the oceans first formed, they were a random mix of elements and compounds that obeyed the laws of physics and chemistry. In a relatively short geological time (about 500 million years), life had emerged. Laboratory simulations indicate that the emergence of life was far too fast to have come from random combinations of compounds, even if driven by lightning and heat. There must be an underlying ability of the complex system to organize itself, resulting in the self-replication we recognize as life. Living entities, even at the unicellular level, are highly organized and systematic. Systems of living organisms are themselves complex adaptive systems. The grandest of these evolved into the biological system we have today, leaving traces in the geological record of steps taken along the way.

Complexity as a discipline examines complex systems, how they adapt and evolve, looking for similarities with other complex adaptive systems. Can, for example, parallels be drawn between biological evolution and the evolution of *economic systems*? Economic systems do emerge quickly, they show tendencies for self-organization, they are complex (in the number and

types of transactions), and they adapt and evolve. Biological systems do all the same types of things. There are other examples of complex adaptive systems being studied for fundamental similarities. *Cultures* show signs of adaptation and evolution. The comparison of different cultural evolutions may bear fruit as well as comparisons to biological evolution. *Science* also is a complex system of human interactions, like culture and economics, that adapts to new information and political pressure, and evolves, usually becoming more organized rather than less. Those who study *creative thinking* also see parallels with complex systems. Humans sometimes organize almost random pieces of information, often subconsciously while doing other things, and come up with brilliant creative insights. The development of *language* is another complex adaptive system that may show similar tendencies. *Artificial intelligence* is an overt attempt to devise an adaptive system that will self-organize and evolve in the same manner as an intelligent living being learns. These are a few of the broad range of topics being studied by those who investigate complexity. There are now institutes, journals, and meetings, as well as popularizations of the emerging topic of complexity.

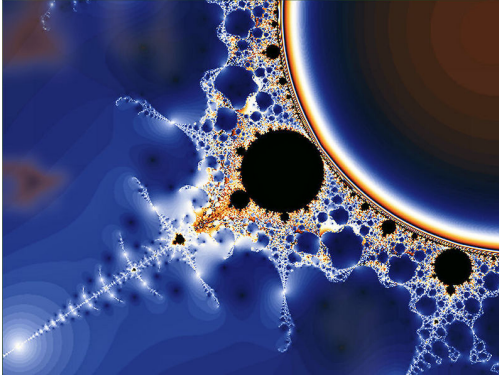
In traditional physics, the discipline of complexity may yield insights in certain areas. Thermodynamics treats systems on the average, while statistical mechanics deals in some detail with complex systems of atoms and molecules in random thermal motion. Yet there is organization, adaptation, and evolution in those complex systems. Non-equilibrium phenomena, such as heat transfer and phase changes, are characteristically complex in detail, and new approaches to them may evolve from complexity as a discipline. Crystal growth is another example of self-organization spontaneously emerging in a complex system. Alloys are also inherently complex mixtures that show certain simple characteristics implying some self-organization. The organization of iron atoms into magnetic domains as they cool is another. Perhaps insights into these difficult areas will emerge from complexity. But at the minimum, the discipline of complexity is another example of human effort to understand and organize the universe around us, partly rooted in the discipline of physics.

A predecessor to complexity is the topic of chaos, which has been widely publicized and has become a discipline of its own. It is also based partly in physics and treats broad classes of phenomena from many disciplines.

Chaos is a word used to describe systems whose outcomes are extremely sensitive to initial conditions. The orbit of the planet Pluto, for example, may be chaotic in that it can change tremendously due to small interactions with other planets. This makes its long-term behavior impossible to predict with precision, just as we cannot tell precisely where a decaying Earth satellite will land or how many pieces it will break into. But the discipline of chaos has found ways to deal with such systems and has been applied to apparently unrelated systems. For example, the heartbeat of people with certain types of potentially lethal arrhythmias seems to be chaotic, and this knowledge may allow more sophisticated monitoring and recognition of the need for intervention.

Chaos is related to complexity. Some chaotic systems are also inherently complex; for example, vortices in a fluid as opposed to a double pendulum. Both are chaotic and not predictable in the same sense as other systems. But there can be organization in chaos and it can also be quantified. Examples of chaotic systems are beautiful fractal patterns such as in [\[link\]](#). Some chaotic systems exhibit self-organization, a type of stable chaos. The orbits of the planets in our solar system, for example, may be chaotic (we are not certain yet). But they are definitely organized and systematic, with a simple formula describing the orbital radii of the first eight planets *and* the asteroid belt. Large-scale vortices in Jupiter's atmosphere are chaotic, but the Great Red Spot is a stable self-organization of rotational energy. (See [\[link\]](#).) The Great Red Spot has been in existence for at least 400 years and is a complex self-adaptive system.

The emerging field of complexity, like the now almost traditional field of chaos, is partly rooted in physics. Both attempt to see similar systematics in a very broad range of phenomena and, hence, generate a better understanding of them. Time will tell what impact these fields have on more traditional areas of physics as well as on the other disciplines they relate to.



This image is related to the Mandelbrot set, a complex mathematical form that is chaotic. The patterns are infinitely fine as you look closer and closer, and they indicate order in the presence of chaos. (credit: Gilberto Santa Rosa)



The Great Red Spot on Jupiter is an example of self-organization in a complex and chaotic system. Smaller vortices in Jupiter's atmosphere

behave chaotically, but
the triple-Earth-size spot
is self-organized and
stable for at least
hundreds of years. (credit:
NASA)

Section Summary

- Complexity is an emerging field, rooted primarily in physics, that considers complex adaptive systems and their evolution, including self-organization.
- Complexity has applications in physics and many other disciplines, such as biological evolution.
- Chaos is a field that studies systems whose properties depend extremely sensitively on some variables and whose evolution is impossible to predict.
- Chaotic systems may be simple or complex.
- Studies of chaos have led to methods for understanding and predicting certain chaotic behaviors.

Conceptual Questions

Exercise:

Problem:

Must a complex system be adaptive to be of interest in the field of complexity? Give an example to support your answer.

Exercise:

Problem: State a necessary condition for a system to be chaotic.

Glossary

complexity

an emerging field devoted to the study of complex systems

chaos

word used to describe systems the outcomes of which are extremely sensitive to initial conditions

High-temperature Superconductors

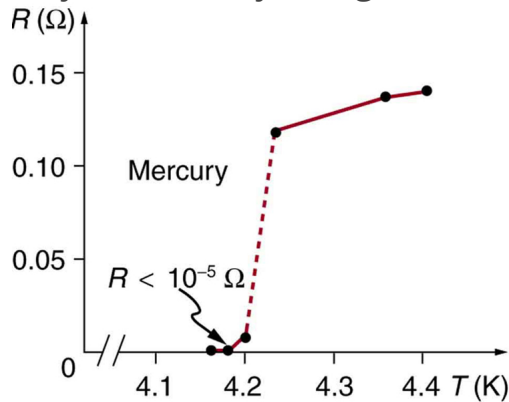
- Identify superconductors and their uses.
- Discuss the need for a high- T_c superconductor.

Superconductors are materials with a resistivity of zero. They are familiar to the general public because of their practical applications and have been mentioned at a number of points in the text. Because the resistance of a piece of superconductor is zero, there are no heat losses for currents through them; they are used in magnets needing high currents, such as in MRI machines, and could cut energy losses in power transmission. But most superconductors must be cooled to temperatures only a few kelvin above absolute zero, a costly procedure limiting their practical applications. In the past decade, tremendous advances have been made in producing materials that become superconductors at relatively high temperatures. There is hope that room temperature superconductors may someday be manufactured.

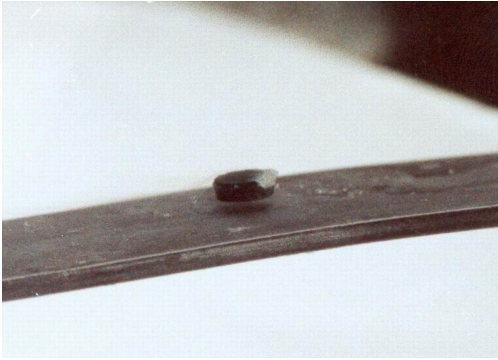
Superconductivity was discovered accidentally in 1911 by the Dutch physicist H. Kamerlingh Onnes (1853–1926) when he used liquid helium to cool mercury. Onnes had been the first person to liquefy helium a few years earlier and was surprised to observe the resistivity of a mediocre conductor like mercury drop to zero at a temperature of 4.2 K. We define the temperature at which and below which a material becomes a superconductor to be its **critical temperature**, denoted by T_c . (See [\[link\]](#).) Progress in understanding how and why a material became a superconductor was relatively slow, with the first workable theory coming in 1957. Certain other elements were also found to become superconductors, but all had T_c s less than 10 K, which are expensive to maintain. Although Onnes received a Nobel prize in 1913, it was primarily for his work with liquid helium.

In 1986, a breakthrough was announced—a ceramic compound was found to have an unprecedented T_c of 35 K. It looked as if much higher critical temperatures could be possible, and by early 1988 another ceramic (this of thallium, calcium, barium, copper, and oxygen) had been found to have $T_c = 125$ K (see [\[link\]](#).) The economic potential of perfect conductors saving electric energy is immense for T_c s above 77 K, since that is the temperature of liquid nitrogen. Although liquid helium has a boiling point

of 4 K and can be used to make materials superconducting, it costs about \$5 per liter. Liquid nitrogen boils at 77 K, but only costs about \$0.30 per liter. There was general euphoria at the discovery of these complex ceramic superconductors, but this soon subsided with the sobering difficulty of forming them into usable wires. The first commercial use of a high temperature superconductor is in an electronic filter for cellular phones. High-temperature superconductors are used in experimental apparatus, and they are actively being researched, particularly in thin film applications.



A graph of resistivity versus temperature for a superconductor shows a sharp transition to zero at the critical temperature T_c . High temperature superconductors have verifiable T_c s greater than 125 K, well above the easily achieved 77-K temperature of liquid nitrogen.

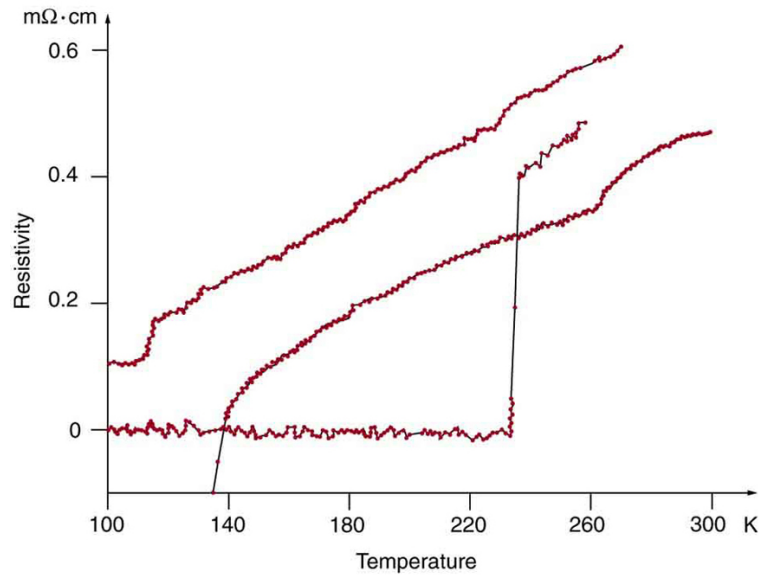


One characteristic of a superconductor is that it excludes magnetic flux and, thus, repels other magnets. The small magnet levitated above a high-temperature superconductor, which is cooled by liquid nitrogen, gives evidence that the material is superconducting. When the material warms and becomes conducting, magnetic flux can penetrate it, and the magnet will rest upon it.
(credit: Saperaud)

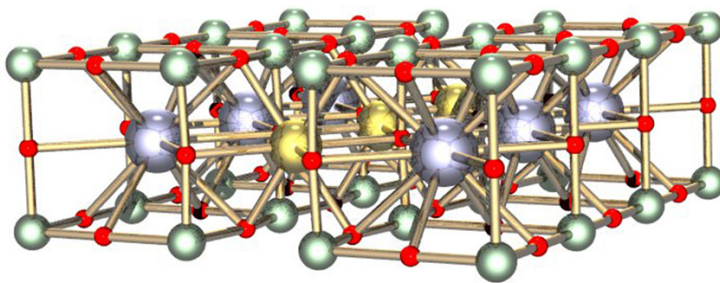
The search is on for even higher T_c superconductors, many of complex and exotic copper oxide ceramics, sometimes including strontium, mercury, or yttrium as well as barium, calcium, and other elements. Room temperature (about 293 K) would be ideal, but any temperature close to room temperature is relatively cheap to produce and maintain. There are persistent reports of T_c s over 200 K and some in the vicinity of 270 K. Unfortunately, these observations are not routinely reproducible, with

samples losing their superconducting nature once heated and re-cooled (cycled) a few times (see [\[link\]](#).) They are now called USOs or unidentified superconducting objects, out of frustration and the refusal of some samples to show high T_c even though produced in the same manner as others. Reproducibility is crucial to discovery, and researchers are justifiably reluctant to claim the breakthrough they all seek. Time will tell whether USOs are real or an experimental quirk.

The theory of ordinary superconductors is difficult, involving quantum effects for widely separated electrons traveling through a material. Electrons couple in a manner that allows them to get through the material without losing energy to it, making it a superconductor. High- T_c superconductors are more difficult to understand theoretically, but theorists seem to be closing in on a workable theory. The difficulty of understanding how electrons can sneak through materials without losing energy in collisions is even greater at higher temperatures, where vibrating atoms should get in the way. Discoverers of high T_c may feel something analogous to what a politician once said upon an unexpected election victory—“I wonder what we did right?”



(a)



(b)

(a) This graph, adapted from an article in *Physics Today*, shows the behavior of a single sample of a high-temperature superconductor in three different trials. In one case the sample exhibited a T_c of about 230 K, whereas in the others it did not become superconducting at all. The lack of reproducibility is typical of forefront experiments and prohibits definitive conclusions. (b) This colorful diagram shows the complex but systematic nature of the lattice structure of a high-temperature superconducting

ceramic. (credit: en:Cadmium,
Wikimedia Commons)

Section Summary

- High-temperature superconductors are materials that become superconducting at temperatures well above a few kelvin.
- The critical temperature T_c is the temperature below which a material is superconducting.
- Some high-temperature superconductors have verified T_c s above 125 K, and there are reports of T_c s as high as 250 K.

Conceptual Questions

Exercise:

Problem:

What is critical temperature T_c ? Do all materials have a critical temperature? Explain why or why not.

Exercise:

Problem:

Explain how good thermal contact with liquid nitrogen can keep objects at a temperature of 77 K (liquid nitrogen's boiling point at atmospheric pressure).

Exercise:

Problem:

Not only is liquid nitrogen a cheaper coolant than liquid helium, its boiling point is higher (77 K vs. 4.2 K). How does higher temperature help lower the cost of cooling a material? Explain in terms of the rate of heat transfer being related to the temperature difference between the sample and its surroundings.

Problem Exercises

Exercise:

Problem:

A section of superconducting wire carries a current of 100 A and requires 1.00 L of liquid nitrogen per hour to keep it below its critical temperature. For it to be economically advantageous to use a superconducting wire, the cost of cooling the wire must be less than the cost of energy lost to heat in the wire. Assume that the cost of liquid nitrogen is \$0.30 per liter, and that electric energy costs \$0.10 per kW·h. What is the resistance of a normal wire that costs as much in wasted electric energy as the cost of liquid nitrogen for the superconductor?

Solution:

Equation:

$$0.30 \, \Omega$$

Glossary

Superconductors

materials with resistivity of zero

critical temperature

the temperature at which and below which a material becomes a superconductor

Some Questions We Know to Ask

- Identify sample questions to be asked on the largest scales.
- Identify sample questions to be asked on the intermediate scale.
- Identify sample questions to be asked on the smallest scales.

Throughout the text we have noted how essential it is to be curious and to ask questions in order to first understand what is known, and then to go a little farther. Some questions may go unanswered for centuries; others may not have answers, but some bear delicious fruit. Part of discovery is knowing which questions to ask. You have to know something before you can even phrase a decent question. As you may have noticed, the mere act of asking a question can give you the answer. The following questions are a sample of those physicists now know to ask and are representative of the forefronts of physics. Although these questions are important, they will be replaced by others if answers are found to them. The fun continues.

On the Largest Scale

1. *Is the universe open or closed?* Theorists would like it to be just barely closed and evidence is building toward that conclusion. Recent measurements in the expansion rate of the universe and in CMBR support a flat universe. There is a connection to small-scale physics in the type and number of particles that may contribute to closing the universe.
2. *What is dark matter?* It is definitely there, but we really do not know what it is. Conventional possibilities are being ruled out, but one of them still may explain it. The answer could reveal whole new realms of physics and the disturbing possibility that most of what is out there is unknown to us, a completely different form of matter.
3. *How do galaxies form?* They exist since very early in the evolution of the universe and it remains difficult to understand how they evolved so quickly. The recent finer measurements of fluctuations in the CMBR may yet allow us to explain galaxy formation.
4. *What is the nature of various-mass black holes?* Only recently have we become confident that many black hole candidates cannot be explained by other, less exotic possibilities. But we still do not know much about

how they form, what their role in the history of galactic evolution has been, and the nature of space in their vicinity. However, so many black holes are now known that correlations between black hole mass and galactic nuclei characteristics are being studied.

5. *What is the mechanism for the energy output of quasars?* These distant and extraordinarily energetic objects now seem to be early stages of galactic evolution with a supermassive black-hole-devouring material. Connections are now being made with galaxies having energetic cores, and there is evidence consistent with less consuming, supermassive black holes at the center of older galaxies. New instruments are allowing us to see deeper into our own galaxy for evidence of our own massive black hole.
6. *Where do the γ bursts come from?* We see bursts of γ rays coming from all directions in space, indicating the sources are very distant objects rather than something associated with our own galaxy. Some γ bursts finally are being correlated with known sources so that the possibility they may originate in binary neutron star interactions or black holes eating a companion neutron star can be explored.

On the Intermediate Scale

1. *How do phase transitions take place on the microscopic scale?* We know a lot about phase transitions, such as water freezing, but the details of how they occur molecule by molecule are not well understood. Similar questions about specific heat a century ago led to early quantum mechanics. It is also an example of a complex adaptive system that may yield insights into other self-organizing systems.
2. *Is there a way to deal with nonlinear phenomena that reveals underlying connections?* Nonlinear phenomena lack a direct or linear proportionality that makes analysis and understanding a little easier. There are implications for nonlinear optics and broader topics such as chaos.
3. *How do high- T_c superconductors become resistanceless at such high temperatures?* Understanding how they work may help make them more practical or may result in surprises as unexpected as the discovery of superconductivity itself.

4. *There are magnetic effects in materials we do not understand—how do they work?* Although beyond the scope of this text, there is a great deal to learn in condensed matter physics (the physics of solids and liquids). We may find surprises analogous to lasing, the quantum Hall effect, and the quantization of magnetic flux. Complexity may play a role here, too.

On the Smallest Scale

1. *Are quarks and leptons fundamental, or do they have a substructure?* The higher energy accelerators that are just completed or being constructed may supply some answers, but there will also be input from cosmology and other systematics.
2. *Why do leptons have integral charge while quarks have fractional charge?* If both are fundamental and analogous as thought, this question deserves an answer. It is obviously related to the previous question.
3. *Why are there three families of quarks and leptons?* First, does this imply some relationship? Second, why three and only three families?
4. *Are all forces truly equal (unified) under certain circumstances?* They don't have to be equal just because we want them to be. The answer may have to be indirectly obtained because of the extreme energy at which we think they are unified.
5. *Are there other fundamental forces?* There was a flurry of activity with claims of a fifth and even a sixth force a few years ago. Interest has subsided, since those forces have not been detected consistently. Moreover, the proposed forces have strengths similar to gravity, making them extraordinarily difficult to detect in the presence of stronger forces. But the question remains; and if there are no other forces, we need to ask why only four and why these four.
6. *Is the proton stable?* We have discussed this in some detail, but the question is related to fundamental aspects of the unification of forces. We may never know from experiment that the proton is stable, only that it is very long lived.
7. *Are there magnetic monopoles?* Many particle theories call for very massive individual north- and south-pole particles—magnetic

- monopoles. If they exist, why are they so different in mass and elusiveness from electric charges, and if they do not exist, why not?
8. *Do neutrinos have mass?* Definitive evidence has emerged for neutrinos having mass. The implications are significant, as discussed in this chapter. There are effects on the closure of the universe and on the patterns in particle physics.
 9. *What are the systematic characteristics of high- Z nuclei?* All elements with $Z = 118$ or less (with the exception of 115 and 117) have now been discovered. It has long been conjectured that there may be an island of relative stability near $Z = 114$, and the study of the most recently discovered nuclei will contribute to our understanding of nuclear forces.

These lists of questions are not meant to be complete or consistently important—you can no doubt add to it yourself. There are also important questions in topics not broached in this text, such as certain particle symmetries, that are of current interest to physicists. Hopefully, the point is clear that no matter how much we learn, there always seems to be more to know. Although we are fortunate to have the hard-won wisdom of those who preceded us, we can look forward to new enlightenment, undoubtedly sprinkled with surprise.

Section Summary

- On the largest scale, the questions which can be asked may be about dark matter, dark energy, black holes, quasars, and other aspects of the universe.
- On the intermediate scale, we can query about gravity, phase transitions, nonlinear phenomena, high- T_c superconductors, and magnetic effects on materials.
- On the smallest scale, questions may be about quarks and leptons, fundamental forces, stability of protons, and existence of monopoles.

Conceptual Questions

Exercise:

Problem:

For experimental evidence, particularly of previously unobserved phenomena, to be taken seriously it must be reproducible or of sufficiently high quality that a single observation is meaningful. Supernova 1987A is not reproducible. How do we know observations of it were valid? The fifth force is not broadly accepted. Is this due to lack of reproducibility or poor-quality experiments (or both)? Discuss why forefront experiments are more subject to observational problems than those involving established phenomena.

Exercise:**Problem:**

Discuss whether you think there are limits to what humans can understand about the laws of physics. Support your arguments.

Atomic Masses

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^1H	1.007 825	99.985%	
	Deuterium	2	^2H or D	2.014 102	0.015%	
	Tritium	3	^3H or T	3.016 050	β^-	12.33 y
2	Helium	3	^3He	3.016 030	$1.38 \times 10^{-4}\%$	
		4	^4He	4.002 603	$\approx 100\%$	
3	Lithium	6	^6Li	6.015 121	7.5%	
		7	^7Li	7.016 003	92.5%	
4	Beryllium	7	^7Be	7.016 928	EC	53.29 d
		9	^9Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{13}N	13.005 738	β^+	9.96 min
		14	^{14}N	14.003 074	99.63%	
		15	^{15}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y
		23	^{23}Na	22.989 767	100%	
		24	^{24}Na	23.990 961	β^-	14.96 h
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		31	^{31}Si	30.975 362	β^-	
15	Phosphorus	31	^{31}P	30.973 762	100%	
		32	^{32}P	31.973 907	β^-	14.28 d
16	Sulfur	32	^{32}S	31.972 070	95.02%	
		35	^{35}S	34.969 031	β^-	87.4 d
17	Chlorine	35	^{35}Cl	34.968 852	75.77%	
		37	^{37}Cl	36.965 903	24.23%	
18	Argon	40	^{40}Ar	39.962 384	99.60%	
19	Potassium	39	^{39}K	38.963 707	93.26%	
		40	^{40}K	39.963 999	0.0117%, EC, β^-	$1.28 \times 10^9 \text{ y}$
20	Calcium	40	^{40}Ca	39.962 591	96.94%	
21	Scandium	45	^{45}Sc	44.955 910	100%	
22	Titanium	48	^{48}Ti	47.947 947	73.8%	
23	Vanadium	51	^{51}V	50.943 962	99.75%	
24	Chromium	52	^{52}Cr	51.940 509	83.79%	
25	Manganese	55	^{55}Mn	54.938 047	100%	
26	Iron	56	^{56}Fe	55.934 939	91.72%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
27	Cobalt	59	^{59}Co	58.933 198	100%	
		60	^{60}Co	59.933 819	β^-	5.271 y
28	Nickel	58	^{58}Ni	57.935 346	68.27%	
		60	^{60}Ni	59.930 788	26.10%	
29	Copper	63	^{63}Cu	62.939 598	69.17%	
		65	^{65}Cu	64.927 793	30.83%	
30	Zinc	64	^{64}Zn	63.929 145	48.6%	
		66	^{66}Zn	65.926 034	27.9%	
31	Gallium	69	^{69}Ga	68.925 580	60.1%	
32	Germanium	72	^{72}Ge	71.922 079	27.4%	
		74	^{74}Ge	73.921 177	36.5%	
33	Arsenic	75	^{75}As	74.921 594	100%	
34	Selenium	80	^{80}Se	79.916 520	49.7%	
35	Bromine	79	^{79}Br	78.918 336	50.69%	
36	Krypton	84	^{84}Kr	83.911 507	57.0%	
37	Rubidium	85	^{85}Rb	84.911 794	72.17%	
38	Strontium	86	^{86}Sr	85.909 267	9.86%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		88	^{88}Sr	87.905 619	82.58%	
		90	^{90}Sr	89.907 738	β^-	28.8 y
39	Yttrium	89	^{89}Y	88.905 849	100%	
		90	^{90}Y	89.907 152	β^-	64.1 h
40	Zirconium	90	^{90}Zr	89.904 703	51.45%	
41	Niobium	93	^{93}Nb	92.906 377	100%	
42	Molybdenum	98	^{98}Mo	97.905 406	24.13%	
43	Technetium	98	^{98}Tc	97.907 215	β^-	$4.2 \times 10^6 \text{ y}$
44	Ruthenium	102	^{102}Ru	101.904 348	31.6%	
45	Rhodium	103	^{103}Rh	102.905 500	100%	
46	Palladium	106	^{106}Pd	105.903 478	27.33%	
47	Silver	107	^{107}Ag	106.905 092	51.84%	
		109	^{109}Ag	108.904 757	48.16%	
48	Cadmium	114	^{114}Cd	113.903 357	28.73%	
49	Indium	115	^{115}In	114.903 880	95.7%, β^-	$4.4 \times 10^{14} \text{ y}$
50	Tin	120	^{120}Sn	119.902 200	32.59%	
51	Antimony	121	^{121}Sb	120.903 821	57.3%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
52	Tellurium	130	^{130}Te	129.906 229	33.8%, β^-	$2.5 \times 10^{21}\text{y}$
53	Iodine	127	^{127}I	126.904 473	100%	
		131	^{131}I	130.906 114	β^-	8.040 d
54	Xenon	132	^{132}Xe	131.904 144	26.9%	
		136	^{136}Xe	135.907 214	8.9%	
55	Cesium	133	^{133}Cs	132.905 429	100%	
		134	^{134}Cs	133.906 696	EC, β^-	2.06 y
56	Barium	137	^{137}Ba	136.905 812	11.23%	
		138	^{138}Ba	137.905 232	71.70%	
57	Lanthanum	139	^{139}La	138.906 346	99.91%	
58	Cerium	140	^{140}Ce	139.905 433	88.48%	
59	Praseodymium	141	^{141}Pr	140.907 647	100%	
60	Neodymium	142	^{142}Nd	141.907 719	27.13%	
61	Promethium	145	^{145}Pm	144.912 743	EC, α	17.7 y
62	Samarium	152	^{152}Sm	151.919 729	26.7%	
63	Europium	153	^{153}Eu	152.921 225	52.2%	
64	Gadolinium	158	^{158}Gd	157.924 099	24.84%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
65	Terbium	159	^{159}Tb	158.925 342	100%	
66	Dysprosium	164	^{164}Dy	163.929 171	28.2%	
67	Holmium	165	^{165}Ho	164.930 319	100%	
68	Erbium	166	^{166}Er	165.930 290	33.6%	
69	Thulium	169	^{169}Tm	168.934 212	100%	
70	Ytterbium	174	^{174}Yb	173.938 859	31.8%	
71	Lutecium	175	^{175}Lu	174.940 770	97.41%	
72	Hafnium	180	^{180}Hf	179.946 545	35.10%	
73	Tantalum	181	^{181}Ta	180.947 992	99.98%	
74	Tungsten	184	^{184}W	183.950 928	30.67%	
75	Rhenium	187	^{187}Re	186.955 744	62.6%, β^-	$4.6 \times 10^{10}\text{y}$
76	Osmium	191	^{191}Os	190.960 920	β^-	15.4 d
		192	^{192}Os	191.961 467	41.0%	
77	Iridium	191	^{191}Ir	190.960 584	37.3%	
		193	^{193}Ir	192.962 917	62.7%	
78	Platinum	195	^{195}Pt	194.964 766	33.8%	
79	Gold	197	^{197}Au	196.966 543	100%	

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		198	^{198}Au	197.968 217	β^-	2.696 d
80	Mercury	199	^{199}Hg	198.968 253	16.87%	
		202	^{202}Hg	201.970 617	29.86%	
81	Thallium	205	^{205}Tl	204.974 401	70.48%	
82	Lead	206	^{206}Pb	205.974 440	24.1%	
		207	^{207}Pb	206.975 872	22.1%	
		208	^{208}Pb	207.976 627	52.4%	
		210	^{210}Pb	209.984 163	α, β^-	22.3 y
		211	^{211}Pb	210.988 735	β^-	36.1 min
		212	^{212}Pb	211.991 871	β^-	10.64 h
83	Bismuth	209	^{209}Bi	208.980 374	100%	
		211	^{211}Bi	210.987 255	α, β^-	2.14 min
84	Polonium	210	^{210}Po	209.982 848	α	138.38 d
85	Astatine	218	^{218}At	218.008 684	α, β^-	1.6 s
86	Radon	222	^{222}Rn	222.017 570	α	3.82 d
87	Francium	223	^{223}Fr	223.019 733	α, β^-	21.8 min
88	Radium	226	^{226}Ra	226.025 402	α	$1.60 \times 10^3 \text{ y}$

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	$1.41 \times 10^{10}\text{y}$
91	Protactinium	231	^{231}Pa	231.035 880	α	$3.28 \times 10^4\text{y}$
92	Uranium	233	^{233}U	233.039 628	α	$1.59 \times 10^3\text{y}$
		235	^{235}U	235.043 924	0.720%, α	$7.04 \times 10^8\text{y}$
		236	^{236}U	236.045 562	α	$2.34 \times 10^7\text{y}$
		238	^{238}U	238.050 784	99.2745%, α	$4.47 \times 10^9\text{y}$
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	$2.41 \times 10^4\text{y}$
95	Americium	243	^{243}Am	243.061 375	α , fission	$7.37 \times 10^3\text{y}$
96	Curium	245	^{245}Cm	245.065 483	α	$8.50 \times 10^3\text{y}$
97	Berkelium	247	^{247}Bk	247.070 300	α	$1.38 \times 10^3\text{y}$
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d

Atomic Number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 min
105	Dubnium	262	^{262}Db	262.113 760	α , fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α , fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
109	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

Atomic Masses

Selected Radioactive Isotopes

Decay modes are α , β^- , β^+ , electron capture (EC) and isomeric transition (IT). EC results in the same daughter nucleus as would β^+ decay. IT is a transition from a metastable excited state. Energies for $\beta^\pm$ decays are the maxima; average energies are roughly one-half the maxima.

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)
^3H	12.33 y	β^-	0.0186	100%		
^{14}C	5730 y	β^-	0.156	100%		
^{13}N	9.96 min	β^+	1.20	100%		
^{22}Na	2.602 y	β^+	0.55	90%	γ	1.27
^{32}P	14.28 d	β^-	1.71	100%		
^{35}S	87.4 d	β^-	0.167	100%		
^{36}Cl	$3.00 \times 10^5 \text{y}$	β^-	0.710	100%		
^{40}K	$1.28 \times 10^9 \text{y}$	β^-	1.31	89%		
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373
						0.618
^{45}Ca	165 d	β^-	0.257	100%		
^{51}Cr	27.70 d	EC			γ	0.320
^{52}Mn	5.59d	β^+	3.69	28%	γs	1.33
						1.43
^{52}Fe	8.27 h	β^+	1.80	43%		0.169
						0.378
^{59}Fe	44.6 d	$\beta^- \text{s}$	0.273	45%	γs	1.10
			0.466	55%		1.29
^{60}Co	5.271 y	β^-	0.318	100%	γs	1.17
						1.33
^{65}Zn	244.1 d	EC			γ	1.12

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)
^{67}Ga	78.3 h	EC			γ s	0.0933
						0.185
						0.300
						others
^{75}Se	118.5 d	EC			γ s	0.121
						0.136
						0.265
						0.280
						others
^{86}Rb	18.8 d	β^- s	0.69	9%	γ	1.08
			1.77	91%		
^{85}Sr	64.8 d	EC			γ	0.514
^{90}Sr	28.8 y	β^-	0.546	100%		
^{90}Y	64.1 h	β^-	2.28	100%		
$^{99\text{m}}\text{Tc}$	6.02 h	IT			γ	0.142
$^{113\text{m}}\text{In}$	99.5 min	IT			γ	0.392
^{123}I	13.0 h	EC			γ	0.159
^{131}I	8.040 d	β^- s	0.248	7%	γ s	0.364
			0.607	93%		others
			others			
^{129}Cs	32.3 h	EC			γ s	0.0400
						0.372
						0.411
						others
^{137}Cs	30.17 y	β^- s	0.511	95%	γ	0.662
			1.17	5%		

Isotope	$t_{1/2}$	DecayMode(s)	Energy(MeV)	Percent		γ -Ray Energy(MeV)
^{140}Ba	12.79 d	β^-	1.035	$\approx 100\%$	γ s	0.030
						0.044
						0.537
						others
^{198}Au	2.696 d	β^-	1.161	$\approx 100\%$	γ	0.412
^{197}Hg	64.1 h	EC			γ	0.0733
^{210}Po	138.38 d	α	5.41	100%		
^{226}Ra	$1.60 \times 10^3 \text{ y}$	α s	4.68	5%	γ	0.186
			4.87	95%		
^{235}U	$7.038 \times 10^8 \text{ y}$	α	4.68	$\approx 100\%$	γ s	numerous
^{238}U	$4.468 \times 10^9 \text{ y}$	α s	4.22	23%	γ	0.050
			4.27	77%		
^{237}Np	$2.14 \times 10^6 \text{ y}$	α s	numerous		γ s	numerous
			4.96 (max.)			
^{239}Pu	$2.41 \times 10^4 \text{ y}$	α s	5.19	11%	γ s	7.5×10^{-5}
			5.23	15%		0.013
			5.24	73%		0.052
						others
^{243}Am	$7.37 \times 10^3 \text{ y}$	α s	Max. 5.44		γ s	0.075
			5.37	88%		others
			5.32	11%		
			others			

Selected Radioactive Isotopes

Useful Information

This appendix is broken into several tables.

- [\[link\]](#), Important Constants
- [\[link\]](#), Submicroscopic Masses
- [\[link\]](#), Solar System Data
- [\[link\]](#), Metric Prefixes for Powers of Ten and Their Symbols
- [\[link\]](#), The Greek Alphabet
- [\[link\]](#), SI units
- [\[link\]](#), Selected British Units
- [\[link\]](#), Other Units
- [\[link\]](#), Useful Formulae

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^8 \text{ m/s}$	$3.00 \times 10^8 \text{ m/s}$
G	Gravitational constant	$6.67408(31) \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$	$6.67 \times 10^{-11} \text{ N} \cdot \text{m}^2/\text{kg}^2$
N_A	Avogadro's number	$6.02214129(27) \times 10^{23}$	6.02×10^{23}
k	Boltzmann's constant	$1.3806488(13) \times 10^{-23} \text{ J/K}$	$1.38 \times 10^{-23} \text{ J/K}$
R	Gas constant	$8.3144621(75) \text{ J/mol} \cdot \text{K}$	$8.31 \text{ J/mol} \cdot \text{K} = 1.99 \text{ cal/mol} \cdot \text{K} =$
σ	Stefan-Boltzmann constant	$5.670373(21) \times 10^{-8} \text{ W/m}^2 \cdot \text{K}$	$5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}$
k	Coulomb force constant	$8.987551788... \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$	$8.99 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2$
q_e	Charge on electron	$-1.602176565(35) \times 10^{-19} \text{ C}$	$-1.60 \times 10^{-19} \text{ C}$
ϵ_0	Permittivity of free space	$8.854187817... \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$	$8.85 \times 10^{-12} \text{ C}^2/\text{N} \cdot \text{m}^2$
μ_0	Permeability of free space	$4\pi \times 10^{-7} \text{ T} \cdot \text{m/A}$	$1.26 \times 10^{-6} \text{ T} \cdot \text{m/A}$
h	Planck's constant	$6.62606957(29) \times 10^{-34} \text{ J} \cdot \text{s}$	$6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

Important Constants^[footnote]

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.10938291(40) \times 10^{-31}\text{kg}$	$9.11 \times 10^{-31}\text{kg}$
m_p	Proton mass	$1.672621777(74) \times 10^{-27}\text{kg}$	$1.6726 \times 10^{-27}\text{kg}$
m_n	Neutron mass	$1.674927351(74) \times 10^{-27}\text{kg}$	$1.6749 \times 10^{-27}\text{kg}$
u	Atomic mass unit	$1.660538921(73) \times 10^{-27}\text{kg}$	$1.6605 \times 10^{-27}\text{kg}$

Submicroscopic Masses^[footnote]

Stated values are according to the National Institute of Standards and Technology Reference on Constants, Units, and Uncertainty, www.physics.nist.gov/cuu (accessed May 18, 2012). Values in parentheses are the uncertainties in the last digits. Numbers without uncertainties are exact as defined.

Sun	mass	$1.99 \times 10^{30}\text{kg}$
	average radius	$6.96 \times 10^8\text{m}$
	Earth-sun distance (average)	$1.496 \times 10^{11}\text{m}$
Earth	mass	$5.9736 \times 10^{24}\text{kg}$
	average radius	$6.376 \times 10^6\text{m}$
	orbital period	$3.16 \times 10^7\text{s}$

Epsilon	Ε	ε	Lambda	Λ	λ	Rho	Ρ	ρ	Psi	Ψ	ψ
Zeta	Ζ	ζ	Mu	Μ	μ	Sigma	Σ	σ	Omega	Ω	ω

The Greek Alphabet

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$N = kg \cdot m/s^2$	newton
	Energy	$J = kg \cdot m^2/s^2$	joule
	Power	$W = J/s$	watt
	Pressure	$Pa = N/m^2$	pascal
	Frequency	$Hz = 1/s$	hertz
	Electronic potential	$V = J/C$	volt
	Capacitance	$F = C/V$	farad
	Charge	$C = s \cdot A$	coulomb
	Resistance	$\Omega = V/A$	ohm

	Entity	Abbreviation	Name
	Magnetic field	$T = N/(A \cdot m)$	tesla
	Nuclear decay rate	$Bq = 1/s$	becquerel

SI Units

Length	1 inch (in.) = 2.54 cm (exactly)
	1 foot (ft) = 0.3048 m
	1 mile (mi) = 1.609 km
Force	1 pound (lb) = 4.448 N
Energy	1 British thermal unit (Btu) = 1.055×10^3 J
Power	1 horsepower (hp) = 746 W
Pressure	$1 \text{ lb/in}^2 = 6.895 \times 10^3$ Pa

Selected British Units

Length	1 light year (ly) = 9.46×10^{15} m
	1 astronomical unit (au) = 1.50×10^{11} m
	1 nautical mile = 1.852 km
	1 angstrom($\text{\AA}$) = 10^{-10} m
Area	1 acre (ac) = 4.05×10^3 m ²
	1 square foot (ft ²) = 9.29×10^{-2} m ²
	1 barn (<i>b</i>) = 10^{-28} m ²
Volume	1 liter (<i>L</i>) = 10^{-3} m ³

	1 U.S. gallon (gal) = $3.785 \times 10^{-3} \text{ m}^3$
Mass	1 solar mass = $1.99 \times 10^{30} \text{ kg}$
	1 metric ton = 10^3 kg
	1 atomic mass unit (u) = $1.6605 \times 10^{-27} \text{ kg}$
Time	1 year (y) = $3.16 \times 10^7 \text{ s}$
	1 day (d) = 86,400 s
Speed	1 mile per hour (mph) = 1.609 km/h
	1 nautical mile per hour (naut) = 1.852 km/h
Angle	1 degree ($^\circ$) = $1.745 \times 10^{-2} \text{ rad}$
	1 minute of arc ($'$) = 1/60 degree
	1 second of arc ($''$) = 1/60 minute of arc
	1 grad = $1.571 \times 10^{-2} \text{ rad}$
Energy	1 kiloton TNT (kT) = $4.2 \times 10^{12} \text{ J}$
	1 kilowatt hour ($\text{kW} \cdot \text{h}$) = $3.60 \times 10^6 \text{ J}$
	1 food calorie (kcal) = 4186 J
	1 calorie (cal) = 4.186 J
	1 electron volt (eV) = $1.60 \times 10^{-19} \text{ J}$
Pressure	1 atmosphere (atm) = $1.013 \times 10^5 \text{ Pa}$
	1 millimeter of mercury (mm Hg) = 133.3 Pa
	1 torricelli (torr) = 1 mm Hg = 133.3 Pa
Nuclear decay rate	1 curie (Ci) = $3.70 \times 10^{10} \text{ Bq}$

Other Units

Circumference of a circle with radius r or diameter d	$C = 2\pi r = \pi d$
Area of a circle with radius r or diameter d	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius r	$A = 4\pi r^2$

Volume of a sphere with radius r

$$V = \frac{4}{3} \pi r^3$$

Useful Formulae

Glossary of Key Symbols and Notation

In this glossary, key symbols and notation are briefly defined.

Symbol	Definition
any symbol	average (indicated by a bar over a symbol— e.g., $\bar{v}$ is average velocity)
$^{\circ}\text{C}$	Celsius degree
$^{\circ}\text{F}$	Fahrenheit degree
//	parallel
$\perp$	perpendicular
$\propto$	proportional to
$\pm$	plus or minus

Symbol	Definition
0	zero as a subscript denotes an initial value
α	alpha rays
α	angular acceleration
α	temperature coefficient(s) of resistivity
β	beta rays
β	sound level
β	volume coefficient of expansion
β^{-}	electron emitted in nuclear beta decay
β^{+}	positron decay
γ	gamma rays

Symbol	Definition
γ	surface tension
$\gamma = 1/\sqrt{1 - v^2/c^2}$	a constant used in relativity
Δ	change in whatever quantity follows
δ	uncertainty in whatever quantity follows
ΔE	change in energy between the initial and final orbits of an electron in an atom
ΔE	uncertainty in energy
Δm	difference in mass between initial and final products
ΔN	number of decays that occur
Δp	change in momentum

Symbol	Definition
Δp	uncertainty in momentum
ΔPE_g	change in gravitational potential energy
$\Delta \theta$	rotation angle
Δs	distance traveled along a circular path
Δt	uncertainty in time
Δt_0	proper time as measured by an observer at rest relative to the process
ΔV	potential difference
Δx	uncertainty in position
ϵ_0	permittivity of free space
η	viscosity

Symbol	Definition
θ	angle between the force vector and the displacement vector
θ	angle between two lines
θ	contact angle
θ	direction of the resultant
θ_b	Brewster's angle
θ_c	critical angle
κ	dielectric constant
λ	decay constant of a nuclide
λ	wavelength
λ_n	wavelength in a medium

Symbol	Definition
μ_0	permeability of free space
μ_k	coefficient of kinetic friction
μ_s	coefficient of static friction
ν_e	electron neutrino
π^+	positive pion
π^-	negative pion
π^0	neutral pion
ρ	density
ρ_c	critical density, the density needed to just halt universal expansion
ρ_{fl}	fluid density

Symbol	Definition
ρ_{obj}	average density of an object
ρ/ρ_{w}	specific gravity
τ	characteristic time constant for a resistance and inductance (RL) or resistance and capacitance (RC) circuit
τ	characteristic time for a resistor and capacitor (RC) circuit
τ	torque
Υ	upsilon meson
Φ	magnetic flux
ϕ	phase angle
Ω	ohm (unit)
ω	angular velocity

Symbol	Definition
A	ampere (current unit)
A	area
A	cross-sectional area
A	total number of nucleons
a	acceleration
a_B	Bohr radius
a_c	centripetal acceleration
a_t	tangential acceleration
AC	alternating current
AM	amplitude modulation

Symbol	Definition
atm	atmosphere
B	baryon number
B	blue quark color
B	antiblue (yellow) antiquark color
b	quark flavor bottom or beauty
B	bulk modulus
B	magnetic field strength
B_{int}	electron's intrinsic magnetic field
B_{orb}	orbital magnetic field
BE	binding energy of a nucleus—it is the energy required to completely disassemble it into separate protons and neutrons

Symbol	Definition
BE/A	binding energy per nucleon
Bq	becquerel—one decay per second
C	capacitance (amount of charge stored per volt)
C	coulomb (a fundamental SI unit of charge)
C_p	total capacitance in parallel
C_s	total capacitance in series
CG	center of gravity
CM	center of mass
c	quark flavor charm
c	specific heat

Symbol	Definition
c	speed of light
Cal	kilocalorie
cal	calorie
COP_{hp}	heat pump's coefficient of performance
COP_{ref}	coefficient of performance for refrigerators and air conditioners
$\cos \theta$	cosine
$\cot \theta$	cotangent
$\csc \theta$	cosecant
D	diffusion constant
d	displacement

Symbol	Definition
d	quark flavor down
dB	decibel
d_i	distance of an image from the center of a lens
d_o	distance of an object from the center of a lens
DC	direct current
E	electric field strength
ε	emf (voltage) or Hall electromotive force
emf	electromotive force
E	energy of a single photon
E	nuclear reaction energy

Symbol	Definition
E	relativistic total energy
E	total energy
E_0	ground state energy for hydrogen
E_0	rest energy
EC	electron capture
E_{cap}	energy stored in a capacitor
Eff	efficiency—the useful work output divided by the energy input
Eff _C	Carnot efficiency
E_{in}	energy consumed (food digested in humans)
E_{ind}	energy stored in an inductor

Symbol	Definition
E_{out}	energy output
e	emissivity of an object
e^+	antielectron or positron
eV	electron volt
F	farad (unit of capacitance, a coulomb per volt)
F	focal point of a lens
F	force
F	magnitude of a force
F	restoring force
F_{B}	buoyant force

Symbol	Definition
F_c	centripetal force
F_i	force input
$\mathbf{F}_{\text{net}}$	net force
F_o	force output
FM	frequency modulation
f	focal length
f	frequency
f_0	resonant frequency of a resistance, inductance, and capacitance (RLC) series circuit
f_0	threshold frequency for a particular material (photoelectric effect)

Symbol	Definition
f_1	fundamental
f_2	first overtone
f_3	second overtone
f_B	beat frequency
f_k	magnitude of kinetic friction
f_s	magnitude of static friction
G	gravitational constant
G	green quark color
$\bar{G}$	antigreen (magenta) antiquark color

Symbol	Definition
g	acceleration due to gravity
g	gluons (carrier particles for strong nuclear force)
h	change in vertical position
h	height above some reference point
h	maximum height of a projectile
h	Planck's constant
hf	photon energy
h_i	height of the image
h_o	height of the object
I	electric current

Symbol	Definition
I	intensity
I	intensity of a transmitted wave
I	moment of inertia (also called rotational inertia)
I_0	intensity of a polarized wave before passing through a filter
I_{ave}	average intensity for a continuous sinusoidal electromagnetic wave
I_{rms}	average current
J	joule
J/Ψ	Joules/psi meson
K	kelvin
k	Boltzmann constant

Symbol	Definition
k	force constant of a spring
K_{α}	x rays created when an electron falls into an $n = 1$ shell vacancy from the $n = 3$ shell
K_{β}	x rays created when an electron falls into an $n = 2$ shell vacancy from the $n = 3$ shell
kcal	kilocalorie
KE	translational kinetic energy
KE + PE	mechanical energy
KE_e	kinetic energy of an ejected electron
KE_{rel}	relativistic kinetic energy
KE_{rot}	rotational kinetic energy
KE	thermal energy

Symbol	Definition
kg	kilogram (a fundamental SI unit of mass)
L	angular momentum
L	liter
L	magnitude of angular momentum
L	self-inductance
ℓ	angular momentum quantum number
L_{α}	x rays created when an electron falls into an $n = 2$ shell from the $n = 3$ shell
L_e	electron total family number
L_{μ}	muon family total number
L_{τ}	tau family total number

Symbol	Definition
L_f	heat of fusion
L_f and L_v	latent heat coefficients
L_{orb}	orbital angular momentum
L_s	heat of sublimation
L_v	heat of vaporization
L_z	z - component of the angular momentum
M	angular magnification
M	mutual inductance
m	indicates metastable state
m	magnification

Symbol	Definition
m	mass
m	mass of an object as measured by a person at rest relative to the object
m	meter (a fundamental SI unit of length)
m	order of interference
m	overall magnification (product of the individual magnifications)
$m(^AX)$	atomic mass of a nuclide
MA	mechanical advantage
m_e	magnification of the eyepiece
m_e	mass of the electron
m_ℓ	angular momentum projection quantum number

Symbol	Definition
m_n	mass of a neutron
m_o	magnification of the objective lens
mol	mole
m_p	mass of a proton
m_s	spin projection quantum number
N	magnitude of the normal force
N	newton
N	normal force
N	number of neutrons
n	index of refraction

Symbol	Definition
n	number of free charges per unit volume
N_A	Avogadro's number
N_r	Reynolds number
$N \cdot m$	newton-meter (work-energy unit)
$N \cdot m$	newtons times meters (SI unit of torque)
OE	other energy
P	power
P	power of a lens
P	pressure
p	momentum

Symbol	Definition
p	momentum magnitude
p	relativistic momentum
$\mathbf{p}_{\text{tot}}$	total momentum
$\mathbf{p}'_{\text{tot}}$	total momentum some time later
P_{abs}	absolute pressure
P_{atm}	atmospheric pressure
P_{atm}	standard atmospheric pressure
PE	potential energy
PE _{el}	elastic potential energy
PE _{elec}	electric potential energy

Symbol	Definition
PE_s	potential energy of a spring
P_g	gauge pressure
P_{in}	power consumption or input
P_{out}	useful power output going into useful work or a desired, form of energy
Q	latent heat
Q	net heat transferred into a system
Q	flow rate—volume per unit time flowing past a point
$+Q$	positive charge
$-Q$	negative charge

Symbol	Definition
q	electron charge
q_p	charge of a proton
q	test charge
QF	quality factor
R	activity, the rate of decay
R	radius of curvature of a spherical mirror
R	red quark color
R	antired (cyan) quark color
R	resistance
R	resultant or total displacement

Symbol	Definition
R	Rydberg constant
R	universal gas constant
r	distance from pivot point to the point where a force is applied
r	internal resistance
$r_{\perp}$	perpendicular lever arm
r	radius of a nucleus
r	radius of curvature
r	resistivity
r or rad	radiation dose unit
rem	roentgen equivalent man

Symbol	Definition
rad	radian
RBE	relative biological effectiveness
RC	resistor and capacitor circuit
rms	root mean square
r_n	radius of the n th H-atom orbit
R_p	total resistance of a parallel connection
R_s	total resistance of a series connection
R_s	Schwarzschild radius
S	entropy
S	intrinsic spin (intrinsic angular momentum)

Symbol	Definition
S	magnitude of the intrinsic (internal) spin angular momentum
S	shear modulus
S	strangeness quantum number
s	quark flavor strange
s	second (fundamental SI unit of time)
s	spin quantum number
s	total displacement
$\sec \theta$	secant
$\sin \theta$	sine
s_z	z-component of spin angular momentum

Symbol	Definition
T	period—time to complete one oscillation
T	temperature
T_c	critical temperature—temperature below which a material becomes a superconductor
T	tension
T	tesla (magnetic field strength B)
t	quark flavor top or truth
t	time
$t_{1/2}$	half-life—the time in which half of the original nuclei decay
$\tan \theta$	tangent
U	internal energy

Symbol	Definition
u	quark flavor up
u	unified atomic mass unit
u	velocity of an object relative to an observer
u'	velocity relative to another observer
V	electric potential
V	terminal voltage
V	volt (unit)
V	volume
v	relative velocity between two observers
v	speed of light in a material

Symbol	Definition
$\mathbf{v}$	velocity
$\mathbf{v}$	average fluid velocity
$V_B - V_A$	change in potential
$\mathbf{v}_d$	drift velocity
V_p	transformer input voltage
V_{rms}	rms voltage
V_s	transformer output voltage
$\mathbf{v}_{\text{tot}}$	total velocity
v_w	propagation speed of sound or other wave
$\mathbf{v}_w$	wave velocity

Symbol	Definition
W	work
W	net work done by a system
W	watt
w	weight
w_{fl}	weight of the fluid displaced by an object
W_{c}	total work done by all conservative forces
W_{nc}	total work done by all nonconservative forces
W_{out}	useful work output
X	amplitude
X	symbol for an element

Symbol	Definition
${}^Z_A X_N$	notation for a particular nuclide
x	deformation or displacement from equilibrium
x	displacement of a spring from its undeformed position
x	horizontal axis
X_C	capacitive reactance
X_L	inductive reactance
x_{rms}	root mean square diffusion distance
y	vertical axis
Y	elastic modulus or Young's modulus
Z	atomic number (number of protons in a nucleus)

Symbol	Definition
Z	impedance